COORDINATION ON THE MISO INTERFERENCE CHANNEL USING THE VIRTUAL SINR FRAMEWORK

Randa Zakhour and David Gesbert

Mobile Communications Department
EURECOM
06560 Sophia Antipolis, France
{zakhour, gesbert}@eurecom.fr

ABSTRACT

This paper addresses the problem of coordination on the interference channel (IC), which has attracted a lot of attention in the research community recently. More precisely, coordinated beamforming is considered, in a multi-cell/link environment where base stations equipped with multiple antennas each attempt to serve a separate user despite the interference generated by the other bases. With single antenna users, this corresponds to the so-called MISO IC considered among others in [1, 2]. In this paper, we propose a distributed approach for designing the beamforming vectors to be used at the transmitters, which relies only on local channel state information (CSI) at each base. The technique exploits a metric which is reminiscent of the virtual uplink method proposed in [1]. We demonstrate analytically the optimality of the proposed approach in terms of achieving the outer bound of the rate region in certain cases. We conduct simulations showing the gains for general settings.

1. INTRODUCTION

The introduction of multiple antennas at transmitters and receivers in communication systems promises great improvements in terms of error resilience and rates achieved. Depending on how much channel state information (CSI) is available at the terminals involved, different degrees of such gains may be achieved in single link transmission, as well as in multiple access (MAC, corresponding to a cellular uplink) and broadcast (BC, corresponding to cellular downlink) channels. In scenarios involving multiple transmitters and receivers, such as a multi-cell scenario, the performance attained will depend on how much information may be shared at the nodes involved. Thus, if either all transmitters or all receivers share their entire data and as a result perform joint transmission or joint decoding respectively, the situation will be equivalent to a BC and a MAC, respectively, for which interference mitigation is well understood. However, if this is not the case (i.e. a distributed optimization scenario where the exchange of CSI among transmitters is limited), then an interference channel (IC) is obtained. This is the situation considered in this paper, as sharing data may put too much strain on the backhaul of the system. More precisely we deal with the downlink direction and propose a transmission strategy based on the so-called ”virtual SINR framework” (explained below).

Assuming each transmitter has multiple antennas and each receiver a single antenna, the setting is the MISO interference channel, considered for example in [1, 2] (the more general MIMO IC, which corresponds to receivers also having multiple antennas, is considered in [3, 4], among others). In particular, [2] and subsequent publications [5, 6] of the same authors have focused on the case of two transmitters and full CSI at the transmitters (CSIT). Considering the scenario from the viewpoint of game theory, with transmitters as players, a parametrization of the Pareto boundary of the rate region was found, and different algorithms suggested for finding different points on the boundary. [7] provides a parametrization of the Pareto boundary in a more general case.

Here we argue that it may not be reasonable to assume that all the CSI is shared by all transmitters, and consider the case where each transmitter has local channel CSI knowledge: it only knows the channel between itself and all receivers that are within its range. In a TDD system, this information may be gained from those users’ transmission in the uplink. If reciprocity may not be assumed, one could consider that each receiver feeds back his full CSI to his serving base which is partially shared with other base stations, thereby saving on signaling. This scenario has been tackled in [8] where an iterative method is proposed to achieve rates at all receivers involved that are higher than those achieved without cooperation. In contrast, what we develop in the present work is a one-shot algorithm. Given the local information at each transmitter, we propose a simple transmission scheme based on having each transmitter maximize what we refer to as a virtual SINR. For certain choices of parameters, the virtual SINR can be seen as the SINR achieved in the uplink if the same fil-

This work was funded in part by ETRI Korea, the European project COOPCOM and the French project ORMAC.

ters were used, in the TDD case, or in the virtual uplink (see [1]) in case there is no actual reciprocity.

Organization The rest of the paper is organized as follows. Section 2 defines the system model and performance measures considered. Section 3 introduces the virtual SINR framework. The approach of maximizing a virtual SINR at each transmitter is justified by relating it to uplink-downlink duality and more importantly to the full-CSIT results for the two-link case, in Section 4 below. Based on the given analysis, Section 5 states the proposed algorithm. Simulations in Section 6 show the value of the proposed algorithm in realistic scenarios when more links are considered.

Notation Throughout what follows we use the following common notation. $\mathbb{E}$ denotes statistical expectation. $\mathbb{C}$ denotes the complex number field. Boldface lowercase letters are used to denote vectors, and boldface uppercase denote matrices. $\mathcal{CN}(m, \sigma^2)$ is the probability distribution of a circularly symmetric complex Gaussian random variable of mean m and variance σ^2.

2. SYSTEM MODEL

We consider the MISO interference channel where K transmitters (e.g. base stations in a cellular system) with $N_t \geq 2$ antennas each, each communicate with a single receiver (mobile terminal) having a single antenna. This is illustrated in Figure 1 for $K = 2$, $N_t = 3$.

