Competitive unlicensed spectrum sharing with partial information on slow fading channels

Xiao Lei[1], K. Avrachenkov[2], L. Cottatellucci[1], and A. Garnaev[3]

[1]Eurecom, Mobile Communications, Sophia Antipolis, France
[2]Inria, Sophia Antipolis Cedex, France
[3]Saint Petersburg State University, Russia
laura.cottatellucci,lei.xiao@eurecom.fr
K.Avrachenkov@sophia.inria.fr,agarnaev@rambler.ru

Abstract. We consider a slow fading multiuser environment with primary and secondary users. The secondary users have only partial knowledge of the channel and are subjected to transmitted power constraints by the primary users. Their communications are intrinsically affected by outage events. We propose and analyze two algorithms for joint rate and power allocation. In one algorithm, the secondary transmitters cooperate to maximizing a common utility function accounting for the total throughput of the network. In a second approach based on a game framework, the secondary users aim at maximizing selfishly their own utilities. The latter approach shows better fairness properties at the expenses of some global performance loss compared to the optimum cooperative approach.

1 Introduction

Spectrum sharing in cognitive radio [1] enables an efficient use of the scarce frequency spectrum by allowing the coexistence of licensed and unlicensed users in the same spectrum. In a typical scenario where independent systems do not cooperate and there is no centralized authority to handle the network access for secondary users, distributed algorithms to share the available resources play a key role. Game theory offers a natural framework to construct distributed algorithms. The literature in the field is copious and an exhaustive overview exceeds the scope of this work. The interested reader can refer to [2, 3]. However, the spectrum allocation problem among secondary users has been neglected for the case of practical and theoretical interest when the channel is slow fading and the secondary users have only a partial channel state information. In these conditions, the secondary users suffer from outage events and a certain level of information loss need to be tolerated. In this work we propose two algorithms, one based on game theory and the other on optimization for the joint power and rate allocation among secondary users. We analyze the characteristics of the game approach in terms of existence and multiplicity of the Nash equilibria in the extreme regimes of high and low noise plus interference from primary users. In the former case, a closed form expression for the Nash equilibrium is provided. In the latter case, criteria for the convergence of a best response algorithm are discussed. The optimization approach is also analyzed in the two above mentioned regimes and closed form expressions for the resource allocation are provided.

Due to space constraints, the proofs of the theoretical results are omitted in this paper.

2 System Model

We consider a multiuser environment consisting of two secondary users and a primary user sharing the same time and frequency band but transmitting to independent receivers. The independent systems do not cooperate and no centralized authority is assumed to handle the network access for secondary users. The secondary users compete for the shared bandwidth constrained by a target quality of service for the primary user. We denote by $\mathcal{S}_*$ and $\mathcal{D}_*$ the transmitter and the receiver of the primary user while $\mathcal{S}_i$ and $\mathcal{D}_i$, for $i = 1, 2$ denote the source and the destination of the secondary users, respectively. Let $\mathcal{I} = \{*, 1, 2\}$. We denote by g_p, with $p \in \mathcal{I}$, the channel attenuation of the direct link $\mathcal{S}_p - \mathcal{D}_p$. The channel attenuations of the interfering links between transmitter $\mathcal{S}_p$ and receiver $\mathcal{D}_q$, with $p \neq q$ and $p, q \in \mathcal{I}$ are denoted by h_{pq}. The rate and the power of the signal transmitted by $\mathcal{S}_p$ are R_p and P_p, respectively. The primary system broadcasts the transmitted power on the signaling channel such that all the transmitters know it, or, alternatively, the transmitters estimate it. The channel is fading and each link fades independently with statistics known at the secondary sources and given by a probability density function $\Theta_{pq}(h_{pq})$, with $p, q \in \mathcal{I}$. We assume also that the fading is Rayleigh distributed and $\Theta_{H_{pq}}(h_{pq}) = \frac{1}{\sigma_{pq}^2}\exp(p_{pq}/\sigma_{pq}^2)$. The sources transmit Gaussian symbols and the received signals at the destinations are impaired by white additive Gaussian noise with variance N_0.

