Distributed Beamforming Coordination in Multicell MIMO Channels

Invited Paper

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Abstract—Coordination in a multi-cell/link environment has been attracting a lot of attention in the research community recently. In this paper, we consider the problem of coordinated beamforming where base stations (BS) equipped with multiple antennas attempt to serve a separate user each despite the interference generated by the other bases. We propose a framework for a distributed optimization of the beamformers at each base, where distributed is defined as using "local CSIT" only. We present and compare two distributed approaches (one iterative and another direct approach) which have in common the optimization of the beamformers as a combination of so-called *egoistic* and *altruistic* solutions for this problem. We provide the intuitions behind these approaches and some theoretical grounds for optimality in certain cases. Performance is finally illustrated through numerical simulations.

I. INTRODUCTION

The problem of coordinating multiple co-channel interfering transmitters for improving the aggregate capacity finds important applications in the context of cellular networks with full resource reuse, cognitive radios, and spectrum sharing. Having multiple antennas at transmitters and receivers provides a powerful framework for coordination of the transmission and promises great improvements in terms of error resilience and rates achieved. The gains depend, however, on i) the number of transmitters and receivers, ii) how much they are allowed to cooperate and iii) the channel state information (CSI) available at each. In scenarios involving multiple interfering transmitterreceiver pairs, if either all transmitters or all receivers share their entire data and as a result perform joint transmission or joint decoding respectively, the situation will be conceptually equivalent to a broadcast channel (BC) and a multiple access channel (MAC). In these cases interference mitigation is well understood. However, if this is not the case (i.e. a distributed optimization scenario where the exchange of data and CSI among transmitters is limited), then an interference channel (IC) results. It is this latter situation which we consider here, since sharing data may put too much strain on the backhaul of the system in a cellular system, or cause unwanted interference in an ad-hoc peer-to-peer scenario.

Note that the MISO-IC setting, assuming each transmitter has multiple antennas and each receiver a single antenna, was previously considered for example in [1], [2] while the more general MIMO IC, which corresponds to receivers also having multiple antennas, is considered in [3], [4], among others. [2] and subsequent publications [5], [6] of the same authors have focused on the case of two transmitters and full CSI at the transmitters (CSIT). These authors consider the problem from the viewpoint of game theory, with transmitters as players; in this case a parametrization of the Pareto boundary of the rate region was found, and different algorithms suggested for finding different points on the boundary, particularly the maximum sum rate point. [7] considers a parametrization of the Pareto boundary in the more general case. In contrast, [3] focuses on the Nash Equilibrium for the MIMO case, where each transmitter optimizes his transmit covariance to achieve the best possible rate, given that no cooperation takes place.

Most of the work above assumes a central knowledge of CSI. In practice, for obvious scalability reasons, it is highly desirable that each transmitter optimize its precoder based on local channel knowledge only. In this case, it is reasonable to assume that it knows the channel between itself and all receivers that are within its range, either through user feedback or, in a TDD system, from those users' transmission in the uplink. In this paper we propose two approaches dealing with MISO and MIMO interference channels, under constraint of distributed optimization of the beamformers (i.e. local CSIT at each transmitter).

The techniques build up on the fact, recently brought up in [5], [6], that Pareto-optimal (i.e. reaching the boundary of the rate region) beamforming solutions for the two-link MISO-IC take the form of a linear combination between extreme solutions, known as the egoistic and altruistic solutions respectively. Although such solutions are compatible with a distributed optimization, the computation of the optimal linear combination weights presented so far in the literature required centralized CSIT. In this paper we propose two methods for a distributed computation of the combination weights. The first method is presented for the MISO case and relies on the socalled *virtual SINR* framework. The second method is based on the idea of iterative games and can handle the MIMO case.

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This paper presents a unified view of these approaches and some performance comparisons.

Notation: Throughout the paper boldface lowercase letters are used to denote vectors and boldface uppercase for matrices. \mathcal{E} is the expectation operator and I is the identity matrix. \mathcal{CN} denotes a complex circularly symmetric random variable. $\mathbf{V}_{(max)}(\mathbf{A})$ is the eigenvector of A corresponding to the maximum eigenvalue of A; $\mathbf{V}_{(min)}$ denotes the eigenvector corresponding to the minimum eigenvalue.

