Multi-User Diversity Gain for Oblivious and Informed Users in Downlink Channels

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***Abstract*— The absolute gain of multi-user diversity in the context of a downlink channel is the focus of this contribution. Enjoying multi-user diversity gain first requires the channel information availability at the transmitter (CSIT) in a normal downlink (DL) system. Although multi-user diversity gains have been specified well, but the burden of exchange of information required is not rigorously accounted for.**

We analyze a time-division duplex (TDD) broadcast channel with initial assumption of channel information neither at the base station (BS) nor at the users' side. We propose two different but simple transmission strategies which make necessary channel state information (CSI) available at both communicating ends. We give approximate analytical expressions for both strategies which capture well the resource utilization (the cost) and the gain associated to multi-user diversity. This helps us to analyze the absolute gain of the multi-user diversity. The two schemes, the one with Oblivious users and the other with Informed users, are compared for the sum rate and interesting conclusions are drawn.

I. INTRODUCTION

In multiple-antenna broadcast channels if a BS has M transmit antennas and the number of users in the system is K with $K \geq M$, this broadcast channel can support data rates M times larger than a single antenna BS, although all users may have single antenna each in both cases [1], [2], [3]. Apart from the multiplexing gain of M, broadcast channels enjoy another gain due to surplus number of users. The term multi-user diversity was coined by Knopp in [4]. It's obvious that if a BS has M antennas and it has the channel information available from K users where $K > M$, it can choose M best users among those K users. It has been shown [5] that the sum capacity of the Gaussian broadcast channel has a scaling factor with number of users as $M\log(\log(K))$, where K is the total number of users in the system whose channel information is available at the BS. Thus the sum-rate grows unbounded with the number of users.

But these promising advantages of broadcast multiple-input multiple output (MIMO) systems don't come for free. To realize these high throughputs, BS has to transmit to multiple users over the same bandwidth hence all orthogonal transmission schemes will be highly sub-optimal as effectively BS will be transmitting to a single user over a particular resource. The biggest price to pay to achieve the full multiplexing gain of M is that BS must know the forward channel to at least M users [1]. Now to achieve the multi-user diversity gain factor $M\log(\log(K))$ of broadcast channel sum rate, BS should know the channel state information of all of these K users where normally K will be much larger than M.

We make no assumption of channel knowledge on either side but we don't prevent any side (transmitter and receivers) to learn/feedback the channel and subsequently use this information for scheduling/precoding/decoding of data. Inherently channel information is never there and the users (receivers) need to estimate the channels implicitly or explicitly to get channel state information at the receiver (CSIR). In frequency-division duplex (FDD) mode of operation, downlink (forward) channels are normally different from the uplink (reverse) channels. So the users need to feedback their estimated forward channel information on the reverse link. But the acquisition of channel state information at the transmitter (CSIT) gets facilitated when the broadcast channel operates under time-division duplex (TDD) mode. In this case, reciprocity implies that the forward channel matrix is the transpose of the reverse channel matrix [6]. So CSIT can be obtained easily compared to FDD mode by some kind of pilot transmission from user terminals to the BS.

Notation: $\mathbb{E}$ denotes statistical expectation. Lowercase letters represent scalars, boldface lowercase letters represent vectors, and boldface uppercase letters denote matrices. $\mathbf{A}^{\dagger}$ denotes the Hermitian of matrix $\mathbf{A}$.

II. THE PROBLEM STATEMENT

There is an enormous volume of research publications analyzing multi-user diversity in different scenarios.

In [7], Gesbert and Alouini brought about the question of how much feedback multi-user diversity really worth. They showed that by selective feedback and intelligent scheduling, feedback load can be reduced dramatically while still capturing the most part of multi-user diversity gain. In [8], it was shown that with perfect CSIT zero-forcing (ZF) precoding achieves the full multiplexing gain M and the full multi-user diversity gain $M\log(\log(K))$ of the broadcast channel.

In a recent work [9], the authors analyze the trade-off of multi-user diversity and accuracy of channel information at the BS, keeping the total number of feedback bits fixed. They conclude that accurate channel information is more important than having multi-user diversity. But in this work, total feedback load is kept fixed to a certain number of bits and the feedback link is different from the transmission bandwidth.

