

Preconditioned Iterative Inter-Carrier Interference Cancellation for OFDM Reception in Rapidly Varying Channels

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Abstract—The attractiveness of OFDM decreases with the rising of inter-carrier interference in quickly time-varying channels. Classical OFDM low-complex detection is impaired and more elaborated techniques are required to mitigate the need for full matrix equalization. We present here a fresh approach to this subject, introducing novel fast-converging iterative techniques based on preconditioning. Moreover, we interpret windowing under a new perspective in association with the Basis Expansion Modeling of the time-varying channel. We discuss the complexity of the proposed methods, showing that they are still linear to the OFDM block size. We conclude by illustrating their competitive performance by means of numerical simulations.

Index Terms—OFDM, Inter-Carrier Interference, Iterative Interference Cancelling, Basis Expansion Modeling

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM), adopted by numerous existing wireless telecommunication standards, allows for flexible bandwidth allocation and low-complexity transmitter and receiver architectures. However the performance of classical OFDM low complexity receivers is severely impacted by fast time-varying propagation channels causing the rising of inter-carrier interference (ICI).

Those circumstances occur in the presence of high Doppler spread relative to the OFDM symbol rate due to the mobile receiver velocity. In practice, the increased ICI prevents classical OFDM receiver schemes from reliably detecting the desired signal. Hence, more advanced receiver equalization techniques are required to mitigate the effect of the ICI.

Optimal linear ICI equalization techniques generally involve complex full channel matrix inversion. In existing OFDM telecommunication systems, the typical size of the required Discrete Fourier Transform renders such a full channel matrix inversion operation prohibitively complex for practical implementation. Hence, several approaches have been addressed to reduce the complexity while maintaining acceptable performance. To this end, the use of time-domain windowing of the OFDM received signal has been shown to limit the significant span of the ICI, generating *banded* channel transfer matrices. In addition, iterative equalization and detection techniques

have been proposed to further reduce the complexity of the receiver operating in the frequency-domain, see e.g. [1], [2] and references therein, or in the time-domain as in [3], [4].

We introduce here an alternative and original framework for iterative ICI cancellation. Our analysis of the detection performance, the convergence speed, and the complexity provide guidelines to derive novel fast-converging iterative ICI cancellation algorithms. We show that proper *preconditioning* exploiting the inherent structure of the OFDM signal and the ICI yields to nearly optimal detection performance with very fast-converging and affordable complexity iterative algorithms.

II. SIGNAL AND SYSTEM MODEL

We consider the transmission over a time-varying, frequency-selective fading channel with continuous-time impulse response $h(t, \tau) = \sum_m \alpha_m(t) \psi(\tau - \tau_m)$ assumed to obey the wide sense stationary uncorrelated scattering (WSS-US) model [5], where $\psi(\tau)$ represents the equivalent transmit-receive front-end low-pass filter, τ_m represents the p -th path delay, $\alpha_m(t)$ is the time-varying complex channel coefficient associated with the m -th path of the propagation channel respectively. We shall refer to $h[k, l]$ as the corresponding low-pass sampled discrete-time impulse response, and assume $h[k, l]$ to be well-approximated by a finite-impulse response model with a maximum delay spread of L samples. Then we assume a classical OFDM system with cyclic-prefix (CP) of duration $N_{cp} \geq L$ to avoid inter-symbol-interference. By letting N denote the number of sub-carriers the OFDM symbol duration is given by $N_{block} = N + N_{cp}$. The frequency-domain k -th OFDM transmit symbol $\mathbf{s}[k] = [s[kN] \dots s[kN - N + 1]]^T$, where $(\cdot)^T$ denotes transpose, comprising the encoded symbols $s[i]$ at the output of channel encoding, interleaving and mapping onto a finite-symbol constellation \mathcal{S} assumed i.i.d. with unit energy, is modulated by an $N \times N$ discrete-Fourier transform unitary matrix \mathbf{F} so as to obtain $\mathbf{x}[k] = \mathbf{F}^H \mathbf{s}[k]$ where $(\cdot)^H$ denotes Hermitian operator.

