

# RECEIVER DIVERSITY WITH BLIND AND SEMI-BLIND FIR SIMO CHANNEL ESTIMATES

Samir-Mohamad Omar, Dirk T.M. Slock and Oussama Bazzi

**Abstract**—Traditionally, the performance of blind SIMO channel estimates has been characterized in a deterministic fashion, by identifying those channel realizations that are not blindly identifiable. In this paper, we focus instead on the performance of Zero-Forcing (ZF) Linear Equalizers (LEs) or Decision-Feedback Equalizers (DFEs) for fading channels when they are based on (semi-)blind channel estimates. Although it has been known that various (semi-)blind channel estimation techniques have a receiver counterpart that is matched in terms of symbol knowledge hypotheses, we show here that these (semi-)blind techniques and corresponding receivers also match in terms of diversity order: the channel becomes (semi-)blindly unidentifiable whenever its corresponding receiver structure goes in outage. In the case of mismatched receiver and (semi-)blind channel estimation technique, the lower diversity order dominates. Various cases of (semi-)blind channel estimation and corresponding receivers are considered in detail. To be complete however, the actual combination of receiver and (semi-)blind channel estimation lowers somewhat the diversity order w.r.t. the ideal picture.

**Index Terms**—channel estimation, blind, semi-blind, receiver diversity, imperfect channel state information

## I. INTRODUCTION

Consider a linear modulation scheme and single-carrier transmission over a Single Input Multiple Output (SIMO) linear channel with additive white noise. The multiple (sub-channel) outputs will be mainly thought of as corresponding to multiple antennas. After a receive (Rx<sup>1</sup>) filter (possibly noise whitening), we sample the Rx signal to obtain a discrete-time system at symbol rate<sup>2</sup>. When stacking the samples corresponding to multiple Rx antennas in column vectors, the discrete-time communication system is described by

$$\underbrace{\mathbf{y}_k}_{n_r \times 1} = \underbrace{\mathbf{h}[q]}_{n_r \times 1} \underbrace{\mathbf{a}_k}_{1 \times 1} + \underbrace{\mathbf{v}_k}_{n_r \times 1} \quad (1)$$

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Samir-Mohamad Omar, Dirk T.M. Slock are with Mobile Communications Dept., EURECOM, BP 193, 06904 Sophia Antipolis Cdx, France. email: {omar, slock}@eurecom.fr

Oussama Bazzi is with Dept. of Physics and Electronics, Faculty of Sciences I, Lebanese University, Hadath, Beirut, Lebanon. email: obazzi@ul.edu.lb

<sup>1</sup>In this paper, "Rx" stands for "receive" or "receiver" or "reception" etc., and similarly for "Tx" and "transmit", ...

<sup>2</sup>In the case of additional oversampling with integer factor w.r.t. the symbol rate, the Rx dimension would get multiplied by the oversampling factor.

where  $k$  is the symbol (sample) period index,  $n_r$  is the number of Rx antennas. The noise power spectral density matrix is  $S_{\mathbf{v}\mathbf{v}}(z) = \sigma_v^2 I_{n_r}$ ,  $q^{-1}$  is the unit sample delay operator:  $q^{-1} a_k = a_{k-1}$ , and  $\mathbf{h}[z] = \sum_{i=0}^L \mathbf{h}_i z^{-i}$  is the SIMO channel transfer function in the  $z$  domain. The channel delay spread is  $L$  symbol periods. In the Fourier domain we get the vector transfer function  $\mathbf{h}(f) = \mathbf{h}[e^{j2\pi f}]$ .

We introduce the vector containing the SIMO impulse response coefficients<sup>3</sup>  $\mathbf{h} = [\mathbf{h}_0^T \dots \mathbf{h}_L^T]^T$ . Assume the energy normalization  $\text{tr}\{R_{\mathbf{h}\mathbf{h}}\} = n_r$  with  $R_{\mathbf{h}\mathbf{h}} = \text{E}\{\mathbf{h}\mathbf{h}^H\}$ . By default we shall assume the i.i.d. complex Gaussian channel model:  $\mathbf{h} \sim \mathcal{CN}(0, \frac{1}{L+1} I_{n_r(L+1)})$  so that spatio-temporal diversity of order  $n_r(L+1)$  is available (which is the case from the moment  $R_{\mathbf{h}\mathbf{h}}$  is nonsingular). The average per Rx antenna SNR is  $\rho = \sigma_a^2 / \sigma_v^2$ .

Whereas in non-fading channels, the probability of error  $P_e$  decreases exponentially with SNR, for a given symbol constellation, in fading channels the probability of error taking channel statistics into account behaves as  $P_e \sim \rho^{-d}$  for large SNR  $\rho$ , where  $d$  is the diversity order. Also, at high SNR, the  $P_e$  is dominated by the outage probability  $P_o$  and has the same diversity order for a well-designed system. If the data rate  $R$  is adapted with SNR such that we get a normalized rate  $r = \lim_{\rho \rightarrow \infty} \frac{R}{\rho} \in [0, 1]$ , then the diversity becomes  $d(r)$  [1]. For all ZF Rx's considered in this paper, we get the following Diversity-Multiplexing Tradeoff (DMT):  $d(r) = d(0)(1-r)$ . Hence it suffices to limit the diversity analysis to the fixed rate  $R$  case with diversity  $d(0) = d$ .

