On the value of data sharing in constrained-backhaul network MIMO

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***Abstract*—This paper addresses the problem of cooperation in a multicell environment where base stations wish to jointly serve multiple users, under a constrained-capacity backhaul. Such a constraint limits, among other things, data sharing and network MIMO concepts need to be revised accordingly. More precisely, we focus on the downlink, and propose to use the backhaul to transmit several messages to each user: some are common to several transmitters and joint precoding is possible, others are private and only local precoding may be done. For the two-cell setup we derive achievable rate regions, optimizing the corresponding beamforming design. Numerical results show how this added flexibility improves performance.**

I. Introduction

A major issue in several types of wireless networks is that of interference. This problem is especially acute in cellular networks with full reuse of the spectrum across all base stations. In traditional designs, each base station obtains from the backbone the data the signals intended for users in its coverage area, i.e., if one ignores cases of soft handover, data for users is not available at multiple base stations. Recent research rooted in MIMO theory has suggested the benefits of relaxing this constraint, thereby allowing for data to be shared at multiple transmitters so that a giant broadcast MIMO channel results. In such a scenario, multicell processing in the form of joint precoding is realized: this scheme is referred to as network MIMO (a.k.a. multicell MIMO).

Full data sharing subsumes very high capacity backhaul links, which may not be feasible, or even simply desirable, in certain applications. Some previous authors have tackled the problem of joint transmission when the backhaul links between a central unit and the transmitters (the base stations), or amongst the latter, are finite, in which case the resulting multicell channel no longer corresponds to a MIMO broadcast channel, nor does it correspond to the so-called interference channel. Among others, in [3] and [4], joint encoding for the downlink of a cellular system is studied under the assumption that the base stations are connected to a central unit via finite-capacity links. The authors investigate different transmission schemes and ways of using the backhaul capacity in the context of a modified version of Wyner's channel model. One of their main conclusions is that "central encoding with oblivious cells", whereby quantized versions of the signals to be transmitted from each base station, computed at the central unit, are sent over the backhaul links, is shown to be a very attractive option for both ease of implementation and performance, unless high data rate are required. If this is the case, the base stations need to be involved in the encoding, i.e. at least part of the backhaul link should be used for sending the messages themselves not the corresponding codewords.

In [5], an optimization framework, for an adopted backhaul usage scheme, is proposed for the downlink of a large cellular system. A so-called joint transmission configuration matrix is defined: this specifies which antennas in the system serve each user, along with the number of quantization bits, for each antenna, associated with that user. Thus the transmit signal of all users are transmitted centrally and different quantized versions of each user's signal are transmitted to the appropriate base stations: this is similar to the central encoding with oblivious cells scheme in [4], except that a more realistic system model is assumed, and the number of quantization bits per user and per antenna are optimized.

In [6], a more information-theoretic approach is taken and a two-cell setup is considered in which, in addition to links between the network and each base station, a finite-capacity link connects the two multi-antenna base stations: the authors view the thus formed channel as a superposition of an interference channel and a broadcast channel. The backhaul is used to share the data to be jointly transmitted: this could be in the form of the full messages, or of quantized versions of the signals to transmit, depending on whether the data is coming from the network directly or shared over the link between the base stations.

In this work, we also consider a two-cell setup, but limit the backhaul to be between the network and each of the base stations. Some of the questions we try to answer are:

- Given the backhaul constraints, what kind of rates can we expect to achieve?
- How much of the data needs to be shared to achieve these rates? I.e. how useful is network MIMO when backhaul constraints are present?

We thus specify a transmission scheme whereby superposition coding is used to transmit signals to each user: this allows us to formulate a continuum between full message sharing between base stations and the conventional network with single serving base stations; the data rate is in fact split between two distinct forms of data to be received by the users, a private form to be sent by the 'serving' base alone and a common form to be transmitted via multiple bases. We express the corresponding rate region in terms of the backhaul constraints and the beamforming vectors used to carry the different signals, and reduce finding the boundary of said region to solving a set of convex optimization problems. This is in contrast to the schemes in [6] where the nonconvexity of the problem makes it difficult to characterize the optimum beamforming vectors to use, and the suboptimal scheme of maximum ratio transmission is resorted to. We also formulate the problem in a way that both the rates that correspond to conventional transmission (each base station receives the signals for one user only) are accounted for in the backhaul.