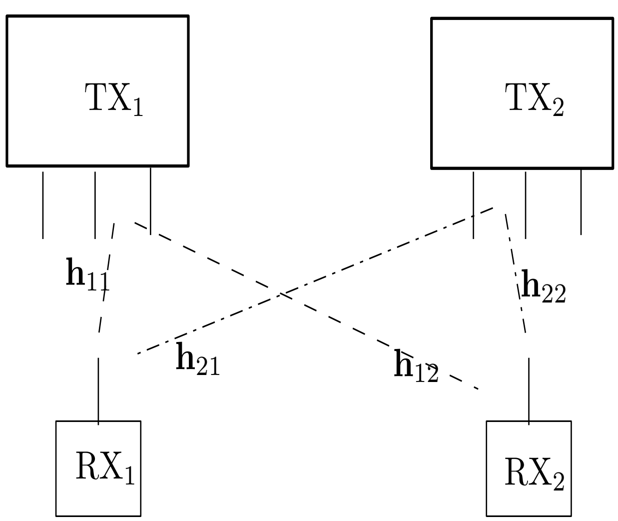

Fig. 1. Scenario considered for $K = 2$, $N_t = 3$. $\mathbf{h}_{11}, \mathbf{h}_{12}$ are known at TX_1, $\mathbf{h}_{21}, \mathbf{h}_{22}$ at TX_2.

We adopt a narrow-band channel model with frequency-flat block fading. Under linear precoding at each transmitter (no joint multibase precoding since BSs do not share the data symbols), the signal transmitted by base station k, $\mathbf{x}_k$ is given by:

$$\mathbf{x}_k = \sqrt{p_k}\mathbf{w}_k s_k \tag{1}$$

where $s_k \sim \mathcal{CN}(0, 1)$ is the symbol being transmitted intended for user k, $\mathbf{w}_k$ is the unit-norm beamforming vector used to carry this symbol and p_k is the transmit power used. A power constraint holds at each transmitter whereby $p_k \leq P$, P being the peak transmit power at each of the base stations. The signal received at user k is given by:

$$y_k = \sum_{j=1}^{K} \sqrt{p_j}\mathbf{h}_{jk}\mathbf{w}_j s_j + n_k \tag{2}$$

where $\mathbf{h}_{jk} \in \mathbb{C}^{N_t}$ is the channel between that user and base station j, $n_k \sim \mathcal{CN}(0, \sigma^2)$ is the noise at the considered receiver. We assume that receivers have full CSI (CSIR) and do not attempt to decode the interfering signals (single-user decoding). Under these assumptions, the rate achieved at user k is given by:

$$R_k = \log_2(1 + \gamma_k) \tag{3}$$

where the SINR γ_k is equal to:

$$\gamma_k = \frac{p_k|\mathbf{h}_{kk}\mathbf{w}_k|^2}{\sigma^2 + \sum_{j\neq k} p_j|\mathbf{h}_{jk}\mathbf{w}_j|^2} \tag{4}$$

The rate region $\mathcal{R}$ is defined as the set of rates that may be achieved simultaneously at the different base stations, given the power constraints at each base station. I.e.:

$$\mathcal{R} = \{(R_1, \ldots, R_K) \in \mathbb{R}_+^K \\ |R_k \text{ as in (3)}, p_k \leq P \forall k \in \{1, \ldots, K\}\} \tag{5}$$

Beamforming under distributed CSIT

In this work, each transmitter's knowledge is limited to the channel between itself and all users [1]. We would like to achieve a set of rates which is as close as possible to the boundary of $\mathcal{R}$, while yielding the best sum rate possible. Moreover, we would like to do so in a distributed fashion, relying only on *locally available* CSI as just defined, which leads us to an optimization problem solved at each base station within the framework of virtual SINR, detailed in the next section.

3. VIRTUAL SINR

In its most general form, a virtual SINR at base station k is defined as the ratio between the useful signal power received

[1]Strictly speaking, each transmitter only needs to know the channels between itself and users that are close enough to suffer from interference.

at its served user and the sum of noise plus a weighted sum of the interference powers it causes at the remaining users. Thus:

$$\gamma_k^{virtual} = \frac{p_k|\mathbf{h}_{kk}\mathbf{w}_k|^2}{\sigma^2 + \sum_{j\neq k}\alpha_{kj}p_k|\mathbf{h}_{kj}\mathbf{w}_k|^2}, \tag{6}$$

where $\alpha_{kj} \in \mathbb{R}_+, j, k = 1, \ldots, K$ are a given set of weights. This can be seen as the SINR achieved on the uplink of a system where at the kth base station, receive vector $\mathbf{w}_k$ is used to process the received signal, mobile station k transmits its signal with power p_k, and mobile $j, \forall j \neq k$ transmit with power $\alpha_{kj}p_k$: the 'virtual uplink' was first introduced in [1] in the context of downlink power control and beamforming in a multicell environment.