In practice also the Linear Equalizer (LE) is often used because of low detection complexity. Also in practice, for both LE and DFE, only a limited degree of non-causality (delay) can be used and the filters are usually of finite length (FIR). Analytical investigations into the diversity for SISO with LEs are much more recent, see [3],[5] for linearly precoded OFDM and [6] for Single-Carrier with Cyclic Prefix (SC-CP). The DMT for various forms of LE and DFE with frequency-selective SIMO channels is investigated in [7]. In [3], it was shown that the introduction of redundant linear precoding in OFDM allows a MMSE-

<sup>3</sup>In this paper,  $*$ ,  $\cdot^T$ , and  $\cdot^H$  denote complex conjugate, transpose and Hermitian (complex conjugate) transpose respectively, and  $\mathbf{h}^\dagger[z] = \mathbf{h}^H[1/z^*]$  denotes the paraconjugate (matched filter). Note that  $\mathbf{h}^\dagger[e^{j2\pi f}] = \mathbf{h}^H(f)$ .

ZF linear block receiver to regain full diversity in the SISO (or SIMO) case. For instance Zero Padding (ZP) introduces redundancy in the time (delay) dimension which allows a LE of inter-symbol interference (ISI) to maintain full diversity: every input symbol can be recovered linearly unless the whole channel impulse response becomes zero. In all the references mentioned above the channel was assumed to be perfectly known at the Rx and in some cases at the Tx too. However, practical receivers must estimate the channel, thereby incurring estimation error that needs to be accounted for in the performance analysis. In [10] we treated the effect of blind channel estimation on the diversity of ZF-LE within the context of SIMO Tx system. However, we focused there more on the effect of the constraint usually used to handle the ambiguity that results from blind channel estimation. In [12] the effect of channel estimation error on the performance of the Viterbi equalizer is studied in a SIMO framework. In [13] the bit-error rate (BER) performance of multilevel quadrature amplitude modulation with pilot-symbol-assisted modulation channel estimation in static and Rayleigh fading channels is derived, both for single branch reception and maximal ratio combining diversity receiver systems. However, in [14] it is shown that the (practical) ML channel estimator preserves the diversity order of MRC (Maximum Ratio Combining), see also [15] for a more thorough analysis.

In this paper we assume the channel to be estimated at the Rx using blind and semi-blind deterministic algorithms and we investigate the effect of the resulting channel estimation error on the diversity achieved by the corresponding equalizers (matched to the channel estimation hypotheses).

## II. OUTAGE ANALYSIS OF SUBOPTIMAL RECEIVER SINRS

A perfect outage occurs when  $\text{SINR} = 0$ . For the Matched Filter Bound (MFB) this can only occur if  $\mathbf{h} = 0$ . For a suboptimal Rx however, the  $\text{SINR} = \text{SINR}(\mathbf{h})$  can vanish for any  $\mathbf{h}$  on the *Outage Manifold*  $\mathcal{M} = \{\mathbf{h} : \text{SINR}(\mathbf{h}) = 0\}$ . At fixed rate  $R$ , the diversity order is the codimension of (the tangent subspace of) the outage manifold, assuming this codimension is constant almost everywhere and assuming a channel distribution with finite positive density everywhere (e.g. Gaussian with non-singular covariance matrix). For example, for the MFB (which only depends on  $\mathbf{h}$ ) the outage manifold is the origin, the codimension of which is the total size of  $\mathbf{h}$ . The codimension is the (minimum) number of complex constraints imposed on the complex elements of  $\mathbf{h}$  by putting  $\text{SINR}(\mathbf{h}) = 0$ . Some care has to be exercised with complex numbers. Valid complex constraints (which imply two real constraints) are such that their number becomes an equal number of real constraints if the channel coefficients were to be real. A constraint on a coefficient magnitude however, which is in principle only one real constraint, counts as a valid complex constraint (at least if the channel coefficient distributions are insensitive to phase changes). For ZF equalizers, consideration of the outage manifold is sufficient. For MMSE equalizers however, a

more complete analysis is required. An actual outage occurs whenever the rate exceeds the capacity,  $\log(1 + \text{SINR}) < R$ , which occurs when  $\mathbf{h}$  lies in the *Outage Shell*, a (thin) shell containing the outage manifold. The thickness of this shell shrinks as the rate increases and depends also on the regularization appearing in MMSE equalizers.

## III. BLIND (B) AND SEMI-BLIND (SB) CHANNEL ESTIMATION AND MATCHED ZF EQUALIZATION

Consider a block Tx system with Rx signal in the time domain [2]

$$\mathbf{Y} = \mathbf{H} \mathbf{A} + \mathbf{V} = \mathbf{H}_K \mathbf{A}_K + \mathbf{H}_U \mathbf{A}_U + \mathbf{V} = \mathcal{A} \mathbf{h} + \mathbf{V} \quad (2)$$

where  $\mathbf{A}$  is the vector of Tx symbols, containing possibly known symbols  $\mathbf{A}_K$  (training/pilots, semi-blind case) and unknown symbols  $\mathbf{A}_U$  (actual data, i.i.d. with variance  $\sigma_a^2$ ).  $\mathbf{H} = \mathbf{H}(\mathbf{h})$  is the channel convolution matrix of which the part  $\mathbf{H}_K$  is affected by  $\mathbf{A}_K$  and the part  $\mathbf{H}_U$  is affected by  $\mathbf{A}_U$ . Due to the commutativity of convolution,  $\mathbf{H} \mathbf{A} = \mathcal{A} \mathbf{h}$  in which  $\mathcal{A} = \mathcal{A}(\mathbf{A}_K, \mathbf{A}_U)$  and  $\mathbf{h}$  contains the vectorized channel impulse response coefficients.  $\mathbf{V}$  is the AWGN with variance  $\sigma_v^2$ . Even though we shall investigate the diversity of receivers due to fading channels, for (semi-)blind channel estimation purposes, the channel  $\mathbf{h}$  is considered a deterministic unknown. In the (semi-)blind techniques considered here, also  $\mathbf{A}_U$  is considered a deterministic unknown.  $\mathbf{H}_U$  and  $\mathcal{A}$  are assumed to have full column rank w.p. 1 when  $\mathbf{h}$  and  $\mathbf{A}$  would be considered random.