II. System Model

The system considered is shown in Figure 1. In this preliminary study, we focus on a two transmitter two receiver setup. Receivers have a single antenna each, whereas transmitters have $N_t \geq 1$ antennas: $\mathbf{h}_{ij}$ is the channel between transmitter j and user i; $\mathbf{h}_i$ is user i's whole channel. We assume a backhaul link of capacity C_i between the central processor (or backbone network) and transmitter i, for $i = 1, 2$: it will be used to transmit the messages for each user. We distinguish between different types of messages:

- private messages, which are known at, and consequently only sent from, one of the transmitters, and
- shared or common messages, which are known to both transmitters and consequently jointly transmitted. Note that this notion of a common message is different from that commonly used in the context of interference channels for example, as they do not correspond to messages to be decoded by both receivers, but rather to messages to be sent by both transmitters.

Assumptions We assume each receiver does single user detection, in the sense that the other user's signal is treated as noise. Moreover, we do not rely on dirty paper coding (DPC) to avoid inter-user interference. Furthermore, full channel state information (CSI) is available at both transmitters, since we want to focus on the cost of sharing data.

Notation In what follows, $\bar{i} = \mod(i, 2) + 1, i = 1, 2$ and is used to denote the other transmitter/receiver depending on the context.

A. Backhaul usage

Let $r_{i,p}$ denote the rate of the private message transmitted from transmitter i to receiver i, and $r_{i,c}$ denote the rate of the shared message intended for receiver i. Thus, his total rate is

$$r_i = r_{i,p} + r_{i,c} \tag{1}$$

The backhaul link to transmitter $i, i = 1, 2$ will be used to transmit the messages (so no quantization is done here) that transmitter i is meant to know, i.e. the private message for receiver i along with both shared messages.

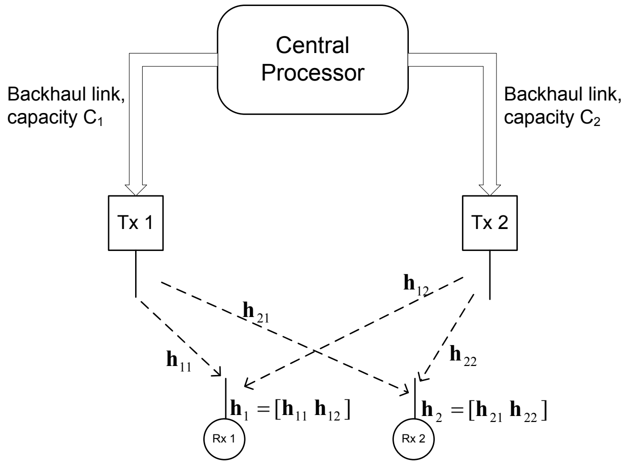

Fig. 1. Constrained backhaul setup.

B. Background: MAC with Common Message

Given our system assumptions, if transmission to user $\bar{i}$ has already been specified, we are left with a MAC with a common message between the two transmitters and user i [1]. Denoting by σ_i^2 the power of the interference, and restricting the transmitted signals to have *rank-1 covariance matrices*, the following rate region is achievable

$$\begin{aligned} r_{i,p} &\leq \log_2 \left(1 + \frac{|\mathbf{h}_{ii}\mathbf{w}_{i,p}|^2}{\sigma_i^2} \right), \\ r_i = r_{i,p} + r_{i,c} &\leq \log_2 \left(1 + \frac{|\mathbf{h}_{ii}\mathbf{w}_{i,p}|^2 + |\mathbf{h}_i\mathbf{w}_{i,c}|^2}{\sigma_i^2} \right), \end{aligned} \tag{2}$$

where the covariance matrix of user i's private message is $\mathcal{C}_{i,p} = \mathbf{w}_{i,p}\mathbf{w}_{i,p}^H$, and that of the common message is $\mathcal{C}_{i,c} = \mathbf{w}_{i,c}\mathbf{w}_{i,c}^H$, and where $\mathbf{w}_{i,p}$ and $\mathbf{w}_{i,c}$ are such that the power constraints at the transmitters are met. Note that Gaussian signaling is optimal for a two-transmitter MAC with a common message (see [2], where this is shown in the context of a MAC with cooperating encoders.).

