Low Complexity Cross-Layer Design for Dense Interference Networks

Sara Akbarzadeh, Laura Cottatellucci, Christian Bonnet

Institut Eurecom, 2229 Route des Cretes, 06904 Sophia Antipolis Cedex, France E-mail: sara.akbarzadeh@eurecom.fr

***Abstract*—We considered a dense interference network with a large number ($K \to \infty$) of transmitter-receiver pairs. Each transmitter is endowed with a finite buffer and accepts packets from an arrival process. Each transmitter-receiver link is a fading vector channel with N diversity paths whose statistics are described by a Markov chain. We investigate *distributed* algorithms for joint admission control, rate and power allocation aiming at maximizing the individual throughput defined as the average information rate successfully received. The decisions are based on the statistical knowledge of the channel and buffer states of the other communication pairs and on the exact knowledge of their own channel and buffer states. In the case of a finite number of communication pairs this problem is computationally extremely intensive with an exponential complexity in the number of users. Assuming that $K, N \to \infty$ with constant ratio the algorithm complexity becomes substantially independent of the number of active communications and grows with the groups of users having distinct asymptotic channel statistics. The cross-layer design is investigated for different kind of decoders at the receiver. The benefits of a cross layer approach compared to a resource allocation ignoring the states of the queues are assessed. The performance loss due to the use of policies designed for asymptotic conditions and applied to networks with a finite number of active communications is studied.**

I. Introduction

In a wireless medium, the users radiate energy and communicate through superposition of each other's transmissions. Thus, the concept of link does not exist intrinsically. This context is extremely challenging but also offers unimaginable possibilities since all the nodes are naturally connected each other. The current generation of wireless networks reduces the problem complexity by using multiple access protocols, introducing a hierarchical network structures, and exploiting the natural attenuation of the medium (frequency reuse), but renounces to benefit from the full connectivity offered by the air interface. Next generation wireless networks aim at exploiting this full connectivity by weakening the notion of central authority (e.g. cognitive radio) or cancelling it completely (e.g. ad hoc networks) without renouncing to the full flexibility and level of services already offered by cellular networks. Then, the focus is on interference networks characterized by interfering communications between multiple sources and multiple destinations. Different levels of cooperations among sources and\or destinations can be envisaged but in general the delocalization of control mechanisms such as rate and power allocation, scheduling, admission control, and routing will play a fundamental role. In decentralized schemes, the decision concerning network parameters (rates and/or powers) and transmission conditions are made by individual nodes based on their local information.

The benefit of cross-layer design and joint optimization of this control mechanisms are well known in centralized communication systems (e.g. [1] and references therein) and can be effectively exploited also in interference networks [2], [3].

The interference channel is currently object of intensive studies both in information and communications theory.

The decentralized cross layer algorithms for resource allocation in interference networks is a complex and intriguing problem since the decision affects many fundamental operational aspects of the network and its resulting performance. Several alternative approaches have been proposed. Two main streams can be identified: (1) schemes based on repeated games and learning dynamics, (2) constrained stochastic games.

The first approach has been extensively applied for power allocation ([5], [6], [7],[8]). Power allocation in interference networks is inherently a repeated process and it is natural to model interactions among users with repeated games. These approaches introduce a learning phase which provides each users with information (intelligence) to make a correct decision. The convergence of the learning dynamics in the repeated game [5] is the main challenge of these schemes. Additionally, they assume slow varying channels. Constrained stochastic games have been applied to decentralized cross layer design for multiple access [2] and interference channels [3]. By assuming statistical knowledge of the channels and the service request processes, constrained stochastic games can be applied to system with fast varying dynamics. However, they rely on a rational behaviour of the players. When applied to networks with a large number of nodes, the former approach incurs in a huge amount of overhead and delay while the latter suffers from an unaffordable complexity.

In this work we focus on the design and analysis of decentralized cross layer algorithms for power and rate allocation, scheduling and admission control for dense interfering networks based on constrained stochastic games with the primarily aim of decreasing their complexity. We assume that the links between transmitter and receiver are characterized by some kind of diversity (e.g. in space, frequency) and we refer to it as vector channels with N diversity paths. Furthermore, we assume that the N diversity paths are random and K, the number of network links, and N tend to infinity with constant ratio. This approach is motivated by the fact that the asymptotic design and analysis of the network in random environments significantly decreases the design complexity and provides insightful analysis results. This model may characterize interference networks with spreading of the transmitted signals based on random signature sequences (similarly to code division multiple access - CDMA - in multiple access networks), or systems with multiple antennas at the receiver, where the randomness is due to channel fading. In such settings, when the number of users and diversity paths grow, fundamental performance measures as capacity and signal to interference and noise ratio (SINR) at the output of a receiver detector converge to deterministic limits.

The performance analysis of various receivers (e.g. matched filters, linear minimum mean square error - LMMSE -, optimal detector), for multiple access vector channels in random environment has been extensively investigated in literature (e.g. [14],[15],[16]). We extend the results to interference channels and apply them to the design and analysis of *distributed* cross layer algorithms in large interference networks[1].

The assumption of large system analysis introduces two fundamental features into the system setting in [3] characterized by a discrete set of decision variables and a discrete set of channel statistics. Firstly, in an interference system with finite number of users and decentralized control mechanisms, a transmission is intrinsically subject to outage since each transmitter is not aware of the interferers' decisions and effects. On the contrary, in large interference channels, the effects of the interferers tends to a deterministic limit regardless of the instantaneous link states. Then, a transmitter can avoid outage events by convenient control algorithms. Secondly, the complexity of the cross layer design algorithms, which increases exponentially with the number of users in [3], scales only with the number of groups of users characterized by the same channel statistics in large systems.

For large interference systems we consider the cross-layer design of rate and power allocation jointly with scheduling and admission control for four different kind of receivers with increasing complexity. Namely, we consider two receivers, one based on linear MMSE detection and the other on optimum detection and subsequent decoding of all users having the same rate and received power. The receivers have only statistical knowledge of the interferers' channel states. A third receiver is based on joint optimum detection and decoding of all users having same received power and rate but with additional knowledge of the interference structure at the receiver. The fourth receiver decodes jointly and optimally all the decodable users while knowing the interference structure.

We compare the performance of the receivers with the designed optimum policies. The mismatch between of performance of the optimum policies for large systems and for finite systems are also assessed.

[1]Hereinafter, we refer to interference networks with number of users and diversity paths growing to infinity with constant ratio as *large interference networks.*

Notations: The boldface capital/small letters are used for matrices/vectors respectively. A superscript for a matrix/row-vector denotes the index of corresponding column-vector/element. A subscript of a matrix/row-vector is the index of corresponding row-vector/element. The probability mass function of a discrete random variable x is denoted by $\mathbb{P}(x)$.

II. System Model

We consider a system consisting of K arbitrary source-destination pairs sharing the same medium, (e.g. ad hoc network). We use the same index for the corresponding transmitter and receiver of a single source-destination pair. The time is uniformly slotted. We assume that one node cannot transmit and receive at the same time. The channels are Rayleigh fading and ergodic within a time slot, while the channel statistics change from a time slot to the following one. Furthermore, codewords are completely transmitted during a single time slot, i.e. the channel is fast fading.

Following the same approach as in [3] we define two sets of discrete variables representing states and actions of each transmitter.

The channel in time slot $t \in \mathbb{N}$ is described by an $K \times K$ matrix $\mathbf{\Sigma}(t)$ whose (j,i) element $\sigma_j^i(t)$ is the average power attenuation of the channel between transmitter j and receiver i during time slot t. Throughout this work, we refer to them as the channel states (CS). The row j includes the states of the channels from the transmitting node j to all the destination nodes. This vector is denoted by $\boldsymbol{\sigma}_j(t)$. The i-th column includes the states of the channels from all the transmitting nodes to the receiver i and it is denoted by the column vector $\boldsymbol{\sigma}^i(t)$. It contains all the CS information necessary to determine the statistics of the signal to interference and noise ratio (SINR) at the destination node i at time slot t. Furthermore, each average power attenuation σ_i^j is modelled as an ergodic Markov chain taking values in the discrete set E of cardinality L. For the sake of notation, we define a bijection between the set E and the set of the natural numbers $\{0,1,...,L-1\}$, $\varphi : E \to \{0,1,...,L-1\}$. Let its inverse be $\psi = \varphi^{-1}$. The Markov chain of σ_j^i is defined by the transition matrix $\boldsymbol{T}(j,i)$ whose (k,ℓ) element $T_k^\ell(j,i)$ is the probability of transition from the CS $\psi(k)$ to the state $\psi(\ell)$. The conditional probability nature of $T_k^\ell(j,i)$ reflects on the fact that $\sum_{\ell=1}^{L} T_k^\ell(j,i) = 1$. We assume throughout that $\boldsymbol{T}(j,i)$ is irreducible and aperiodic as in [2]. The steady CS probability distribution of the channel between transmitter i and destination j is given by the column vector $\boldsymbol{\pi}(i,j)$.