Maximum likelihood (ML) estimation of  $\mathbf{h}$  (with  $\mathbf{A}_U$  as nuisance parameters) leads to the least-squares cost function [8]

$$\min_{\mathbf{h}, \mathbf{A}_U} \|\mathbf{Y} - \mathbf{H} \mathbf{A}\|^2. \quad (3)$$

As this cost function is separable [8], we can first optimize w.r.t.  $\mathbf{A}_U$ , which leads to

$$\hat{\mathbf{A}}_U = (\mathbf{H}_U^H \mathbf{H}_U)^{-1} \mathbf{H}_U^H (\mathbf{Y} - \mathbf{H}_K \mathbf{A}_K). \quad (4)$$

In the semi-blind case ( $\mathbf{A}_K \neq 0$ ), this is a particular form of a MMSE-ZF block DFE, with feedback only from the known symbols  $\mathbf{A}_K$ . Here, the diversity of a DFE will only get analyzed with a matched semi-blind channel estimate, in which the feedback symbols play the role of pilots. In the blind case,  $\mathbf{A}_K = 0$ ,  $\mathbf{H}_U = \mathbf{H}$  and (4) corresponds to a MMSE-ZF block LE. The ML (semi-)blind channel estimate is obtained by minimizing (3) after having plugged in (4), leading to

$$\hat{\mathbf{h}} = \arg \min_{\mathbf{h}} \|P_{\mathbf{H}_U}^\perp (\mathbf{Y} - \mathbf{H}_K \mathbf{A}_K)\|^2 \quad (5)$$

where we introduced the projection matrices  $P_{\mathbf{H}}^\perp = I - P_{\mathbf{H}}$  and  $P_{\mathbf{H}} = \mathbf{H}(\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H$ . Note that the Rx diversity with (semi-)blind channel estimate to be considered here is not restricted to only ML channel estimates however; any other (semi-)blind method that exploits the same information will lead to similar diversity results.

The Fischer Information Matrix (FIM) for the joint estimation of  $\boldsymbol{\theta} = [\mathbf{A}_U^H \mathbf{h}^H]^H$  is

$$FIM_{joint}^{SB} = \frac{1}{\sigma_v^2} [\mathbf{H}_U \mathcal{A}]^H [\mathbf{H}_U \mathcal{A}] . \quad (6)$$

The marginal Cramer-Rao Bound (CRB) for  $\mathbf{A}_U$  (treating  $\mathbf{h}$  as nuisance parameters) is

$$CRB_{\mathbf{A}_U}^{SB} = \sigma_v^2 (\mathbf{H}_U^H P_{\mathcal{A}}^\perp \mathbf{H}_U)^{-1} \quad (7)$$

while for  $\mathbf{h}$  (treating  $\mathbf{A}_U$  as nuisance parameters), it is

$$CRB_{\mathbf{h}}^{SB} = \sigma_v^2 (\mathcal{A}^H P_{\mathbf{H}_U}^\perp \mathcal{A})^{-1} \quad (8)$$

in which the inverses become pseudo-inverses in the blind case or in the semi-blind case with insufficient pilots [9]. On the other hand, if the channel is known (full Channel State Information at the Rx (CSIR)), the CRB for  $\mathbf{A}_U$  becomes

$$CRB_{\mathbf{A}_U}^{CSIR} = \sigma_v^2 (\mathbf{H}_U^H \mathbf{H}_U)^{-1} . \quad (9)$$

The CRB for symbol  $k$  in  $\mathbf{A}_U$  provides a lower bound on the symbol estimation (reception) error variance, which leads to an SINR upper bound

$$\text{SINR}_k = \frac{\sigma_a^2}{(CRB_{\mathbf{A}_U})_{k,k}} \quad (10)$$

In the case of full CSIR, this is not an upper bound but the correct SINR. In the (semi-)blind case, the bound becomes tight at high SNR, which is the regime of interest for diversity analysis. Now, we get  $\text{SINR}_k^{CSIR} = 0$  whenever  $\mathbf{H}_U$  loses full column rank, in which case  $\mathbf{H}_U^H \mathbf{H}_U$  becomes singular. The number of constraints that this loss of column rank imposes on  $\mathbf{h}$  will be the diversity order. This diversity order will be considered in detail for various cases in the further sections.

Now, considering  $\text{SINR}_k^{SB}$  (see (8), (9) also), we get  $\text{SINR}_k^{SB} = 0$  whenever  $\text{SINR}_k^{CSIR} = 0$ . Hence the diversity order of the Rx with (semi-)blind channel estimate will be at most that of the Rx with full CSIR. The Rx signal dimension reduction due to the projection  $P_{\mathcal{A}}^\perp$  on the noise subspace leads to some reduction in diversity order. Note that due to the randomness of  $\mathbf{A}$ , the orientation of the subspaces considered is random. Due to this randomness, the effect of this reduction should become negligible whenever the relative effect of this dimension reduction becomes negligible, namely whenever the ratio of channel delay spread over block length becomes small.