When transmitting at full power, Equation (6) becomes:

$$\gamma_k^{virtual} = \frac{|\mathbf{h}_{kk}\mathbf{w}_k|^2}{\frac{1}{\rho} + \sum_{j\neq k}\alpha_{kj}|\mathbf{h}_{kj}\mathbf{w}_k|^2}, \tag{7}$$

where $\rho = \frac{P}{\sigma^2}$.

As the objective is to have a distributed algorithm which relies only on information local to each base station, we propose that each transmitter solve a virtual SINR maximization problem, which can be stated as follows:

$$\mathbf{w}_k = \arg\max_{\|\mathbf{w}\|^2=1} \frac{|\mathbf{h}_{kk}\mathbf{w}_k|^2}{\frac{1}{\rho} + \sum_{j\neq k}\alpha_{kj}|\mathbf{h}_{kj}\mathbf{w}_k|^2}. \tag{8}$$

This is justified in the following section.

4. ANALYSIS

As first noted in [1], the same rate region may be achieved in the UL (for a reciprocal channel, in the virtual UL otherwise) and DL directions using the same set of vectors for receive and transmit beamforming respectively, but with different power levels in both directions that satisfy the same total power constraint. This is one form of what is referred to as uplink-downlink duality. In what follows, we do not pursue our formulation in the context of duality any further, but use it here to argue that considering virtual SINRs bears some relation to actual SINRs.

4.1. Two-link case

Transmission in the MISO IC may be viewed as a game, where each of the transmitters is a player trying to optimize his rate in some way. One can then define the Pareto boundary of the channel, as the set of Pareto-optimal rate-tuples: thus, a given tuple belongs to the Pareto boundary if it is not possible to increase any rate within that tuple without decreasing at least one of the others. As shown in [2, 7], rates on the Pareto boundary of the MISO interference channel are achieved by transmitting at full power (at least for the case where $N_t \geq K$). Thus, we restrict ourselves to the virtual SINR formulation of (7). We show that, for the two-link case, which is the most well understood, the following two theorems hold.

Theorem 1. *Any point on the Pareto boundary may be attained by solving the virtual SINR optimization problem, as given in* (8)*, for an appropriate choice of* $\alpha_{12}, \alpha_{21} \in \mathbb{R}_+$.

Proof. The proof follows by showing that the same parametrization of the Pareto boundary given in [6] can be retrieved for the beamforming vectors specified in this fashion. Details are given in Appendix A. □

Theorem 2. *The rate pair obtained by beamforming using the solutions to problem* (8) *with* $\alpha_{12} = \alpha_{21} = 1$ *lies on the Pareto boundary of the two-link rate region.*

Proof. See Appendix B. □

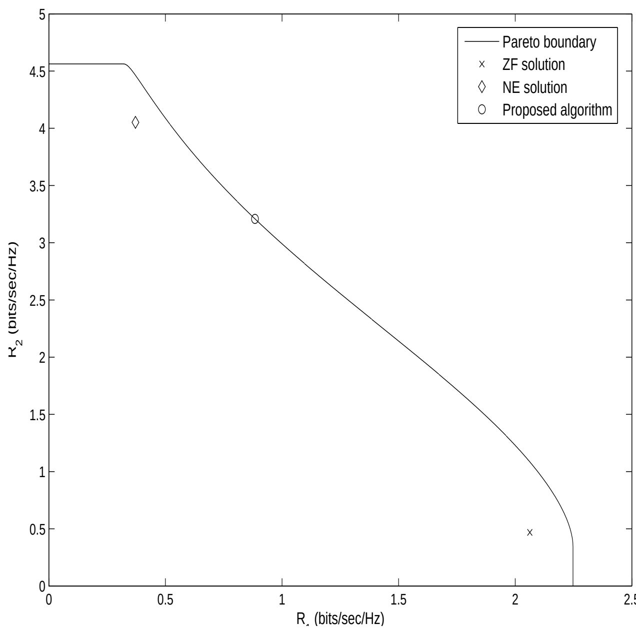

Fig. 2. Pareto rate boundary, MRT, ZF and $\alpha_{12} = \alpha_{21} = 1$ points for a channel instance sampled from a channel with independent identically distributed $\mathcal{CN}(0, 1)$ coefficients, $N_t = 3, K = 2$.