At each node, packets arrive from the upper layer according to an independent and identically distributed arrival process $\gamma_i(t), t \in \mathbb{K}$ with arrival rate λ_i. Here, $\mathbb{P}(\gamma_i(t))$ is the probability of receiving $\gamma_i(t)$ packets at time instant t. The packets have constant length.

Each transmitter is endowed with a buffer of finite length. We denote by B_i the maximum length of the buffer at node i and by $q_i(t)$ number of queuing packets at the beginning of slot t. In the following, we address the variable $q_i(t)$ also as the queue state (QS).

In each time slot, on the basis of the available information at time t transmitter i decides (a) the transmission power level $p_i \in \mathcal{P}_i$, where $\mathcal{P}_i$ is a finite set of nonnegative reals including zero; (b) the number of packets to transmit $\mu_i \in \mathcal{M}_i$, with $\mathcal{M}_i = \{0, 1, ..., M_i\}$ and $M_i \leq B_i$; (c) to accept or reject new packets arriving from upper layers. We denote with $c_i = 1$ and $c_i = 0$ the decision of accepting and rejecting the packets, respectively. Therefore, the action of the node i at time slot t is described by the triplet $d_i(t) = (p_i(t), \mu_i(t), c_i(t))$.

The information available at node i at time t is given by the pair $x_i(t) = (\sigma_i^i(t), q_i(t))$, i.e. the CSs from transmitter i to receiver i and the number of the packets in the queue at the beginning of time slot t (QS). We refer to the pair $x_i(t)$ as the transmitter state (TS). Additionally, each transmitter knows the statistics of the other channels and the statistics of the arrival process in the buffer.

We assume that the link between a source and a destination is a vector channel with equal average power attenuation over all the N paths. A vector channel can model systems with several types of diversity (e.g. spatial diversity if the receivers are equipped with N antennas, frequency diversity if code division multiple access, CDMA, or orthogonal frequency division modulation, OFDM are selected as multiple access schemes).

The complex-valued channel model for receiver i is

$$\boldsymbol{y}^{(i)}[m] = \boldsymbol{S}[m]\boldsymbol{H}^{(i)}\left[\left\lfloor\frac{m}{N}\right\rfloor\right]\boldsymbol{A}\left[\left\lfloor\frac{m}{N}\right\rfloor\right]\boldsymbol{b}[m] + \boldsymbol{w}_i[m] \quad (1)$$

where m is the index for symbol intervals and depends on the frame interval t by the expression $m = tN + p$ with $p = 0, \dots N-1$; $\boldsymbol{y}^{(i)}[m]$ and $\boldsymbol{b}[m]$ are the N-dimensional complex vectors of received signals by node i and transmitted symbols by all nodes, respectively. Here, $\boldsymbol{S}[m]$ is a $K \times N$ complex matrix with zero mean independent and identically distributed (i.i.d.) entries having variance $1/N$. The matrices $\boldsymbol{H}^{(i)}\left[\lfloor\frac{m}{N}\rfloor\right]$ and $\boldsymbol{A}\left[\lfloor\frac{m}{N}\rfloor\right]$ are diagonal with j-th diagonal elements equal to $\sqrt{\sigma_j^i(t)}$ and $\sqrt{p_j(t)}$, respectively. Finally, $\boldsymbol{w}_i$ is the N dimensional complex vector of the additive white Gaussian noise with zero mean and unit variance. We assume that the transmitted signals $b_i[m]$ are i.i.d., with zero mean and unit variance.

In order to model a large interference network as $K \to \infty$, we assume that the transition matrices $\boldsymbol{T}(j,i)$ are taken from a finite set of transition matrices $\mathcal{T} = \{\boldsymbol{T}^{(1)}, \dots, \boldsymbol{T}^{(c)}\}$ and the channel between each transmitter and each receiver is described with probability $\mathbb{P}(\boldsymbol{T}^{(\ell)})$ by the transition matrix $\boldsymbol{T}^{(\ell)}$. The same property holds for each receiver.

If (1) models a CDMA system, the matrix $\boldsymbol{S}[m]$ includes the effects of the spreading sequences with spreading factor N and the randomness of a Rayleigh fading channel. If (1) models K interfering single antennas transmitting to K receivers equipped with multiple antennas (SIMO systems), then the matrix $\boldsymbol{S}[m]$ accounts for the Rayleigh fading. In both cases, the matrix $\boldsymbol{H}^{(i)}\left[\lfloor\frac{m}{N}\rfloor\right]$ models the effects of the pathloss. Eventual coupling effects among the receiving antennas in interfering SIMO systems are neglected in this model.

Throughout this work we will consider a system in the steady state. Thus, we will neglect the symbol interval m when its omission does not cause ambiguity. In the following section, conditions for the convergence to a steady state of the whole system will be detailed.

The probability mass function of the joint action and transmitter state in the steady state of the Markov decision chain is denoted by $\mathbb{P}(a_k, \sigma_k^k, q_k)$. A policy of transmitter k is a deterministic or probabilistic application from the space of TS $\mathcal{X}_k$ to the action space $\mathcal{D}_k$. A probabilistic (or mixed) policy of transmitter k is $u_k(d_k|x_k)$, i.e. the probability that mobile k chooses the action d_k when the state is x_k or equivalently, the conditional probability that user k chooses the action triplet (p_k, μ_k, c_k) conditioned to the transmitter state (σ_k^k, q_k)The class of decentralized policies of mobile k is denoted by $\mathcal{U}_k$. If we assume that the user policies are known, then the probability mass function of $\mathfrak{p}_k^i = p_k\sigma_k^i$, $k = 1 \dots K$, the average received power from transmitter k by receiver i is given by

$$\begin{aligned}\mathbb{P}(\mathfrak{p}_k^i) &= \sum_{\substack{\boldsymbol{\sigma}_k, p_k: \\ p_k\sigma_k^i = \mathfrak{p}_k^i}} \sum_{c_k}\sum_{q_k}\sum_{\mu_k} \mathbb{P}(\boldsymbol{\sigma}_k, q_k)u_k(p_k, \mu_k, c_k|\sigma_k^k, q_k) \\ &= \sum_{\substack{\boldsymbol{\sigma}_k, p_k: \\ p_k\sigma_k^i = \mathfrak{p}_k^i}} \sum_{c_k}\sum_{q_k}\sum_{\mu_k} \mathbb{P}(\sigma_k^k, q_k)\mathbb{P}(\sigma_k^i)u_k(p_k, \mu_k, c_k|\sigma_k^k, q_k)\end{aligned} \quad (2)$$

where the second step is a consequence of the independence of σ_k^k and σ_k^i.