For the sake of the notational simplicity and without loss of generality, we shall drop the time index k in the sequel.

Without accounting for the CP, the received OFDM symbol block can be written as

$$\mathbf{r} = \mathbf{H}\mathbf{F}^H \mathbf{s} + \mathbf{z} \quad (1)$$

where \mathbf{H} represents the $N \times N$ time domain channel convolution matrix, and \mathbf{z} represents a circularly symmetric complex additive white Gaussian noise such that $\mathbf{z} \sim \mathcal{N}_C(\mathbf{0}, \sigma_z^2 \mathbf{I})$.

Since in general $L \ll N$ the channel matrix \mathbf{H} will tend to be *sparse* and *banded*. When the channel is time invariant within an OFDM symbol, \mathbf{H} is circulant and therefore the frequency-domain channel matrix, $\mathbf{F}\mathbf{H}\mathbf{F}^H$, is diagonal.

This characteristic is widely exploited to perform one-tap frequency-domain equalization.

In case of time-varying channel though, \mathcal{H} is no longer circulant and results in a full frequency-domain channel matrix. Thus the classical OFDM equalization approach is highly sub-optimal and more complex equalization is required (see [1], [2] and references therein).

A. Channel BEM Representation

The channel convolution matrix can be reformulated as

$$\mathbf{H} = \sum_{l=0}^{L-1} \mathbf{Q}_l \text{diag}\{\mathbf{h}_l\} \quad (2)$$

where $\mathbf{h}_l = \mathbf{h}[k, l] = [h[kN, l] \dots h[kN - N + 1, l]]^T$ comprises the l -th channel tap time-varying values and \mathbf{Q}_l denotes the corresponding $N \times N$ circulant delay matrix with ones in the l -th lower diagonal and zeros elsewhere, i.e. with elements $[\mathbf{Q}_l]_{ij} = 1$ if $j = (i - l) \bmod N$ and zero otherwise. The vector corresponding to the time-varying evolution of the l -th channel tap can be expressed according to the BEM as follows

$$\mathbf{h}_l = \mathbf{B}\mathbf{v}_l = \sum_{p=0}^{P-1} v_{l,p} \mathbf{b}_p \quad (3)$$

where the $N \times P$ matrix $\mathbf{B} = [\mathbf{b}_0 \mathbf{b}_1 \dots \mathbf{b}_{P-1}]$ denotes the deterministic basis spanned by the P complex vectors \mathbf{b}_p for $p = 0, \dots, P-1$, and $\mathbf{v}_l = [v_{l,0} \dots v_{l,P-1}]^T$ the stochastic coefficients describing the l -th channel tap behavior for the given OFDM block on the P basis functions.

Then, by plugging (2) in (3)

$$\begin{aligned} \mathbf{H} &= \sum_{l=0}^{L-1} \left(\sum_{p=0}^{P-1} v_{l,p} \text{diag}\{\mathbf{b}_p\} \right) \mathbf{Q}_l \\ &= \sum_{p=0}^{P-1} \text{diag}\{\mathbf{b}_p\} \sum_{l=0}^{L-1} v_{l,p} \mathbf{Q}_l \end{aligned} \quad (4)$$

By defining $\mathbf{B}_p = \text{diag}\{\mathbf{b}_p\}$ and summing over the L channel taps, it results

$$\mathbf{H} = \sum_{p=0}^{P-1} \mathbf{B}_p \mathbf{F}^H \mathbf{D}_p \mathbf{F} \quad (5)$$

Then the received signal \mathbf{r} of (1) can be expressed according to the channel BEM as $\mathbf{r} = \sum_{p=0}^{P-1} \mathbf{B}_p \mathbf{F}^H \mathbf{D}_p \mathbf{s} + \mathbf{z}$, $\sum_{l=0}^{L-1} v_{l,p} \mathbf{Q}_l$

being a circulant matrix then $\mathbf{D}_p = \mathbf{F} \sum_{l=0}^{L-1} v_{l,p} \mathbf{Q}_l \mathbf{F}^H$ is a diagonal matrix.