This is illustrated in Figure 2, which also shows the rate pairs corresponding to the Nash Equilibrium (NE) or Maximum Ratio Transmission (MRT) and Zero-Forcing (ZF) solutions, which correspond to the most selfish and the most altruistic strategies, respectively, and whose beamforming vectors are given in equation (15) of Appendix A.

5. PROPOSED ALGORITHM

The performance of the set of precoding vectors obtained in a distributed way by maximizing a virtual SINR at each of the transmitters will depend on the α_{ij}'s selected. Motivated by Theorem 2 above, we propose to set all of these to 1. Thus at base station k:

$$\mathbf{w}_k = \arg \max_{\|\mathbf{w}\|^2=1} \frac{|\mathbf{h}_{kk}\mathbf{w}|^2}{\frac{1}{\rho} + \sum_{j\neq k} |\mathbf{h}_{kj}\mathbf{w}|^2}. \tag{9}$$

5.1. Two-link case: Further Analysis

Theorem 2 states that the rate pair achieved by setting $\alpha_{12} = \alpha_{21} = 1$ lies on the Pareto boundary of the rate region, but says nothing about the achieved sum rate. An idea of the performance is gained by analyzing the SINR's at low and high SNR, and comparing them with the optimal strategies at those extreme regimes, which are known to be the NE solution and the ZF solution, respectively.

The resulting SINR's from applying our algorithm in the two-link case are given by, where the parameters involved are defined in Appendix A:

$$\gamma_i = \rho \frac{(a_i + b_i(1+c_i))^2}{a_i + b_i(1+c_i)^2} \frac{a_{\bar{i}} + b_{\bar{i}}(1+c_{\bar{i}})^2}{(1+c_{\bar{i}})(a_{\bar{i}} + b_{\bar{i}}(1+c_{\bar{i}}))} \tag{10}$$

At the ZF solution:

$$\gamma_i^{ZF} = \rho b_i \tag{11}$$

At the NE solution:

$$\gamma_i^{NE} = \rho \frac{a_i + b_i}{1 + \frac{a_{\bar{i}}}{a_{\bar{i}}+b_{\bar{i}}} c_{\bar{i}}} \tag{12}$$

At low SNR, $1 + c_i \approx 1$, and both γ_i and γ_i^{NE} may be approximated by $\rho(a_i + b_i)$. On the other hand, at high SNR, $\gamma_i \approx \rho b_i = \gamma_i^{ZF}$. Thus, at both extremes, this approach performs as good as the best out of these two schemes.

5.2. Comparison with Full CSIT Case

We would like to have an idea of the loss due to the distributed nature of our algorithm. The simplest way to define loss is in terms of total power consumption: in our algorithm, all transmitters always use full power. Alternatively, if full CSIT was available at all transmitters or these were allowed to share channel information, then it may be possible to achieve the same rates at all users with a lower total transmit power. The corresponding optimization problem may then be formulated as:

$$\text{minimize } \sum_{k=1}^{K} \mathbf{u}_k^H \mathbf{u}_k \tag{13}$$

$$\text{subject to } \mathbf{u}_k^H \mathbf{u}_k \leq P, \quad k = 1, \ldots, K$$

$$\frac{|\mathbf{h}_{kk}\mathbf{u}_k|^2}{\sigma^2 + \sum_{j\neq k} |\mathbf{h}_{jk}\mathbf{u}_j|^2} \geq \gamma_k^{(alg)}, \quad k = 1, \ldots, K$$

Parameter	Value
Path loss model	Cost-231, small/medium city
K	3, 7
Cell radius	1000 m
Transmit antenna gain, G_{tx}	16 dB
Shadowing mean	0 dB
Shadowing variance	10 dB
Receive antenna gain, G_{rx}	6 dB
Edge SNR	0-15 dB

Table 1. Simulation setup parameters

where, in relation to our previous notation, $\mathbf{u}_k = \sqrt{p_k}\mathbf{w}_k$, and $\gamma_k^{(alg)}$ is the SINR achieved at user k when beamforming is done using our algorithm.

This can easily be shown to be a convex optimization problem (see [9] for example), and as such algorithms exist to solve it.

6. NUMERICAL RESULTS

Figure 3 illustrates the performance of our approach in a more realistic scenario, the parameters of which are specified in Table 1. User locations in a cell follow a uniform distribution.

We show the average sum rates achieved and compare them with the selfish scheme corresponding to MRT, and the altruistic scheme corresponding to minimizing the total interference caused to other users. Our scheme clearly surpasses both. As the figure illustrates the 7-cell case, and there are always fewer antennas than that, interference caused can never be eliminated completely and eventually the rates would saturate. However, for the cell edge SNR range considered the performance gains are still quite significant. Note that, when $N_t \geq K$, interference can be eliminated completely and at high SNR the rates achieved with our scheme and the interference minimizing solution would differ by at most a constant (in favor of our scheme).