Let us notice that the empirical eigenvalue distribution of the matrix $\boldsymbol{H}^{(i)}\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{H}^{(i)^H}$ converges to the probability distribution function of the averaged received power $\mathfrak{p}_k^i$, when the system is in the steady state ($t \to +\infty$) and the number of communication flows grows large ($K \to +\infty$)

Additionally, the assumptions on the finite cardinalities of the state and action sets induce a dynamic partition on the set of the K transmitter-receiver pairs for each given receiver i. This partition consists of a finite number of subsets: all the communication pairs having the *same received power* at the receiver i and the *same rate* at a certain time interval belong to the same group. We denote the total number of groups by $N_g^{(i)}$ and $\mathcal{G}_m^{(i)}$ is the m-th group. There exist a bijection between the set of groups $\mathcal{G}_m^{(i)}$ and the set of pairs $(\mathfrak{p}_r^i, \mu_s)$. Let $K_1^{(i)}, K_2^{(i)}, \dots K_{N_g}^{(i)}$, with $\sum_{m=1}^{N_g} K_m^{(i)} = K$, be the cardinality of the sets $\mathcal{G}_1^{(i)}, \mathcal{G}_2^{(i)}, \dots \mathcal{G}_{N_g}^{(i)}$, respectively. Let us notice that, in general, the bijection depends on the block interval. However, when we consider the steady state and $N, K \to +\infty$, with $\frac{K}{N} \to \beta$, the convergence $\frac{K_m^{(i)}}{N} \to \beta_i^{(k)}$, with $\frac{\sum_{m=1}^{N_g} K_m^{(i)}}{N} = \frac{K}{N} = \beta$ holds. For further studies, it is useful to introduce the correlation matrix of the whole transmitted signals $\boldsymbol{R}^{(i)} = \boldsymbol{S}\boldsymbol{H}^{(i)}\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{H}^{(i)^H}\boldsymbol{S}^H$ and $\boldsymbol{R}_{\widehat{\mathcal{G}}^{(i)}}^{(i)}$ the correlation matrix of the signals transmitted by nodes in $\widehat{\mathcal{G}}^{(i)}$ and received by node i. The correlation matrix $\boldsymbol{R}_{\widehat{\mathcal{G}}^{(i)}}^{(i)}$ is obtained by setting $\mathfrak{p}_m^i = 0$ in $\boldsymbol{R}^{(i)}$, for all transmitting nodes not in $\widehat{\mathcal{G}}^{(i)}$. Finally, we define the correlation matrix of the interfering signals to the signals of interest in $\widehat{\mathcal{G}}^{(i)}$, $\boldsymbol{R}^{(i)}_{\sim\widehat{\mathcal{G}}^{(i)}}$. It is obtained from $\boldsymbol{R}^{(i)}$ setting $\mathfrak{p}^i_m = 0$ if the m-th transmitter is in $\widehat{\mathcal{G}}^{(i)}$.

Let us turn to the structure of the receiver at each node.

We will consider different receivers depending on the assumptions we make about (I) the level of knowledge of the interference available at the receiver; (II) the eventual use of a suboptimal receiver based on a preliminary pre-decoding processing (e.g. detection) followed by decoding; and (III) the type of the decoder, i.e. single-user/joint decoder. It is important to note that the aim of receiver k is to decode its own message of interest, i.e. the message transmitted by the corresponding transmitter k. The other messages are decoded if this is beneficial for decoding the message of interest. Based on these observations, we consider four approaches detailed in the following:

SG-MMSE/UIS/SGD (Single Group MMSE detection/ Unknown Interference Structure/Single Group Decoding): In this case we assume that the receiver k has knowledge only of the channel vectors $\sqrt{\mathfrak{p}^k_i}\boldsymbol{s}^k_i$ for the communication flows which have the same received powers and transmission rate of the user of interest k, i.e. the transmitters in $\mathcal{G}^{(k)}_k$, but no knowledge of the others. The interference from the latter communication flows is considered as a white additive Gaussian signal. The receiver first detects the transmitted symbols for all the flows with known vector channels by a linear minimum mean square error (LMMSE) detector. Subsequently, it performs single-group decoding, i.e. it decodes jointly the information streams of the pairs in $\mathcal{G}^{(k)}_k$.

NP/UIS/SGD (No preprocessing/Unknown Interference Structure/Single Group Decoding): This case differs from the previous one only in the fact that no preprocessing of the received signal is performed.

NP/KIS/SGD (No preprocessing/Known Interference Structure/ Single Group Joint Decoding): The receiver k has knowledge of all the vector channels $\sqrt{\mathfrak{p}^k_i}\boldsymbol{s}^k_i$. It decodes jointly the information streams of the single group $\mathcal{G}^{(k)}_m$ it belongs to, i.e. with the same received power as the user of interest $\mathfrak{p}^k_k$ and same rate μ_k. In the decoding it makes use of the knowledge about all the interference structure, i.e. the knowledge of the vector channels of all active streams.

NP/KIS/MGD (No preprocessing/Known Interference Structure/ Multi Group Joint Decoding) All the vector channels of the active transmitters are known to receiver i. Then, receiver i identifies the maximum decodable set of information streams and decode them jointly while taking into account of the interference structure for the users which are not decoded.

Let us notice that the investigated receivers are in order of increasing performance in decoding the information of interest.

In the following we will denote by X_k the information bits (uncoded bits) transmitted by node k, by $X_{\mathcal{V}}$ the information bits transmitted by the transmitting nodes in the set $\mathcal{V}$. Finally, $I(X_{\mathcal{V}};Y^{(k)})$ is the mutual information of the channel transmitting $X_{\mathcal{V}}$ and receiving $\boldsymbol{y}^{(k)}$.

III. Preliminary Useful Tools

In this section we will specialize known results on large multiple access networks and on the rate regions of interference channels to our interference networks with a large number of nodes. Additionally, key remarks will be stated.

A. Some Convergence Results

Let us consider the Markov chain with finite states which characterize the statistics of a channel between a transmitter and a receiver node. If we assume that the Markov chain is irreducible and aperiodic, then there is a unique stationary distribution which describes the steady state. Let us further assume that all the transmitter-receiver channels are described by the same Markov chain. Then,applying the Glivenko-Cantelli theorem (e.g. [10]) the empirical distribution of the channel states in the matrix $\boldsymbol{H}^{(i)}$, for any i, as the system is in the steady state, converges almost surely to the unique stationary distribution of the unique Markov chain. If the policies of all users, $\mathcal{U}_k, k = 1, ..., K,$, are known and identical, then also the empirical distribution mass function of the received powers in the matrix $\boldsymbol{A}\boldsymbol{H}^{(i)}\boldsymbol{H}^{(i)H}\boldsymbol{A}^H$ converges almost surely to the distribution mass function (2). A similar convergence result can be obtained if the channels between a transmitter and a receiver are described by a Markov chain defined by a transition matrix belonging to a finite set with a given distribution $\mathbb{P}(\boldsymbol{T})$

This kind of convergence satisfies the conditions for the applicability of results on random large matrices (see e.g. [11]) which are the key tools to derive the following results.

B. Large System Analysis of the Receivers

The large system analysis of multiple access vector channels with random channel vectors is in [14], [16], [15]. Effects of interference on large network performance are investigated in [12], [13]. The extension of their results to the interference network in Section II is presented here.

Without loss of generality, in the following we will focus on the transmitter-receiver pair 1 and we denote by $\mathcal{G}^{(1)}_1$ the group of all the communication flows with received power at receiver 1 and transmission rate equal to $\mathfrak{p}^1_1$ and $\mu_1 R$, respectively.

In the case of a SG-MMSE/NIS/SGD receiver and the system size grows large with $K, N \to \infty$, $\frac{K}{N} \to \beta$ and $\frac{|\mathcal{G}^{(1)}_1|}{N} \to \beta^{(1)}_1$, the spectral efficiency per chip converges almost surely to [14]

$$\mathcal{C}^{mmse}(\mathrm{SNR}, \beta^{(1)}_1) \to \beta^{(1)}_1 \log_2(1 + \mathrm{SNR} - \frac{1}{4}\mathcal{F}(\mathrm{SNR}, \beta^{(1)}_1) \quad (3)$$

being $\mathcal{F}(x, z) = \left(\sqrt{x(1+\sqrt{z})^2+1} - \sqrt{x(1-\sqrt{z})^2+1}\right)^2$ and SNR the signal to noise ratio accounting in the noise also the interference from other groups, i.e.

$$\mathrm{SNR} = \frac{\mathfrak{p}_1^{(1)}}{1+\sum_{m\in\{2,\ldots,N_g\}}\beta_m^{(1)}\mathfrak{p}_m^{(1)}}. \quad (4)$$

The information stream of the pair in $\mathcal{G}_1^{(1)}$ can be decoded reliably if and only if

$$\mu_1 R \leq \frac{\mathcal{C}^{mmse}(\mathrm{SNR},\beta_1^{(1)})}{\beta_1^{(1)}}. \quad (5)$$

In fact, from the definition of group $\mathcal{G}_1^{(1)}$ and the capacity region of a multiple access channel, the elements of all the information flows in $\mathcal{G}_1^{(1)}$ are reliably decodable if the following infinite conditions are satisfied:

$$\begin{cases} \widetilde{\beta}_1^{(1)}\mu_1 R \leq \mathcal{C}^{mmse}(\mathrm{SNR},\widetilde{\beta}_1^{(1)}) & \text{for } 0<\widetilde{\beta}_1^{(1)}\leq\beta_1^{(1)}, \\ \mu_1 R \leq \log_2(1+\mathrm{SNR}) & \text{for any subset of } \mathcal{G}_1^{(1)} \text{ with} \\ & \text{finite cardinality and } N\to\infty. \end{cases} \quad (6)$$

The condition on the dominant face (5) implies all the infinite other conditions (6) since the term $\frac{\mathcal{C}^{mmse}(\mathrm{SNR},\widetilde{\beta}_1^{(1)})}{\widetilde{\beta}_1^{(1)}}$ is a decreasing function of $\widetilde{\beta}_1^{(1)}$.