3 Problem Statement

We will consider the problem of joint power and rate allocation by the secondary sources under constraints on the quality of service of the primary communication in the case of block fading, i.e. varying channel which is constant in the timeframe of a codeword but independent and identically distributed from a codeword to a codeword.
We assume that all the receivers estimate their respective direct links g_p, $p \in \mathcal{I}$ and broadcast it on the signaling channel such that all the transmitters in the system know them. Furthermore, the secondary sources are supposed to track the interference link from the primary source and exchange this information with the other secondary users, i.e. transmitter $\mathcal{S}_i$, $i \in \{1, 2\}$ has knowledge of both

h_{*1} and h_{*2}. Only statistical knowledge of the reverse link from a secondary transmitter to the primary user is available at the secondary users.

Because of the partial knowledge of the channel by the sources and the assumption of block fading, reliable communications, i.e. with error probability arbitrarily small, are not feasible and outage events may happen. If the source $p \in \mathcal{I}$ transmits at a certain rate with constant transmitted power P_p, an outage event happens if

$$R_p > \log\left(1 + \frac{P_p g_p}{N_0 + P_{q_1} h_{q_1 p} + P_{q_2} h_{q_2 p}}\right), \qquad p, q_1, q_2 = \mathcal{I} \text{ with } i \neq j, \tag{1}$$

and the outage probability of source p depends on the choice of R_p, P_p, P_{q_1} and P_{q_2}. We define the throughput as the average information that can be reliably received by the destination. The throughput is given by

$$T_p(P_p, R_p, P_{q_1}, P_{q_2}) = R_p \Pr\left\{ R_p \leq \log\left(1 + \frac{P_p g_p}{N_0 + P_{q_1} h_{q_1 p} + P_{q_2} h_{q_2 p}}\right)\right\} \tag{2}$$

where $p, q_1, q_2 = \mathcal{I}$ with $p \neq q_1 \neq q_2$, , and $\Pr\{\mathcal{E}\}$ denotes the probability of the event $\mathcal{E}$. We assume that the metric to measure the performance loss of the primary user is the average interference $\sigma_{i*}^2 P_i + \sigma_{j*}^2 P_j$ and the secondary users pay a penalty proportional to the average interference caused to the primary communication.

We study joint power and rate allocation for the secondary communications by applying two different criteria. In the optimization approach the secondary transmitters cooperate to maximize a global utility function accounting for the total throughput of the secondary users and their costs due to the transmitted power and the interference caused to the primary user,

$$\begin{aligned} u\left(P_1, P_2, R_1, R_2\right) &= \textstyle\sum_{i=1, i\neq j}^{2} \left(T_i\left(P_i, R_i, P_j, R_j\right) - C_i P_i - K_i \sigma_{i*}^2 P_i\right) \\ &= \textstyle\sum_{i=1}^{2} \left(R_i F_i(t_i) - C_i P_i - K_i \sigma_{i*}^2 P_i\right). \end{aligned} \tag{3}$$

where $F_i(t_i) = 1 - exp\left(-\frac{t_i}{P_j \sigma_{ji}^2}\right)$, with $t_i = \frac{P_i g_i}{\mathrm{e}^{R_i} - 1} - N_0 - P_* \sigma_{i*}^2$, and C_i and K_i are unit costs per transmitted power and per average generated interference, respectively. Note that the costs C_i and K_i can be interpreted as the Lagrangian multipliers of constraints on the maximum power and maximum average interference to the primary user. Then, the utility function (3) corresponds to the dual function (see e.g. [4]) of a constrained optimization problem with objective function 2.

Alternatively, we investigate the case as the secondary users are rational and selfish and allocate their rates and powers to maximize their own utility functions. The utility function of transmitter $i = 1, 2$ is given by

$$\begin{aligned} u_i(P_i, P_j) &= T_i(P_i, R_i, P_j, P_*) - C_i P_i - K_i(\sigma_{i*}^2 P_i + \sigma_{j*}^2 P_j) \\ &= R_i F_i(t_i) - C_i P_i - K_i(\sigma_{i*}^2 P_i + \sigma_{j*}^2 P_j) \qquad \text{with } i \neq j. \end{aligned} \tag{4}$$

