Adaptive Feedback Rate Control in MIMO Broadcast Systems

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***Abstract*— We consider a MIMO broadcast channel where the channel state information at the transmitter (CSIT), to be used for user scheduling and beamforming, is gained through a limited-rate feedback channel. In view of optimizing the overall spectral efficiency of the system, we propose an adaptive scheme in which the feedback rate is no longer constant but rather optimized as a function of the time-dependent channel quality seen at the user side. One key idea is that, under an average feedback rate constraint, a user ought to provide more feedback at moments when it is more likely to be scheduled. We provide the theoretical grounds for our approach then derive quasi-optimal feedback resource allocation schemes, the performance of which is illustrated through Monte Carlo simulations.**

I. Introduction

Much recent research has focussed on the use of multiple antennas in wireless networks, and on the practical realization of the associated capacity gains. In a system with N_t antennas at the transmitter (base station) and $N \geq N_t$ single-antenna receivers (users), up to N_t streams may be broadcast simultaneously over the channel[1]; the attained rates may be further enhanced by appropriate selection of who to transmit to, i.e. by profiting from multi-user diversity (MUD) [2].

Given the complexity of the dirty paper coding scheme (DPC) required to attain any point within the capacity region of the Gaussian MIMO Broadcast channel with full channel state information at the transmitter (CSIT) (established in [3]), several sub-optimal schemes have been proposed that try to approach important points in said capacity region, including the maximum sum-rate point [4], [5]. The importance of CSIT in achieving capacity, and its cost in terms of overhead needed to feed back the CSI to the transmitter, has also instigated many recent publications dealing with the limited feedback CSIT case (see [6] and references therein). The present work falls under this category of investigation.

Most approaches to multiuser MIMO (MU-MIMO) transmission under partial CSI have centered on linear precoding, especially zero-forcing beamforming (ZFBF), which is much simpler than nonlinear processing (required for implementing the optimal DPC scheme, for example), and still achieves rates quite close to capacity, especially in the large number of users case [7]. Under ZFBF, the fed-back CSI is used to design a zero-forcing (ZF) precoding channel matrix, eliminating inter-user interference when perfect CSI is available. With such a scheme, the same capacity scaling as the optimal one, in terms of multiplexing and MUD gains, is possible *provided the feedback rate scales linearly with SNR in dB* [8], [9], [10].

The strategies for representing CSIT through a limited feedback channel are diverse [6]. In most previous work a time-constant rate is allocated to the feedback channel from the users to the base, where the number of bits used to represent the CSIT is optimized as function of fixed parameters such as the number of users and the average SNR. In recent work [11] the constant feedback rate is split across two feedback stages, one stage for scheduling and another stage for beamforming, where the number of CSIT bits is different for the two stages, yet remains time-constant.

The key intuition behind this paper is that, if each user were subject to an *average* feedback rate constraint, rather than a *peak* constraint, then the resource allocated for feedback at each moment could be optimized as function of the instantaneous channel conditions. In particular, one expects that in a system where the number of users N exceeds the number of antennas at the base N_t, the accuracy with which the user channel must be described to the latter should be made a function of both (i) *the user's channel quality* and (ii) *the probability with which this user will be selected*.

In a way which is reminiscent of power allocation schemes over time-varying channels (e.g. [12]), we propose an adaptive feedback framework where the user self-optimizes his feedback resource over time so as to match an average feedback rate constraint. A 'rate-waterfilling' allocation scheme is obtained for certain conditions, and the result is specialized to the case of $N_t = 2$ antennas and an arbitrary number of users.

Interestingly, the proposed framework can be interpreted as an extension of the on-off feedback scheme proposed in [13] for SISO. In that work, feedback reduction is realized by silencing users for which the SINR estimate is below a certain threshold, while a constant feedback rate is used for users which pass the threshold. Extensions to the MIMO case were given in [14], [15] whereby not only the channel quality

[1]Effectively, for users with independent Rayleigh fading channels up to N_t^2 streams may be transmitted [1]. However the gain obtained from transmitting to more than N_t users is normally limited. Moreover, for many linear transmission schemes such as the zero-forcing scheme assumed here, it is in fact not possible to transmit to more than N_t users at a time.