Let us notice that the effects of interference become deterministic if $\beta_j^{(1)}$ are deterministic.

The derivation of the large system performance for the NP/UIS/SGD receiver follows along similar lines when we observe that the spectral efficiency of the multiple access channel consisting of all the transmitters in $\mathcal{G}_1^{(1)}$ and the reference receiver 1 is given by [14]

$$C^{\mathrm{opt}}(\mathrm{SNR},\beta_1^{(1)}) = \beta_1^{(1)}\log_2(1+\mathrm{SNR}-\frac{1}{4}\mathcal{F}(\mathrm{SNR},\beta_1^{(1)}))$$
$$+\log_2(1+\mathrm{SNR}\beta_1^{(1)}-\frac{1}{4}\mathcal{F}(\mathrm{SNR},\beta_1^{(1)}))-\frac{\log e}{4\mathrm{SNR}}\mathcal{F}(\mathrm{SNR},\beta_1^{(1)})$$

with SNR defined in (4). Then, the information streams of the pairs in $\mathcal{G}_1^{(1)}$ can be decoded reliably by a NP/UIS/SGD receiver if and only if

$$\mu_1 R \leq \frac{\mathcal{C}^{opt}(\mathrm{SNR},\beta_1^{(1)})}{\beta_1^{(1)}}. \quad (7)$$

The performance of an NP/KIS/SGD receiver can be derived by using the fundamental relation on the mutual information

$$\begin{aligned} I(X_{\mathcal{G}_1^{(1)}};Y^{(1)}) &= I(X_{\mathcal{G}^{(1)}};Y^{(1)}) - I(X_{\sim\mathcal{G}_1^{(1)}};Y^{(1)}|X_{\mathcal{G}_1^{(1)}}) \\ &= \log_2\det(\boldsymbol{R}^{(1)}+\boldsymbol{I}) - \log_2\det(\boldsymbol{R}^{(1)}_{\sim\mathcal{G}_1^{(1)}}+\boldsymbol{I}) \end{aligned} \quad (8)$$

where $\boldsymbol{X}_{\mathcal{G}_1^{(1)}}$ denotes the set of transmitted information streams in $\mathcal{G}_1^{(1)}$, $Y^{(1)}$ is the set of the received random signals at receiver 1, and $X_{\sim\mathcal{G}_1^{(1)}}$ is the set of all the information streams transmitted by the nodes in the set $\sim\mathcal{G}_1^{(1)} = \bigcup_{m=2}^{N_g}\mathcal{G}_m^{(1)}$.

In large systems, the spectral efficiency per chip at the receiver 1 when all the transmitted information are decoded (multiple access vector channel) is given by [15], [16]

$$\begin{aligned} \mathcal{C}^{(\mathrm{MAC})}(\mathrm{SNR},\beta_1^{(1)}) = \sum_{m=1}^{N_g}\beta_m^{(1)}\log_2(1+\mathfrak{p}_m^{(1)}\eta^{(1)}) \\ -\log_2\eta^{(1)}+(\eta^{(1)}-1)\log_2\mathrm{e} \end{aligned} \quad (9)$$

being $\eta^{(1)}$ the unique real nonnegative solution of the fixed point equation

$$\eta^{(1)} = \frac{1}{1+\sum_{m=1}^{N_g}\beta_m\frac{\mathfrak{p}_m^{(1)}}{1+\mathfrak{p}_m^{(1)}\eta^{(1)}}}. \quad (10)$$

Then, (8) and (9) yield the spectral efficiency per chip of an NP/KIS/SGD receiver

$$\begin{aligned} \mathcal{C}^{(\mathrm{NP/KIS/SGD})}(\mathrm{SNR},\beta_1^{(1)}) &= \beta_1^{(1)}\log_2(1+\mathfrak{p}_1^{(1)}\eta^{(1)}) \\ &+\sum_{m=2}^{N_g}\beta_m^{(1)}\log_2\left(\frac{1+\mathfrak{p}_m^{(1)}\eta^{(1)}}{1+\mathfrak{p}_m^{(1)}\eta^{(1)}_{\sim\mathcal{G}_1^{(1)}}}\right) \\ &-\log_2\frac{\eta^{(1)}_{\sim\mathcal{G}_1^{(1)}}}{\eta^{(1)}}+(\eta^{(1)}-\eta^{(1)}_{\sim\mathcal{G}_1^{(1)}})\log_2\mathrm{e} \end{aligned} \quad (11)$$

with $\eta^{(1)}$ given in (10) and $\eta^{(1)}_{\sim\mathcal{G}_1^{(1)}}$ satisfying the relation

$$\eta^{(1)}_{\sim\mathcal{G}_1^{(1)}} = \frac{1}{1+\sum_{m=2}^{N_g}\beta_m\frac{\mathfrak{p}_m^{(1)}}{1+\mathfrak{p}_m^{(1)}\eta^{(1)}_{\sim\mathcal{G}_1^{(1)}}}}. \quad (12)$$

Let us consider the multiuser efficiency $\eta^{(1)}$ of the NP/KIS/SGD receiver as a function of $\beta_1^{(1)}$ and observe that it is a decreasing function of $\beta_1^{(1)}$. Then, making use of this property and appealing to similar arguments to the ones adopted for the SG-MMSE/NIS/SGD receiver it can be shown that the a reliable communication is possible if and only if the rate $\mu_1 R$ in $\mathcal{G}_1^{(1)}$ satisfies the conditions on the dominant face of the rate region, i.e.

$$\mu_1 R \leq \frac{\mathcal{C}^{(\mathrm{NP/KIS/SGD})}(\mathrm{SNR},\beta_1^{(1)})}{\beta_1^{(1)}}. \quad (13)$$

Let us consider now a NP/KIS/MGD receiver. We aim to provide necessary and sufficient conditions for a reliable decoding. Let us first observe that for each receiver there exist a unique maximal decodable set of transmitters, i.e. a set of transmitters which are jointly decodable by the receiver and is not a proper subset of any other decodable subset [17]. Furthermore,

THEOREM 1 *[17] A subset $\widehat{\mathcal{G}}^{(1)}\subseteq\mathcal{G}^{(1)}$ is the unique maximal decodable subset at receiver 1 if and only if the transmitters'* *rates satisfy the following inequalities*

$$\begin{cases} \sum_{i\in\overset{o}{\mathcal{G}}^{(1)}} \mu_i R \leq I(X_{\overset{o}{\mathcal{G}}^{(1)}}; Y^{(1)}|X_{\widehat{\mathcal{G}}^{(1)}\backslash\overset{o}{\mathcal{G}}^{(1)}}) & \forall \overset{o}{\mathcal{G}}^{(1)} \subseteq \widehat{\mathcal{G}}^{(1)}, \\ \sum_{i\in\overset{e}{\mathcal{G}}^{(1)}} \mu_i R > I(X_{\overset{e}{\mathcal{G}}^{(1)}}; Y^{(1)}|X_{\widehat{\mathcal{G}}^{(1)}}) & \forall \overset{e}{\mathcal{G}}^{(1)} \subseteq \mathcal{G}^{(1)}\backslash\widehat{\mathcal{G}}^{(1)}. \end{cases} \tag{14}$$

This theorem was derived in [17] for finite sets $\mathcal{G}^{(1)}$ but it can be extended to infinite sets. In this case, conditions (14) consist of infinite inequalities and it is not of practical usefulness. Nevertheless, for our system, the partition of the transmitter-receiver pairs in groups $\mathcal{G}_m^{(i)}$, $m = 1, \ldots, N_g$ can be utilized to reduce the set of conditions (14) to a finite set. In fact, the following properties derive from basic inequalities in information theory: (I) If a receiver is able to decode one transmitter in a group of users with identical received powers and transmission rates, it is able to decode all transmitteed information by the users in the group. Equally, if a receiver is not able to decode jointly all the users with identical received powers and transmission rates it is not able to decode any single transmitted information by one user in the group. (II) If a receiver is able to decode two groups[2] of transmitters, the union is also decodable by the receiver. Thus, also for large systems we can conclude that if a transmitter of a group $\mathcal{G}_m^{(i)}$ belongs to the decodable set any other transmitter belonging to the same group is also decodable and the full set is included in the maximum decodable set [3]. $\mathcal{G}_m^{(i)}$ and (14) reduce to a finite set of conditions.