C. Over the air transmission

In light of the previous subsection, the transmitted signal may be modeled as follows:

$$\begin{aligned} \mathbf{x} = &\begin{bmatrix} \mathbf{w}_{1,c} & \mathbf{w}_{2,c} \end{bmatrix} \begin{bmatrix} s_{1,c} \\ s_{2,c} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_{1,p} \\ \mathbf{0} \end{bmatrix} s_{1,p} \\ &+ \begin{bmatrix} \mathbf{0} \\ \mathbf{w}_{2,p} \end{bmatrix} s_{2,p}, \end{aligned} \tag{3}$$

where $\mathbf{x} \in \mathbb{C}^{2Nt}$ is the transmitted signal, such that the first N_t elements are transmitter 1's transmit signal, the remaining N_t elements are transmitter 2's signal. Though not necessarily optimal, Gaussian signaling is assumed, so that $s_{1,p}, s_{1,c}, s_{2,p}, s_{2,c}$ are all $\mathcal{CN}(0, 1)$. Per base station power constraints $P_i, i = 1, 2$ imply that:

$$\|\mathbf{D}_i\mathbf{w}_{1,c}\|^2 + \|\mathbf{D}_i\mathbf{w}_{2,c}\|^2 + \|\mathbf{w}_{i,p}\|^2 \leq P_i, \tag{4}$$

where $\mathbf{D}_i$ is a matrix whose only non-zero elements are elements $(N_t-1)i+1 : iN_t$ along the diagonal and are equal to 1.

D. Achievable rates

The signal received at receiver i will be given by (see (3)):

$$\begin{aligned} y_i &= \mathbf{h}_i\mathbf{x} + n_i = \begin{bmatrix} \mathbf{h}_{i1} & \mathbf{h}_{i2} \end{bmatrix} \mathbf{x} + n_i \\ &= \mathbf{h}_i\mathbf{w}_{1,c}s_{1,c} + \mathbf{h}_i\mathbf{w}_{2,c}s_{2,c} + \mathbf{h}_{i1}\mathbf{w}_{1,p}s_{1,p} \\ &+ \mathbf{h}_{i2}\mathbf{w}_{2,p}s_{2,p} + n_i \end{aligned} \tag{5}$$

Given our single-user detection (SUD) assumption, user i's rates will satisfy (2) with σ_i^2 given by:

$$\sigma_i^2 = \sigma^2 + \left|\mathbf{h}_{i\bar{i}}\mathbf{w}_{\bar{i},p}\right|^2 + \left|\mathbf{h}_i\mathbf{w}_{\bar{i},c}\right|^2. \tag{6}$$

III. Rate Region

An achievable rate region $\mathcal{R}$ is the set of $(r_1, r_{1,p}, r_2, r_{2,p})$, as specified above, that satisfy the given backhaul and power constraints.

One way to obtain the rate region boundary is to solve the following problem for $\alpha \in [0,1]$, which maximizes the sum rate, subject to a given split between the two users.