II. SYSTEM MODEL

The general scenario we consider is that of the MIMO interference channel where N_c transmitters (for example BSs in the cellular context) each communicate with a single receiver (mobile station (MS)). Each BS has $N_t > 1$ transmit antennas, each MS $N_r \ge 1$ antennas.

We adopt a narrow-band channel model with frequency-flat block fading. Under linear precoding at each of the transmitters (no joint multibase precoding since BSs do not share the data symbols), the signal transmitted by BS k, may be written as:

$$\mathbf{x}_k = \sqrt{p_k} \mathbf{w}_k s_k,\tag{1}$$

where $s_k \sim C\mathcal{N}(0, 1)$ is the symbol being transmitted intended for user k^1 , \mathbf{w}_k is the unit-norm precoding vector used to carry this symbol, so $\|\mathbf{w}_k\| = 1$, $p_k \leq P$, where p_k is the transmit power used, and P is the transmit power available at each BS. The signal received at user k is:

$$\mathbf{y}_k = \sum_{j=1}^{N_c} \sqrt{p_j} \mathbf{H}_{jk} \mathbf{w}_j s_j + \mathbf{n}_k, \qquad (2)$$

where $\mathbf{H}_{jk} \in \mathbb{C}^{N_t \times N_r 2}$ is the channel between that user and BS j, $\mathbf{n}_k \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I})$ is the noise at the considered receiver. We assume that receivers have full CSI (CSIR) and perform single-user detection, i.e. they do not attempt to decode the interfering signal. Denoting by \mathbf{v}_k the receive beamforming vector at user k, the receive signal after receiver processing is given by:

$$\begin{split} \tilde{y}_k &= \mathbf{v}_k^H \left[\sum_{j=1}^{N_c} \sqrt{p_j} \mathbf{H}_{jk} \mathbf{w}_j s_j + \mathbf{n}_k \right] \\ &= \sqrt{p_k} \mathbf{v}_k^H \mathbf{H}_{kk} \mathbf{w}_k s_k + \sum_{j=1, j \neq k}^{N_c} \sqrt{p_j} \mathbf{v}_k^H \mathbf{H}_{jk} \mathbf{w}_j s_j + \tilde{n}_k. \end{split}$$

Assuming unit-norm \mathbf{v}_k , $\tilde{n}_k \sim \mathcal{CN}(0, \sigma_n^2)$.

Given our assumptions, the rate achieved at the user served in cell k is given by:

$$R_k = \log_2(1 + \gamma_k),\tag{3}$$

¹I.e. Gaussian codebooks are used, even though these may be suboptimal for the interference channel.

²Note that in what follows, for the MISO case, the channel matrices reduce to row vectors and lower case will be used to denote the corresponding variables, i.e. \mathbf{h}_{jk} instead of \mathbf{H}_{jk} for example.

where the SINR γ_k is equal to:

$$\gamma_k = \frac{p_k |\mathbf{v}_k^H \mathbf{H}_{kk} \mathbf{w}_k|^2}{\sigma_n^2 + \sum_{j \neq k} p_j |\mathbf{v}_k^H \mathbf{H}_{jk} \mathbf{w}_j|^2}.$$
 (4)

The rate region \mathcal{R} is defined as the set of rates that may be achieved simultaneously at the different BSs, under their individual power constraints (cf. (5)).

$$\mathcal{R} = \{ (R_1, \dots, R_{N_c}) \in \mathbb{R}^{N_c}_+ \\ | R_k \text{ as in } (3), \forall k \in \{1, \dots, N_c\} \} .$$
(5)

III. A DISTRIBUTED FRAMEWORK FOR BEAMFORMING COORDINATION

A. Distributed CSIT

The performance of the above-described system will depend on how much CSI is allowed to be shared between the different nodes involved. Thus if CSI is collected at a central node, or made available at each BS, then the optimal (for the system performance metric of interest) beamforming vectors for each BS, can be computed. This, however, comes at the cost of signaling overhead. Hence, the interest of cooperative distributed approaches, which limit the exchange of information between the transmitters while trying to optimize system performance.