Let us observe that the possible decodable sets for which to verify condition (14) are 2^{N_g}. Because of the exponential complexity of this step, it is of great interest to have low complexity algorithms. An algorithm with polynomial complexity is proposed in [17]. It is based on the submodular function $f(\mathcal{V}, \mathcal{S})$, with $\mathcal{V} \supseteq \mathcal{S} \supseteq \mathcal{G}$, and $\mathcal{G}$ finite set transmitters[4]

$$f(\mathcal{V}, \mathcal{S}) = I(X_{\mathcal{V}}; Y^{(1)}|X_{\mathcal{S}\backslash\mathcal{V}}) - R_{\mathcal{V}} \tag{15}$$

and $R_{\mathcal{V}}$ is the sum of the rates of all the transmitters in $\mathcal{V}$. Note that $f(\mathcal{V}, \mathcal{S})$ is defined also for the empty set $\emptyset$, and $f(\emptyset, \mathcal{S}) = 0$. Additionally, the algorithm exploits well known polynomial time algorithms for the minimization of submodular functions [18], [19]. The application of this approach to a large system is almost straightforward when we determine the maximum decodable set up to a subset with zero measure. Then, if the communication of interest belongs to a set of zero measure,independently whether it is decodable or not.

The polynomial time algorithm to verify whether the information stream of the transmitter-receiver pair of interest in $\mathcal{G}^{(1)}$, with cardinality $|\mathcal{G}^{(1)}| \to \infty$ is decodable or not is detailed below.

ALGORITHM 1 Initial Step:

Set $\mathcal{S} = \bigcup_{\substack{\ell=1 \\ \beta_\ell^{(1)} \neq 0}} \mathcal{G}_\ell^{(1)}$.

Step 2:

Determine the set $\mathcal{V}_{\min} = \bigcup_{\substack{\ell=1 \\ \beta_\ell^{(1)} \neq 0}} \mathcal{G}_{i_\ell}^1 \subseteq \mathcal{S}$ *with minimum cardinality that minimize the submodular function*

$$\widetilde{f}(\mathcal{V}, \mathcal{S}) = \lim_{\substack{K,N\to\infty \\ \frac{K}{N}\to\beta}} \frac{f(\mathcal{V}, \mathcal{S})}{N}$$
$$= \sum_{\substack{\ell=1 \\ \mathcal{G}_\ell^{(1)} \notin \sim\mathcal{S}}}^{N_g} \beta_\ell^{(1)} \log_2 \left(\frac{1 + \mathfrak{p}_\ell^{(1)} \eta_{\sim\mathcal{S}\backslash\mathcal{V}_{\min}}^{(1)}}{1 + \mathfrak{p}_\ell^{(1)} \eta_{\sim\mathcal{S}}^{(1)}} \right)$$
$$\sum_{\substack{\ell=1 \\ \mathcal{G}_\ell^{(1)} \in \mathcal{V}_{\min}}}^{N_g} \beta_\ell^{(1)} \log_2 (1 + \mathfrak{p}_\ell^{(1)} \eta_{\sim\mathcal{S}\backslash\mathcal{V}_{\min}}^{(1)}) + \log_2 \frac{\eta_{\sim\mathcal{S}}^{(1)}}{\eta_{\sim\mathcal{S}\backslash\mathcal{V}_{\min}}^{(1)}}$$
$$+ (\eta_{\sim\mathcal{S}\backslash\mathcal{V}_{\min}}^{(1)} - \eta_{\sim\mathcal{S}}^{(1)}) \log_2 \mathrm{e} - \sum_{\substack{\ell=1 \\ \mathcal{G}_\ell^{(1)} \in \mathcal{V}_{\min}}}^{N_g} \mu_m \beta_m^{(1)} R.$$

with $\eta_{\sim\mathcal{S}\backslash\mathcal{V}_{\min}}^{(1)}$ *and* $\eta_{\sim\mathcal{S}}^{(1)}$ *defined as in (12).*

Step 3

Set $\mathcal{S} \leftarrow \mathcal{S}\backslash\mathcal{V}_{\min}^{(1)}$. *If* $\mathcal{V}_{\min}^{(1)} \neq \emptyset$ *go to step 2.*

Step 4

If $\mathcal{G}_1^{(1)} \subseteq \mathcal{S}$ *then the transmitter-receiver pair 1 is decodable. STOP.*

Step 5

If $\beta_1^{(1)} = 0$ *then set* $\mathcal{V}$ *to a sigleton set containing the transmitter-receiver pair 1 and compute the function*

$$f_0(\mathcal{V}, \mathcal{S}) = \lim_{\substack{K,N\to\infty \\ \frac{K}{N}\to\beta}} f(\mathcal{V}, \mathcal{S})$$
$$= \lim_{\substack{K,N\to\infty \\ \frac{K}{N}\to\beta}} \log_2 \det(\boldsymbol{R}_{\mathcal{S}} + \mathfrak{p}_1^1 \boldsymbol{s}_1^{1^H} \boldsymbol{s}_1^1 + \boldsymbol{I})$$
$$- \log_2 \det(\boldsymbol{R}_{\mathcal{S}} + \boldsymbol{I}) - \mu_1 R = \log_2 \left(1 + \mathfrak{p}_1^1 \eta_{\mathcal{S}}\right) - \mu_1 R$$

with $\eta_{\mathcal{S}}$ *defined as in (12).*

Step 6

If $f_0(\mathcal{V}, \mathcal{S}) \geq 0$ *then the transmitter-receiver pair 1 is decodable otherwise is not decodable. STOP.*

IV. PROBLEM STATEMENT

The utility function for this problem is defined as the individual throughput of each transmission flow, i.e. the average number of information bits transmitted by a source and successfully received by the corresponding destination in the time unit. We are interested in finding the policies $\mathcal{U}_k$ which maximize the individual throughput with some constraints while using one of the receivers described in Section II. With this aim, we investigate the problem introduced in [3] under the assumption that $\frac{K}{N} \to \beta > 0$ and β finite. We make use of

[2]Each group consists of users having same received powers and transmission rate.

[3]These properties hold thanks to the existence and uniqueness of the maximum decodable set [17] and the fact that all users in the same set have the same transmitted and received power.

[4]Note that the function $f(\mathcal{V}; \mathcal{S}) \geq 0$ if the sum rate of the information transmitted by nodes in $\mathcal{V}$ is lower than the mutual information over the channel between the nodes in $\mathcal{V}$ and the receiver when all the information transmitted by nodes in $\mathcal{S}\backslash\mathcal{V}$ is known at the receiver and the information transmitted by the nodes in $\mathcal{G}^{(1)}\backslash\mathcal{S}$ is treated as noise.

mathematical results on random matrices successfully utilized in the analysis of several large systems.

In the rest of this section we introduce the throughput optimization problem as a stochastic game defined for the interference network under investigation.

At each time slot, a node chooses its action without having a global view of the channel states and the other users' interference. There is no coordination among transmitters' actions and only local information is available at each node. Therefore, in the general case, for any choice (p_i, μ_i), there is no guaranty that the μ_i transmitted packets can be received correctly when the TS is x_i.

