

Performance of Different User Selection Algorithms for Transmit Power Minimization

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Abstract—In multi-user communication from one base station (BS) to multiple users, the problem of minimization of transmit power to achieve some target guaranteed performance (rates) at users has been well investigated in literature. Similarly various user selection algorithms have been proposed and analyzed when BS has to transmit to a subset of the users in the system, mostly for the objective of the sum rate maximization.

We study the joint problem of the minimization of the transmit power at the BS with user selection. The general analytical results for the average transmit power required to meet guaranteed performance at the users' side are difficult to obtain even without user selection due to joint optimization required over beamforming vectors and power allocation scalars. Nevertheless, we characterize analytically the average transmit power required to meet guaranteed performance with various users selection algorithms, namely semi-orthogonal user selection (SUS), norm-based user selection (NUS) and angle-based user selection (AUS), in the limiting case when only 2 users are selected for simultaneous transmission. The SUS performs better than other presented selection algorithms due to its user selection mechanism.

I. INTRODUCTION

In multi-antenna downlink (DL) systems, the maximization of the sum rate has been the most widely studied problem. Conditioned upon the availability of perfect channel state information (CSI), the capacity region is known and hence the optimal and a wide variety of sub-optimal (but less complicated) transmission strategies have been treated and analyzed. In many practical wireless systems, maximizing the throughput may not be the primary objective. A very important design objective for multi-antenna multi-user systems is to achieve a particular link quality over all links with minimum transmission power which is equivalent to achieving certain signal-to-interference-and-noise ratios (SINR) or data rates over corresponding links. This problem, in some sense, is the dual problem of the sum rate maximization under a fixed power constraint. Certainly from an operator's perspective, the minimization of the average transmit power to achieve these SINR targets is of prime importance.

The problem of minimization of downlink transmit power required to meet users' SINR constraints by joint optimization of transmit beamforming (BF) and power allocation was solved in [1] and [2]. They showed the interesting duality of uplink (UL) and DL channels for this problem. For Gaussian multi-user channels (either UL or DL), they showed that the problem of the minimization of transmit power corresponding to certain SINR targets bears a relatively simple solution due

to the added structure which may be exploited by successive interference cancellation (SIC) in UL and by dirty paper coding (DPC) based encoding for known interference in DL channels and the results were presented in [3], [4] and [2]. The optimal BF turns out to be the MMSE solution and power allocation is done to raise the SINR level to the target SINR.

The performance of different user selection algorithms for transmit power minimization was studied in [5]. The Gaussian multi-user systems were analyzed without exploiting the extra system structure through SIC or DPC. For the case of 2 users transmitted simultaneously, analytical expressions were obtained for minimum average transmit power required for guaranteed rates with norm-based user selection (NUS) and angle-based user selection (AUS).

We study the problem of average transmit power minimization to meet users' SINR constraints in conjunction with user scheduling. In this Gaussian multi-user system, we make use of SIC in the UL channel or DPC based encoding in the DL channel. This problem formulation gives twofold advantage over [5]: first no iterations are required to compute the optimal BF vectors and power allocation scalars, and second less average power is required at the transmitter to satisfy the same SINR constraints at the users' side. For the case of two users transmitted simultaneously, we derive analytical results for the minimum average transmit power required with semi-orthogonal user selection (SUS), NUS and AUS. To find the minimum average transmit power to achieve certain SINR constraints when users are selected through SUS is one of the novelties of this work. We compare the performance of these user selection algorithms in terms of minimum average transmit power required to satisfy users' SINR constraints. It turns out that NUS and AUS are strictly sub-optimal when compared with SUS.

This contribution is organized as follows. Section II describes the system model. Section III gives a brief overview of the problem of transmit power minimization without user selection. In section IV, certain user selection algorithms are reviewed for which later we analyze the performance. The main results of the paper, the analytical expressions for the minimum average transmit power when two users are simultaneously transmitted, are presented in section V along with performance comparison. The concluding remarks appear in section VI.

Notation: \mathbb{E} denotes statistical expectation. Lowercase letters represent scalars, boldface lowercase letters represent vectors,

and boldface uppercase letters denote matrices. \mathbf{A}^\dagger denotes the Hermitian of matrix \mathbf{A} .