Figure 4 illustrates the power loss due to the distributed nature of our scheme, again for the 7-cell case. More precisely, for different number of antennas, the minimum power needed, under the individual power constraints at each base station, to achieve the same rates as those achieved by our distributed algorithm is computed and the figure illustrates the difference between this power and the power consumed by our scheme as a percentage of the total power available across the system. As the number of antennas at each transmitter increases, the difference decreases: this is because with more degrees of freedom afforded by the higher number of antennas, even in our distributed algorithm, more efficient power use is done automatically thereby reducing the benefit of centralized knowledge, at least for achieving the same rates as our algorithm. From our other simulations, not shown here, for the 3-cell case, for $N_t \geq 3$, this difference is almost negligible, which leads us to conjecture that we are quite close,

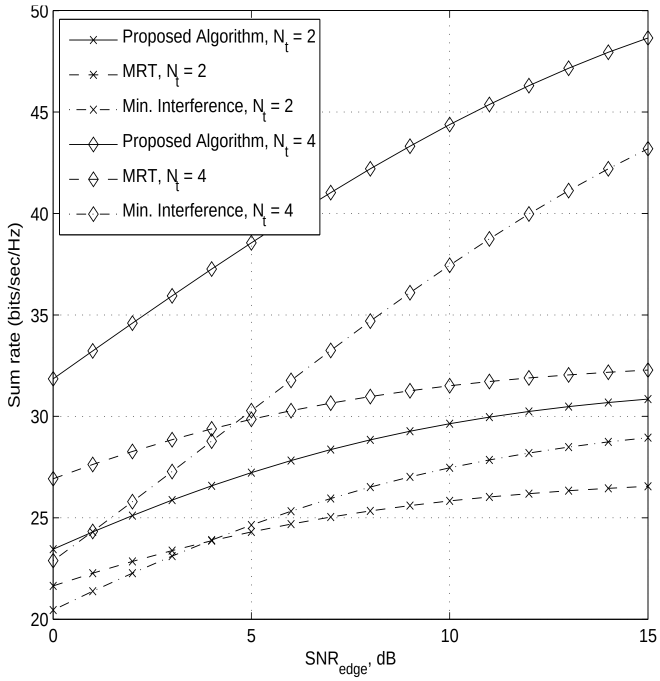

Fig. 3. Sum rates vs. cell-edge SNR for $N_t = 2, 4$, for the 7-cell case.

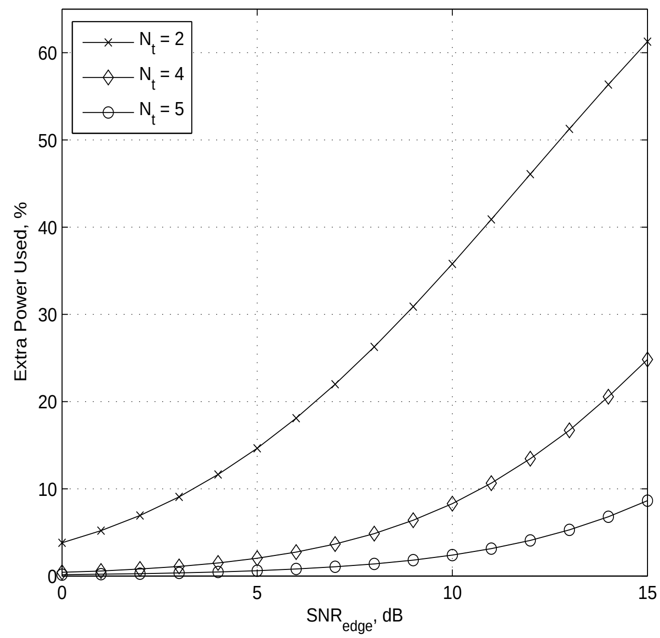

Fig. 4. Extra power consumed due to the distributed nature of our scheme vs. cell-edge SNR for $N_t = 2, 4, 5$, for the 7-cell case.

if not on, the Pareto boundary in this case, for most channel instances.

7. CONCLUSION

In this paper, a distributed coordinated beamforming approach, based on partial CSI at each BS, was proposed. Analytical justification for the algorithm was given for the case of two base stations, and numerical simulations illustrated its performance for more general cases.

A. PROOF OF THEOREM 1

To simplify expressions, in what follows $\bar{i}$ is used to denote the 'other' user/base station index (i.e., $\bar{i} = \mod(i, 2) + 1$, for $i \in \{1, 2\}$).