However, for large interference networks, as $N, K \to \infty$ and $\frac{K}{N} \to \beta$, the total interference impairing user i can be replaced by a deterministic value. Therefore, during a block time t, $\mu_i(t)$ packets can be transmitted successfully by source i if the conditions derived in Section III for the achievable rates on the interference channel are satisfied. Namely, if an SG-MMSE/NIS/SGD receiver is adopted , the power and transmission rate are such that (5) is satisfied. For an NP/UIS/SGD receiver, condition (7) needs to be fulfilled. Condition (13) is required for reliable communications when NP/KIS/SGD receivers are utilized. Conditions for reliable communications over a system based on NP/KIS/MGD receivers are provided in (14) or, equivalently, in Algorithm 1.

Let $\mathbb{P}(\mu_k(t) R \,\text{achievable}\, | x_k^k = \chi_0)$ be the probability of receiving correctly $\mu_k(t)$ transmitted packets at block time t, conditioned to $x_k(0) = \chi_0$, the initial state of user k. This probability depends on the choice of the receiver although it is not explicitly expressed by the adopted notation.

The average throughput for source k is

$$\limsup_{T \to +\infty} \frac{1}{T} \sum_{t=0}^{T-1} E\{\mathbb{P}\left(\mu_k(t) R | x_k^k(0) = \chi_0\right) \mu_k(t) R\} \tag{16}$$

where the expectation is conditioned to $x_k^k(0)$, the initial TS of user k.

For physical and QoS reasons the transmitters are subjected to constraints on the average transmitted powers and on the average queue length. More specifically, the average power of transmitter k is constrained to a maximum value $\overline{p}_k$ and the following upper bound is enforced

$$\limsup_{T \to +\infty} \frac{1}{T} \sum_{t=0}^{T-1} E\{p_k(x_k(t), d_k(t)) | x_k(0) = \chi_0\} \le \overline{p}_k \tag{17}$$

where $p_k(x_k(t), d_k(t))$ is the power, eventually zero, transmitted by the source k at time instant t when the action triplet $d_k(t)$ is selected. The expectation is conditioned to the initial TS $x_k(0) = \chi_0$ of transmitter k. Similarly, in order to keep the average delay of the packets limited, the average queue length is constrained by the following bound:

$$\limsup_{T \to +\infty} \frac{1}{T} \sum_{t=0}^{T-1} E\{q_k(t) | x_k(0) = \chi_0\} \le \overline{q}_k \tag{18}$$

where $\overline{q}_k$ is maximum allowed average queue and the expectation is conditioned to $x_k(0) = \chi_0$.

V. Game in Large Symmetric Interference Networks

In this section we restrict our investigation to a large symmetric interference network. A large symmetric interference network is characterized by the fact that all the channels are characterized by the same Markov chain and the statistically identical processes for the arrival processes. Additionally, their action sets and the constraint parameters are identical. Equivalently, in a large symmetric interference network all the users have the same objectives and constraints. In such a case, an optimal policy is identical for all users. Furthermore, the distributions of the received powers are equal for all users.

Therefore, here on, we omit the user index and generalized our analysis to any transmitter-receiver pair. We denote by κ the cardinality of the product set $\mathcal{K} = \mathcal{X} \times \mathcal{D} = \{(x, d) : x = (\sigma, q) \in \mathcal{X}, d = (p, \mu, c) \in \mathcal{D}\}$ and by $< x, d >_n$ the n-th element of $\mathcal{K}$. In the asymptotic case, the other users' policies will influence the payoff function only through the asymptotic distribution of the received powers. If we denote this probability by $\mathbb{P}(\mathfrak{p})$, the payoff function is

$$c(x, d, \mathbb{P}(\mathfrak{p})) = \mu R \, 1(\mu R \,\text{achievable}; \mathbb{P}(\mathfrak{p})) \tag{19}$$

where $1(\cdot)$ is the indicator function. The payoff function can be computed for each given pair in $\mathcal{K}$ and $\mathbb{P}(\mathfrak{p})$ from conditions (5), (7), (13) or (14) according to the adopted decoding method.

Let $z = z(x, a)$ be the joint probability that the transmitter performs action a while being in state x. It can be expressed by the column vector $\boldsymbol{z} = (z_1, z_2, \ldots z_\kappa)^T$. Then, for a given received power distribution, the payoff ρ is given by the linear form

$$\rho(\mathbb{P}(\mathfrak{p})) = \sum_{<x,d> \in \mathcal{K}} c(x, d, \mathbb{P}(\mathfrak{p})) z_n. \tag{20}$$

Therefore the constrained optimization problem defined in (16)-(18) can be expressed as follows

$$\max_{z(x,d)} \sum_{x \in \mathcal{X}} \sum_{d \in \mathcal{D}} z(x, d) \mu R 1(\mu R \,\text{achievable}; \mathbb{P}(\mathfrak{p})) \tag{21a}$$

Subject to:

$$\sum_{x \in \mathcal{X}} \sum_{d \in \mathcal{D}} z(x, d)[\delta_r(x) - P_{xdr}] = 0 \quad \forall r \in \mathcal{X} \tag{21b}$$

$$\sum_{x \in \mathcal{X}} \sum_{d \in \mathcal{D}} p(x, d) z(x, d) \le \overline{p} \tag{21c}$$

$$\sum_{x \in \mathcal{X}} \sum_{d \in \mathcal{D}} q z(x, d) \le \overline{q} \tag{21d}$$

$$z(x, d) = 0 \quad \text{if } q \le \mu \tag{21e}$$

$$z(x, d) \ge 0; \; \forall (x, d) \in \mathcal{K}; \quad \sum_{(x,d) \in \mathcal{K}} z(x, d) = 1 \tag{21f}$$

where P_{xdr} is the probability to move from state x to state r when action d is performed. $\delta_r(x)$ is a delta function which is equal to 1 where $x = r$ and zero for other values of x. Additionally, (21b) guarantees that the graph of the obtained MDP is closed; (21c)-(21d) correspond to the constraints (17)-(18), respectively; (21e) eliminates the invalid pairs in $\mathcal{K}$ such that the number of packets to be sent is not higher than the number of packets in the queue.

Note that if the distribution $\mathbb{P}(\mathfrak{p})$ had been known (20) would have reduced to a linear equation and the optimal $\boldsymbol{z} = \boldsymbol{z}^*$ would have been solution of a linear program.

The optimal policy $u^*(d|x)$ of a transmitter can be immediately derived from $\boldsymbol{z}^*$ in the steady state of the MDC system by the relation $u(d|x) = \frac{z^*(x,d)}{\sum_{d' \in \boldsymbol{d}} z^*(x,d')}$.

In a large symmetric network an equilibrium for the network is achieved when all the transmitters adopt the same policy $u(d|x)$ or $z(x, d)$. Since the probability of the received powers $\mathbb{P}(\mathfrak{p})$ depends on $u(d|x)$, then the game (21) is intrinsically nonlinear and difficult to solve. Thus we propose a best response algorithm as solution of the game. We choose arbitrarily a policy for all the infinite transmitters except the reference pair 1. Based on such a policy it is possible to determine the probability of the received powers at receiver 1 by (2). Then, the new probability mass function $\mathbb{P}(\mathfrak{p})$ is utilize to solve the linear problem defined in (21). This procedure can be iterated. If the algorithm converges the solution is a Nash equilibrium.

VI. Numerical Results

In this section, we consider two methods for resource allocation. The first method is the cross-layer method proposed in this work and denoted shortly CL. The second method is the conventional resource allocation ignoring the state of queues. It is denoted shortly as Conv. We use the setting of a symmetric large interference network with parameters detailed in Table I for the comparisons presented here.

We compare the performance of the optimal game strategies, at receiver i, while using the three classes of receivers described in Section II, namely (SG-MMSE/UIS/SIG), (NP/KIS/SGD), (NP/KIS/MGD). For the sake of brevity, we address the approaches as $Am - r$ where $m \in \{CL, Conv\}$ and $r \in \{(\mathrm{SG-MMSE/UIS/SIG}), (\mathrm{NP/KIS/SGD}), (\mathrm{NP/KIS/MGD})\}$.