II. SYSTEM MODEL

The system, we consider, consists of a BS having M transmit antennas and K single-antenna user terminals. In the DL, the signal received by k -th user can be expressed as

$$y_k = \mathbf{h}_k^\dagger \mathbf{x} + n_k, \quad k = 1, 2, \dots, K \quad (1)$$

where $\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_K$ are the channel vectors of users 1 through user K with $\mathbf{h}_k \in \mathbb{C}^{M \times 1}$, $\mathbf{x} \in \mathbb{C}^{M \times 1}$ denotes the signal transmitted by the BS and n_1, n_2, \dots, n_K are independent complex Gaussian additive noise terms with zero mean and variance σ^2 . We denote the concatenation of the channels by $\mathbf{H}_F = [\mathbf{h}_1 \mathbf{h}_2 \dots \mathbf{h}_K]$, so \mathbf{H}_F is $K \times M$ forward channel matrix with k -th row equal to the channel of the k -th user (\mathbf{h}_k^\dagger). The channel is assumed to be block fading having coherence length of T symbol intervals. The entries of the forward channel matrix \mathbf{H}_F are i.i.d. complex Gaussian with zero mean and unit variance. The CSI at the transmitter is assumed to be perfectly known.

We suppose that each user has the same SINR constraint of γ . If K_s out of K users are selected for transmission during each coherence interval, the channel input \mathbf{x} can be written as $\mathbf{x} = \bar{\mathbf{V}} \mathbf{P}^{1/2} \mathbf{u}$, where $\bar{\mathbf{V}} \in \mathbb{C}^{M \times K_s}$ denotes the beamforming matrix with normalized columns, \mathbf{P} is $K_s \times K_s$ diagonal power allocation matrix with positive real entries and $\mathbf{u} \in \mathbb{C}^{K_s \times 1}$ is the vector of zero-mean unit-variance Gaussian information symbols. Hence, $\mathbb{E}[\text{Tr}(\mathbf{P})]$ is the average transmit power which can be minimized by optimizing over the beamforming matrix $\bar{\mathbf{V}}$ and the power allocation matrix \mathbf{P} . We select this minimum average transmit power as the performance metric and study the performance of various user selection algorithms when users' SINR constraints (γ) have to be satisfied.

III. OVERVIEW OF TRANSMIT POWER MINIMIZATION PROBLEM

The signal received by k -th user can be written as

$$\begin{aligned} y_k &= \mathbf{h}_k^\dagger \bar{\mathbf{V}} \mathbf{P}^{1/2} \mathbf{u} + n_k, \quad k = 1, 2, \dots, K_s \\ &= \sqrt{p_k} \mathbf{h}_k^\dagger \bar{\mathbf{v}}_k u_k + \sum_{\substack{j=1 \\ j \neq k}}^{K_s} \sqrt{p_j} \mathbf{h}_k^\dagger \bar{\mathbf{v}}_j u_j + n_k, \end{aligned} \quad (2)$$

where the second term represents the interference contribution at k -th user due to beams meant for other selected users. The SINR of k -th user can be written as

$$\text{SINR}_k = \frac{p_k |\mathbf{h}_k^\dagger \bar{\mathbf{v}}_k|^2}{\sum_{\substack{j=1 \\ j \neq k}}^{K_s} p_j |\mathbf{h}_k^\dagger \bar{\mathbf{v}}_j|^2 + \sigma^2}. \quad (3)$$

Without user selection, the problem of optimization of beamforming vectors and power allocation matrix was solved in [2] and [1] using the UL-DL duality (see Section 4.3 and 5.2

in [2]). They gave iterative algorithms to obtain the optimal beamforming vectors and the optimal power allocation for each user. The optimal beamforming vectors corresponding to a particular (sub-optimal) power allocation are obtained, then power allocations are updated corresponding to these beamforming vectors. This process is repeated till both converge to their optimal values. Unfortunately general closed form expressions for transmit power required to achieve SINR targets don't exist due to intricate inter-dependence of beamforming vectors and power allocations, as is evident from eq. (3).

For Gaussian multi-user systems (the case of interest), the extra structure allows the use of SIC in UL or DPC based encoding in the DL. This permits to obtain the optimal BF vectors and power assignments using back substitution without any iteration. Although iterations are not required in this scenario, yet beamforming vector and power allocation of one user depend upon the BF vectors and power assignments of already treated users, hence closed form results are possible only when two users are transmitted simultaneously. If both of the users have the same SINR target γ (for relatively large γ), the minimum instantaneous transmit power required is given by the expression from Section 5.2 of [2].

$$p_r(\mathbf{h}_1, \mathbf{h}_2) = \sigma^2 \gamma \left(\frac{1}{\|\mathbf{h}_1\|^2} + \frac{1}{\|\mathbf{h}_2\|^2 \sin^2(\theta_{12})} \right), \quad (4)$$

where θ_{12} is the angle between the channel vectors of the two users.