4 Game-Based Resource Allocation

Power and rate allocation for the two secondary transmitters can be formulated as a game $\mathcal{G} = \{\mathcal{S},\mathcal{U},\mathcal{P}\}$, where $\mathcal{S}$ is the set of players (the two secondary transmitters), $\mathcal{U} = \{u_1, u_2\}$ is the utility functions defined in (4), and $\mathcal{P}$ is the strategy set defined by $\mathcal{P} \equiv \{(P_1, R_1), (P_2, R_2)|P_1, P_2, R_1, R_2 > 0\}$. The power and rate allocation of the secondary transmitter is obtained as an equilibrium point of the system. When both transmitters aims at maximizing their utility function, on equilibrium point is the Nash equilibrium defined by the allocation strategy $(P_1^*, R_1^*, P_2^*, R_2^*)$ such that

$$u_1\left(P_1^*, R_1^*, P_2^*, R_2^*\right) \geq u_1\left(P_1, R_1, P_2^*, R_2^*\right) \quad \text{for} \quad \forall P_1, R_1 \in \mathbb{R}_{++}$$

$$u_2\left(P_1^*, R_1^*, P_2^*, R_2^*\right) \geq u_2\left(P_1^*, R_1^*, P_2, R_2\right) \quad \text{for} \quad \forall P_2, R_2 \in \mathbb{R}_{++}$$

where $\mathbb{R}_{++}$ denotes the set of positive reals.

Nash equilibria of the game $\mathcal{G}$ necessarily satisfy the following system of equations

$$\frac{\partial u_i}{\partial P_i} = \frac{R_i g_i}{(\mathrm{e}^{R_i}-1)P_j\sigma_{ji}^2} - C_i - K_i\sigma_{i*}^2 = 0$$

$$\frac{\partial u_i}{\partial R_i} = F_i(t_i) - \frac{R_i P_i g_i \mathrm{e}^{R_i}}{(\mathrm{e}^{R_i}-1)^2}F_i^{'}(t_i) = 0$$

or equivalently,

$$1 - \exp\left(-\frac{t_i}{P_j\sigma_{ji}^2}\right) = P_i\left(C_i + K_i\sigma_{i*}^2\right)\frac{\mathrm{e}^{R_i}}{\mathrm{e}^{R_i}-1} \tag{5}$$

$$\frac{1}{P_j\sigma_{ji}^2}\exp\left(\frac{-t_i}{P_j\sigma_{ji}^2}\right) = \frac{\left(C_i + K_i\sigma_{i*}^2\right)\left(e^{R_i}-1\right)}{R_i g_i} \tag{6}$$

From (5) and (6) we obtain

$$P_i\frac{\mathrm{e}^{R_i}}{\mathrm{e}^{R_i}-1}\left(C_i + K_i\sigma_{i*}^2\right) + \frac{(\mathrm{e}^{R_i}-1)\sigma_{ji}^2(C_i + K_i\sigma_{i*}^2)}{R_i g_i} = 1 \tag{7}$$

which yields

$$P_i = \left(1 - \frac{(C_i + K_i\sigma_{i*}^2)P_j\sigma_{ji}^2}{R_i g_i}(\mathrm{e}^{R_i}-1)\right)\frac{(\mathrm{e}^{R_i}-1)}{(C_i + K_i\sigma_{i*}^2)\mathrm{e}^{R_i}} \quad i = \{1,2\}\,. \tag{8}$$

A solution of the 4-equation system (5)-(6) is a Nash equilibrium if it satisfies the conditions

$$\frac{\partial^2 u_i}{\partial R_i^2} < 0, \frac{\partial^2 u_i}{\partial P_i^2} < 0, H = \frac{\partial^2 u_i}{\partial R_i^2}\frac{\partial^2 u_i}{\partial P_i^2} - \left(\frac{\partial_{u_i}^2}{\partial P_i \partial R_i}\right)^2 > 0. \tag{9}$$

It is straightforward to verify that the utility function is not concave in R_i. Hence, the results of N-concave games can not be applied here. Additionally,the analysis of the general case results complex. In order to get additional insights into the system behavior, we consider firstly the following extreme cases : (1) the interference from the primary user and the noise tend to zero, (*secondary-user interference limited regime*), (2) the noise is much higher than the transmitted power (*high noise regime*).

Secondary-User Interference Limited Regime When $N_0+P_* h_{*i}$ is negligible compared to the interference, the payoff function is still given by (4), with $t_i = \frac{P_i g_i}{\mathrm{e}^{R_i}-1}$.