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but also the accuracy of the quantized channel direction is considered in the feedback decision process. This contrasts with our approach where the adaptive process tweaks the quantization accuracy according to the channel quality seen by the user and, importantly, according to the probability of selection that the user estimates for itself. Another extension of this opportunistic approach is taken in [16], where a scalable feedback protocol is proposed, whereby several rounds of threshold-based feedback are done to determine the best user set. The protocol is designed in such a way that the number of feedback slots is bounded.

Regarding the quantization scheme used in this paper, we build on the line of work of [9], [10], [17] where the quantization scheme focuses on the direction information while the channel quality indicator (CQI) (in our case the channel norm, but often some estimate of the achievable SINR) is assumed unquantized (or quantized with fixed, high accuracy). As our aim is to keep feedback bounded, one could consider extending the work to incorporate the CQI quantization in the adaptation process.

The performance gain of our adaptive feedback framework over a non-adaptive one is illustrated with Monte Carlo simulations.

We note that throughout our discussions we make the common assumption of perfect channel state information at the receiver. We do not take training on the downlink into consideration. These matters have been tackled in the recent publication [18].

Notation: $\mathbb{E}$ denotes statistical expectation. $\mathbb{C}^n$ represents the n-dimensional complex space. Boldface lowercase letters denote vectors, and boldface uppercase matrices. $f_x(.)$ gives the probability density function (pdf) of random variable x, and $F_x(.)$ its cumulative density function (cdf). The probability of an event A occuring is denoted by $Pr[A]$. The l^2-norm of vector $\mathbf{x}$ is denoted as $\|\mathbf{x}\|$, and $\tilde{\mathbf{x}} \triangleq \frac{\mathbf{x}}{\|\mathbf{x}\|}$. Finally, $\log(.)$ is the natural logarithm.

II. System Model

We consider a multi-antenna Gaussian broadcast channel, where a transmitter equipped with N_t antennas communicates with $N \geq N_t$ single-antenna receivers. The latter are assumed to have perfect channel knowledge. The received signal at user k, denoted $y_k \in \mathbb{C}$ can be written as:

$$y_k = \mathbf{h}_k \mathbf{x} + n_k \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ is the transmitted signal vector, $\mathbf{h}_k \in \mathbb{C}^{1 \times N_t}$ and and $n_k \in \mathbb{C}$ represent the channel vector and the noise at the kth user, respectively. The noise vector consists of i.i.d. zero mean unit variance complex Gaussian random variables (r.v.'s), $\mathcal{CN}(0,1)$. Furthermore, we assume a block-fading channel and focus on the ergodic sum rate as system performance measure.

A. User Selection and Precoding Scheme

ZFBF with uniform power allocation is the adopted precoding scheme. Thus, the transmitted signal $\mathbf{x}$ is given by:

$$\mathbf{x} = \sqrt{\frac{P}{N_t}} \mathbf{W} \mathbf{s} \tag{2}$$

where $\mathbf{W} \in \mathbb{C}^{N_t \times N_t}$ is a zero-forcing matrix having unit-norm column vectors, $\mathbf{s}$ contains the symbols from N_t independently generated zero-mean unit-variance Gaussian codewords, and P is the average total transmit power. The scheduling scheme tries to maximize the sum rate achieved by ZFBF to N_t users, based on the fed-back CSI: in the optimal case, this is done through exhaustive search over all groups of N_t users; a suboptimal scheme would use a greedy algorithm such as those of [7], [21], and may actually schedule fewer users.

B. CSI and Quantization

For our derivations, we assume the N receivers have i.i.d. Rayleigh fading channels, and that CSI consists of feeding back one scalar (the channel norm), and a quantized version of the channel direction as was done previously in [9], [10], [17], where we use $\tilde{\mathbf{h}}$ to denote the true direction and $\hat{\mathbf{h}}$ to denote its quantized version.