In [10] the authors had given a very innovative scheme coined as Random Beam Forming (RBF) where only a few bits of feedback are required from every user and the sum rate was shown to converge to the sum capacity, obtainable through dirty paper coding (DPC) [11], but for RBF gains to be valid, the number of users in the system should be extremely large.

Although there are many contributions which analyze multi-user diversity but mostly attention is focused on maximizing the downlink sum rate with a fixed feedback load. The true parameter of interest for multi-user systems is **absolute gain of multi-user diversity** which is the gain in DL sum rate due to multi-user diversity minus the UL feedback load required to inform the BS about the channels of large number of users so that BS can prioritize good users among them. A very simple example showing the importance of this absolute gain would be the RBF transmission scheme which requires as few as $\log(M)$ bits of feedback plus a scalar from each user in the system, but considering the fact that RBF requires the presence/feedback of asymptotically large number of users, the absolute gain would become questionable.

Another very important aspect which often gets overlooked in the analysis of multi-user systems is the consideration of channel coherence time. The channels in practice have finite coherence time and when multi-user transmission strategies are devised, there is possibility that channel has sufficiently changed during the multiple preliminary training and feedback intervals and channel information attained during these phases has become meaningless. So one has to pay close attention to channel coherence time while developing these transmission strategies.

We analyze the cost incurred and the benefit attainable of multi-user diversity in a more suitable and meaningful fashion. To make this task tractable, we simplify the problem by selecting TDD broadcast channel and assuming that perfect reciprocity holds. TDD channel with reciprocity assumption simplifies the acquisition of CSIT as pilot transmission from users to BS acts as channel feedback. So we have a fixed bandwidth available, a BS having M transmit antennas and K single antenna users. Now this fixed bandwidth can be used for UL/DL data transmission or training/feedback. The objective would be to maximize the DL sum rate over varying degrees of multi-user diversity gain and the difference of optimized rate from sum rate with no multi-user diversity gain (feedback from M users) would reveal the absolute gain of multi-user diversity.

Two transmission strategies are given in this contribution, in the first one the users who feedback are chosen independent of their channel realizations (hence termed as **Oblivious Users**). In the second scheme, the users learn their channel information first and then only good users (in terms of channel norm) feedback (hence termed as **Informed Users**). To let the users decide whether to feedback or not is more of a system level issue which could depend upon the application requirements and the services offered by the system [9].

III. SYSTEM MODEL

The system we consider consists of one BS having M transmit antennas and K single-antenna user terminals. In the DL, the signal received by k-th user can be expressed as

$$y_k = \mathbf{h}_\mathbf{k}^\dagger \mathbf{x} + n_k, \qquad k = 1, 2, \ldots, K \tag{1}$$

where $\mathbf{h_1}, \mathbf{h_2}, \ldots, \mathbf{h_K}$ are the channel vectors of users 1 through user K with $\mathbf{h_k} \in \mathbb{C}^{M\times 1}$ ($\mathbb{C}^{M\times 1}$ denotes the M-dimensional complex space), $\mathbf{x} \in \mathbb{C}^{M\times 1}$ denotes the M-dimensional signal transmitted by the BS and $n_1, n_2, \ldots, n_K$ are independent complex Gaussian additive noise terms with zero mean and unit variances. We denote the concatenation of the channels by $\mathbf{H}_\mathbf{F}^\dagger = [\mathbf{h_1}\mathbf{h_2}\cdots\mathbf{h_K}]$, so $\mathbf{H_F}$ is the $K \times M$ forward channel matrix with k-th row equal to the channel of the k-th user ($\mathbf{h}_\mathbf{k}^\dagger$). The input must satisfy a transmit power constraint of P i.e., $\mathbb{E}[||\mathbf{x}||^2] \leq P$.

The channel is assumed to be block fading having coherence length of T symbol intervals where fading remains the same, with independent fading from one block to the next [12]. The entries of the forward channel matrix $\mathbf{H_F}$ are independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance. We don't impose unrealistic assumptions of the presence of CSIR or CSIT. So initially all receivers and BS transmitter are oblivious of the channel realization in each block.