Proposition 1. *When the interference from the primary user and the noise tend to zero, the Nash equilibrium of game $\mathcal{G}$ satisfy the system of equations*

$$x_1 = \kappa_1 f(x_2) \tag{10}$$

$$x_2 = \kappa_2 f(x_1) \tag{11}$$

where $x_i = \frac{g_i}{(C_i+\kappa_i\sigma_{i*}^2)P_j\sigma_{ji}^2}$, $\kappa_i = \frac{(C_i+K_i\sigma_{i*}^2)g_j}{(C_j+K_j\sigma_{j*}^2)\sigma_{ij}^2}$, $i,j \in 1,2,\ i \neq j$ *and*

$$f(x) = \left(1 - \frac{\mathrm{e}^{R(x)}-1}{xR(x)}\right)^{-1}\left(1-\mathrm{e}^{-R(x)}\right)^{-1} \tag{12}$$

for $1 < x < \infty$*. In (*12*),* $R(x)$ *is the unique positive solution of the equation*

$$1 - \frac{xR}{\mathrm{e}^R - 1}\exp\left(-\frac{x}{\mathrm{e}^R} + \frac{\mathrm{e}^R-1}{R\mathrm{e}^R}\right) = 0 \tag{13}$$

such that

$$-x + \frac{\mathrm{e}^R-1}{R} \neq 0 \tag{14}$$

Let (x_1^0, x_2^0) *be solutions of system (*12*). The corresponding Nash equilibrium is given by*

$$P_1 = \frac{g_2}{(C_2+K_2\sigma_{2*}^2)x_2^0\sigma_{12}^2}, \quad R_1 = R(x_1^0),$$
$$P_2 = \frac{g_1}{(C_1+K_1\sigma_{1*}^2)x_1^0\sigma_{21}^2}, \quad R_2 = R(x_2^0).$$

Remarks Note that the solution $\overline{R}(x)$ to

$$\frac{\mathrm{e}^R-1}{R} = x$$

is also a solution to (13). It is possible to verify that such a solution corresponds to a minimizer of the utility function. The solution $R(x_j)$ to (13) is the rate which maximizes the utility function corresponding to the transmit power of the other transmitter $P_i = \frac{g_j}{(C_j+K_j\sigma_{j*}^2)x_j\sigma_{ji}^2}$. It lies in the interval $\left(0, \overline{R}(x_j)\right)$ and we refer to it as the *best response in terms of rate* of player j to strategy P_i of player i. Similarly, $\kappa_i f(x_j)$ is the *best response in terms of power* of users to the strategy P_i of its opponent.

The solution (x_1^0, x_2^0) to system (10) depends on the system parameters only through the constants κ_1 and κ_2. The existence and uniqueness of Nash equilibrium for the class of systems considered in Proposition I reduces to the analysis of the solution of system (10) and depends on the system via x_1 and x_2. The solution to equation (13) can be effectively approximated by $R(x) \approx 0.8\log(x)$. Then, the function $f(x)$ is approximated by

$$f(x) \approx \left(1 - \frac{\mathrm{e}^{0.8\log(x)} - 1}{x0.8\log(x)}\right)^{-1} \left(1 - \mathrm{e}^{-0.8\log(x)}\right)^{-1}.$$

The following proposition provides sufficient conditions for the existence of a Nash equilibrium.

Proposition 2. *When the interference from the primary user and the noise tend to zero, a Nash equilibrium of the game $\mathcal{G}$ exists if*

$$(\kappa_1 - 1)(\kappa_2 - 1) > 0$$

with κ_i defined in Proposition 1.