$$\begin{aligned} &\text{max. } r \\ &\text{s.t. } r_1 \geq \alpha r \\ &\quad r_2 \geq (1-\alpha) r \\ &\quad r_1 + r_2 - r_{2,p} \leq C_1, \quad r_1 + r_2 - r_{1,p} \leq C_2 \\ &\quad r_i \leq \log_2\left(1 + \frac{\left|\mathbf{h}_{ii}\mathbf{w}_{i,p}\right|^2 + \left|\mathbf{h}_i\mathbf{w}_{i,c}\right|^2}{\sigma^2 + \left|\mathbf{h}_{i\bar{i}}\mathbf{w}_{\bar{i},p}\right|^2 + \left|\mathbf{h}_i\mathbf{w}_{\bar{i},c}\right|^2}\right), i = 1,2, \\ &\quad r_{i,p} \leq \log_2\left(1 + \frac{\left|\mathbf{h}_{ii}\mathbf{w}_{i,p}\right|^2}{\sigma^2 + \left|\mathbf{h}_{i\bar{i}}\mathbf{w}_{\bar{i},p}\right|^2 + \left|\mathbf{h}_i\mathbf{w}_{\bar{i},c}\right|^2}\right), i = 1,2, \\ &\quad \|\mathbf{w}_{i,p}\|^2 + \|\mathbf{D}_i\mathbf{w}_{i,c}\|^2 + \|\mathbf{D}_i\mathbf{w}_{\bar{i},c}\|^2 \leq P_i, i = 1,2 \end{aligned} \tag{7}$$

We solve this problem using a bisection method.

1) $r_{min} = 0, r_{max} = C_1 + C_2$
2) Repeat until $r_{max} - r_{min} < \epsilon$
   a) Set $r = (r_{min} + r_{max})/2$
   b) Determine feasibility of r: this is detailed in subsection III-A below.
   c) If feasible, $r_{min} = r$, else $r_{max} = r$.

A. Establishing feasibility of a given rate

Assume sum rate r and α to be fixed. Thus, $r_1 = \alpha r$, $r_2 = (1-\alpha)r$. Establishing feasibility of a given rate pair hinges on the following two remarks:

- For r_i to be supported, it cannot possibly exceed C_i, and
- Sharing information whenever possible outperforms not doing so. Thus if a rate pair is not achievable for the minimum possible private message rates, it is not achievable at all. Given the backhaul constraints, the minimum possible private message rate $r_{i,p}$, $i = 1,2$ is given by:

$$(r_{i,p})_{min} = \min(r_i, \max(0, r_1 + r_2 - C_{\bar{i}})). \tag{8}$$

How to establish whether a given rate tuple $(r_1, r_{1,p}, r_2, r_{2,p})$ and determine a beamforming scheme to achieve it is specified in section III-B below. If this procedure yields a valid solution for rate tuple $\left(r_1, (r_{1,p})_{min}, r_2, (r_{2,p})_{min}\right)$, then r is feasible [1].

B. Feasibility of $(r_1, r_{1,p}, r_2, r_{2,p})$

Assume r_1, r_2, $r_{1,p}$ and $r_{2,p}$ are fixed. Solve

$$\begin{aligned} &\text{min. } \sum_{i=1}^{2}\left[\|\mathbf{w}_{i,c}\|^2 + \|\mathbf{w}_{i,p}\|^2\right] \\ &\text{s.t. } 2^{r_i} - 1 \leq \frac{\left|\mathbf{h}_{ii}\mathbf{w}_{i,p}\right|^2 + \left|\mathbf{h}_i\mathbf{w}_{i,c}\right|^2}{\sigma^2 + \left|\mathbf{h}_{i\bar{i}}\mathbf{w}_{\bar{i}}\right|^2 + \left|\mathbf{h}_i\mathbf{w}_{\bar{i},c}\right|^2}, i = 1,2, \\ &\quad 2^{r_{i,p}} - 1 \leq \frac{\left|\mathbf{h}_{ii}\mathbf{w}_{i,p}\right|^2}{\sigma^2 + \left|\mathbf{h}_{i\bar{i}}\mathbf{w}_{\bar{i}}\right|^2 + \left|\mathbf{h}_i\mathbf{w}_{\bar{i},c}\right|^2}, i = 1,2, \\ &\quad \|\mathbf{D}_i\mathbf{w}_{i,c}\|^2 + \|\mathbf{D}_i\mathbf{w}_{\bar{i},c}\|^2 + \|\mathbf{w}_{i,p}\|^2 \leq P_i, i = 1,2. \end{aligned}$$

We can transform the above problem into an equivalent convex optimization problem.