The only assumption we make about the CDI quantization codebooks, is that some amount of ('controlled') randomization exists, so that one can claim that different users' quantized directions are independent of each other, and that on average the fed-back quantized directions are uniformly distributed on the unit sphere in $\mathbb{C}^{N_t}$, as are the actual channel directions. Random vector quantization (RVQ) [19], used in the simulations, satisfies this assumption, as each user has different and independently generated codebooks, a codebook of size 2^b consisting of 2^b unit-vectors independently sampled from the isotropic distribution on said sphere. The corresponding quantization error is defined as $\sin^2 \epsilon$, where $\epsilon \triangleq \angle(\tilde{\mathbf{h}}, \hat{\mathbf{h}})$, is the angle between the true and quantized channel directions, and its cdf is upper-bounded by [20]:

$$F_{\sin^2 \epsilon}(x) = \begin{cases} \delta^{1-N_t} x^{N_t - 1} & 0 \leq x \leq \delta \\ 1 & x > \delta \end{cases} \tag{3}$$

where $\delta \triangleq 2^{-b/(N_t-1)}$. This distribution will be used in our derivations below.

III. Adaptive Feedback Rate Allocation

As noted in the introduction, under an average feedback rate constraint, it makes sense for a given user to quantize its channel more accurately if[2] (i) it is more likely to be scheduled, and (ii) its channel quality is better, i.e. if the associated expected rate is higher. In doing so, a user maximizes its expected rate, given its local knowledge (its instantaneous CSI, the channel state statistics of the other users, the number of users in the system and the transmit power at the base station).

[2]Note that these conditions are interrelated, since the base station tries to maximize the sum rate.

Letting S denote the event of being scheduled, the expected rate may be expressed as:

$$\mathbb{E}R = \int_0^\infty Pr[S|\alpha = a]\mathbb{E}[R|\alpha = a]f_\alpha(a)da, \qquad (4)$$

where $\alpha \triangleq \|\mathbf{h}\|^2$, and possible dependencies on the channel direction were dismissed for the following reasons:

- In the considered channel model, channel norm and channel direction have independent distributions, and channel direction is isotropically distributed on the unit sphere in $\mathbb{C}^{N_t}$.
- As users are independent and do not share state information, and channel directions and their quantized versions are isotropically distributed (cf. section II-B), the probability of a particular user being scheduled conditioned on its current channel state information will be a function of the channel norm alone. Similar arguments may be used to justify removing the dependency of the expected rate given current channel state.

Thus, the objective is to maximize a user's expected rate as given in (4), subject to an average feedback rate constraint $\bar{B}$ per user (since users have i.i.d. distributed channels, this is the same as solving the problem for a system-wide average feedback rate of $N\bar{B}$):

$$\int_0^\infty b(a)f_\alpha(a)da = \bar{B}, \qquad (5)$$

$b(a)$, corresponding to the number of feedback bits as a function of channel norm squared a. Clearly our objective is to determine the best function $b(a)$, which is done below. To simplify notation, we define the following functions of channel norm a and associated feedback bit rate $b(a)$:

$$g(a, b(a)) \triangleq \mathbb{E}[R|\alpha = a] \qquad (6)$$

and

$$P_S(a) \triangleq Pr[S|\alpha = a] \qquad (7)$$

A. 'Water-filling' Solution

In order to tackle the above optimization problem, we begin by relaxing the constraint on $b(a)$ being an integer-valued function. Moreover, we assume the following:

Assumption 1: $g(a, b(a))$ is a concave function of $b(a)$. This is not necessarily obvious but will turn out to be met by our estimate of $g(a, b(a))$.

This makes the problem a (functional) convex optimization problem, and guarantees an optimum solution.

The corresponding Lagrangian is given by:

$$\begin{aligned} L &= \mathbb{E}R - \lambda\left(\int_0^\infty b(a)f_\alpha(a)da - \bar{B}\right) + \int_0^\infty \nu(a)b(a)da \\ &= \int_0^\infty da\Big[P_S(a)g(a, b(a))f_\alpha(a) \\ &\quad - \lambda(b(a) - \bar{B})f_\alpha(a) + \nu(a)b(a)\Big] \end{aligned} \qquad (8)$$

where λ and $\nu(a) \geq 0, a \in [0, \infty)$ are the Lagrange multipliers associated with the average feedback rate and the positivity of $b(a)$ constraints, respectively.

