

Bounds on Optimal End-to-End Distortion of MIMO Links

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Abstract—For transmitting a continuous-amplitude source, quadratic distortion is the primary performance metric. Its lower bound decreases with channel capacity which can be increased by multiple antennas. In this paper, assuming the transmitter perfectly knows the instantaneous channel rate by feedback from the receiver and a white thermal noise source is transmitted over a quasi-static Rayleigh fading AWGN channel, we give the compact analytic expression of the tight lower bound on the optimal end-to-end mean quadratic distortion at any SNR, in terms of antenna numbers, the ratio of source-bandwidth to channel-bandwidth (SCBR), the ratio of signal power to noise power (SNR) and the source power. We analyze the expression of the lower bound to obtain the SNR exponent and the SNR coefficient at the asymptotically high SNR. Results indicate that the optimal distortion does necessarily decrease with both antenna numbers for any SCBR, the commutation between the transmit antenna number and the receive antenna number can affect the optimal distortion, and the frequency-selectivity can benefit the optimal distortion.

I. INTRODUCTION

The end-to-end distortion is the primary performance metric for continuous-amplitude (analog) source transmission. Shannon has pointed out that increasing channel capacity can benefit the quadratic distortion (mean square error) in [1]. In MIMO links, channel capacity can be increased by multiple antennas [2]. Intuitively, the systems are supposed to profit on end-to-end distortion by putting more antennas either at the transmitter or at the receiver.

The existence of the SNR exponent of the optimal mean quadratic distortion at the asymptotically high SNR has been proposed and proved in [3]–[5]. In [6], considering source-channel coding, Caire and Narayanan have given the upper bound a_{ub} on the SNR exponent of mean quadratic distortion under the assumption of no time interleaving. It is a function of antenna numbers (M, N) and the ratio of source bandwidth to channel bandwidth, W_s/W_c . They present that there exists a tradeoff between the distortion SNR exponent upper-bound a_{ub} and W_s/W_c , which looks like an interesting partner to the tradeoff between the diversity order and the multiplexing gain in digital transmission [7].

Their resulting formula shows that when W_s/W_c is enough low, $a_{ub} = MN$, which is the same as the maximum diversity order in [8], whereas when W_s/W_c is enough high, $a_{ub} = 2\min\{M, N\}W_c/W_s$, which is unrelated to $\max\{M, N\}$. This conclusion makes us wonder how MIMO links can profit by increasing $\max\{M, N\}$ in the high W_s/W_c regime.

Let us suppose at the asymptotically high SNR,

$$D_{lb}(\rho, \eta) \sim c^*(\rho, \eta)\rho^{-a_{ub}(\eta)} \quad (1)$$

where $\eta = W_s/W_c$ and ρ is the SNR per receive antenna. We conjecture that in the case of large η (high SCBR), by increasing $\max\{M, N\}$, MIMO systems can profit on the coefficient $c^*(\rho, \eta)$ though not on the exponent $a_{ub}(\eta)$. Then, if we know the explicit expression of $D_{lb}(\rho, \eta)$, we would be able to clarify what the coefficient $c^*(\rho, \eta)$ is and thereby relate it to (M, N) , which could demonstrate our conjecture. Fortunately, on the basis of Shannon's theory, we find the compact analytic expression of $D_{lb}(\rho, \eta)$ and thus what $c^*(\rho, \eta)$ and $a_{ub}(\eta)$ are. The $a_{ub}(\eta)$ derived by our method is exactly the same as Caire and Narayanan have claimed. And, the behavior of $c^*(\rho, \eta)$ corresponds to our conjecture. Therefore, in the high SNR regime, the coefficient c^* can be an evaluating measurement other than the exponent a_{ub} .

II. SYSTEM MODEL AND EXISTING RESULTS

Consider a frequency-flat block-fading MIMO channel with M inputs and N outputs represented by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (2)$$

where \mathbf{x} is an M -length column vector denoting transmitted symbols, \mathbf{y} is an N -length column vector denoting received symbols, \mathbf{n} is an N -length column vector denoting noises and \mathbf{H} is an N -by- M matrix denoting the channel. We assume all elements in \mathbf{n} are zero-mean i.i.d. complex random variables with variance σ_n^2 and all elements in \mathbf{H} are i.i.d. $\mathcal{CN}(0, 1)$.