General conditions for the uniqueness of the Nash equilibrium are difficult to determine analytically. Let us observe that in general a system with noise and interference from the primary source that tend to zero may have more than one Nash equilibrium. Let us consider the two systems corresponding to the two pairs of coefficients $\kappa_1^{(1)} = \kappa_2^{(1)} = 1.05$ and $\kappa_1^{(2)} = \kappa_2^{(2)} = 2$. The two curves $x_2 = \kappa_1^{(1)} f(x_1)$, for $i = \{1, 2\}$ cross each other in $x_1 = x_2$. Additionally, the curve $x_2 = \kappa_1^{(1)} f(x_1)$ has two asymptotics in $x_1 = 1$ and $x_2 = 1$. Then, by observing Figure 1 , it becomes apparent that the curves with $\kappa_1^{(1)} = \kappa_2^{(1)} = 1.05$ will cross again for high x_1 and x_2 values. In contrast, the curves with $\kappa_1^{(1)} = \kappa_2^{(1)} = 2$ will diverge from each other, and these crossing points correspond to Nash equilibria, it is worth noticing that for $x_1 \gg 1$, $x_2 \approx 1$, (and for $x_2 \gg 1$, $x_1 \approx 1$). Then, from a telecommunication point of view, it is necessary to question whether the model for $N_0 + P_* h_{*i} \to 0$ is still applicable. In fact, in such a case, $P_1 \ll \frac{g_1}{C_1 \sigma_{12}^2}$, but also $P_1 \gg N_0 + P_* h_{*1}$ has to be satisfied because of the system model assumptions. Typically, the additional Nash equilibria with some $\kappa_i \approx 1$ are not interesting from a physical point of view since the system model assumptions are not satisfied.

By numerical simulations, we could observe that games with multiple Nash equilibrium exist for a very restricted range of system parameters, more specifically for $1 \leq \kappa_i \leq 1.1$.

Proposition 1 suggests also an iterative algorithm for computing Nash equilibrium based on the best response. Choose an arbitrary point $x_1^{(0)}$ and compute

the corresponding value $x_2^{(0)} = \kappa_1 f(x_1^{(0)})$. From a practical point of view, this is equivalent to choose arbitrarily the transmitted power $P_2^{(0)} = \frac{g_1}{\sigma_{21}^2 x_1^{(0)}(C_1+\kappa_1\sigma_{1*}^2)}$ for transmitter 2 and determine the power allocation for user 1 which maximizes its utility function. The optimum power allocation for user 1 is $P_1^{(0)} = \frac{g_2}{\sigma_{12}^2 x_2^{(0)}(C_2+\kappa_2\sigma_{2*}^2)}$. We shortly refer to $P_1^{(0)}$ as the best response of user 1 to user 2. Then, by using $x_2^{(0)}$ it is possible to compute $x_1^{(1)} = \kappa_2 f\left(x_2^{(0)}\right)$, the best response of user 2 to user 1. By iterating on the computation of the best responses of user 1 and user 2 we can obtain resource allocations closer and closer to the Nash equilibrium and converge to the Nash equilibrium. We refer to this algorithm as the best response algorithm.
The best response algorithm is very appealing for its simplicity. Nevertheless, its convergence is not guaranteed. This issue is illustrated in Figure 1. Let us consider the interference channel with $\kappa_1 = \kappa_2 = 1.05$ and the corresponding solid and dashed curves $x_2 = f(x_1)$ and $x_1 = f(x_2)$. The Nash equilibrium exists and is unique but the best response algorithm diverges from the Nash equilibrium even for choices of the initial point arbitrarily close to the Nash equilibrium but different from it. Numerical results show that if κ_1 and κ_2 are both greater than 1.2, the best response algorithm always converges to a Nash equilibrium.
Analytically, it is possible to prove the following Proposition.

Proposition 3. *For sufficiently large κ_1 and κ_2, the fixed point iterations*

$$\begin{cases} x_1^{(k+1)} &= \kappa_1 f(x_2^{(k)}), \\ x_2^{(k+1)} &= \kappa_2 f(x_1^{(k)}), \end{cases} \tag{15}$$

converge.

In fact, large values of κ_1 and κ_2 correspond to a realistic situation for system where the noise plus the interference from the primary source are negligible compared to the transmitted powers of the secondary users.

High noise regime Let us turn to the case when noise is much higher than the transmitted power, $P_i g_i \ll N_0 + P_* h_{*i}$. The throughput can be approximated by

$$\begin{aligned} \overline{T}_i\left(P_i, R_i, P_j, P_*\right) &= R_i \mathrm{Pr}\left\{R_i \leq \frac{P_i g_i}{N_0+P_j h_{ji}+P_* h_{*i}}\right\} \\ &= R_i \mathrm{Pr}\left\{h_{ji} \leq \frac{1}{P_j}\left(P_i \frac{g_i}{R_i} - N_0 - P_* h_{*i}\right)\right\} \end{aligned} \tag{16}$$