IV. REVIEW OF USER SELECTION ALGORITHMS

In this section, we briefly state how different user selection algorithms operate.

A. Norm-Based User Selection (NUS)

In NUS, users are selected based only upon their channel strengths. So K users are sorted in descending order of their channel norm values, and first K_s users are selected for transmission.

B. Angle-Based User Selection (AUS)

The user selection criterion in AUS is the mutual orthogonality of their channel vectors. The first user is selected which has the largest channel norm. The second user is selected as the one which is the most orthogonal to this user, without any regard to its channel strength. This process is repeated till K_s users have been selected.

C. Semi-Orthogonal User Selection (SUS)

The user selection metric for SUS is the combination of the channel strength and its spatial orthogonality w.r.t. the other users. The first chosen user is the one with the largest channel norm. The second chosen user is the one whose projection on the null space of the first user has the largest norm. This process is repeated till K_s users get selected. Interested readers can find the details of this algorithm in [6] or [7].

V. AVERAGE TRANSMIT POWER WITH USER SELECTION

In this section, first we give the main results of this contribution, some analytical expressions for the minimum average transmit power required to achieve certain SINR targets at users' side when these users have been selected following different user selection algorithms. Later, we compare the performance of these user selection algorithms.

Theorem 1 (Minimum Average Transmit Power): Consider a DL system having a BS equipped with M transmit antennas and K single antenna users, each having an SINR constraint of γ , and $K_s = 2$ users are selected for simultaneous transmission in each block. The minimum average transmit power with NUS and AUS denoted as p_N and p_A respectively is given by

$$p_N = \gamma\sigma^2 \left(K\alpha_{M,K-1} - (K-2) - \frac{1}{M-2} \right) \alpha_{M,K}. \quad (5)$$

$$p_A = \gamma\sigma^2 \left(\frac{1}{K-1} \left(\frac{K}{M-1} - \alpha_{M,K} \right) + \frac{(M-1)(K-1)\alpha_{M,K}}{(M-1)(K-1)-1} \right). \quad (6)$$

Now a lower bound of the average transmitted power with SUS denoted as p_S is given by

$$p_S = \gamma\sigma^2 (\alpha_{M,K} + K\alpha_{M-1,K-1} - (K-1)\alpha_{M-1,K}), \quad (7)$$

where $\alpha_{M,K}$ is a constant solely governed by M and K and is given by

$$\alpha_{M,K} = \int_0^\infty K \frac{e^{-x} x^{M-2}}{\Gamma(M)} G(M, x)^{K-1} dx, \quad (8)$$

where $G(M, x)$ is the regularized Gamma function [8]. For the case of two selected users, a useful lower bound on the average transmit power required to achieve SINR targets (performance upper bound) is given by

$$p_L = \gamma\sigma^2 \left(K\alpha_{M,K-1} - (K-1) \left(1 - \frac{M-1}{(M-1)(K-1)-1} \right) \alpha_{M,K} \right) \quad (9)$$

Proof: The proof outline is given in Appendix B using some known results from Appendix A. ■

A. Performance Comparison for $K_s = 2$ Selected Users

The plot of minimum average transmit power required to attain specific SINR targets γ versus number of antennas at the BS appears in Fig. 1. We remark that SUS performs better than the other user selection schemes but with the increase in the number of transmit antennas, NUS also performs very well. The reason comes from the fact that with the increase in the number of transmit antennas, users' channels start becoming (close to) spatially orthogonal and furthermore, due to difference $(M - K_s)$ very good beamforming vectors can be chosen to cause very small interference for other users.