- If $r_{i,p} \equiv 0$ or $r_i \equiv r_{i,p}$, we can reduce the problem as follows:
  - If $r_{i,p} \equiv 0$, the corresponding constraint becomes redundant, and $\mathbf{w}_{i,p} = 0$.
  - If $r_i \equiv r_{i,p}$, then $\mathbf{w}_{i,c} = \mathbf{0}$ at the optimum and we can remove the constraint corresponding to r_i.

  In both cases, the remaining constraint can be transformed into a second-order cone program [7], [8], [9].
- Otherwise, the problem is reformulated as follows. Consider the inequalities related to user i's rates. Imposing the decoding order to be common message, then private message , both inequalities must be met with equality at the optimum. Combining these two equations, we get:

$$\frac{2^{r_{i,p}} - 1}{2^{r_i} - 2^{r_{i,p}}}\left|\mathbf{h}_i\mathbf{w}_{i,c}\right|^2 = \left|\mathbf{h}_{ii}\mathbf{w}_{i,p}\right|^2. \tag{9}$$

  Further noting that $\mathbf{h}_i\mathbf{w}_{i,c}$ and $\mathbf{h}_{ii}\mathbf{w}_{i,p}$ being real does not restrict the solution, we can transform our original problem into a convex optimization problem [7], [8], [9]:

$$\begin{aligned} &\text{min. } \sum_{i=1}^{2}\left[\|\mathbf{w}_{i,c}\|^2 + \|\mathbf{w}_{i,p}\|^2\right] \\ &\text{s.t. } \sqrt{2^{r_{i,p}} - 1}\left\|\begin{bmatrix} \sigma & \mathbf{h}_{i\bar{i}}\mathbf{w}_{\bar{i},p} & \mathbf{h}_i\mathbf{w}_{\bar{i},c} \end{bmatrix}\right\| \leq \mathbf{h}_{ii}\mathbf{w}_{i,p}, i = 1,2 \\ &\quad \sqrt{\frac{2^{r_i} - 2^{r_{i,p}}}{2^{r_{i,p}} - 1}}\mathbf{h}_{ii}\mathbf{w}_{i,p} = \mathbf{h}_i\mathbf{w}_{i,c}, i = 1,2 \\ &\quad \|\mathbf{D}_i\mathbf{w}_{i,c}\|^2 + \|\mathbf{D}_i\mathbf{w}_{\bar{i},c}\|^2 + \|\mathbf{w}_{i,p}\|^2 \leq P_i, i = 1,2 \end{aligned}$$

IV. Numerical Results

Figures 2 and 3 show the rate region for different values of the backhaul, for two different instances of the channels with $N_t = 1$. We let $C_1 = C_2 = C$, i.e. similar size backhaul links between the central processor/network and each of the

[1] Note that in our simulations, since not sharing messages yields a simpler beamforming scheme, we first check for the feasibility of rate tuple (r_1, r_1, r_2, r_2).

transmitters. For $N_t = 1$, the sum rate of the interference channel (IC) with SUD is known to be maximized by having the transmitters being either off or transmitting at full power. For the first channel instance shown, the maximum sum rate is achieved by transmitter 1 transmitting at full power and transmitter 2 being off, whereas in the second instance both transmitters transmit at full power. Moreover, in this second case, the IC rate region corresponds to a larger portion of the network MIMO region. When C is low, the same rate region is achieved in both cases. As it increases, the difference between the two setups becomes quite significant.

Finally, Figure 4 compares the maximum average sum rates achieved for $\alpha = .5$ ($r_1 = r_2$) and different channel statistics. Let $\mathbf{h}_{ij} \sim \mathcal{CN}(0, \sigma_{ij}^2 \mathbf{I}_{N_t})$, then the curves marked with x have $\sigma_{ij}^2 = 1$, for $i, j = 1, 2$, whereas those marked with $\triangle$ have $\sigma_{ii}^2 = 1$, and $\sigma_{i\bar{i}}^2 = .5, i = 1, 2$. Note that for lower $\sigma_{i\bar{i}}^2$, higher IC rates are achieved but lower network MIMO rates when the backhaul constraints are ignored. The situation is not as clear-cut when it is. The figure also shows how much of the rates achieved correspond to private messages alone.