The utility function is given by

$$v_i = R_i \left(1 - \exp\left(-\frac{1}{P_j \sigma_{ji}^2 \left(P_i \frac{g_i}{R_i} - N_0 - P_* h_{*i}\right)}\right)\right) - C_i P_i - K_i(\sigma_{i*}^2 P_i + \sigma_{j*}^2 P_j)$$

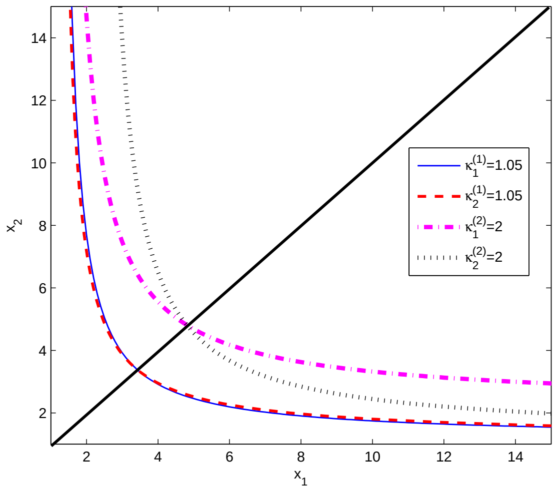

Fig. 1. Graphical investigation of convergence of the best response algorithm in the interference limited regime

for $i = 1, 2$ Correspondingly, we consider the game $\overline{\mathcal{G}} = \{\mathcal{S}, \mathcal{V}, \mathcal{P}\}$, where the set of players and policies coincide with the corresponding sets in $\overline{\mathcal{G}}$ while the utility function set consists of the functions (16). The joint rate and power allocation for selfish secondary transmitters is given by Nash equilibrium of game $\mathcal{G}$. The following proposition states the conditions for the existence and uniqueness of a Nash equilibrium in $\overline{\mathcal{G}}$ and provides the equilibrium point.

Proposition 4. *Game $\overline{\mathcal{G}}$ admits Nash equilibrium if and only if*

$$\frac{g_i}{C_i + K_i \sigma_{i*}^2} > N_0 + P_* h_{i*}, \quad i = 1, 2.$$

If above condition is satisfied, $\overline{\mathcal{G}}$ has the unique equilibrium $\left((R_i^*, P_i^*), (R_j^*, P_j^*)\right)$ where P_i^* and P_j^* are the unique roots of the equations

$$\left(1 - \ln\left(\frac{Q_j P_i \sigma_{ij}^2}{g_j}\right)\right) P_i \sigma_{ij}^2 = \frac{g_j}{Q_j} - N_0 - \sigma_{j_*^2}$$

and

$$\left(1 - \ln\left(\frac{Q_i P_j \sigma_{ji}^2}{g_i}\right)\right) P_j \sigma_{ji}^2 = \frac{g_i}{Q_i} - N_0 - \sigma_{i_*^2}$$

in the intervals $\left(0, \frac{g_j}{Q_j\sigma_{ij}^2}\right)$ *and* $(0, \frac{g_i}{Q_i\sigma_{ji}^2})$ *respectively, being* $Q_i = C_i + K_i\sigma_{i*}^2$. *Also,*

$$R_i = \frac{P_i g_i Q_i}{g_i - P_j\sigma_{ji}^2 Q_i} \quad \textit{and} \quad R_j = \frac{P_j g_j Q_j}{g_j - P_i\sigma_{ij}^2 Q_j}.$$

General case Let us consider now the general case, when the noise, the powers of interferences and the transmitted powers are of the same order of magnitude. A Nash equilibrium necessarily satisfies the system of equations (6) and (8). Substituting (8) in (6) yields

$$1 - \frac{x_i R_i}{\mathrm{e}^{R_i} - 1}\exp\left(-\frac{x_i}{\mathrm{e}^{R_i}} + \frac{\mathrm{e}^{R_i} - 1}{R_i \mathrm{e}^{R_i}} + n_i\right) = 0 \quad i = 1, 2 \tag{17}$$

with $n_i = \frac{N_0 + P_* h_{*i}}{P_j\sigma_{ji}^2}$. Equations (8) and (17) provide an equivalent system to be satisfied by Nash equilibrium. In order to determine a Nash equilibrium we can proceed as in the case of the secondary-interference limited regime. Observe that, in this case, (17) depends on the system parameters and the other player strategy not only via x_i but also via n_i. Then, the general analysis feasible for any communication system in the secondary interference limited regime is no longer possible and the existence and multiplicity of a Nash equilibrium should be studied independently for each communication system. In the following, we detail guidelines for this analysis.