In the paradigm of matched (semi-)blind channel estimate and Rx considered so far, the channel estimation and the data reception are based on the same data block. However, simulations show that the diversity to be analyzed does not change when the channel estimation and data reception are performed on disjoint data blocks, where the Rx for one data block is constructed with the channel estimate from a different data block (see the next section also). This would indicate that the diversity effect of the (semi-)blind channel estimate dominates.

#### IV. GENERAL TREATMENT OF THE CASE OF NON-MATCHED RECEIVERS

The channel impulse response  $\mathbf{h}$  can be decomposed into its estimate  $\hat{\mathbf{h}}$  and its estimation error  $\tilde{\mathbf{h}}$ :  $\mathbf{h} = \hat{\mathbf{h}} + \tilde{\mathbf{h}}$ . In the (semi-)blind case,  $\hat{\mathbf{h}}$  represents the channel estimate in which possible ambiguities have been resolved. This channel decomposition leads to the following signal model

$$\mathbf{Y} = \hat{\mathbf{H}} \mathbf{A} + \tilde{\mathbf{H}} \mathbf{A} + \mathbf{V} = \hat{\mathbf{H}}_K \mathbf{A}_K + \hat{\mathbf{H}}_U \mathbf{A}_U + \mathbf{Z} \quad (11)$$

with  $\hat{\mathbf{H}} = \mathbf{H}(\hat{\mathbf{h}})$ ,  $\tilde{\mathbf{H}} = \mathbf{H}(\tilde{\mathbf{h}})$ , and where  $\mathbf{Z} = \tilde{\mathbf{H}} \mathbf{A} + \mathbf{V} = \mathcal{A} \tilde{\mathbf{h}} + \mathbf{V}$  has covariance matrix  $R_{\mathbf{Z}\mathbf{Z}} = E_{\mathbf{A}_U} \mathcal{A} R_{\tilde{\mathbf{h}}\tilde{\mathbf{h}}} \mathcal{A}^H + \sigma_v^2 I$  (if we assume that the channel estimate is obtained from data independent of the  $\mathbf{Y}$  considered here, to make  $\tilde{\mathbf{h}}$  and  $\mathbf{V}$  independent). If we treat  $\mathbf{Z}$  as Gaussian noise that is independent of  $\hat{\mathbf{h}}$  and  $\mathbf{A}_U$ , then we get a capacity (or mutual information (MI)) lower bound (that is fairly tight). The correlations in  $R_{\mathbf{Z}\mathbf{Z}}$  depend on the correlations  $R_{\tilde{\mathbf{h}}\tilde{\mathbf{h}}}$  in the channel estimation error, but may get suppressed by the averaging over  $\mathbf{A}_U$ , depending on the structure of  $\mathcal{A}$ . As far as the independence of  $\mathbf{Z}$  and  $\mathbf{A}_U$  is concerned, this independence is correct if we estimate the channel from one Rx block and use that channel to detect the symbols in a different Rx block (with independent data). In any case, considering outage probability, the MI lower bound leads to a diversity order upper bound.

Whereas the considerations so far pave the way to consider arbitrary Rx structures, in what follows we shall again focus on matched Rx structures (but applied to different data blocks). Thus, a MMSE-ZF ( $\delta = 0$ ) or MMSE ( $\delta = 1$ ) LE/DFE output is obtained as

$$\hat{\mathbf{A}}_U = (\hat{\mathbf{H}}_U^H R_{\mathbf{Z}\mathbf{Z}}^{-1} \hat{\mathbf{H}}_U + \delta \sigma_a^{-2} I)^{-1} \hat{\mathbf{H}}_U^H R_{\mathbf{Z}\mathbf{Z}}^{-1} (\mathbf{Y} - \hat{\mathbf{H}}_K \mathbf{A}_K) \quad (12)$$

with resulting error covariance matrix

$$R_{\tilde{\mathbf{A}}_U \tilde{\mathbf{A}}_U} = (\hat{\mathbf{H}}_U^H R_{\mathbf{Z}\mathbf{Z}}^{-1} \hat{\mathbf{H}}_U + \delta \sigma_a^{-2} I)^{-1} . \quad (13)$$

At least, this expression becomes correct at high SNR, where we can limit the expression to first order terms in  $\sigma_v^2$  and where  $\tilde{\mathbf{H}} \mathbf{A}$  and  $\mathbf{A}_U$  become decorrelated as  $\tilde{\mathbf{h}}$  becomes linear in the noise. The resulting SINR for symbol  $k$  in  $\mathbf{A}_U$  then is

$$\text{SINR}_k^{MMRx} = \frac{\sigma_a^2}{(R_{\tilde{\mathbf{A}}_U \tilde{\mathbf{A}}_U})_{k,k}} - \delta . \quad (14)$$