In our setting, we assume that CS varies according to a Markov chain with the following transition probabilities: $T_0^0(i,j) = \frac{1}{2}, T_0^1(i,j) = \frac{1}{2}, T_{L-1}^{L-1}(i,j) = \frac{1}{2}, T_{L-1}^{L-2}(i,j) = \frac{1}{2}; (2 \leq k \leq L-2) T_k^k(i,j) = \frac{1}{3}, T_k^{k-1}(i,j) = \frac{1}{3}, T_k^{k+1}(i,j) = \frac{1}{3}$. This means that at each time slot the channel preserves its state or changes by one unit. The packet arrival process is described by a Poisson distribution with average rate $\lambda_i = 1$. In our simulations, we assume that the possible rates are multiple of $R = \frac{1}{2}$.

We perform a two-level admission control; one is defined by our offline policy and set the variable c_i to 1/0 corresponding to the acceptance/rejection decision. However, as we only use one admission control flag c_i for all the possible number of packet arrivals, there exist situations where the remaining space of the queue is less than the number of packets arrived at the time. The second (realtime) control is needed in order to drop the packets when the queue is full.

The algorithm in Section V converges for all the classes of receivers. The optimal policies are in general not unique and depend on the policy initializing the algorithm.

The optimal policies in Figure 1, are obtained in high SNR regime. This figure shows the equilibrium policies obtained by the proposed algorithm for the three classes of receivers. The action index is presented in abscissa while the state index is represented in ordinate. The state index addresses the pair of CS and QS. The indexing approach is presented in Table II. Similarly, Table III describes the mapping between action indices and the triplets (μ_i, p_i, c_i).

Interestingly, the optimal policies of the *large interference network* studied in this paper have the following decoupling property: (I) decision on μ_i is not affected by the CSs and is an increasing function of the QS, and (II) the power level is independent from the queue level and only a function of CS. This property is specific of large interference networks and it does not hold in the general case of interference networks with finite users [3].

For all three classes of receivers, the optimal policy does not transmit packets when the channel is in the worst situation. For two other channel states, namely medium and good, the decision on μ_i is a non-decreasing function of QS. The optimal policies for ACL-(SG-MMSE/UIS/SGD) yield transmissions with lower rates compared to the two other receivers. ACL-(NP/KIS/MGD) yields a number of transmitted packets not lower than the ACL-(NP/KIS/SGD) receiver at the same power.

At high SNR, the policies of the ACL-(NP/KIS/MGD) receiver yield transmission at the maximum allowed rate whenever the channel state of the transmitter is nonzero. In other words, the optimal rate is limited by the discrete rate set. In contrast, for the other two receivers the optimal rates are limited by the interference and they show an interference limited behavior. This observation helps us in a better understanding of the saturation behavior of the receivers in the following Figure 2-4.

Figure 2 shows the performance of optimal policies in our cross-layer approach for three classes of receivers. This figure shows the throughput obtained by each class of receivers versus the energy per bit per noise level, Eb/N_0. The simulations are done on a range of noise variances from -30dB to 0dB. The value of the throughput here is obtained through averaging the data rates (bits/s/Hz) of the equilibrium policies of our proposed algorithm over all transmitter states. To be compliant with the definition of throughput, the energy per bit per noise level is obtained by the same averaging function.

As the value of energy per bit per noise level increases, all receivers enter into a saturation mode. For the ACL-(NP/KIS/MGD) receiver, this behavior results from the fact that the optimal rate is limited by the discrete rate set. In contrast, for the other two cases, the throughput is interference

title	β	B_i	L	M_i	$\|\mathcal{P}_i\|$	$\overline{p}_i$	$\overline{q}_i$
CL	2	5	3	5	4	1	2
Conv	2	–	3	5	4	1	2

Table I
NETWORK PARAMETERS

state index	0 1 2 3 4 5 6 ... 17
queue state	0 0 0 1 1 1 2 ... 5
channel state	0 1 2 0 1 2 0 ... 2

Table II
LABELLING OF STATES

action index	0 1 2 3 4 5 6 7 8 9 ... 48
Num of packets	0 0 0 0 0 0 0 0 1 1 1 ... 5
power level	0 0 1 1 2 2 3 3 0 0 1... 3
accept/reject	0 1 0 1 0 1 0 1 0 1 0... 1

Table III
LABELLING OF POLICIES

	TP	Outage Rate	Drop rate
policy of asymptotic problem	0.6	0.38	0.09
policy adapted to the finite problem	0.61	0.36	0.09

Table IV
COMPARISON BETWEEN THE PERFORMANCE OF THE EQUILIBRIUM POLICY OBTAINED FOR THE ASYMPTOTIC PROBLEM AND THE ONE ADAPTED TO A 2-FLOW NETWORK [3] IN A NETWORK WITH 2 ACTIVE COMMUNICATIONS

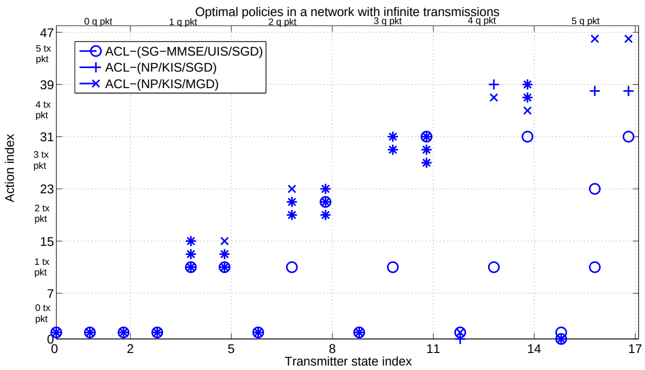

Figure 1. Policies in a network with infinite transmissions

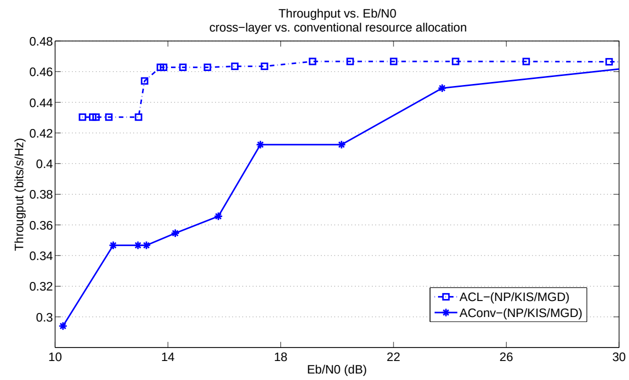

Figure 3. Throughput vs Eb/N0, Cross-layer vs. Conventional mechanisms

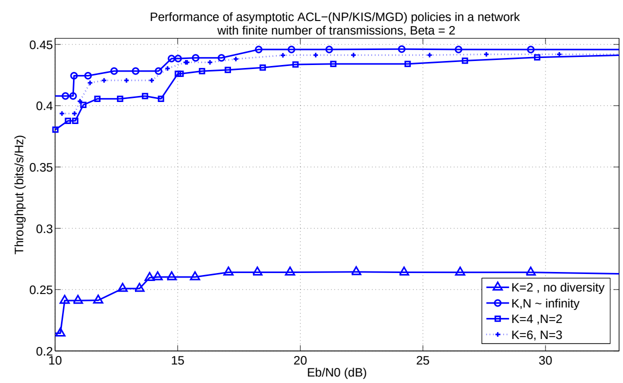

Figure 4. Throughput vs Eb/N0, performance of asymptotic ACL-(NP/KIS/MGD)in a network of finite transmissions

limited.

Figure 3 compares the performances of cross-layer and conventional mechanisms while using the best receiver, namely ACl-(NP/KIS/MGD) and AConv-(NP/KIS/MGD). At the first glance, we can observe that in the conventional approach more power is consumed for sending a given packet. Indeed, the policies in this case are decided regardless of the queue states. Consequently, there exist cases where the power is adjusted to satisfy a certain rate while there is not enough data in the queue to provide that rate. In such cases, the remaining data in the queue is sent with a power level higher than needed.

Figure 4 represents the performance of the optimal policies obtained for the asymptotic case in networks with finite transmissions. We can observe that using the policies obtained from the asymptotic problem, even when the number of transmitter is very low, e.g. $K = 4$, the finite network performs almost as well as the large interference network. For $K = 8$ the performance of a finite network attains the asymptotic one.