From (17), it is possible to determine the best response in terms of rate of transmitter i to policy P_j of transmitter j. Conditions for the existence of such best response are detailed in the following statement.

Proposition 5. *Equation (17) admits positive roots if and only if*

$$1 - x_i \mathrm{e}^{-x_i + 1 + n_i} > 0. \tag{18}$$

If (18) is satisfied, (17) admits a single positive root in the interval $(0, \log x_i)$, *which corresponds to the best response in terms of rate to policy* P_j *of user* j.

From the best responses in terms of rate, it is straightforward to determine the best response in terms of powers for the two players.

5 Optimum Joint Rate and Power Allocation

In this section, we study the joint rate and power allocation when both the secondary users cooperate to maximize the utility function.
In this case we assume that the strategy set is defined by[1]

$$\overline{\mathcal{P}} = \{(P_1, R_1), (P_2, R_2) | P_1, P_2, R_1, R_2 \geq 0\}.$$

We consider again the two extreme regimes when the noise and the interference generated by the primary user is very high and when it is very low. In both cases we show that the optimum resource allocation privileges a single secondary user transmission. The following two propositions state the results.

[1] Note that $\overline{\mathcal{P}}$ is the closure of the open strategy set $\mathcal{P}$ of game $\mathcal{G}$.

Proposition 6. *Let us assume that the noise plus the interference from the primary user are very high compared to the power transmitted by the secondary transmitter, or equivalently,* $\frac{g_i}{C_i} > N_0 + P_* h_{*i}$ *and* $\frac{g_i}{C_i} \approx N_0 + P_* h_{*i}$, $i = 1, 2$. *Then, if*

$$\log \frac{g_i}{C_i(N_0 + P_* h_{*i}) + C_i\,(N_0 + P_* h_{*i})} > \log \frac{g_j}{C_j(N_0 + P_* h_{*j}) + C_j\,(N_0 + P_* h_{*j})} \qquad i, j = 1, 2 \quad i \neq j \quad (19)$$

transmitter i *transmits at power* $P_i = \frac{1}{g_i}\left(\frac{g_i}{C_i} - N_0 - P_* h_{*i}\right)$ *and rate* $R_i = \log\left(\frac{g_i}{C_i(N_0 + P_* h_{i*})}\right) \approx \frac{g_i}{C_i(N_0 + P_i)}$, *and the transmitter* j *is silent, i.e.* $P_j = R_j = 0$.

Similarly, for the noise and interference from the primary user negligible compared to the interference from the secondary user the following result holds.

Proposition 7. *Let us assume that the noise plus the interference from the primary user are very low while the potential interference from the secondary source could be substantially higher, i.e,* $N_0 + P_* h_{*1} \to 0$ *and* $\frac{\sigma_{21}^2}{C_2} \gg 0$ *for transmitter 1 and* $N_0 + P_* h_{*2} \to 0$ *and* $\frac{\sigma_{12}^2}{C_1} \gg 0$ *for transmitter 2. There does not exist an optimum allocation strategy for both* $P_1, P_2 > 0$. *If* (19) *is satisfied, transmitter* i *transmits at power and rate*

$$P_i = \frac{1}{g_i}\left(\frac{g_i}{C_i} - N_0 - P_* h_{*i}\right) \approx \frac{1}{C_i} \quad \text{and } R_i = \log\left(\frac{g_i}{C_i(N_0 + P_* h_{*i})},\right)$$

respectively, while transmitter j *stays silent.*

Note that both under the conditions of Proposition 6 and 7, a decision on the optimum resource allocation would require knowledge of both h_{*1} and h_{*2} at both secondary transmitters. A distributed resource allocation approach requires an exchange of information between transmitter 1 and transmitter 2, which has been introduced in the system model.

Closed form resource allocation strategies for the general case are not available and numerical constrained optimization is necessary.