In [3]–[5], it has been proposed and proved that there exists an SNR exponent of the optimal quadratic distortion at the asymptotically high SNR,

$$a = - \lim_{\rho \rightarrow +\infty} \frac{\log D(\rho)}{\rho} \quad (3)$$

where $D(\rho)$ is the optimal quadratic distortion and ρ is the SNR per receive antenna.

In [6], by applying Varadhan integral lemma and the Wishart probability density function (pdf), Caire and Narayanan have derived the upper bound $a_{ub}(\eta)$ on the optimal mean quadratic distortion SNR exponent for analog source transmission over a slowly varying fading MIMO channel,

$$a_{ub}(\eta) = \sum_{i=1}^m \min \left\{ \frac{2}{\eta}, 2i - 1 + |M - N| \right\}, \quad (4)$$

where $m = \min\{M, N\}$, and $\eta = W_s/W_c$ with W_s the number of source samples per second and W_c the number of channel uses per second. η is termed *spectral efficiency* in their work. For avoiding confusion with other existing meanings for the term *spectral efficiency*, e.g. bit/Hz/s, in our work, we call it *SCBR (the ratio of source bandwidth to channel bandwidth)*. In the remainder, we shall show that we obtain the same upper bound as (4) by an alternative derivation method, along with other distortion-related results.

III. PRELIMINARIES

Lemma 1: Define a m -by- m Hankel matrix $\mathbf{H}(x)$ whose (i, j) -th entry is in the form of $c_{i+j}x^{\min\{a, i+j\}}$ where c_{i+j} is a non-zero constant, $x, a \in \mathbb{R}^+$ and $1 \leq i, j \leq m$. Then,

$$\lim_{x \rightarrow 0} \frac{\log \det \mathbf{H}(x)}{\log x} = \sum_{i=1}^m \min\{a, 2i\}. \quad (5)$$

Proof: Omitted due to space limit. ■

Lemma 2: Define a m -by- m Hankel matrix $\mathbf{H}(x)$ whose (i, j) -th entry is in the form of $c_{i+j}x^{i+j}$ where c_{i+j} is a non-zero constant, $x \in \mathbb{R}^+$ and $1 \leq i, j \leq m$. Then, each elementary product from $\mathbf{H}(x)$ has the same power $m(m+1)$.

Proof: Omitted due to space limit. ■

Lemma 3: Define a m -by- m Hankel matrix \mathbf{H} whose (i, j) -th entry is $\Gamma(a+i+j-1)$ where $1 \leq i, j \leq m$ and $a \in \mathbb{R}$. Then the determinant

$$\det \mathbf{H} = \prod_{k=1}^m \Gamma(k)\Gamma(a+k). \quad (6)$$

Proof: Omitted due to space limit. ■

Lemma 4: Define a m -by- m Hankel matrix \mathbf{H} whose (i, j) -th entry is $\Gamma(a+i+j-1)\Gamma(b-i-j+1)$ where $1 \leq i, j \leq m$, $m \geq 2$ and $a, b \in \mathbb{R}$. Then the determinant

$$\det \mathbf{H} = \Gamma(a+1)\Gamma(b-1)\Gamma^{m-1}(a+b) \cdot \prod_{k=2}^m \Gamma(k)\Gamma(a+k) \frac{\Gamma(b-2k+2)\Gamma(b-2k+1)}{\Gamma(a+b-k+1)\Gamma(b-k+1)}. \quad (7)$$

Proof: Omitted due to space limit. ■

Lemma 5: Define a m -by- m Hankel matrix $\mathbf{H}(x)$ as follows. (i, j) is the entry index, $1 \leq i, j \leq m$. For $a \in \mathbb{R}^+$, $b+2 \in \mathbb{Z}^+$, $a-b \in [2, 2m]$, in the case where $a-b$ is an even number, the $(\frac{a-b}{2}, \frac{a-b}{2})$ -th entry of $\mathbf{H}(x)$ is $c_{a-b}x^{-a} \log x$ while other entries are in the form of $\theta_{i+j}x^{-\min\{a, b+i+j\}}$ with θ_{i+j} a non-zero constant; in the case of odd number, $([\frac{a-b}{2}], [\frac{a-b}{2}]+1)$ -th and $([\frac{a-b}{2}]+1, [\frac{a-b}{2}])$ -th entries are $\theta_{a-b}x^{-a} \log x$ while other entries are in the form of $\theta_{i+j}x^{-\min\{a, b+i+j\}}$, where $[x]$ rounds x to the nearest integer towards zero; in other cases, all entries in $\mathbf{H}(x)$ are in the form of $\theta_{i+j}x^{-\min\{a, b+i+j\}}$.