The corresponding Euler-Lagrange equation is given by:

$$f_\alpha(a)P_S(a)\frac{\partial g(a, b(a))}{\partial b(a)} + \nu(a) - \lambda f_\alpha(a) = 0 \qquad (9)$$

Rewriting in terms of $\nu'(a) = \frac{\nu(a)}{f_\alpha(a)}$, this becomes:

$$P_S(a)\frac{\partial g(a, b(a))}{\partial b(a)} + \nu'(a) - \lambda = 0 \qquad (10)$$

Assumption 2: $g(a, b(a))$ is an increasing function of $b(a)$. This is quite natural, since quantizing more accurately would lead to better interference cancelation, and consequently higher average rate.

This implies that $\lambda > 0$. Moreover the concavity of $g(a, b(a))$ in $b(a)$ means that $\frac{\partial g(a,b(a))}{\partial b(a)}$ is maximum at $b(a) = 0$. Denoting by $\lambda^*, \nu^*(a)$ and $b^*(a)$ the values of the different variables at the optimum, the following holds:

If

$$P_S(a) \left.\frac{\partial g(a, b(a))}{\partial b(a)}\right|_{b(a)=0} < \lambda^* \qquad (11)$$

then $b^*(a) = 0$.

Assumption 3: $P_S(a)$ and $\left.\frac{\partial g(a,b(a))}{\partial b(a)}\right|_{b(a)=0}$ are increasing functions of a. $P_S(a)$ being an increasing function is also intuitive since the higher the channel norm, the higher (at least under perfect CSIT) the probability of being scheduled.

This implies that the left-hand side of the above equation is an increasing function of a, so that a threshold value of a, which will be denoted a_{thresh}, exists, such that $b^*(a) = 0$ for $a \leq a_{thresh}$ whereas otherwise $b^*(a)$ solves

$$P_S(a) \left.\frac{\partial g(a, b(a))}{\partial b(a)}\right|_{b(a)=b^*(a)} = \lambda^* \qquad (12)$$

It thus becomes possible to solve the problem numerically, via the bisection method for example, provided $P_S(a)$ and $g(a, b(a))$ can be evaluated.

B. Conditional Expected Rate

$$g(a, b(a)) = \mathbb{E}\left[\log\left(1 + \frac{\frac{P}{N_t}a|\tilde{\mathbf{h}}\mathbf{v}|^2}{1 + \frac{P}{N_t}a\sum_{j=1}^{N_t-1}|\tilde{\mathbf{h}}\mathbf{u}_j|^2}\right)|a, b(a)\right] \qquad (13)$$

where $\tilde{\mathbf{h}}$ is the channel direction vector, $\mathbf{v}$ is the beamforming vector corresponding to the user's data, $\mathbf{u}_j$ are the remaining beamforming vectors, and conditioning is over α being equal to a, and the associated feedback rate being equal to $b(a)$.

An upper bound on the above rate, which will be used to approximate (13), is obtained by assuming perfectly orthogonal (in their quantized channels, that is) users may be found, so that a user's dedicated beamforming vector is aligned with its quantized channel direction and interfering beamforming vectors are orthogonal to it (see [10] for example), and using the distribution in (3):

$$\begin{aligned} & g_{upper}(a, b(a)) \\ & = \mathbb{E}_{\epsilon(b(a))}\left[\log\left(1+\frac{\frac{Pa}{N_t}\cos^2\epsilon(b(a))}{1+\frac{Pa}{N_t}\sin^2\epsilon(b(a))}\right)\right] \\ & = \log\left(1+\frac{Pa}{N_t}\right)-\left(1+\frac{(-1)^{N_t}}{x_a^{N_t-1}}\right)\log\left(1+x_a\right) \\ & + \sum_{i=0}^{N_t-2}\frac{(-1)^i}{x_a^i}\frac{1}{N_t-1-i}, \end{aligned} \tag{14}$$

where for the sake of compactness, $x_a \triangleq \frac{Pa2^{-b(a)/(N_t-1)}}{N_t}$.