We can conclude that channel BEM shows to be very useful as the operation of convolution by the time-varying channel can be efficiently implemented with $\mathcal{O}(N \log_2 N)$ operations.

III. LINEAR EQUALIZATION

In this section we briefly recall (Linear) Minimum-Mean-Square-Error (L-MMSE), Zero Forcing (ZF), and Matched Filter (MF) equalization.

Letting $\mathcal{H} = \mathbf{H}\mathbf{F}^H$, we have for the estimated OFDM transmitted sequence

$$\hat{\mathbf{s}}_{\text{MMSE}} = \left(\mathcal{H}^H \mathcal{H} + \sigma_z^2 \mathbf{I} \right)^{-1} \mathcal{H}^H \mathbf{r} \quad (6)$$

$$\hat{\mathbf{s}}_{\text{ZF}} = \left(\mathcal{H}^H \mathcal{H} \right)^{-1} \mathcal{H}^H \mathbf{r} \quad (7)$$

$$\hat{\mathbf{s}}_{\text{MF}} = \mathcal{H}^H \mathbf{r} \quad (8)$$

with the MMSE, ZF, and MF linear equalization respectively.

In the assumption of *perfect* knowledge of the channel and of its second order statistics, the MMSE and ZF estimates (6) and (7) entail the inversion of a full matrix in general requiring complexity orders of $\mathcal{O}(N^3)$ or $\mathcal{O}(N^2)$ order when classical techniques are used, such as *Gauss-Jordan elimination* or *Cholesky decomposition* (exploiting the Hermitian nature of $\mathcal{H}^H \mathcal{H}$) respectively [6]. Instead, iterative techniques can be adopted to avoid a full matrix inversion thus reducing the receiver equalization complexity as detailed in the following.

IV. ITERATIVE ICI CANCELLATION

A wide range of iterative techniques have been proposed in the literature to solve linear systems of equation, see e.g. [7]. For a given technique the overall complexity depends on the number of operations per iteration stage times the number of iterations necessary to achieve the estimation accuracy required for the target sequence detection performance. In view of these considerations the speed of convergence is a primary aspect driving the design of an iterative equalization algorithm. Considering a generic linear system of equations of the form

$$\mathbf{A}\mathbf{x} = \mathbf{b} \quad (9)$$

where the vector \mathbf{x} is the sequence to be estimated, \mathbf{b} is the observation vector, and the matrix \mathbf{A} is the input-output transfer matrix, which we assume to be full-rank with dimension $N \times N$ in the scope of our treatment, then for any iterative estimation method, the convergence of the sequence estimates $\hat{\mathbf{x}}^{(k)} \rightarrow \mathbf{x}$ is governed by the spectral properties of the matrix \mathbf{A} . A commonly used metric for those spectral properties is the *condition-number* (CN) $\kappa(\mathbf{A}) = \|\mathbf{A}\| \|\mathbf{A}^{-1}\|$. The closer $\kappa(\mathbf{A})$ is to 1, the faster a given iterative algorithm will converge. In particular, the equalization problems (6) and (7) can be expressed in the form of (9)

$$\left(\mathcal{H}^H \mathcal{H} + \sigma_z^2 \mathbf{I} \right) \hat{\mathbf{s}}_{\text{MMSE}} = \mathcal{H}^H \mathbf{r} \quad (10)$$

$$\left(\mathcal{H}^H \mathcal{H} \right) \hat{\mathbf{s}}_{\text{ZF}} = \mathcal{H}^H \mathbf{r} \quad (11)$$

In light of the above, the convergence of an iterative approach to the solution of both problems will depend on $\kappa(\mathcal{H}^H\mathcal{H}) = \kappa(\mathcal{H})^2$ in the high SNR ($\sigma_z^2 \rightarrow 0$) regimes. In the case of the OFDM system under analysis the matrix \mathcal{H} is in general full-rank. Then the ZF problem (11) can be equivalently expressed as follows

$$\mathcal{H}s_{\text{ZF}} = \mathbf{r} \quad (12)$$

whose CN is $\kappa(\mathcal{H})$, and since $\kappa(\mathcal{H}) \leq \kappa(\mathcal{H}^H\mathcal{H})$, an iterative algorithm applied to (12) will generally converge faster than if applied to (10) and (11).