For the positive integer $l = \lfloor \frac{a-b}{2} \rfloor$, $\mathbf{H}(x)$ can be partitioned as

$$\mathbf{H}(x) = \begin{pmatrix} \mathbf{A}(x) & \mathbf{B}(x) \\ \mathbf{B}^T(x) & \mathbf{C}(x) \end{pmatrix} \quad (8)$$

with the l -by- l submatrix $\mathbf{A}(x)$ and the $(m-l)$ -by- $(m-l)$ submatrix $\mathbf{C}(x)$.

When $x \rightarrow +\infty$,

$$\det \mathbf{A}(x) \sim \theta_A(x)x^{-l(l+b+1)}, \quad (9)$$

$$\det \mathbf{C}(x) \sim \theta_C \cdot x^{-a(m-l)}. \quad (10)$$

Then,

$$\det \mathbf{H}(x) \sim \theta_A(x)\theta_C x^{-[l(l+b+1)+a(m-l)]}, \quad (11)$$

Proof: Omitted due to space limit. ■

Lemma 6: Define

$$f(n) = \prod_{k=1}^m \frac{\Gamma(n-m-a+k)}{\Gamma(n-k+1)}, \quad (12)$$

$$g(n) = n^{am} \prod_{k=1}^m \frac{\Gamma(n-m-a+k)}{\Gamma(n-k+1)}, \quad (13)$$

$$0 < m \leq n, \quad 0 < a \leq n-m+1.$$

Then $f(n)$ and $g(n)$ are monotonically decreasing.

Proof: Omitted due to space limit. ■

IV. MAIN RESULTS

We assume that a white thermal noise source is to be transmitted, and systems are working on “short” frames due to strict time delay constraint, that is, time-interleaving is impossible to be done and no time diversity can be exploited. The transmitter is supposed to perfectly know the instantaneous channel rate R_c which can be fed back by the receiver as a real scalar.

Theorem 1 (Mean Quadratic Distortion Lower Bound):

The optimal mean quadratic distortion is tightly lower bounded by

$$D_{lb}(\rho, \eta) = \frac{P_s \det \mathbf{G}(\rho, \eta)}{\prod_{k=1}^m \Gamma(n-k+1)\Gamma(m-k+1)}. \quad (14)$$

with $m = \min\{M, N\}$, $n = \max\{M, N\}$, and $\mathbf{G}(\rho, \eta)$ a m -by- m Hankel matrix whose (i, j) -th entry is

$$g_{ij}(\rho, \eta) = \left(\frac{\rho}{M}\right)^{-d_{ij}} \Gamma(d_{ij}) \Psi\left(d_{ij}, d_{ij}+1 - \frac{2}{\eta}; \frac{M}{\rho}\right) \quad (15)$$

where ρ is the signal-to-noise ratio per receiver antenna, η is the ratio of source bandwidth to channel bandwidth (SCBR), $d_{ij} = i+j+n-m-1$, $1 \leq i, j \leq m$, and $\Psi(a, b; x)$ is the Ψ function in [9, pp. 257, 261].

Proof:

Under the assumption that the transmitter only knows the instantaneous channel rate R_c , the optimal covariance matrix of the transmitted vector \mathbf{x} at the transmitter is supposed to be a para-identity matrix $\frac{P_t}{M} \mathbf{I}_M$, where P_t is the transmit power constraint and \mathbf{I}_M is the M -by- M identity matrix. Thus, given by [2], the mutual information per channel use is

$$\mathcal{I}(\mathbf{x}; \mathbf{y}) = \log \det(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{H}^\dagger) \quad (18)$$

where ρ is the SNR per receive antenna, P_t/σ_n^2 .