From Theorem 1 from [6], for the two-link case, any point on the Pareto boundary is achievable with the beamforming strategies:

$$\mathbf{w}_i(\lambda_i) = \frac{\lambda_i \mathbf{w}_i^{NE} + (1-\lambda_i)\mathbf{w}_i^{ZF}}{\|\lambda_i \mathbf{w}_i^{NE} + (1-\lambda_i)\mathbf{w}_i^{ZF}\|}, i = 1, 2 \qquad (14)$$

for some $0 \le \lambda_i \le 1$, where

$$\mathbf{w}_i^{NE} = \frac{\mathbf{h}_{ii}^H}{\|\mathbf{h}_{ii}\|} \quad \text{and} \quad \mathbf{w}_i^{ZF} = \frac{\Pi_{i\bar{i}}^{\perp}\mathbf{h}_{ii}^H}{\|\Pi_{i\bar{i}}^{\perp}\mathbf{h}_{ii}^H\|} \qquad (15)$$

are the NE or MRT and ZF solutions, respectively. $\mathbf{\Pi}_{i\bar{i}}^{\perp}$ is the projection matrix onto the null space of $\mathbf{h}_{i\bar{i}}$.

We show that the rate region achieved by this parametrization of the Pareto boundary can also be achieved by varying the α's in their feasible region ($\mathbb{R}_+$) and maximizing the corresponding virtual SINRs.

Maximizing the virtual SINR of (7):

$$\mathbf{w}_i = \arg\max_{\|\mathbf{w}\|^2=1} \frac{|\mathbf{h}_{ii}\mathbf{w}|^2}{1/\rho + \alpha_{i\bar{i}}|\mathbf{h}_{i\bar{i}}\mathbf{w}|^2}. \qquad (16)$$

Proposition 1. *The solution of problem* (16) *can be written as:*

$$\mathbf{w}_i = \sqrt{\zeta_i}\frac{\Pi_{i\bar{i}}\mathbf{h}_{ii}^H}{\|\Pi_{i\bar{i}}\mathbf{h}_{ii}^H\|} + \sqrt{1-\zeta_i}\frac{\Pi_{i\bar{i}}^{\perp}\mathbf{h}_{ii}^H}{\|\Pi_{i\bar{i}}^{\perp}\mathbf{h}_{ii}^H\|} \qquad (17)$$

where $0 \le \zeta_i \le 1, i = 1, 2$.

Proof. Similar to that of Proposition 1 in [6]. □

Define:

$$a_i = \|\Pi_{i\bar{i}}\mathbf{h}_{ii}^H\|^2$$
$$b_i = \|\Pi_{i\bar{i}}^{\perp}\mathbf{h}_{ii}^H\|^2$$
$$c_i = \rho\|\mathbf{h}_{i\bar{i}}\|^2 \qquad (18)$$

Proposition 2. ζ_i *that solves* (16) *is given by:*

$$\zeta_i = \frac{a_i}{a_i + b_i(1+\alpha_{i\bar{i}}c_i)^2} \qquad (19)$$

Proof. With $\mathbf{w}_i$ as in (17),

$$|\mathbf{h}_{ii}\mathbf{w}_i|^2 = \left|\sqrt{\zeta_i}\frac{\mathbf{h}_{ii}\Pi_{i\bar{i}}\mathbf{h}_{ii}^H}{\|\Pi_{i\bar{i}}\mathbf{h}_{ii}^H\|} + \sqrt{1-\zeta_i}\frac{\mathbf{h}_{ii}\Pi_{i\bar{i}}^{\perp}\mathbf{h}_{ii}^H}{\|\Pi_{i\bar{i}}^{\perp}\mathbf{h}_{ii}^H\|}\right|^2$$
$$= \left(\sqrt{a_i\zeta_i} + \sqrt{b_i(1-\zeta_i)}\right)^2 \quad (20)$$

Similarly,

$$|\mathbf{h}_{i\bar{i}}\mathbf{w}_i|^2 = \zeta_i\|\mathbf{h}_{i\bar{i}}\|^2 \quad (21)$$

Thus the virtual SINR is equal to:

$$\gamma_i^{virtual} = \frac{\rho\left(\sqrt{a_i\zeta_i} + \sqrt{b_i(1-\zeta_i)}\right)^2}{1+\alpha_{i\bar{i}}\zeta_i c_i} \quad (22)$$

One can easily verify that this ratio is maximized for the value specified in (19). □

Proposition 3. *In terms of the NE and ZF beamforming vectors,* (17) *can be rewritten as:*

$$\mathbf{w}_i = \frac{\lambda_i\mathbf{w}_i^{NE} + (1-\lambda_i)\mathbf{w}_i^{ZF}}{\|\lambda_i\mathbf{w}_i^{NE} + (1-\lambda_i)\mathbf{w}_i^{ZF}\|} \quad (23)$$

where

$$\lambda_i = \frac{1}{\sqrt{\frac{a_i}{a_i+b_i}}\left(\frac{1}{\zeta_i}-1\right) + \left(1-\sqrt{\frac{b_i}{a_i+b_i}}\right)} \quad (24)$$