IV. TRANSMISSION SCHEMES

In the sub-sections which follow, we describe our two transmission schemes with oblivious users and with informed users. For both of the schemes, we get approximate analytical expressions of sum rate which capture the gain of multi-user diversity and the cost incurred. The validity for both of the expressions is shown through simulations and they help us to maximize the sum rate by optimizing over the users who feedback.

A. Scheme with OBLIVIOUS USERS

We give a simple transmission scheme where both BS and all the users get necessary channel state information. We call this "Oblivious" as users who feedback are independent of their channel realizations. This scheme divides the coherence length of T symbol intervals in three phases, 1) uplink training, 2) downlink training and 3) coherent data transmission. The first phase is the uplink training phase where K_{obl} of the K users present in the system transmit pilots to the BS. Because of TDD mode, this phase is equivalent to a feedback phase so when BS estimates users' channels, it recovers the forward channels of the users who feedback. The length of this training/feedback phase is denoted by T_1 where $T_1 \geq K_{obl}$. Here we assume that each user is peak power constrained with P_u. For this uplink training, the use of orthogonal training

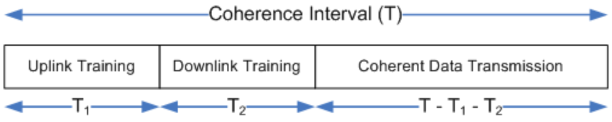

Fig. 1. Transmission Phases with Oblivious Users

sequences by all users is very attractive because in that case all users can transmit simultaneously to the BS with their full power without interfering with each other. Details have already been given in [13], so we just give here the estimation error of one channel coefficient at the BS

$$\sigma^2_{obl} = \mathbb{E}[|\mathbf{H}_{ij} - \hat{\mathbf{H}}_{ij}|^2] = \frac{1}{P_u T_1 + 1}. \tag{2}$$

This training interval length $T_1 \geq K_{obl}$ is basically the price to pay to achieve multi-user diversity gain as it reduces the time left for coherent transmission of data.

We adopt ZF precoding at the BS preceded by the semi-orthogonal user selection (SUS) algorithm of [8]. In ZF precoding, unit-norm beamforming vector for k-th selected user (denoted as $\bar{\mathbf{v}}_{\mathbf{k}}$), is selected such that it is orthogonal to the channel vectors of all other selected users. Hence with perfect CSIT, each user will receive only the beam directed to it and no multi-user interference will be experienced. For the case in hand, where the BS has imperfect estimate of the channel matrix, there is some residual interference. If we represent ZF beamforming matrix by $\bar{\mathbf{V}} = [\bar{\mathbf{v}}_{\mathbf{1}} \bar{\mathbf{v}}_{\mathbf{2}} \cdots \bar{\mathbf{v}}_{\mathbf{M}}]$, the transmitted signal $\mathbf{x}$ becomes $\mathbf{x} = \bar{\mathbf{V}}\mathbf{u}$ and the signal received by k-th selected user (1) can be expressed as

$$\begin{aligned} y_k &= \mathbf{h}_{\mathbf{k}}^{\dagger} \bar{\mathbf{V}} \mathbf{u} + n_k \\ &= \mathbf{h}_{\mathbf{k}}^{\dagger} \bar{\mathbf{v}}_{\mathbf{k}} u_k + \sum_{j \neq k} \mathbf{h}_{\mathbf{k}}^{\dagger} \bar{\mathbf{v}}_{\mathbf{j}} u_j + n_k, \end{aligned} \tag{3}$$

where $\mathbf{u}$ is the data vector with u_k data intended for k-th selected user.

The second phase is the DL training phase where the BS transmits pilots so that scheduled users estimate their corresponding effective channels. It was shown in [13] that only one symbol interval is sufficient to let the M selected users learn their effective scalar channels ($\mathbf{h}_{\mathbf{k}}^{\dagger} \bar{\mathbf{v}}_{\mathbf{k}}$ for user k). Moreover with the fact that BS is able to transmit with sufficient power reducing the estimation error, we assume that selected users are able to estimate their effective scalar channels perfectly.

When this second phase ends, both sides of the broadcast channel have necessary channel state information, although CSIT is imperfect. Thus starting from a broadcast channel with no CSIT and no CSIR, reaching up to the third data phase, we have a broadcast channel with imperfect CSIT and CSIR. In this third data phase, we adopt independent data transmission with equal power allocation P/M to finally selected M users.