In this work, each transmitter's knowledge is limited to the channel between itself and all users ³, i.e. transmitter j knows $\mathbf{H}_{jk}, k = 1, \dots, N_c$.

Our main performance metric is the sum rate achieved across the system. However, maintaining fairness among users in terms of individual rates is also important. Another desired performance feature is our ability to reach the rate region boundary.

Given the distributed nature of the problem, one can think of BSs as players in a game, the strategies they choose to follow affect the global system performance. The objective is for each BS to find a "good" (in terms of our performance metrics) strategy to play.

B. Beamforming Coordination

The distributed approaches we adopt can be related, as will become clear later on, to some extreme beamforming strategies and previous results obtained for the full CSIT case for the two-link MISO interference channel.

1) Extreme Beamforming Strategies: Assuming no power control (justified below), i.e. each transmitter always uses full power, two extreme beamforming strategies arise: an *egoistic* strategy, where the interference generated is completely ignored and the focus is on maximizing the useful signal received at one's own user, and an *altruistic* strategy where the main focus is on trying to reduce the interference caused to others. Remarkably both these strategies are consistent with the distributed CSIT assumption above.

³Strictly speaking, each transmitter only needs to know the channels between itself and users that are close enough to suffer from interference.

1) Maximum Ratio Transmission (Egoistic strategy): In this case, the transmit beamforming vector at transmitter *k* is given by:

$$\mathbf{w}_{k}^{MRT} = \frac{\mathbf{h}_{kk}^{H}}{\|\mathbf{h}_{kk}\|}.$$
 (6)

2) Interference Minimizing Transmission (Altruistic strategy): If enough antennas are available at the transmitter, zero-forcing (ZF) of the generated interference is possible and the optimal ZF beamforming vector is given by:

$$\mathbf{w}_{k}^{INT} = \frac{\mathbf{\Pi}^{\perp,k} \mathbf{h}_{kk}^{H}}{\|\mathbf{\Pi}^{\perp,k} \mathbf{h}_{kk}^{H}\|}$$
(7)

where $\mathbf{\Pi}^{\perp,k}$ is the projection matrix onto the null space of the channels being interfered. Otherwise, in order to minimize the total interference caused, \mathbf{w}_k^{INT} can be selected to be the right singular vector corresponding to the smallest singular value of the aggregate interfered channel $\mathbf{H}_{-k} = [\mathbf{h}_{k1} \dots \mathbf{h}_{kk-1} \mathbf{h}_{kk+1} \dots \mathbf{h}_{kN_c}]$.

2) Previous Results on the Parametrization of the MISO Interference Channel with full CSIT: [2], [5], [6] show the following main result, which will be of some importance in our later derivations, for the case of two transmitters under full CSIT.

Theorem 1 (reproduced from [6]): Any point on the Pareto boundary is achievable with the beamforming strategies

$$\mathbf{w}_{i}(\lambda_{i}) = \frac{\lambda_{i} \mathbf{w}_{i}^{NE} + (1 - \lambda_{i}) \mathbf{w}_{i}^{ZF}}{\|\lambda_{i} \mathbf{w}_{i}^{NE} + (1 - \lambda_{i}) \mathbf{w}_{i}^{ZF}\|}, i = 1, 2$$
(8)

for some $0 \le \lambda_i \le 1$.

In the above theorem, $\mathbf{w}_i^{NE} = \mathbf{w}_i^{MRT}$ and $\mathbf{w}_i^{ZF} = \mathbf{w}_i^{INT}$ from (6)-(7).

Thus Pareto optimal beamformers belong to the set:

$$\mathcal{S} \equiv \left\{ \mathbf{w} \in \mathbb{C}^{N_t} | \mathbf{w} = \alpha \mathbf{w}_i^{MRT} + \beta \mathbf{w}_i^{INT}, \alpha, \beta \in \mathbb{R}_+, \\ \| \mathbf{w} \| = 1 \right\}.$$
(9)

It was also shown that:

Proposition 1 ([7]): Along the Pareto boundary of the MISO-IC, and $N_t \ge N_c$, full power must be used at each transmitter.