Proof. $\mathbf{w}_i$, as expressed by (17), can be rewritten in terms of $\mathbf{w}_i^{NE}$ and $\mathbf{w}_i^{ZF}$ as:

$$\mathbf{w}_i = \sqrt{\zeta_i}\sqrt{\frac{a_i+b_i}{a_i}}\mathbf{w}_i^{NE} + \left[\sqrt{1-\zeta_i} - \sqrt{\zeta_i}\sqrt{\frac{b_i}{a_i}}\right]\mathbf{w}_i^{ZF} \quad (25)$$

We need to show that this is in fact of the form given in (23), i.e. that the following equalities hold for some λ_i:

$$\frac{\lambda_i^2}{\lambda_i^2\frac{a_i}{a_i+b_i} + \left(1-\lambda_i+\lambda_i\sqrt{\frac{b_i}{a_i+b_i}}\right)^2} = \zeta_i\frac{a_i+b_i}{a_i}, \text{and}$$
$$\frac{1-\lambda_i}{\sqrt{\lambda_i^2\frac{a_i}{a_i+b_i} + \left(1-\lambda_i+\lambda_i\sqrt{\frac{b_i}{a_i+b_i}}\right)^2}} = \sqrt{1-\zeta_i} - \sqrt{\zeta_i}\sqrt{\frac{b_i}{a_i}}$$

where we replaced the denominator of (23) by its value in terms of the parameters defined in (18).

One can verify that λ_i as given by (24) above satisfies both these equations. □

Combining propositions 2 and 3, we complete the proof. Plugging (19) into (24), we get:

$$\lambda_i = \frac{1}{\alpha_{i\bar{i}}c_i\sqrt{\frac{b_i}{a_i+b_i}}+1} \quad (26)$$

Clearly this is a decreasing function of $\alpha_{i\bar{i}}$. It is easy to check that for $\alpha_{i\bar{i}} = 0$, $\lambda_i = 1$ and that as $\alpha_{i\bar{i}} \to \infty$, $\lambda_i \to 0$.

B. PROOF OF THEOREM 2

For any $\mathbf{w}_1, \mathbf{w}_2$ of the form (17), one can show that SINRs are given by:

$$\gamma_i = \rho\frac{(\sqrt{a_i\zeta_i} + \sqrt{b_i(1-\zeta_i)})^2}{1+\zeta_{\bar{i}}c_{\bar{i}}} \quad (27)$$

A rate pair is Pareto optimal if one cannot increase one of the rates without necessarily decreasing the other. Note that any point on Pareto boundary has to have the corresponding (ζ_1, ζ_2) pair in the region defined by $\zeta_i \in \left[0, \frac{a_i}{a_i+b_i}\right], i = 1, 2$: this is so since for higher ζ_i it is always possible to achieve higher useful signal at user i while causing less interference at user $\bar{i}$ (cf. (27)).

Denote by $\gamma_i^{1,1}$ the SINR values achieved by setting $\alpha_{12} = \alpha_{21} = 1$. To show that the corresponding rates belong to the Pareto boundary, we solve the following optimization problem:

$$\begin{aligned} &\text{maximize } \gamma_1 \\ &\text{such that } 0 \le \zeta_i \le \frac{a_i}{a_i+b_i}, \quad i = 1, 2 \\ &\gamma_2 \ge \gamma_2^{1,1} \end{aligned} \quad (28)$$

This can be formalized as the following convex optimization problem:

$$\begin{aligned} &\text{minimize } -t \\ &\text{such that } 0 \le \zeta_i \le \frac{a_i}{a_i+b_i}, \quad i = 1, 2 \\ &t \ge 0 \\ &\gamma_2^{1,1}(1+\zeta_1 c_1) - \rho\left(\sqrt{a_2\zeta_2} + \sqrt{b_2(1-\zeta_2)}\right)^2 \le 0 \\ &t(1+\zeta_2 c_2) - \rho\left(\sqrt{a_1\zeta_1} + \sqrt{b_1(1-\zeta_1)}\right)^2 \le 0 \end{aligned} \quad (29)$$

This problem is strictly feasible and consequently Slater's condition for strong duality holds [10].