Fig. 2 plots the curves of the minimum average transmit power versus the number of users for a fixed number of transmit antennas. SUS again performs very close to the optimal (obtained by exhaustive search) but we remark that NUS does not behave very well in this scenario because it

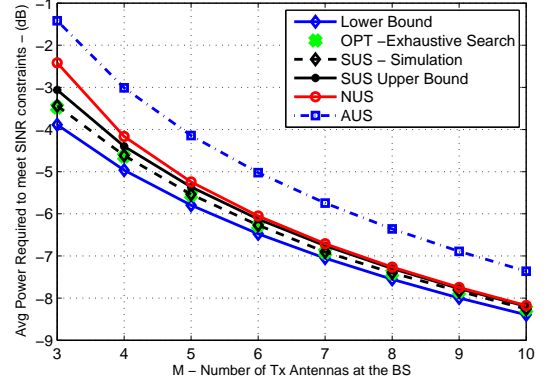


Fig. 1. Min. Avg. Transmit Power vs. M for $K = 10$, $K_s = 2$, $\gamma = 10$ dB, $\sigma^2 = 0.1$. Curves show that SUS is the best strategy and follows closely the power lower bound. NUS also performs close to SUS with increasing number of transmit antennas.

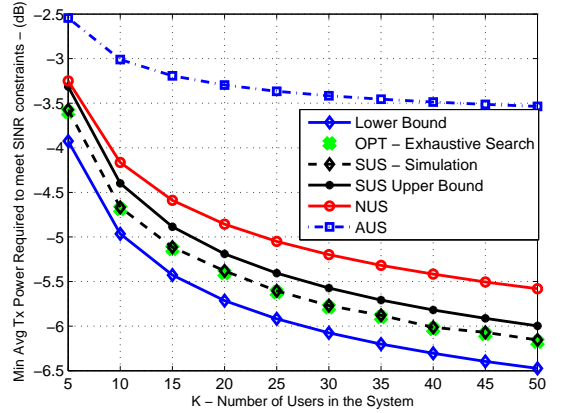


Fig. 2. Min. Avg. Transmit Power vs. Nb. of Users for $M = 4$, $K_s = 2$, $\gamma = 10$ dB, $\sigma^2 = 0.1$. Curves show that SUS performs the best and NUS becomes sub-optimal when number of users increases.

just chooses users with good channel norms without paying any attention to their spatial orthogonality which may affect significantly the interference observed by the selected users.

B. Performance with $K_s > 2$ Selected Users

We have plotted the minimum average transmit power required to achieve certain SINR targets versus the number of transmit antennas and versus the number of system users in Fig. 3 and Fig. 4 respectively, for the already treated user selection algorithms. We observe the same behavior as observed in the case of 2 selected users. For large number of transmit antennas, both SUS and NUS perform very close to the optimal, even AUS performs well. And for a fixed number of transmit antennas at the BS when the number of users present in the system increases, the performance of NUS degrades substantially as it pays no attention to the inter-user spatial separation. The results show that SUS, thanks to its selection mechanism, performs very close to the optimal, for any set of system parameters.

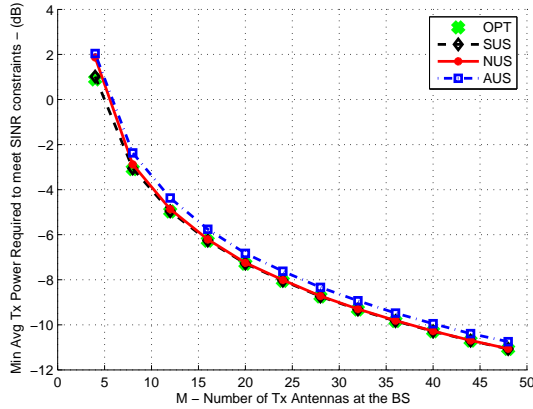


Fig. 3. Min. Avg. Transmit Power vs. M for $K = 8$, $K_s = 4$, $\gamma = 10$ dB, $\sigma^2 = 0.1$. Curves show that SUS is the best strategy and follows closely the power lower bound. NUS also becomes optimal for a reasonably large number of transmit antennas.

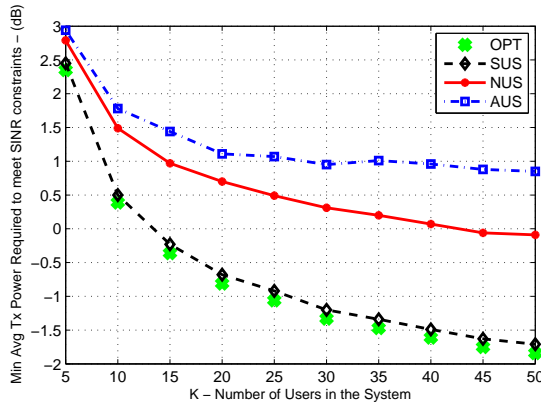


Fig. 4. Min. Avg. Transmit Power vs. Nb. of Users for $M = 4$, $K_s = 4$, $\gamma = 10$ dB, $\sigma^2 = 0.1$. Curves show that SUS performs the best and NUS becomes strictly sub-optimal when number of users increases.