IV. PROPOSED DISTRIBUTED BEAMFORMING STRATEGIES

Given Proposition 1, we ignore power control from now on and assume all $p_k = P, \forall k = 1, ..., N_c$, though this may be suboptimal, for single-antenna receivers at least, when $N_t < N_c$, i.e. when there are at least as many interferers as antennas at a given BS. Under this setup, we adopt two different approaches to distributed coordinated beamforming, a first approach presented for the MISO case solely, and another more general approach for MIMO cases.

A. MISO case: Virtual SINR framework

Given the local information at each transmitter, we propose a simple transmission scheme based on having each transmitter maximize what we refer to as a virtual SINR. This essentially corresponds to balancing between the desired signal power generated and the noise plus interference generated at other users.

Under full power transmission, a virtual SINR at base station k is defined as:

$$\gamma_k^{virtual} = \frac{|\mathbf{h}_{kk}\mathbf{w}_k|^2}{\frac{1}{\rho} + \sum_{j \neq k} \alpha_{kj} |\mathbf{h}_{kj}\mathbf{w}_k|^2},$$
(10)

where $\alpha_{kj} \in \mathbb{R}_+, j, k = 1, \ldots, K$ are a given set of weights, and $\rho = \frac{P}{\sigma_n^2}$. This can be seen as the SINR achieved on the uplink of a system where at the *k*th base station, receive vector \mathbf{w}_k is used to process the received signal, mobile station *k* transmits its signal with power *P*, and mobile stations $j, \forall j \neq k$ transmit with power $\alpha_{ij}P$: the notion of a virtual uplink was first introduced in [1] in the context of downlink power control and beamforming in a multicell environment.

1) Two-link case Analysis: For the two-link case, the following proposition holds and allows us to relate the solution to the set S in (9).

Proposition 2: Maximizing the virtual SINRs in (10) for any $\alpha_{12}, \alpha_{21} \in [0, \infty)$ yields beamforming vectors of the form (8), with:

$$\lambda_{i} = \frac{1}{\rho \alpha_{i\bar{i}} \| \mathbf{h}_{i\bar{i}} \|^{2} \frac{\| \mathbf{\Pi}_{i\bar{i}}^{\pm} \mathbf{h}_{ii} \|}{\| \mathbf{h}_{i\bar{i}} \|} + 1},$$
(11)

where $\overline{i} = \mod(i, 2) + 1, i = 1, 2$, and $\Pi_{i\overline{i}}^{\perp} = \mathbf{I} - \frac{\mathbf{h}_{i\overline{i}}^{H}\mathbf{h}_{i\overline{i}}}{\|\mathbf{h}_{i\overline{i}}\|^{2}}$. *Proof:* Details in [8].

It is easy to see that any point covered by the parametrization in Theorem 1, equivalently any point in set S, can be reached for appropriate α_{12} and α_{21} . But which pair of α 's to use and still have a distributed solution? The following theorem provides a hint.

Theorem 2: The rate pair attained with full-power transmission and precoding using \mathbf{w}_k 's that maximize $\gamma_k^{virtual}$ in (10) with $\alpha_{12} = \alpha_{21} = 1$, lies on the Pareto boundary of the rate region.

Proof: Details in [8]. The proof relies on a version of the parametrization given in Theorem 1.

2) Proposed Algorithm: This leads us to propose the following distributed algorithm: transmit with full power and with precoding vector \mathbf{w}_k given by:

$$\mathbf{w}_{k} = \arg \max_{\|\mathbf{w}\|^{2}=1} \frac{|\mathbf{h}_{kk}\mathbf{w}|^{2}}{\frac{1}{\rho} + \sum_{j \neq k} |\mathbf{h}_{kj}\mathbf{w}|^{2}}.$$
 (12)

This is a generalized eigenvalue problem. Its solution is thus the unit-norm right eigenvector corresponding to the largest (and only non-zero in this case) generalized eigenvalue of the matrices $\mathbf{h}_{kk}^{H}\mathbf{h}_{kk}$ and $\frac{1}{a}\mathbf{I} + \sum_{j \neq k} \mathbf{h}_{kj}^{H}\mathbf{h}_{kj}$.