Practically,  $R_{\mathbf{Z}\mathbf{Z}}$  is not known because it depends on the true channel through  $R_{\tilde{\mathbf{h}}\tilde{\mathbf{h}}} = R_{\tilde{\mathbf{h}}\tilde{\mathbf{h}}}(\mathbf{h})$ . However, at high SNR, one can equivalently use  $R_{\tilde{\mathbf{h}}\tilde{\mathbf{h}}}(\hat{\mathbf{h}})$ . A different complication arises when the channel gets estimated and the Rx symbols get detected from the same Rx signal block. In that case the expression for  $R_{\mathbf{Z}\mathbf{Z}}$  needs to be modified in order to account for the correlation between  $\tilde{\mathbf{h}}$  and  $\mathbf{V}$ . Finally, one has to admit that accounting for  $R_{\mathbf{Z}\mathbf{Z}}$  in the Rx as in (12) complicates the Rx quite a bit. To avoid all these complications, one could consider the simplified Rx

$$\hat{\mathbf{A}}_U^s = (\hat{\mathbf{H}}_U^H \hat{\mathbf{H}}_U + \delta \frac{\sigma_v^2}{\sigma_a^2} I)^{-1} \hat{\mathbf{H}}_U^H (\mathbf{Y} - \hat{\mathbf{H}}_K \mathbf{A}_K) \quad (15)$$

which corresponds to ignoring  $\tilde{\mathbf{H}} \mathbf{A}$  and hence using  $R_{\mathbf{Z}\mathbf{Z}} = R_{\mathbf{V}\mathbf{V}} = \sigma_v^2 I$ . Now further neglecting  $\tilde{\mathbf{H}} \mathbf{A}$  leads to a symbol estimation error covariance matrix lower bound  $R_{\tilde{\mathbf{A}}_U \tilde{\mathbf{A}}_U}^s = \sigma_v^2 (\tilde{\mathbf{H}}_U^H \tilde{\mathbf{H}}_U + \delta \frac{\sigma_v^2}{\sigma_a^2} I)^{-1}$  and to a corresponding SINR upper bound

$$\text{SINR}_k^{MMRxs} = \frac{\sigma_a^2}{\left(R_{\tilde{\mathbf{A}}_U \tilde{\mathbf{A}}_U}^s\right)_{k,k}} - \delta. \quad (16)$$

A perhaps more accurate approximation would be  $R_{\tilde{\mathbf{A}}_U \tilde{\mathbf{A}}_U}^s = (\tilde{\mathbf{H}}_U^H \tilde{\mathbf{H}}_U + \delta \frac{\sigma_v^2}{\sigma_a^2} I)^{-1} (\tilde{\mathbf{H}}_U^H R_{\mathbf{Z}\mathbf{Z}} \tilde{\mathbf{H}}_U + \delta \frac{\sigma_v^4}{\sigma_a^2} I) (\tilde{\mathbf{H}}_U^H \tilde{\mathbf{H}}_U + \delta \frac{\sigma_v^2}{\sigma_a^2} I)^{-1}$ . Our simulations show that these approximate equalizers (15) achieve the same diversity order as those of (12), be it in terms of outage using the SINR in (16) (with either of the two approximate expressions for  $R_{\tilde{\mathbf{A}}_U \tilde{\mathbf{A}}_U}^s$ ) or (14), or in terms of probability of error of these Rxs with QAM transmission. Indeed, according to the various expressions for  $\text{SINR}_k^{MMRxs}$ , an outage should occur whenever  $\tilde{\mathbf{H}}_U$  loses full column rank and/or  $R_{\mathbf{Z}\mathbf{Z}}$  explodes (because  $R_{\tilde{\mathbf{h}} \tilde{\mathbf{h}}}^s$  explodes). In the simulations shown in [10], we worked with (12) except for the case of FIR.

## V. FIXING THE SCALAR AMBIGUITY IN THE BLIND CASE

The blind channel estimate  $\hat{\mathbf{h}}$  can only be determined up to a scalar  $\alpha$  and to make it comparable to the true channel (or to use it in a Rx), this ambiguity needs to be fixed to obtain the final estimate  $\hat{\mathbf{h}} = \hat{\mathbf{h}} \alpha$ . As we shall see (see [10] also), the way by which we resolve the scalar ambiguity has a major effect on the diversity achieved by the receiver. In this paper we deal with three different constraints namely, Linear (Lin) constraint, Least-Squares (LSq) constraint and Fixing one-tap (FOT) constraint. Admittedly, these fixings are rather theoretical. In practice, one needs to consider differential modulation (see [10]) or a semi-blind approach.

### A. Linear (Lin) Constraint

Generally, the cost function of any blind deterministic channel estimation can be represented by  $\hat{\mathbf{h}}^H \mathbf{Q} \hat{\mathbf{h}}$  where possibly  $\mathbf{Q} = \mathbf{Q}(\hat{\mathbf{h}})$ . To resolve the scalar ambiguity we can minimize this cost function subject to a linear constraint as follows:  $\min_{\hat{\mathbf{h}} \mathbf{h} = \mathbf{h}^H \hat{\mathbf{h}}} \|\hat{\mathbf{h}}^H \mathbf{Q} \hat{\mathbf{h}}\|^2$ . Applying the Lagrange multiplier we get:

$$\hat{\mathbf{h}} = \frac{\mathbf{h}^H \mathbf{h}}{\mathbf{h}^H \mathbf{Q}^{-1} \mathbf{h}} \mathbf{Q}^{-1} \mathbf{h}. \quad (17)$$

This constraint yields  $\tilde{\mathbf{h}} \perp \mathbf{h}$  and leads to the minimal CRB. Normally, the CRB is defined as the inverse of the FIM while for a singular FIM with the linear constraint considered here, the corresponding CRB is the pseudo-inverse of the FIM [11].