Iterative techniques can greatly take advantage from appropriate *preconditioning* to reduce the CN and to allow faster convergence. The iterative methods is then applied an equivalent *preconditioned* linear system derived from (9) into $\mathbf{P}\mathbf{A}\mathbf{x} = \mathbf{P}\mathbf{b}$ with \mathbf{P} being the preconditioning matrix and such that $\kappa(\mathbf{A}) \geq \kappa(\mathbf{P}\mathbf{A}) \geq 1$ with $\mathbf{P}\mathbf{A} = \mathbf{1}$ if $\mathbf{P}^{-1} = \mathbf{A}$.

Many preconditioning techniques exist [7]. Among those, a simple, straightforward method is the *Jacobi* preconditioning where \mathbf{P} is chosen to be diagonal and such that $\text{diag}\{\mathbf{P}^{-1}\} = \text{diag}\{\mathbf{A}\}$ if $[\mathbf{A}]_{ii} \neq 0$ for $i = 1, \dots, N$. The *Jacobi* preconditioning suggests that the preconditioning operation consists in approximately solving the problem of inverting matrix \mathbf{A} and transform the original problem into a better conditioned one.

A. Preconditioned Iterative ZF Equalization

In light of the above, in [3] a relevant approach to the ZF iterative ICI cancellation problem is proposed. The described method consists of a *diagonally pre-conditioned* ZF iterative algorithm. The pre-conditioner is made of a diagonal matrix whose elements are exactly the diagonal matrix of the inverse of the frequency-domain channel matrix $\text{diag}\{\mathbf{P}\} = \text{diag}\left\{\left(\mathbf{F}\mathbf{H}\mathbf{F}^H\right)^{-1}\right\}$ and the weighting coefficients are computed as $w_k = (-1)^k \binom{K}{k+1}$ with K being the total number of iterations. Notice that the stage \mathcal{H} is realized using channel BEM as of equation (5) and a *polynomial*-basis functions. Interestingly, the complexity of this approach is linear to the OFDM block size N . The performance of the ZF diagonal pre-conditioned iterative ICI cancellation method [3] can be improved in several respects. First the diagonal pre-conditioning although low-complexity yields to an increased CN with respect to $\kappa(\mathcal{H})$. Secondly it is inherently sub-optimal with respect to the MMSE since attempting to approximate the global ZF solution.

In the following, we address faster converging iterative ICI techniques approaching the MMSE optimal detection performance for a comparable complexity.

B. BEM-MMSE Preconditioned Iterative ICI Cancellation

In this section, we approximate the global MMSE optimal solution iterative techniques combining different forms of *local* MMSE pre-conditioning and combining based on BEM structure. As for the method presented in section IV-A, the

channel BEM allows us to derive here expressions for an improved pre-conditioner yet with affordable complexity.

Indeed, the channel BEM can be exploited at the receiver side and interpreted as a *multiple* windowing of the received signal where the windowing functions correspond to the conjugate of the basis \mathbf{B}_p . Let the output of each *windowing*-branch vector be defined as the projection of the received signal onto the p -th basis function $\mathbf{y}_p = \mathbf{F}\mathbf{B}_p^H\mathbf{r}$, then the *expanded* observation vector of the received signal is obtained by stacking each windowing-branch vector in a $PN \times 1$ vector as

$$\mathbf{y} = \left[\mathbf{B}_0\mathbf{F}^H\mathbf{B}_1\mathbf{F}^H \dots \mathbf{B}_{P-1}\mathbf{F}^H \right]^H \mathbf{r} = \mathbf{U}\mathbf{r} \quad (13)$$