We are interested in getting an expression for the achievable sum rate of this broadcast channel which captures the gain and the cost associated with multi-user diversity. In pursuit of this, the SINR of k-th user can be written as

$$\text{SINR}_k = \frac{\frac{P}{M} ||\mathbf{h}_{\mathbf{k}}||^2 |\bar{\mathbf{h}}_{\mathbf{k}}^{\dagger} \bar{\mathbf{v}}_{\mathbf{k}}|^2}{1 + \frac{P}{M} ||\mathbf{h}_{\mathbf{k}}||^2 \sum_{j \neq k} |\bar{\mathbf{h}}_{\mathbf{k}}^{\dagger} \bar{\mathbf{v}}_{\mathbf{j}}|^2}, \tag{4}$$

where the channel vector $\mathbf{h}_{\mathbf{k}}$ is split in its norm $||\mathbf{h}_{\mathbf{k}}||$ and the unit norm direction vector $\bar{\mathbf{h}}_{\mathbf{k}}$.

The factor $|\bar{\mathbf{h}}_{\mathbf{k}}^{\dagger} \bar{\mathbf{v}}_{\mathbf{k}}|^2$ which appears in the signal power of the SINR of k-th user has a limited effect [14],[3]. The factor $||\mathbf{h}_{\mathbf{k}}||^2$ was shown to grow as $\log(K)$ in [10] using the results from order statistics for asymptotically large K and was shown that it holds well for practically large K. The interference contribution coming from the j-th beam, $|\bar{\mathbf{h}}_{\mathbf{k}}^{\dagger} \bar{\mathbf{v}}_{\mathbf{j}}|^2$ was shown to depend solely upon the imperfections in the BS estimates of users' channels in [13]. Combining all these facts and slight approximations in the SINR of k-th user, the achievable rate per symbol time of k-th user can be approximated as

$$R_k \approx \log \left(1 + \frac{\frac{P}{M} \log(K_{obl})}{1 + \frac{P}{M} \log(K_{obl}) \frac{1}{P_u K_{obl} + 1}} \right), \tag{5}$$

where we have used σ_1^2 as $1/(P_u K_{obl} + 1)$, i.e. we have restricted the training length equal to the number of users who feedback ($T_1 = K_{obl}$).

Our approximations have removed the dependence of the order of users in the user-selection process, hence accounting for the fact there are M users being served in each coherence interval, the sum rate of this broadcast channel can be approximated as

$$R_{sum} = M \log \left(1 + \frac{\frac{P}{M} \log(K_{obl})}{1 + \frac{P}{M} \log(K_{obl}) \frac{1}{P_u K_{obl} + 1}} \right). \tag{6}$$

If we account for the loss in coherence interval T due to feedback (training) phase of length T_1 and the restriction of $T_1 = K_{obl}$, then the sum rate for this scheme of oblivious users becomes

$$R_{sum}^{obl} = \frac{T - K_{obl}}{T} M \log \left(1 + \frac{\frac{P}{M} \log(K_{obl})}{1 + \frac{P}{M} \log(K_{obl}) \frac{1}{P_u K_{obl} + 1}} \right). \tag{7}$$

B. Scheme with INFORMED USERS

This scheme also consists of transmission phases through which both the BS and all users get necessary channel information. We call this "Informed" as users who feedback are no more random. They are the users with best channel norm for the current channel coherence block. This scheme divides the coherence length of T symbol intervals in four phases, 1) initial downlink training, 2) uplink training, 3) downlink training and 4) coherent data transmission.

Fig. 2. Transmission Phases with Informed Users

In the first phase, termed as initial downlink training, BS transmits pilots to users based upon which all users estimate their corresponding channel vectors. As BS has M antennas so this training interval length is lower bounded by M, giving $T_{iDL} \geq M$. As this training interval length is independent of the number of users in the system K and as we assumed for the oblivious user case that BS can transmit with sufficient power to provide very good estimates, we don't take into account the estimation error during this phase but we subtract $T_{iDL} = M$ from the coherence length.