### B. Least-Squares (LSq) Constraint

In this case the minimization process is done in two steps. First:  $\min_{\|\hat{\mathbf{h}}\|=1} \hat{\mathbf{h}}^H \mathbf{Q} \hat{\mathbf{h}}$  to get  $\hat{\mathbf{h}} = V_{\min}(\mathbf{Q})$ , where  $V_{\min}$

represents the eigenvector that corresponds to the minimum eigenvalue. Then the scalar ambiguity is resolved by least squares as follows:  $\min_{\alpha} \|\mathbf{h} - \alpha \hat{\mathbf{h}}\|^2$ . After some manipulation we get the following solution:

$$\hat{\mathbf{h}} = \frac{\hat{\mathbf{h}}^H \mathbf{h}}{\|\hat{\mathbf{h}}\|^2} \hat{\mathbf{h}} = P_{\hat{\mathbf{h}}} \mathbf{h} \quad (18)$$

so that  $\tilde{\mathbf{h}} \perp \hat{\mathbf{h}}$  which is a well known feature of LS estimation. Also with this constraint, the corresponding CRB is the pseudo-inverse of the FIM. As a result, both the Linear and Least-Squares constraints lead to the same diversity order. Either of these constraints will be assumed in the further discussion of diversity in the blind case.

### C. Fixing One Tap (FOT) Constraint

Now we minimize the cost function by considering wlog. that the first tap of the channel on the first Rx antenna is known:  $e_1^H \mathbf{h} = 1$  with  $e_1^H = [1 0 \dots 0]$ ,  $\min_{e_1^H \hat{\mathbf{h}}=1} \hat{\mathbf{h}}^H \mathbf{Q} \hat{\mathbf{h}}$ .

Applying the Lagrange multiplier we get:

$$\hat{\mathbf{h}} = \mathbf{Q}^{-1} e_1 \frac{e_1^H \mathbf{h}}{e_1^H \mathbf{Q}^{-1} e_1}. \quad (19)$$

It is obvious from (19) that for  $\hat{\mathbf{h}}$  to vanish it is sufficient that  $e_1^H \mathbf{h}$  gets very small. Hence the diversity achieved is one regardless of the Rx used:  $d^{FOT} = 1$ . This may in part explain the bad performance of blind channel estimation algorithms using this constraint.

## VI. ZF EQUALIZATION IN SINGLE CARRIER CYCLIC PREFIX (SC-CP) SYSTEMS

The diversity of LE for SC-CP systems has been studied in [6] for the SISO case with i.i.d. Gaussian channel elements, fixed rate  $R$  and block size  $N = L+1$ . The LE DMT for SIMO SC-CP systems appears in [7]. Consider a block of  $N$  symbol periods preceded by a cyclic prefix (CP) of length  $L$  (as a result of the CP insertion, actual rates are reduced by a factor  $\frac{N}{N+L}$ , which is ignored here in what follows). The channel input-output relation over one block can be written as

$$\mathbf{Y} = \mathbf{H} \mathbf{A} + \mathbf{V} = \mathbf{A} \mathbf{h} + \mathbf{V}; \quad (20)$$

where  $\mathbf{Y} = \mathbf{Y}_k = [\mathbf{y}_k^T \mathbf{y}_{k+1}^T \dots \mathbf{y}_{k+N-1}^T]^T$  etc.  $\mathbf{H}$  is a banded block-circulant matrix (see (13) in [7]) and  $\mathbf{A} = \mathbf{A}' \otimes \mathbf{I}_{n_r}$  where  $\mathbf{A}'$  is a toeplitz matrix filled with the elements of  $\mathbf{A}$ . Now apply an  $N$ -point DFT (with matrix  $F_N$ ) to each subchannel received signal, then we get

$$\underbrace{F_{N,n_r} \mathbf{Y}}_{\mathbf{U}} = \underbrace{F_{N,n_r} \mathbf{H} F_N^{-1}}_{\mathcal{H}} \underbrace{F_N \mathbf{A}}_{\mathbf{X}} + \underbrace{F_{N,n_r} \mathbf{V}}_{\mathbf{W}} \quad (21)$$

where  $F_{N,n} = F_N \otimes I_{n_r}$  (Kronecker product:  $A \otimes B = [a_{ij} B]$ ),  $\mathcal{H} = \text{blockdiag}\{\mathbf{h}_0, \dots, \mathbf{h}_{N-1}\}$  with  $\mathbf{h}_n = \mathbf{h}(f_n)$ , the  $n_r \times 1$  channel transfer function at tone  $n$ :  $f_n = \frac{n}{N}$ , at which we have

$$\mathbf{u}_n = \mathbf{h}_n \mathbf{x}_n + \mathbf{w}_n. \quad (22)$$

The  $\mathbf{x}_n$  components are i.i.d. and independent of the i.i.d.  $\mathbf{w}_n$  components with  $\sigma_x^2 = N \sigma_a^2$ ,  $\sigma_w^2 = N \sigma_v^2$ .