Given the BEM representation of equation (5), we estimate the symbol $s[n]$ at sub-carrier n by adopting *local* MMSE Finite-Impulse-Response (FIR) filter \mathbf{f}_n across tones for all the basis output. Exploiting the particular structure of ICI in the channel BEM representation, one can limit the complexity of a full *per-tone* equalization across all sub-carriers, by properly selecting a subset of the elements of vector \mathbf{y} as $\bar{\mathbf{y}}_n = \mathbf{S}_n\mathbf{y}$ with \mathbf{S}_n being a $L_{\text{FIR}} \times PN$ selection matrix obtained by extracting L_{FIR} rows of the identity matrix \mathbf{I}_{PN} optimally exploiting the structure of \mathbf{U} for a given L_{FIR} and sub-carrier n , $\hat{s}[n] = \mathbf{f}_n^T \bar{\mathbf{y}}_n$.

Therefore, the *per-tone* MMSE filter coefficients are computed such that $\mathbf{f}_n^T = \mathbb{E}\{s[n]\bar{\mathbf{y}}_n^H\} \mathbf{R}_{\bar{\mathbf{y}}_n}^{-1} \bar{\mathbf{y}}_n$ where $\mathbf{R}_{\bar{\mathbf{y}}_n} \bar{\mathbf{y}}_n = \mathbb{E}\{\bar{\mathbf{y}}_n\bar{\mathbf{y}}_n^H\}$ which gives

$$\mathbf{f}_n^T = \mathbf{1}_n \mathcal{H}^H \mathbf{U}^H \mathbf{S}_n^T \left[\mathbf{S}_n \mathbf{U} \left(\mathcal{H} \mathcal{H}^H + \sigma_z^2 \mathbf{I} \right) \mathbf{U}^H \mathbf{S}_n^T \right]^{-1} \quad (14)$$

with $\mathbf{1}_n$ being the $1 \times N$ vector containing 1 in n -th position and 0 elsewhere. It is noteworthy mentioning that the above expression stems from the multiplication of a $1 \times L_{\text{FIR}}$ vector $\mathbb{E}\{s(n)\bar{\mathbf{y}}_n^H\}$ and $L_{\text{FIR}} \times L_{\text{FIR}}$ inverse matrix of $\mathbf{R}_{\bar{\mathbf{y}}_n} \bar{\mathbf{y}}_n$ which varies across sub-carriers.

By applying *local* MMSE filtering, we are able to reduce the computational requirements to $\mathcal{O}(L_{\text{FIR}}^3)$ at the expense of an increased number of iterations depending on the target performance.

All the filters coefficients can be stacked in a *sparse* filter matrix

$$\mathbf{G} = \left[\mathbf{S}_0^T \mathbf{f}_0 \mathbf{S}_1^T \mathbf{f}_1 \dots \mathbf{S}_{N-1}^T \mathbf{f}_{N-1} \right]^T \quad (15)$$

The matrix resulting from the product of $\mathbf{G}\mathbf{U}$ can therefore be seen a *improved* BEM-MMSE pre-conditioner of \mathcal{H} . Moreover, the complexity associated to the filtering operation is proportional to $P(N + N \log_2 N)$.

Indeed, this approach achieves considerably better preconditioning than the one previously presented relying on *diagonal* preconditioning.

This novel approach can be directly plugged into the method described in section IV-A to give the polynomial iterative receiver depicted in figure 1 whose performance are considerably improved, as shown in the simulation results of section V, but yet of affordable complexity as the original method.