The derivative with respect to $b(a)$ is given by:

$$\frac{\log 2}{x_a^{N_t-1}}\left[(-1)^{N_t-1}\log(1+x_a)-\sum_{k=1}^{N_t-1}(-1)^k\frac{x_a^{N_t-k}}{N_t-k}\right] \tag{15}$$

and can be shown to be nonnegative.

One can also show that the second-order derivative is non-positive, thus proving concavity in $b(a)$. It can also be verified that $\left.\frac{\partial g_{upper}(a,b(a))}{\partial b(a)}\right|_{b(a)=0}$ is increasing in a, thus justifying the use of the solution presented in the previous section.

C. Conditional Scheduling Probability

The probability of a user being scheduled given its instantaneous channel norm, $P_S(a)$, has yet to be specified. The exact function will depend on the actual scheduling scheme used. Three possible estimates were considered; in order of decreasing accuracy (and of decreasing complexity), they correspond to:

1) assuming the scheduler uses the Semiorthogonal User Group (SUS) algorithm of [7]; this is a greedy user selection algorithm. At stage i, $i \leq N_t$, users which are semiorthogonal to the already selected group are eligible for selection; among those, the one with the highest channel norm, after projection onto the null space of already selected users' channels, is chosen. Semiorthogonality is defined as: two unit-norm column vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ are semiorthogonal if $|\mathbf{x}_1^{\mathrm{H}}\mathbf{x}_2|^2 \leq \eta$, where $\eta < 1$ needs to be specified to the algorithm.
2) assuming users are selected according to the greedy user selection scheme used in the greedy zero-forcing dirty-paper algorithm of [21]. The algorithm is equivalent to the SUS algorithm with no semiorthogonality constraint ($\eta = 1$).
3) ignoring any orthogonality constraints in the scheduling process (thereby creating a stronger bias towards higher channel norms), and approximating $P_S(a)$ by the probability of having a channel norm among the N_t highest.

$$P_S(a) \approx \sum_{i=0}^{N_t-1}\binom{N}{i}\left(F_\alpha(a)\right)^{N-i}\left(1-F_\alpha(a)\right)^i \tag{16}$$

Given the complexity of the first two estimates, the corresponding $P_S(a)$ were obtained for the $N_t = 2$ case only. Details may be found in the appendix.

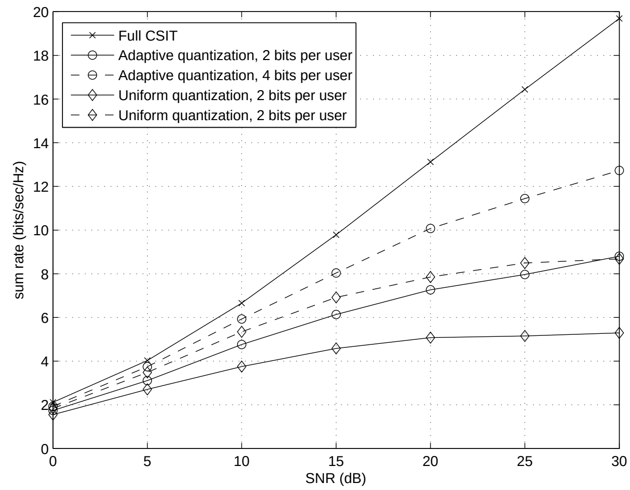

Fig. 1. Achievable sum rate for N=5 users, and average feedback rates per user of 3 and 5 bits

IV. Simulation Results

The proposed scheme was tested for a $N_t = 2$ system. As results for $\eta < 1$ ($\eta = 0.3, 0.4$) were quite close to those of the much simpler $\eta = 1$ case, we only show results for the second and third estimates of section III-C. Thus the feedback rate adaptation algorithm was made to use these approximations, the water-filling solution determined, then numbers of bits were rounded down to ensure they are integers, while still respecting the feedback rate constraint. Figure 1 shows the achievable rates for average bits per user of 2 and 4 bits, both for the adaptive scheme ($\eta = 1$) and for a constant feedback rate.