### A. Blind Channel Estimation

The Rx matched to blind channel estimation is the ZF LE. In the case of full CSIR, the SINR is given by (9), (10). In this case  $\mathbf{H}_U = \mathbf{H}$  is block circulant and loses column rank when  $\mathbf{h}_n = 0$ , i.e. when there is a complete fade on one of the tones, which represents  $n_r$  constraints on  $\mathbf{h}$ . So in this case simultaneously the ZF LE fades and the channel becomes unidentifiable. Hence, the full CSIR diversity is  $n_r$ . In the case of the LE with blind channel estimate, we need to consider (7), (10). As mentioned earlier, the combination of the blind channel estimate in the LE Rx leads to some Rx dimension and hence some diversity loss due to  $P_{\mathbf{A}}^\perp$ . As a result we can state that

$$d_{SC-CP}^{B-ZF} \leq d_{SC-CP}^{CSIR-ZF} = n_r \quad (23)$$

where the inequality becomes an equality as  $\frac{L+1}{N} \rightarrow 0$ . In the case of full CSIR, the SINR is identical for all symbols in the block. The SINR becomes position dependent in the blind case. We have investigated via simulations the dependence of the diversity order on the symbol position but did not find any. Also replacing the per symbol MSE by an average over the block led to the same diversity.

### B. Semi-Blind Channel Estimation

We consider here  $M$  consecutive pilot symbols in the time domain. For the symbol following the  $M$  pilots, the block DFE Rx configuration is exactly that a classical DFE with feedback length  $M$ . It has been shown in [7] that the diversity for such a full CSIR DFE is  $d = n_r(1 + \min\{M, L\})$ . Hence we conclude

$$d_{SC-CP}^{B-ZF} \leq d_{SC-CP}^{SB-ZF} \leq d_{SC-CP}^{CSIR-ZF} = n_r(1 + \min\{M, L\}). \quad (24)$$

In this case the inequality is not only due to channel estimate-Rx coupling for a finite block length as in the blind case, but possibly also depends on the distribution of the  $M$  pilots over the block, as simulations reveal (see further).

## VII. ZF EQUALIZATION IN OFDM SYSTEMS

Whereas for SC-CP the Tx symbols are  $\mathbf{A}$  in time domain, in OFDM the symbols are in  $\mathbf{X}$  in frequency domain. The same block processing formulas remain valid, if considered in frequency domain. In OFDM, the channel is flat at every tone and transmission at different tones is decoupled. As a result, we get for the blind case

$$d_{OFDM}^{B-ZF} \leq d_{OFDM}^{CSIR-ZF} = n_r. \quad (25)$$

In the semi-blind case, pilots are now placed in the frequency domain. If we introduce  $\mathcal{H}\mathbf{X} = \mathcal{X}\mathbf{h}$  then we get

$$CRB_{\mathbf{X}_v}^{SB} = \sigma_w^2 (\mathcal{H}_U^H P_{\mathcal{X}}^\perp \mathcal{H}_U)^{-1} \quad (26)$$

where  $\mathcal{H}_U$  is obtained from  $\mathcal{H}$  by eliminating the columns corresponding to the pilot positions. The pilots have no incidence on reception, only on channel estimation. As a result we get

$$d_{OFDM}^{B-ZF} \leq d_{OFDM}^{SB-ZF} \leq d_{OFDM}^{CSIR-ZF} = n_r. \quad (27)$$

## VIII. ZF FIR/NON-CP EQUALIZATION

For time domain FIR equalization of length  $N$ , the block signal Tx model can be derived from the SC-CP case in (20) by considering a SC-CP block length of  $N+L$  and removing the  $L$  first Rx samples in the block.  $\mathbf{H}$  is now replaced by a  $Nn_r \times (N+L)$  banded block Toeplitz matrix  $\overline{\mathbf{H}}$  which can be obtained from the block circulant  $\mathbf{H}$  by removing the  $L$  top block rows, and  $\mathbf{A}$  is replaced by  $\overline{\mathbf{A}}$  containing  $N+L$  symbols. For a ZF FIR LE with full CSIR and  $N > L$ , it was shown in [7] that the diversity is  $d = n_r - 1$ . There is a diversity order loss of 1 compared to the SC-CP case because now the LE SINR fades or the channel becomes blindly unidentifiable whenever  $\mathbf{h}[z]$  has a zero anywhere in the  $z$ -plane, as opposed to a zero at  $N$  discrete points on the unit circle as for the SC-CP case. So the constraint on  $\mathbf{h}$  is that the  $n_r - 1$  other subchannels have a zero equal to a (any) zero of the first subchannel, which is  $n_r - 1$  constraints. As a result, we get for the FIR ZF LE with matching blind channel estimate

$$d_{FIR}^{B-ZF} \leq d_{FIR}^{CSIR-ZF} = n_r - 1. \quad (28)$$

For the semi-blind case, we can expect similarly  $d_{FIR}^{SB-ZF} \leq d_{SC-CP}^{SB-ZF}$ .

## IX. SIMULATIONS

The probability of error for Tx symbols drawn from an 8PSK constellation is simulated, averaged over  $1e^6$  Monte-Carlo runs of AWGN noise, symbols and i.i.d. Rayleigh fading channel realizations. We consider Tx block length  $N = 20$ ,  $n_r = 3$  Rx antennas, and channel memory  $L = 1$ . For the case of comparable outage probability  $\Pr(O)$  considerations, we assume a rate  $R = \frac{N}{N+L} \log(K)$ , where  $K = 8$  is the constellation size. In Fig. 1 we have simulated  $\Pr(O)$  (via (7), (10)) for the SC-CP and FIR scenarios with blind channel estimation. It is obvious that in both scenarios  $d \approx 2$ . This result confirms our interpretation that blind channel estimation leads to a loss in the diversity order of a ZF-LE. In Fig. 2 we simulate all the Tx scenarios considered in this paper but this time with semi-blind channel estimation. We assume that  $M = 4$  training symbols (pilots) are inserted at the beginning of each block. We observe that SC-CP attains a full diversity order of  $d = 6$  while OFDM achieves  $d = 3$  (full spatial diversity) and the Non-CP case achieves just 2.5 (less than full spatial diversity). However, in Fig. 3 we reduce  $M$  to 2 SC-CP and OFDM, but  $M = 3$  for Non-CP (1 at the beginning of each block and 2 at the end). For SC-CP we get  $d = 4$  while for OFDM  $d = 3$  only. This result reveals the effect of the length of the training sequence (pilots) used on the diversity order achieved. On the other hand, for the Non-CP case with the distributed 3 training symbols, we get  $d = 4$  which is higher than in the previous simulation even though 4 training symbols were inserted at the beginning of the block. This shows the necessity to distribute the training sequence over both edges of the Tx block to achieve higher diversity orders for the Non-CP case.