VI. CONCLUSIONS

We have studied the performance of various user selection algorithms in terms of the minimization of average transmit power required to satisfy specific SINR targets at users' side. Closed form expressions of the average transmit power for the three user selection algorithms, namely SUS, NUS and AUS, were derived when only two users are selected for simultaneous transmission. SUS, which has been shown to behave close to optimal for the sum rate maximization, shows very attractive performance in this dual problem setting of transmit power minimization to achieve specific SINR targets.

APPENDIX A - SOME USEFUL DISTRIBUTIONS

In this appendix, we give some useful cumulative distribution functions (CDF) for which probability density functions (PDF) can be computed by simple differentiation. Most of these are known relations, others have been computed using the tools from order statistics [9] and were also given in [5].

As users' channels are spatially i.i.d. Gaussian, the CDF of $\|\mathbf{h}_i\|^2$ for any i is χ^2 distributed with $2M$ degrees of freedom whose CDF is

$$F_i(M; x) = G(M, x), \quad (10)$$

where G denotes the regularized Gamma function [8]. Below we give the CDFs for the largest and second largest order statistics. The CDF of the user having the largest channel norm among K i.i.d. users distributed as $F_i(M; x)$ is

$$F_1(M, K; x) = F_i(M; x)^K. \quad (11)$$

The CDF of the user having the second largest channel norm among K i.i.d. users is

$$F_2(M, K; x) = K F_i(M; x)^{K-1} - (K-1) F_i(M; x)^K. \quad (12)$$

Similarly from [5], the distribution of any random user which does not have the largest norm can be specified as

$$F_i(M, K; x) = \frac{K}{K-1} F_i(M; x) - \frac{1}{K-1} F_i(M; x)^K. \quad (13)$$

The CDF of the angle between any two randomly distributed M -dimensional channel vectors is given by

$$F_{\theta_i}(M; x) = [\sin(x)]^{2M-2}. \quad (14)$$

This can be computed based upon the fact that the squared cosine of the angle between two random vectors is β distributed. Similarly the distribution of the largest angle between one user and any other user in a system of K users, is the maximum of $K-1$ i.i.d. angles distributed as $F_{\theta_i}(M; x)$, hence the CDF for this largest angle statistic is given by

$$F_{\theta_1}(M, K; x) = F_{\theta_i}(M; x)^{K-1} = [\sin(x)]^{2(M-1)(K-1)}. \quad (15)$$

APPENDIX B - PROOF OF THEOREM 1

The instantaneous transmit power required to meet SINR constraint of γ for the two selected users, by using optimal beamforming vectors and power allocation assignments, given in eq. (4), is

$$p_r(\mathbf{h}_1, \mathbf{h}_2) = \sigma^2 \gamma \left(\frac{1}{\|\mathbf{h}_1\|^2} + \frac{1}{\|\mathbf{h}_2\|^2 \sin^2(\theta_{12})} \right). \quad (16)$$

θ_{12} represents the angle between the channel vectors of two selected users. The objective is to compute the average transmit power when these two users have been selected using various user selection algorithms, namely NUS, AUS and SUS.

A. Norm-Based User Selection

For NUS, the users are chosen as described in section IV. Hence the squared norm of the first selected user is distributed as $F_1(M, K; x)$ and that of the second user as $F_2(M, K; x)$. As these users are selected based only upon their channel norms, the angle between their channel vectors θ_{12} is distributed as the angle between any two random vectors and hence has the CDF of $F_{\theta_i}(M; x)$.

For the case of two users, we have two ways to perform SIC (considering UL) or DPC based encoding (considering

DL). It's known that for the objective of the minimization of transmit power, the weaker user should be the one which gets decoded without interference [10]. For this optimal ordering, \mathbf{h}_1 should be the weaker user (2nd strongest, distributed as $F_2(M, K; x)$ and gets decoded with no interference) and \mathbf{h}_2 should be the strongest user distributed as $F_1(M, K; x)$ facing some interference. Hence average transmit power is given by

$$p_N = \sigma^2 \gamma \left(\mathbb{E} \frac{1}{\|\mathbf{h}_1\|^2} + \mathbb{E} \frac{1}{\|\mathbf{h}_2\|^2} \mathbb{E} \frac{1}{\sin^2(\theta_{12})} \right). \quad (17)$$

Computing all these expectations, we get the result.