Based upon the channel information the users acquire in the initial DL training interval, K_{inf} best users w.r.t. the channel norm are selected for feedback. The next three transmission phases are exactly similar as those for transmission scheme with oblivious users. In the second phase of uplink training, the K_{inf} users with the largest channel norms feedback their channel information to the BS. Based upon this channel information, BS uses SUS algorithm to further select M best users and computes corresponding ZF beamforming vectors. Then in the third phase of downlink training, BS transmits through these ZF beamforming vectors so that selected users estimate their corresponding effective scalar channels. We neglect the burden of this phase for the reasons mentioned in Oblivious user scheme. The last phase is the coherent data transmission phase with equal power allocated to M independent streams.

To get the approximate sum rate expression, following the same steps as performed in the Oblivious users' setting, the sum rate can be shown to be

$$R_{sum} = M \log\left(1 + \frac{\frac{P}{M}\log(K)}{1 + \frac{P}{M}\log(K)\frac{1}{P_u K_{inf}+1}}\right), \quad (8)$$

where K is the total number of users in the system and K_{inf} is the number of users who actually feedback (chosen based upon the strength of their channels).

If we account for the loss in coherence interval T due to initial downlink training $T_{iDL} = M$, feedback (training) phase of length T_1 and the restriction of $T_1 = K_{inf}$, then the sum rate becomes

$$R_{sum}^{inf} = \frac{T - M - K_{inf}}{T} M \log\left(1 + \frac{\frac{P}{M}\log(K)}{1 + \frac{P}{M}\log(K)\frac{1}{P_u K_{inf}+1}}\right). \quad (9)$$

So this expression carries the gain of multi-user diversity taking into account the resource usage for training/feedback phases for the strategy where users are first informed of their channels.

C. Validity of Approximate Sum Rate Expressions

As the expressions for both schemes involve similar approximations, it's enough to show the validity for one of the schemes. We establish the validity of the sum rate expression for Oblivious users and to see how closely it captures the behavior of sum rate with different system parameters, we compare it with actual sum rate. We use eq. (6) as it will prove the validity of the sum rate expression for any coherence length T. Actual sum rate curves have been obtained by replicating transmitter-receiver chains and doing Monte-Carlo simulations.

Fig. 3 shows the plots of the sum rate versus SNR. Uplink power constraint for each user has been selected to be 10dB. There are 100 users in the system who feedback and the BS is equipped with 4 antennas. The curves for actual rate and the approximate rate show that approximate expression well captures the growth of the sum rate with SNR and it saturates almost at the same SNR level as the actual sum rate. This saturation of the sum rate is caused by the imperfect BS estimates based upon which ZF BF vectors are computed (shown for digital FB in [3] and for analog feedback in [13]).

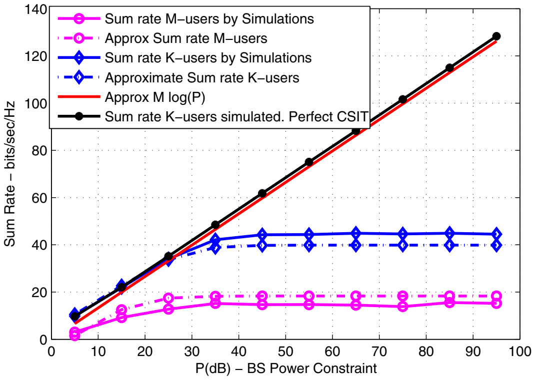

Fig. 3. Sum Rate versus SNR

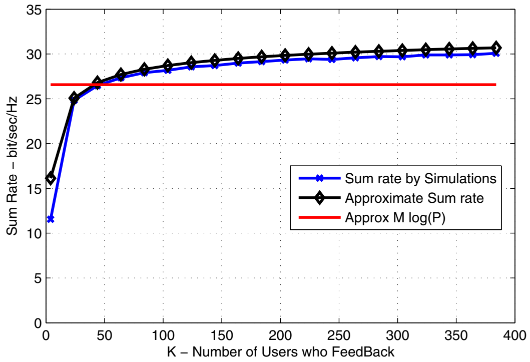

Fig. 4. Sum Rate versus Nb of Users

Fig. 4 shows the plots of the sum rate with varying number of user K. Again 4 antenna BS with 20 dB power constraint, has users available with power constraint of 10 dB each. These curves also show that approximate expression is able to capture fully the multi-user diversity gain. Thus with increase in number of users it shows the same growth as shown by the actual sum rate.