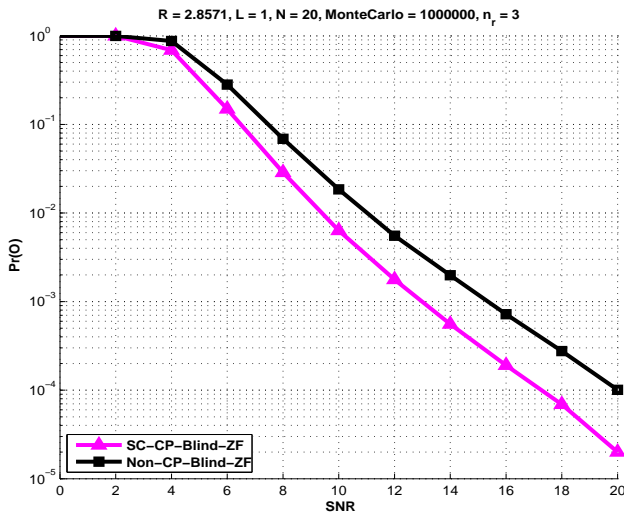


Fig. 1. Probability of outage vs. SNR for SC-CP and FIR (Non-CP) Tx scenarios with blind channel estimation and ZF-LE.

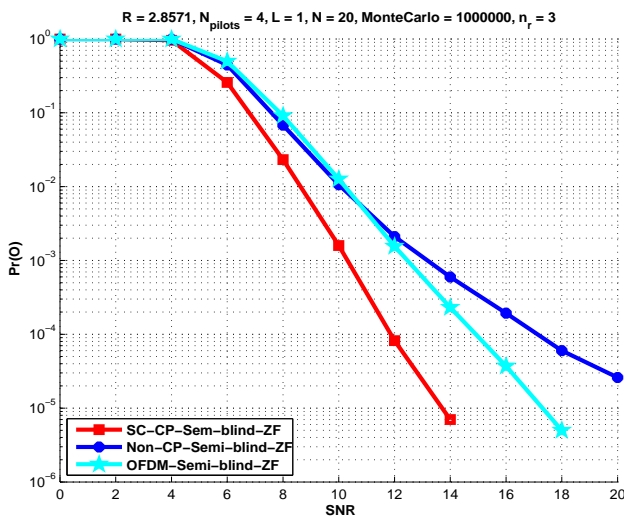


Fig. 2. Probability of outage vs. SNR for SC-CP, OFDM and FIR (Non-CP) Tx scenarios with semi-blind channel estimation and ZF block DFE with 4 pilots at the start of the Tx block.

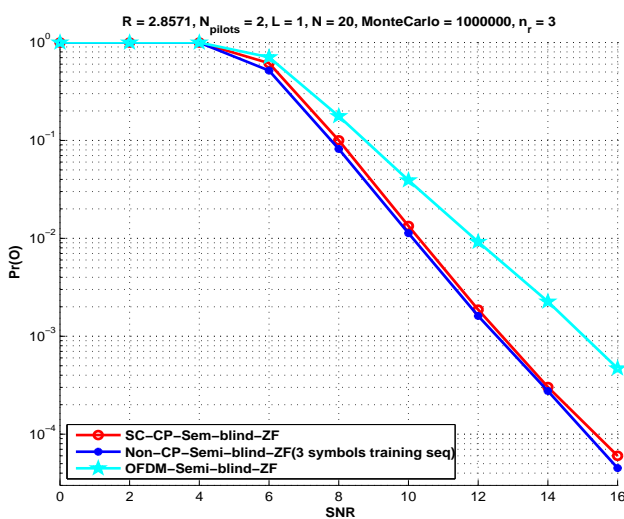


Fig. 3. Probability of outage vs. SNR for SC-CP, OFDM and FIR (Non-CP) Tx scenarios with semi-blind channel estimation and ZF-DFE where 3 pilots are used for the case of Non-CP and 2 pilots for SC-CP or OFDM.

## X. CONCLUSIONS

In this paper we have analyzed the diversity order of MMSE-ZF Linear and Decision-Feedback Equalization for frequency-selective SIMO channels, with the receivers being constructed from matching (semi-)blind channel estimates. The matching is furthermore interpreted here in a strict sense in which both the symbols and the channel get estimated on the basis of the same block of data. We have seen that matching leads essentially to the same diversity order for the receivers considered, built from (semi-)blindly estimated channels or from the true channel. For finite block lengths, the combination of receivers and channel estimates leads to some diversity reduction that requires further investigation. The effect of the positioning of pilot symbols also requires further investigation, as also the analysis of non-matching scenarios,

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