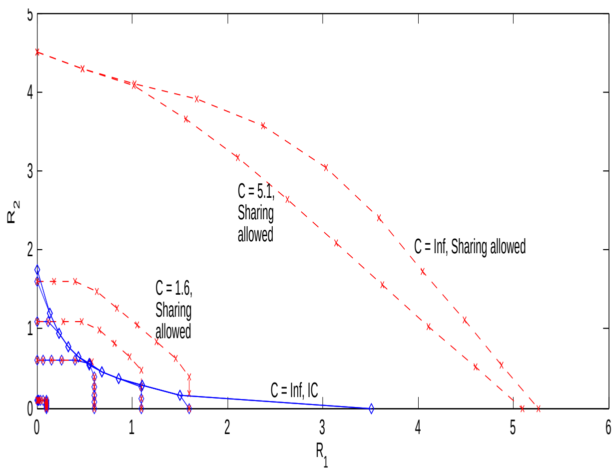

Fig. 2. Sample Rate Region, for $N_t = 1$, SNR = 10dB, and different backhaul rates $C_1 = C_2 = C$. 'x' denotes the scheme proposed, '$\diamondsuit$' the IC.

V. Conclusion

In this paper, we proposed to use the backhaul capacity to convey different types of messages: private messages transmitted from the serving base station, and common messages jointly transmitted from several base stations. A corresponding achievable rate region was characterized and simulations have shown that unless both interference and backhaul capacity are relatively low, the benefit of data sharing is quite significant.

Acknowledgment

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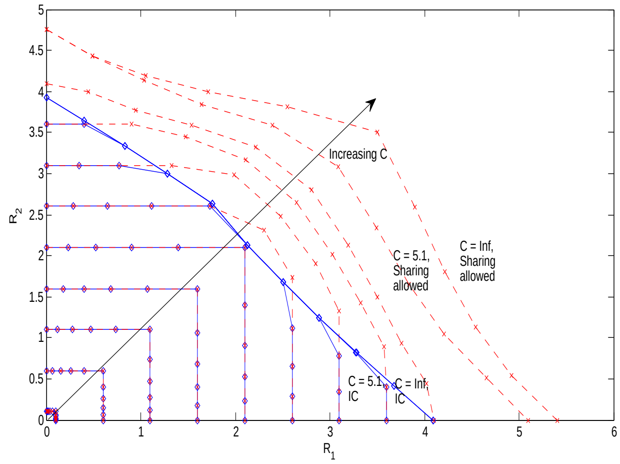

Fig. 3. Sample Rate Region, for $N_t = 1$, SNR = 10dB, and different backhaul rates $C_1 = C_2 = C$. 'x' denotes the scheme proposed, '$\diamondsuit$' the IC.

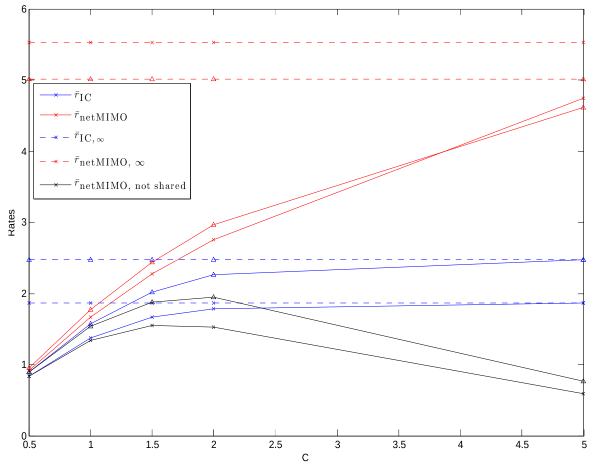

Fig. 4. Average Max Min Rates vs. Backhaul, for $N_t = 1$, SNR = 10dB,

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