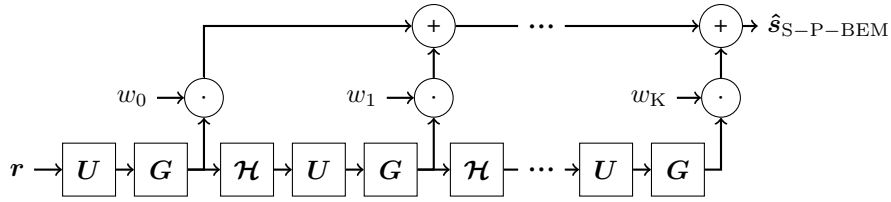


Fig. 1. BEM-MMSE preconditioned iterative receiver

C. BEM-MMSE Parallel Interference Cancellation

The channel BEM representation can be effectively exploited to perform time-domain PIC detection. By setting $\mathcal{H}_p = \mathbf{B}_p \mathbf{F}^H \mathbf{D}_p$ and $\mathcal{H} = \sum_{k=0}^{P-1} \mathcal{H}_k = \mathbf{H} \mathbf{F}^H$, let $\mathcal{H}_0 = \mathcal{H} - \mathcal{H}_0$ represent the *time-varying* part of the channel matrix assuming an orthogonal-polynomial basis, the coefficients of the PIC filtering matrix $\hat{\mathbf{G}}$ are computed according to a modified formula assuming perfect cancellation of the ICI: $\hat{\mathbf{G}} = \mathcal{H}_0^H [\mathbf{U}_0 (\mathcal{H}_0 \mathcal{H}_0^H + \sigma_z^2 \mathbf{I})]^{-1}$ where $\hat{\mathbf{G}}$ is a diagonal matrix. Figure 2 shows the block diagram of the PIC receiver using *hard-decisions* as non-linear decision criterion.

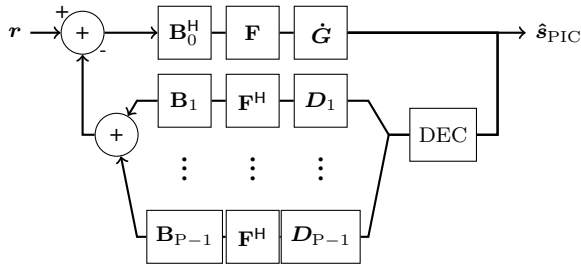


Fig. 2. Time-domain PIC iterative decoder

V. SIMULATION RESULTS

We compare the methods proposed in this paper by means of Monte Carlo simulations assuming a cyclic prefixed OFDM setup with $N = 128$ sub-carriers, a multi-path channel with $L = 4$ with uniform power delay profile and Jake's Doppler spectrum [5] with normalized Doppler frequency of 0.1 with respect to the sub-carrier spacing. We assume an orthogonal-polynomial BEM channel with $P = 2$ sufficient enough to approximate the time-varying channel. The performance are measured in terms of bit-error-rate of uncoded QPSK modulated transmitted sequences. The SNR is defined as the ratio $1/\sigma_z^2$. The methods presented in the paper are evaluated for BEM-MMSE preconditioning filtering lengths of $L_{\text{FIR}} = 3$ and $L_{\text{FIR}} = 5$ for a number of iterations $K = 3$. For all simulation results presented in figures 3–4, the Preconditioned ZF Iterative (P-ZF) of [3] is drastically improved by the use of BEM-MMSE preconditioning (P-BEM) and the PIC iterative receiver provides a good trade-off in terms of performance and complexity.

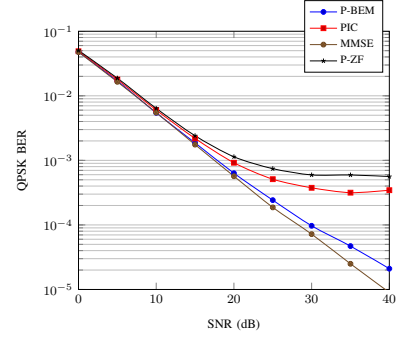


Fig. 3. Performance of iterative methods with 3 iterations, $L_{\text{FIR}} = 5$

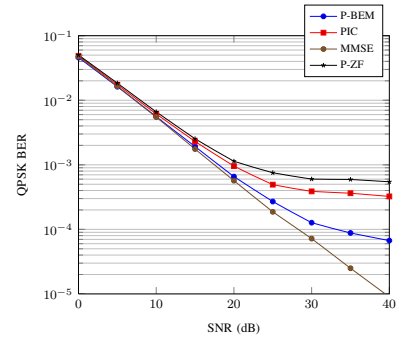


Fig. 4. Performance of iterative methods with 3 iterations, $L_{\text{FIR}} = 3$

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