Let $\mu_i, i = 1, \ldots, 3$ be the Langrange multipliers associated with the positivity constraints, $\xi_i, i = 1, 2$ the Lagrange multipliers associated with the upper bounds on the ζ_i, and $\lambda_i, i = 1, 2$ the Lagrange multipliers associated with the SINR constraints, the corresponding Karush-Kuhn-Tucker

(KKT) conditions [10] are given by:

$$
\begin{aligned}
&-1-\mu_3+\lambda_2(1+\zeta_2 c_2)=0\\
&-\mu_1+\xi_1+\lambda_1\gamma_2^{1,1}c_1\\
&\quad=\lambda_2\rho\left(a_1-b_1+\sqrt{a_1b_1}\left(\sqrt{\frac{1-\zeta_1}{\zeta_1}}-\sqrt{\frac{\zeta_1}{1-\zeta_1}}\right)\right)\\
&-\mu_2+\xi_2+\lambda_2 t c_2\\
&\quad=\lambda_1\rho\left(a_2-b_2+\sqrt{a_2b_2}\left(\sqrt{\frac{1-\zeta_2}{\zeta_2}}-\sqrt{\frac{\zeta_2}{1-\zeta_2}}\right)\right)\\
&\quad\mu_1,\mu_2,\mu_3,\lambda_1,\lambda_2,\xi_1,\xi_2\geq 0\\
&\quad\mu_i\zeta_i=0,\quad \xi_i\left[\zeta_i-\frac{a_i}{a_i+b_i}\right]=0, i=1,2\\
&\quad\mu_3 t=0\\
&\quad\lambda_1\left[\gamma_2^{1,1}\left(1+\zeta_1c_1\right)-\rho\left(\sqrt{a_2\zeta_2}+\sqrt{b_2(1-\zeta_2)}\right)^2\right]=0\\
&\quad\lambda_2\left[t\left(1+\zeta_2c_2\right)-\rho\left(\sqrt{a_1\zeta_1}+\sqrt{b_1(1-\zeta_1)}\right)^2\right]=0
\end{aligned}
\tag{30}
$$

For $\zeta_i, i=1,2$ given by (19), with $\alpha_{i\bar{i}}=1$, one can verify that these values, together with the values of t and the Lagrange multipliers given in equation (31) below provide a consistent solution of the KKT conditions. This guarantees optimality. Noting that the optimal value of problem (28) is indeed that achieved by our algorithm completes the proof.

$$
\begin{aligned}
&\mu_1=\mu_2=\mu_3=0, \xi_1=\xi_2=0,\\
&\lambda_2=\frac{1}{1+\zeta_2c_2},\\
&\lambda_1=\frac{1}{1+\zeta_2c_2}\frac{(a_1+b_1(1+c_1))^2(a_2+b_2(1+c_2)^2)}{(a_2+b_2(1+c_2))^2(a_1+b_1(1+c_1)^2)},\\
&t=\gamma_1^{1,1}.
\end{aligned}
\tag{31}
$$

C. REFERENCES

[1] F. Rashid-Farrokhi, K.J.R. Liu, and L. Tassiulas. Transmit beamforming and power control for cellular wireless systems. *IEEE Journal on Selected Areas in Communications*, 16:1437–1450, October 1998.

[2] E.G. Larsson and E.A. Jorswieck. The MISO interference channel: Competition versus collaboration. In *Proc. Allerton Conference on Communication, Control and Computing*, September 2007.

[3] G. Scutari, D. P. Palomar, and S. Barbarossa. Asynchronous iterative water-filling for gaussian frequency-selective interference channels. *IEEE Transactions on Information Theory*, 54:2868–2878, July 2008.

[4] V. R. Cadambe and S. A. Jafar. Interference alignment and the degrees of freedom for the K user interference channel. *IEEE Transactions on Information Theory*, 54:3425–3441, August 2008.

[5] E.G. Larsson and E.A. Jorswieck. Competition versus cooperation on the MISO interference channel. *IEEE Journal on Selected Areas in Communications*, 26:1059–1069, September 2008.

[6] E.A. Jorswieck and E.G. Larsson. The MISO interference channel from a game-theoretic perspective: A combination of selfishness and altruism achieves Pareto optimality. In *Proc. Int'l Conf. Acoustics, Speech and Sig. Proc. (ICASSP)*, Las Vegas, March 31 - April 4 2008.

[7] E.A. Jorswieck and E.G. Larsson. Complete characterization of the Pareto boundary for the MISO interference channel. *IEEE Transactions on Signal Processing*, 56:5292–5296, October 2008.

[8] K. Ming Ho and D. Gesbert. Spectrum sharing in multiple antenna channels: A distributed cooperative game theoretic approach. In *Proc. IEEE International Symposium on Personal, Indoor, Mobile Radio Communications (PIMRC)*, Cannes, 15-18 September 2008.

[9] Wei Yu and T. Lan. Transmitter optimization for the multi-antenna downlink with per-antenna power constraints. *IEEE Transactions on Signal Processing*, 55:2646–2660, June 2007.

[10] S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, first edition, 2004.