B. General MIMO case: Iterative Beamforming

In [9], an iterative algorithm, termed *Distributive Bargaining Solution (DBS)*, was proposed for the MISO IC whereby each transmitter selects its beamforming vector as a linear combination of its MRT and ZF solutions, i.e. as a member of set S; the Pareto boundary is approached by gradually changing the combination coefficients (and consequently the direction of the beamforming vectors) in each iteration so that *every* link would have a higher transmission rate. To extend this to the MIMO scenario, we start by revisiting the concepts of egoistic and altruistic solutions.

1) Egoistic Solution: The Egoistic solution of BS $k, k = 1, ..., N_c$, is to maximize, given its local knowledge, its user's expected received SINR, γ_k . I.e., each BS finds:

$$\mathbf{w}_{k}^{(ego)} = \arg\max_{\mathbf{w}_{k}} \mathcal{E}_{\mathbf{H}_{kj}, j \neq k} \left\{ |\mathbf{v}_{k}^{H} \mathbf{H}_{kk} \mathbf{w}_{k}|^{2} \right\},$$
(13)

which constitutes an intuitive generalization of the MRT solution in the MISO case (cf. (6)). We assume an SINR-maximizing MMSE receiver v_k to be used, given by:

$$\mathbf{v}_k = \mathbf{C}_{Rk}^{-1} \mathbf{H}_{kk} \mathbf{w}_k, \qquad (14)$$

where $C_{Rk} = \sum_{j=1, j \neq k}^{N_c} \mathbf{H}_{kj} \mathbf{w}_j \mathbf{w}_j^H \mathbf{H}_{kj}^H + \sigma_n^2 \mathbf{I}$, is the covariance matrix of the received interference and noise at MS k.

For independent identically distributed (iid) $\mathcal{CN}(0,1)$ channel coefficients, the following theorem holds.

Theorem 3: The optimal Egoistic solution (cf. (13)), for iid $\mathcal{CN}(0,1)$ channel coefficients, is to transmit along the eigenvector corresponding to the maximum eigenvalue of $\mathbf{H}_{kk}\mathbf{H}_{kk}^{H}$. Thus:

$$\mathbf{w}_{k}^{(ego)} = \mathbf{V}_{(max)}(\mathbf{H}_{kk}\mathbf{H}_{kk}^{H})$$
(15)

and \mathbf{v}_k is defined in (14).

Proof: The proof is provided in [10].

2) Altruistic Solution: Given its local CSI knowledge, BS k minimizes the sum of the expected interference it causes at the other MS's. The latter is only one of the possible optimization metrics, employed here for tractability. interference it generates to other MS $j, j \neq k$. Thus, BS k finds:

$$\mathbf{w}_{k}^{(alt)} = \arg\min_{\mathbf{w}_{k}} \mathcal{E}_{\mathbf{H}_{jj},\mathbf{H}_{ji},i\neq j,k} \sum_{j=1,j\neq k}^{N_{c}} |\mathbf{v}_{j}^{H}\mathbf{H}_{jk}\mathbf{w}_{k}|^{2}.$$
 (16)

As before, each MS k uses a MMSE receiver given by (14).

In the two-cell scenario, for i.i.d. $\mathcal{CN}(0,1)$ channel coefficients, one can easily obtain the optimal transmit beamformer $\mathbf{w}_{k}^{(alt)}$.

Theorem 4: With only 2 cells, for i.i.d. $\mathcal{CN}(0,1)$ channel coefficients, the optimal altruistic strategy is $\mathbf{w}_k^{(alt)} = \mathbf{V}^{(min)}(\mathbf{H}_{ik}^H\mathbf{H}_{jk})$.

Proof: We only provide a sketch of the proof. A more detailed version is provided in reference [10].

The optimal transmit beamformer for BS k as defined in (16) can be shown to simplify to:

$$\mathbf{w}_{k}^{(alt)} = \arg\min_{\mathbf{w}_{k}} \frac{\sigma_{n}^{-4}x}{(1+\sigma_{n}^{-2}x)^{2}}, \quad k = 1, 2,$$
 (17)

where $x = \mathbf{w}_k^H \mathbf{H}_{jk}^H \mathbf{H}_{jk} \mathbf{w}_k$, $j \neq k$. The expression is minimum when x is minimum, thereby yielding the result.