B. Angle-Based User Selection

For AUS, the first selected user is the strongest user whose squared norm is distributed as $F_1(M, K; x)$ and the second user is the one making the largest angle with the first user. The squared norm of the second selected user is distributed as the squared norm of the any random user which is not the user with the largest norm and hence the CDF is $F_i(M, K; x)$. The angle between the channel vectors of two selected users assumes the distribution of the largest order angle statistic among K users and hence the CDF is given by $F_{\theta_1}(M, K; x)$. Now following the optimal ordering, \mathbf{h}_2 is the strongest user distributed as $F_1(M, K; x)$. The average transmit power for this user selection is given by

$$p_A = \sigma^2 \gamma \left(\mathbb{E} \frac{1}{\|\mathbf{h}_1\|^2} + \mathbb{E} \frac{1}{\|\mathbf{h}_2\|^2} \mathbb{E} \frac{1}{\sin^2(\theta_{12})} \right). \quad (18)$$

This will give the result given in Theorem 1.

C. Semi-Orthogonal User Selection

In SUS, the first user is selected with the largest channel norm but the second selected user is the one whose projection on the null space of the first user has the largest norm. Let's take \mathbf{h}_1 as the user with the largest norm, whose squared norm is distributed as $F_1(M, K; x)$. Now $\mathbf{h}_2 \sin(\theta_{12})$ is the projection of the vector \mathbf{h}_2 on the null space of \mathbf{h}_1 . To compute the expectation over this projected squared norm, we make use of [11, Lemma 3], which was also used in [6, Appendix III]. The term $\|\mathbf{g}_2\|^2 \triangleq \|\mathbf{h}_2\|^2 \sin^2(\theta_{12})$ is the maximum of the $K-1$ channel norms orthogonalized w.r.t. \mathbf{h}_1 . Following [11], we can orthogonalize all channel vectors w.r.t. an arbitrary vector so for each of them the squared norm is χ^2 distributed with $2(M-1)$ degrees of freedom and hence the distribution is given by $F_i(M-1; x)$. Now the second largest norm of these orthogonalized vectors will be

$$\|\hat{\mathbf{g}}_2\|^2 = 2^{\text{nd}} \max \|\hat{\mathbf{g}}_i\|^2, i = 1, \dots, K \quad (19)$$

whose distribution is given by $F_2(M-1, K; x)$. Lemma 3 in [11] shows that statistically $\|\hat{\mathbf{g}}_2\|^2$ is smaller than $\|\mathbf{g}_2\|^2$. Hence for SUS, an upper bound of the average transmit power can be computed by replacing \mathbf{g}_2 with $\hat{\mathbf{g}}_2$ as

$$p_S = \sigma^2 \gamma \left(\mathbb{E}_{F_1(M, K; x)} \frac{1}{\|\mathbf{h}_1\|^2} + \mathbb{E}_{F_2(M-1, K; x)} \frac{1}{\|\hat{\mathbf{g}}_2\|^2} \right). \quad (20)$$

Once the expectations computed in this expression, we get the SUS result of Theorem 1.

D. Performance Upper Bound

To compute a lower bound on the minimum average transmit power required to satisfy SINR targets of γ , the performance upper bound, we assume that the two selected users have the two largest norms as in NUS with CDFs as $F_1(M, K; x)$ and $F_2(M, K; x)$ and the angle between their channel vectors is the largest angle possible as in AUS, distributed as $F_{\theta_1}(M, K; x)$. Hence with optimal ordering, the lower bound on the average transmit power can be obtained by computing the following expression

$$p_L = \sigma^2 \gamma \left(\mathbb{E}_{F_2} \frac{1}{\|\mathbf{h}_1\|^2} + \mathbb{E}_{F_1} \frac{1}{\|\mathbf{h}_2\|^2} \mathbb{E}_{F_{\theta_1}} \frac{1}{\sin^2(\theta_{12})} \right). \quad (21)$$

ACKNOWLEDGMENTS

Eurecom's research is partially supported by its industrial members: BMW Group, Bouygues Telecom, Cisco, Hitachi, ORANGE, SFR, Sharp, STMicroelectronics, Swisscom, Thales. The research work leading to this paper has also been partially supported by the European Commission through the FP7 ICT Network of Excellence NewCom++ and project WHERE, and by the French ANR project APOGEE.

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