Finally, we compare the performance of the policy adapted to a finite network of 2 communication flows (obtained in [3]) with the one of the asymptotic problem. The performance measures here are: (i) Throughput (TP), i.e. the number of packets per time slot correctly decoded by the receiver, (ii) Outage rate, i.e. the fraction of transmitted packets which can not be decoded correctly, (iii) Drop rate, i.e. the fraction of arriving packets from upper layers which are rejected due to admission control. The value of the performance metrics for both policies are represented in Table IV. We can observe that in a network of 2 communication flows the policy obtained through the asymptotic problem, performs almost as well as the one adapted to this finite network. Therefore, also for a 2-flow network one can choose the less complex problem, i.e. the asymptotic one, for obtaining good policies.

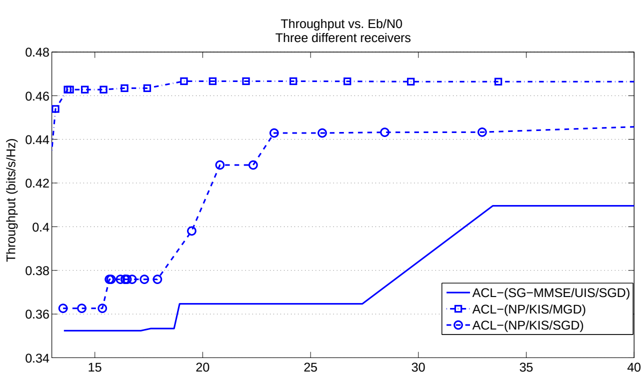

Figure 2. Throughput vs Eb/N0 for three different receivers

VII. Conclusions

In the current work, we considered a dense interference network with a large number ($K \to \infty$) of transmitter-receiver pairs. We investigated *distributed* algorithms for joint admission control, rate, and power allocation aiming at maximizing the individual throughput. The decisions are based on the statistical knowledge of the channel and buffer states of the other communication pairs and on the exact knowledge of their own channel and buffer states.

We considered different receivers depending on the assumptions we make about (I) the level of knowledge of the interference available at the receiver; (II) the eventual use of a suboptimal receiver based on a preliminary pre-decoding processing (e.g. detection) followed by decoding; and (III) the type of decoder, i.e. single-user/joint decoder.

In a finite framework, this problem presents an extremely high complexity when the number of users and/or transmitter states grows above a very limited range (e.g. 2, 3 users!). This makes distributed cross layer approaches very intensive. The asymptotic approach of large interference networks enables a sizable complexity reduction. More specifically, the complexity does not scale with the number of users but with the number of groups of users having identical statistics. The problem has an especially low complexity in the practical case of symmetric networks.

The optimal policies obtained with the asymptotic approach can be effectively applied in finite interference networks. In fact, we studied the performance loss due to the application of policies designed for asymptotic conditions in network with a finite number of active communications. We observed that even for a network containing 4 active communications, the performance of finite networks almost attains the one of large interference networks. Similar results are obtained for the converse comparison. We compare the performance of a finite network when an asymptotic approximation of the policies is adapted with the one obtained with policies tailored to the finite networks [3]. Even for the most challenging case of a network with 2 communication flows, the optimal policy of the asymptotic problem performs almost as well as the policy adapted to the network.

We further investigated the benefits of a cross layer approach compared to a conventional resource allocation ignoring the states of the queues. In the conventional approach more power is consumed for sending a given amount of data as there exist cases where the power is allocated to satisfy a certain rate although there is not enough data in the queue to achieve that rate. To neglect the state of the queue causes a relevant performance loss since the power is not efficiently allocated.

Interestingly, the optimal policy of the large interference network studied in this paper presents interesting decoupling properties. More specifically, the rate is an increasing function of the queue state only while the allocated power is a function of the channel state only.

References

[1] L. Georgiadis, M. J. Neely, and L. Tassiulas, Resource allocation and cross-layer control in wireless networks , *Foundations and Trends in Networking*, 2006.

[2] E. Altman, K. Avratchenkov, N. Bonneau, M. Debbah, R. El-Azouzi, D.S. Menasche, Constrained stochastic games in wireless networks, *Global Telecommunications Conference*, pp. 315-320, Nov. 2007.

[3] S. Akbarzadeh, L. Cottatellucci, E. Altman, and C. Bonnet, Distributed communication control mechanisms for ad-hoc networks, *IEEE International Conference on Communications*, June 2009.

[4] M. J. Neely, E. Modiano, and Ch. E. Rohrs, Dynamic power allocation and routing for time-varying wireless networks, *IEEE Journal on Selected Areass in Communications*, VOL. 23, n. 1, pp. 89-103, Jan. 2005.

[5] Ch. Long, Q. Zhang, Bo Li, Huilong Yang, and Xinping Guan, Non-cooperative power control for wireless ad hoc networks with repeated games, *IEEE Journal on Selected Areas in Communications*, VOL. 25, n. 6, pp. 1101-1112 Aug. 2007.

[6] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, Efficient power control via pricing in wireless data networks, *IEEE Transactions on Communications*, VOL. 50, n. 2, Feb. 2002.

[7] J. Huang, R. A. Berry, and M. L. Honig, Distributed interference compensation for wireless networks, *IEEE Transactions on Communications*, VOL. 24, n. 5, pp. 1074- 1084, May 2006.

[8] E. Altman, Z. Altman, S-Modular games and power control in wireless networks, *IEEE Transactions on Automatic Control*, VOL. 48, NO. 5, pp. 839- 842, May 2003.

[9] D. N. C. Tse, O. Zeitouni, Linear multiuser receivers in random environments, *IEEE Transactions on Information Theory*, VOL. 46, n. 1, pp. 171-205, Jan. 2000.

[10] Patrick Billingsley, Probability and Measure, John Wiley & Sons, 1995, series *Wiley in probability and mathematical statistics*, 3rd ed.

[11] J. W. Silverstein and Z.D. Bai, On the empirical distribution of eigenvalues of a class of large dimensional random matrices, *Journal of Multivariate Analysis*, VOL. 54, pp. 175-192, 1995.

[12] B. M. Zaidel, S. Shamai and S. Verdú, Multicell uplink spectral efficiency of coded DS-CDMA with random signatures, *IEEE Journal on Selected Areas in Communications*, VOL. 19, n. 8, pp. 1556–1569, Aug. 2001.

[13] D. Aktas, M. N. Bacha, J. Evans, and S. Hanly, Scaling results on the sum capacity of cellular networks with MIMO links, *IEEE Transactions on Information Theory*, VOL. 52, n. 7, pp. 3264-3274, Jul. 2006.

[14] S. Verdu,Sh. Shamai, Spectral efficiency of CDMA with random spreading, *IEEE Transactions of Information Theory*, VOL. 45, n. 2, pp. 622-640 March. 1999.

[15] D. Tse, S. V. Hanly, Linear multiuser receivers: effective interference, effective bandwidth and user capacity, *IEEE Transactions of Information Theory*, VOL. 45, n. 2, pp. 641-657, March. 1999.

[16] Sh. Shamai, S. Verdu, The impact of frequency-flat fading on the spectral efficiency of CDMA, *IEEE Transactions of Information Theory*, VOL. 47, n. 4, pp. 1302-1327, May. 2001.

[17] A. S. Mtahari, A. Khandani, To decode the interference or to consider it as noise, *arxiv:0711/0711.3176v1*, Nov. 2007.

[18] A. Schrijver, A comninatorial algorithm minimizing submodular functions in strongly polynomial time, *Journal of Combinatorial Theory*, VOL. 80, pp. 346-355 ,2000.

[19] S. Iwata, L. Fleischer, and S. Fujishige, A comninatorial strongly polynomial algorithm for minimizing submodular functions, *Journal of ACM*, vol. 48, pp. 761-777 ,2001.

[20] E. Altman, K. Avratchenkov, N. Bonneau, M. Debbah, R. El-Azouzi, and D. Sadoc Menasche, Constrained cost-coupled stochastic games with independent state processes, *arXiv:cs/0703099v1 [cs.IT]*,2006.

[21] A. Lozano, A. M. Tulino, and S. Verdu, Optimum power allocation for parallel gaussian channels with arbitrary input distributions, *IEEE Transactions of Information Theory*,vol. 52, No. 7, pp. 3033- 3051, July 2006.