For a more general case, the altruistic solution of multicell scenario is simply stated as:

$$\mathbf{w}_{k}^{(alt)} = \mathbf{V}^{(min)} \left(\sum_{j \neq k}^{N_{c}} \mathbf{H}_{jk} \mathbf{H}_{jk}^{H} \right)$$
(18)

and $\mathbf{v}_k^{(alt)}$ is defined in (14).

3) The Distributive Bargaining Solution (DBS): The DBS algorithm introduced in [9] for the MISO case is summarized below.

Denote the beamforming vector of transmitter *i* in iteration j by $\mathbf{w}_i(j)$. Intuitively, it is reasonable to initialize the beamforming vectors $\mathbf{w}_i(0)$ to be the egoistic solutions $\mathbf{w}_i^{(ego)}$ because users start off with a non-cooperative setting. However they can also be initialized in a joint altruistic setting (see [9]). At iteration j,

- At transmitter *i*, the beamforming vector is updated as follows: w_i(j) = w_i(j-1)+δ_w(j) and w_i(j) → w_i(j) ||w_i(j)|| where δ_w(j) is computed based on the 1-bit feedback sent by its receiver at the previous iteration.
- At receiver i, the MMSE receiver beamforming vector v_i(j) is computed using the new w_k(j), ∀k, as in (14). MS i then computes its rate r_i^(j) (cf. (3)), and reports back to its transmitter a single bit of feedback to inform it of its satisfaction: a '1' for an increment of data rates (i.e. r_i^(j) > r_i^(j-1)), a '0' otherwise.

A stop condition is implemented so that the beamformer trajectory is halted as near as possible to the Pareto boundary. Many options may be considered. A reasonable and intuitive stopping condition is that a transmitter stops cooperating and terminates the algorithm when it encounters a decrement of transmission rate. In other words, transmitter i, $1 \le i \le N_c$ stops cooperating if

$$r_i^{(j)} < r_i^{(j-1)}. (19)$$

V. SIMULATION RESULTS

The performance of the proposed distributed coordinated beamforming approaches was tested, in terms of average sum rates, for a relatively realistic scenario whose parameters are specified in table I. User locations in each cell follow a uniform distribution.

A. MISO Scenarios

The average sum rates achieved using the virtual SINR approach and the DBS scheme are compared to each other and to the egoistic and the altruistic schemes described in Section III-B.1.

Figure 1 illustrates the performance for 3 antennas at each BS, and 3 cells in the setup. In the curves shown as in all the simulations conducted for the MISO case, the virtual SINR scheme outperforms the others in terms of average sum rates achieved. Note that for this layout, as soon as there is



Fig. 1. Sum rates vs. cell-edge SNR, MISO case.

more than 2 antennas per base station, zero-forcing can be performed and for all but the egoistic strategy, linear increase of the sum rate with the cell-edge SNR in dB is observed. When the number of antennas available does not allow for interference cancellation, saturation occurs as SNR increases.

B. MIMO Scenarios

The sum rate performance of the proposed DBS scheme is compared to that of the altruistic and egoistic solutions as defined earlier.

In Figure 2, the sum rate vs. N_t is plotted at SNR=15dB. Note that the performance gap between the proposed scheme and the egoistic and altruistic solutions increases with N_t because of the increased dimension or degrees of freedom.



Fig. 2. Sum rates versus N_t at 15dB cell edge SNR.

In Figure 3, the sum rate performance is plotted for increas-

ing N_c . The altruistic solution performs better than the egoistic solution as interference increases. Although the amount of interference increases, the proposed scheme outperforms and maintains a constant performance gap with respect to the altruistic and egoistic solutions.



Fig. 3. Sum rates versus N_c at 15dB cell edge SNR.

VI. CONCLUSION

In this paper, two different distributed coordinated beamforming approaches, based on partial CSI at each BS, both of which can be brought back to formulating the solution as a linear combination of an egoistic and an altruistic solution, were proposed. Their performance was illustrated via numerical simulations. The virtual SINR framework outperforms the iterative beamforming algorithm but the latter has the advantage that it lends itself to the MIMO case.

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