6 Numerical Result

In this section, we assess the performance of the proposed algorithms and compare them. The resource allocation has a complex dependency on several system parameters, e.g. noise, interference from the primary user, channel gains, costs. We first investigate the performance of the game resource allocation on the system parameters. We consider a system with parameters $\sigma_{12}^2 = \sigma_{21}^2 = 0.1$ and $g_1 = g_2 = 1$. Figure 2 shows the throughput attained by the game based algorithm for increasing costs $Q_i = C_i + K_i \sigma_{i*}^2$ and $Q_i = Q_j$. As expected, in the general case, an increase of the costs implies a decrease of the achievable

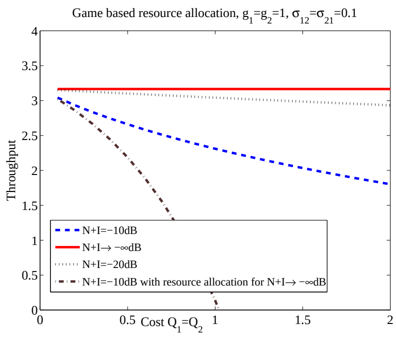

Fig. 2. Throughput attained by Nash equilibria versus costs $Q_1 = Q_2$ for different values of the noise plus interference from the primary user.

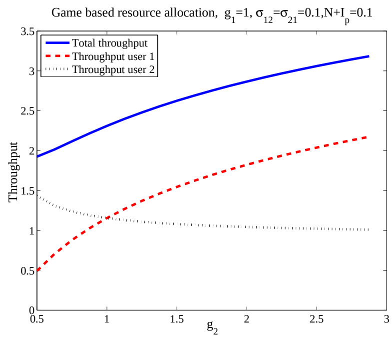

Fig. 3. Throughput of the two secondary users and total throughput attained by Nash equilibria versus user 2 channel attenuation.

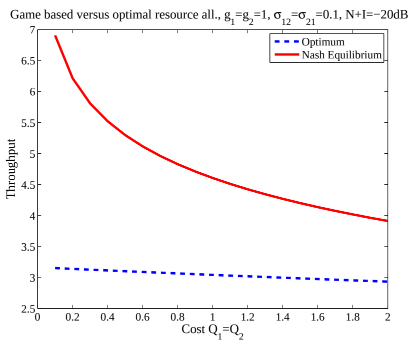

Fig. 4. Throughput versus costs $Q_1 = Q_2$. Comparison between the throughput attained by Nash equilibria or by optimum resource allocation.

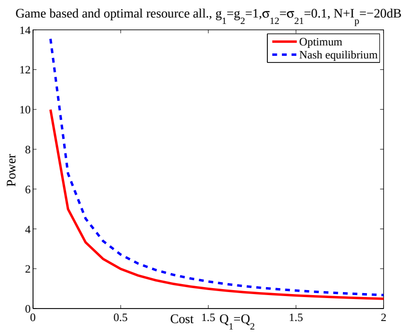

Fig. 5. Transmitted power versus costs $Q_1 = Q_2$. Comparison between the resources allocated by Nash equilibria or by optimum resource allocation.

throughput. The solid line in Figure 2 shows the throughput in the secondary interference limited regime. In this case the system performance is completely independent of the channel cost. At first glance, this bahaviour could appear surprising. However, it is a straightforward consequence of Proposition 1 when we observe that the best responses depend on the costs only via the ratio Q_1/Q_2. The dependency of the throughput on the costs becomes more and more relevant when $N + I_p$, the noise and the interference from the primary user, increases. Finally, the dashed dotted line in Figure 2 shows the degradation in terms of throughput, when the presence of $N + I_p$ is neglected in the resource allocation but $N + I_p = -10dB$. Figure 3 illustrates the dependency of the throughput on the channel attenuation g_2 of user 2 for the following set of parameters: $\sigma_{12}^2 = \sigma_{21}^2 = 0.1$, $N + I_p = -10dB$, $Q_1 = Q_2 = 1$. For increasing values of g_2, the total throughput decreases because of the increased interference of user 2 on user 1. Note that for game based resource allocation the users access simultaneously to the channel while the optimum resource allocation privileges a time sharing policy.

Figure 4 and 5 compared the game based resource allocation to the optimum one. They show the throughput and the power, respectively, as function of the costs. For very low values of $N+I_b$ and low costs, the optimum resource allocation outperforms significantly the game based approach at the expenses of fairness. In fact, the former assigns the spectrum to a single user. The performance loss at the Nash equilibrium decreases as the costs increases.

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