V. OPTIMAL USERS TO FEEDBACK

Once the approximate sum rate expressions have been validated, we can use them to maximize the sum rate by optimizing over how much feedback load should be there in terms of the number of feeding back users. Thus for the scheme with oblivious users our objective function is

$$K_{obl}^{opt} = \arg\max_{K_{obl}} R_{sum}^{obl}. \quad (10)$$

And for the scheme where users are informed first of their channel realizations, the objective function would be

$$K_{inf}^{opt} = \arg\max_{K_{inf}} R_{sum}^{inf}. \tag{11}$$

The analytical solutions for the above two equations do not seem probable but they can be solved numerically to get the optimal value of feedback load in terms of K_{obl} and K_{inf}.

A. Optimal Feedback Load vs. DL SNR

First we see how the optimal number of users (feeding back) for the two schemes should scale with SNR. For this we plot the graph of the optimal number of oblivious and informed users versus SNR in Fig. 5 and also plot corresponding sum rates achieved by using that optimal number of users corresponding to both schemes for each value of SNR in Fig. 6. For comparison, we plot the sum rate when only M users feedback their channel information to the BS. We take T=1000, per user power constraint is 10dB and M=4. For the scheme with informed users, the number of users in the system is taken to be 200.

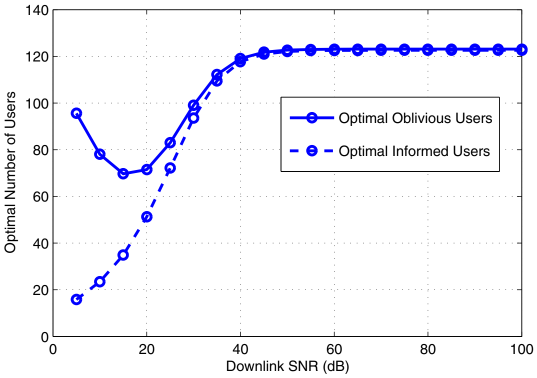

Fig. 5. Optimal Users of two schemes versus SNR

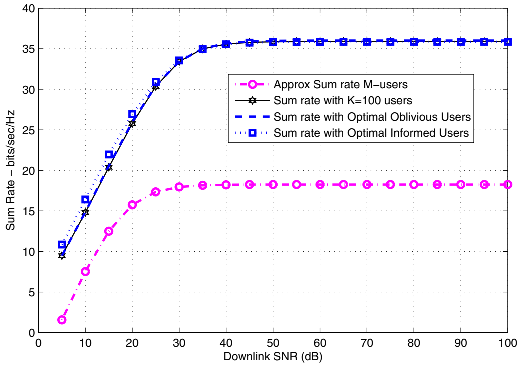

Fig. 6. Sum Rate of two schemes with Optimal Users versus SNR

It's evident that the gains of multi-user diversity are undeniable because the sum rate with only feedback from M users is much less than the sum rate of optimal number of users' feedback available with the same system parameters. The saturation of the sum rate because of imperfect CSIT depicted in Fig. 6 has been explained in [13] and [3].

The behavior of the curves of optimal number of users feeding back for two schemes versus SNR is not very straight forward. At high SNR, both schemes require feedback from a large number of users. Because here system is becoming interference limited, so feedback load which causes to reduce the interference increases with SNR but it saturates as increase in feedback interval causes to reduce the time for data transmission. At low SNR both curves show very different behavior. The reason is at low SNR, system is basically noise limited and multi-user diversity factor is very important hence only the users with strong channels should be scheduled. In Informed users scheme, only strong users feedback so it does not require feedback from a large number of users but the scheme with Oblivious users (where users feedback independent of their channel realizations) requires feedback from a large number of users to select users with strong channel norms.

Although the optimal number of users changes drastically with the change in DL SNR and the optimal feeding back users in two schemes differ significantly for lower to medium SNR values but the sum rates obtained for two schemes with optimal feedback load overlap completely. At low to medium SNR values, informed user strategy gives slightly higher rate but this difference is minor. We have also plotted the sum rate when 100 users feedback in each coherence interval for both schemes. This curve also overlaps fully the sum rates of two schemes with optimal feedback load (Fig. 6). It indicates that for a fixed channel coherence length, a fixed reasonable value of feeding back users (normally much larger than M) can capture multi-user diversity significantly.