The benefit of adaptive quantization is more poignant when more users are in the system, as shown in figure 2, where with an average of 4 feedback bits per user, the sum rate is almost able to keep up with the full CSIT case, whereas uniform quantization leads to the achievable sum rate getting saturated as expected (see [8] for example). The figure also shows that using a relatively inaccurate estimate for the scheduling probability, which ignores semiorthogonality constraints, provides rate improvements which are quite close to estimates that try to take these constraints into consideration.

V. Conclusion

An adaptive scheme in which the feedback rate is optimized as a function of the channel quality was proposed, and an algorithm to implement it was established for Rayleigh fading i.i.d. users in the system. Simulation results have illustrated the associated performance gains. Future work will aim at extending the results to a more general channel model, and to more general quantization schemes.

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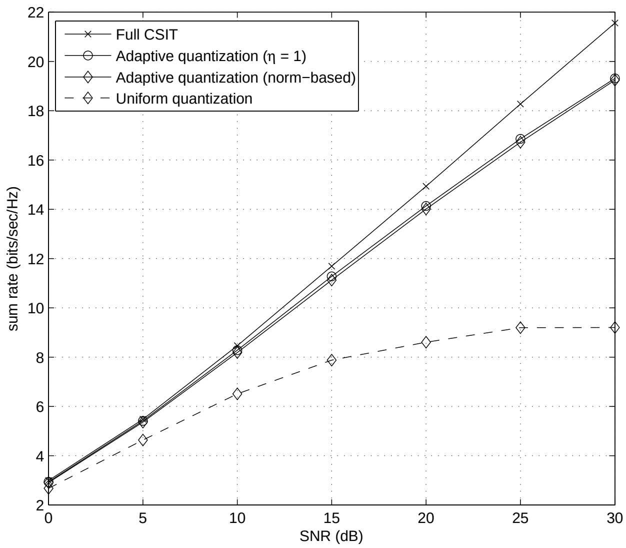

Fig. 2. Achievable sum rate for N=20 users, and average feedback rates per user of 4 bits.

APPENDIX

For any greedy scheduling algorithm, $P_S(a)$ can be written as:

$$P_S(a) = \sum_{i=1}^{N_t} Pr[S_i|\alpha = a], \tag{17}$$

where S_i denotes the event of being the user selected at the i^{th} stage of the scheduling algorithm.

From order statistics, $Pr[S_1|\alpha = a] = (F_\alpha(a))^{N-1}$ for all the estimates considered. On the other hand, finding closed-form solutions for $Pr[S_i|\alpha = a], i > 1$ is much more tedious when channel directions are incorporated in the analysis.

For $N_t = 2$, for the SUS algorithm, $Pr[S_2|\alpha = a]$ is the probability of not having the best channel norm, of being semiorthogonal to the user who does, and of having, among all semiorthogonal users, the best channel norm projected on that user's channel. After some simplications, and plugging in the expressions for the channel norm and quantization error distributions for $N_t = 2$, this probability may be expressed as:

$$\begin{aligned} & Pr[S_2|\alpha = a] \\ & = (N-1)\sum_{k=0}^{N-2}\binom{N-2}{k}(1-\eta)^{N-2-k} \\ & . \left[\int_a^\infty a' e^{-a'}\left[1-(1+a')e^{-a'}\right]^{N-2-k} P_k(a,a')da'\right] \end{aligned} \tag{18}$$

where

$$P_k(a,a') = \frac{1}{a}\int_{(1-\eta)a}^{a}\left(U(x,a')\right)^k dx \tag{19}$$

and

$$\begin{aligned} U(x,a') &= \eta - e^{-x} \\ &+ \begin{cases} (1-\eta)e^{-\frac{x}{1-\eta}} & x < (1-\eta)a' \\ e^{-a'}\left[(1+a')(1-\eta)-x\right] & \text{otherwise} \end{cases} \end{aligned}$$

Equation (18) may be evaluated numerically. Note that for the $\eta = 1$ case, the summation reduces to its last term.

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