B. Optimal Feedback Load vs. Channel Coherence Time

We now analyze how the optimal feedback load (in terms of the number of users) behaves with the increase in coherence interval. So we plot two graphs, one showing the optimal feedback load of users versus coherence interval (Fig. 7) for different power constraints on the BS and the other showing the sum rates corresponding to the optimal feedback versus coherence interval (Fig. 8). Here BS has $M = 4$ antennas, its power constraint is 20 dB and each user has the power constraint of 10 dB. For the scheme with Informed users, the total number of users in the system is $K = 500$.

The curves of the optimal number of users versus channel coherence length show almost linear increase for both schemes. The optimal number of users feeding back is more for scheme with Oblivious users than in the case of Informed users. This behavior can be anticipated from Fig. 5. Contrary to the sum rate versus SNR curves where a single suitable number of users feeding back captures the full multi-user diversity gain, here it is difficult to find one such number of users (feeding back) capturing multi-user diversity gain for a large range of channel coherence length. Thus the number of users who feedback must scale up with the increase in

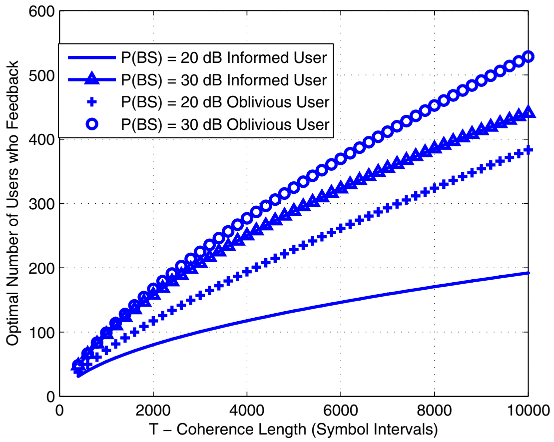

Fig. 7. Optimal Users versus Coherence Length

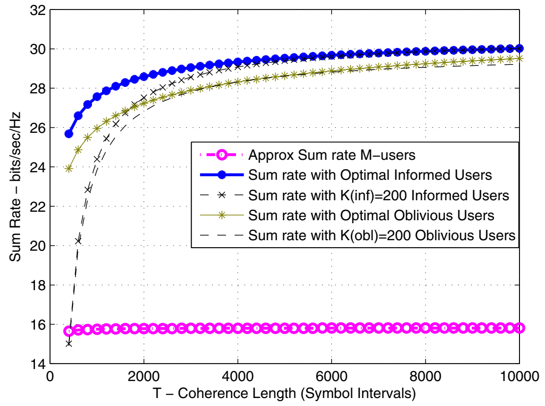

Fig. 8. Sum Rate with Optimal Users versus Coherence Length for BS power constraint of 20 dB

channel coherence length. Sum rate curves have been plotted at $P = 20$ dB so Informed user scheme performs betters as can be guessed from Fig. 6. But at high DL SNRs, this difference vanishes completely.

Remark 1: In the second transmission scheme which involves Informed users, we select just the strongest users who train the BS about their channels. Strictly speaking, this is impractical but the underlying idea is to see how much feedback load (how many users) should be there to maximize the sum rate. In practice those many users can be made to feedback on the average by intelligent selection of a threshold with which users compare their channel strength locally as detailed in [7].

VI. CONCLUSIONS

We studied the absolute multi-user diversity gain with achievable sum rate for a broadcast channel with no assumption of channel knowledge at any communicating end. We gave two simple transmission strategies which provide necessary CSI to both the BS and the users. Simple approximate analytical expressions for both schemes were obtained which capture the cost and the gain associated to multi-user diversity. They enable us to maximize the sum rate by optimizing over the amount of feedback involved. It turns out that for a fixed coherence length, a reasonably large number of feeding back peak power constrained users can capture fully the multi-user diversity gain over any SNR. Moreover it turns out that the scheme where the users are made to feedback based only upon the channel norm strength gives a slight gain over the scheme where the feeding back users are chosen independent of their channel realizations at low to moderate DL SNR values vanishing for higher SNRs.

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