Assume an AWGN channel of bandwidth W_c is used at $2W_c$ channel uses per second as a time-discrete channel [10, pp. 248]. For the block-fading case, the block time-length is supposed to be μ seconds. Hence, the channel rate (bit/block) is

$$R_c = 2\mu W_c \mathcal{I} = 2\mu W_c \log \det(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{H}^\dagger). \quad (19)$$

Given by Shannon in [1], the source information rate required by reproducing a white thermal noise source at the receiver with the quadratic distortion d

$$R_s = W_s \log \frac{P_s}{d}. \quad (20)$$

In the scenario of no time interleaving to be done, $\mu R_s \leq R_c$. Hence, the lower bound on the quadratic distortion

$$d_{lb}(\rho, \eta) = P_s \det(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{H}^\dagger)^{-\frac{2}{\eta}}, \quad (21)$$

with η the ratio of source bandwidth to channel bandwidth (SCBR) of the source-channel encoder,

$$\eta \triangleq W_s / W_c. \quad (22)$$

This lower bound is achievable according to [1].

Then, the tight lower bound on the mean quadratic distortion with respect to \mathbf{H}

$$D_{lb}(\rho, \eta) = P_s \mathbb{E}_H [\det(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{H}^\dagger)^{-\frac{2}{\eta}}]. \quad (23)$$

Invoking [9], [11], [12], this lower bound can be written in the compact analytic form as Theorem 1 presents. ■

Theorem 2 (Distortion SNR Exponent Upper Bound):

At the asymptotically high SNR, there exists an optimal distortion SNR exponent $a^*(\eta)$ which is upper bounded by

$$\begin{aligned} a_{ub}(\eta) &= - \lim_{\rho \rightarrow +\infty} \frac{\log D_{lb}(\rho, \eta)}{\log \rho} \\ &= \sum_{i=1}^m \min \left\{ \frac{2}{\eta}, 2i - 1 + |M - N| \right\}. \end{aligned} \quad (24)$$

□

Proof: Observe matrix entries $g_{ij}(\rho, \eta)$ in (15). When ρ is large, M/ρ is small. As $d_{ij} + 1 - \frac{2}{\eta}$ is a real number, we can thus use results in [9, pp. 262] as Table I shows. Following (15), when ρ is large,

$$g_{ij}(\rho, \eta) \sim A_{ij}(\rho, \eta) \rho^{-a_{ij}}, \quad (25)$$

where

$$\lim_{\rho \rightarrow +\infty} \frac{\log A_{ij}(\rho, \eta)}{\log \rho} = 0 \quad (26)$$

and

$$a_{ij} = \min \left\{ \frac{2}{\eta}, d_{ij} \right\} = \min \left\{ \frac{2}{\eta}, n - m + i + j - 1 \right\}. \quad (27)$$

Consequently, at the asymptotically high SNR,

$$\det \mathbf{G}(\rho, \eta) \sim A(\rho, \eta) \rho^{-a_{ub}(\eta)}, \quad (28)$$

where

$$\lim_{\rho \rightarrow +\infty} \frac{\log A(\rho, \eta)}{\log \rho} = 0. \quad (29)$$

By Lemma 1,

$$a_{ub}(\eta) = \sum_{i=1}^m \min \left\{ \frac{2}{\eta}, 2i - 1 + |M - N| \right\}. \quad (30)$$

■

Theorem 3 (Distortion SNR Coefficient): Define the SNR coefficient $c^*(\rho, \eta)$ for the optimal end-to-end distortion at the asymptotically high SNR as

$$D_{lb}(\rho, \eta) \sim c^*(\rho, \eta) \rho^{-a_{ub}(\eta)} \quad (31)$$

where

$$\lim_{\rho \rightarrow \infty} \frac{\log c^*(\rho, \eta)}{\log \rho} = 0. \quad (32)$$

Define two functions $\kappa_l(\beta, t)$ and $\kappa_h(\beta, t)$ as (16) and (17) at the bottom of this page, for $\beta \in \mathbb{R}^+$ and $t \in \{0, \mathbb{Z}^+\}$.

$c^*(\rho, \eta)$ is given as follows,

1. For $2/\eta \in (0, n - m + 1)$, termed *the high SCBR state*, the corresponding SNR coefficient is

$$c^*(\eta) = P_s M^{a_{ub}} \frac{\kappa_h(\frac{2}{\eta}, m)}{\prod_{k=1}^m \Gamma(n - k + 1) \Gamma(m - k + 1)}, \quad (33)$$

and monotonically decreases with n .

TABLE I
 $\Psi(a, c; x)$ FOR SMALL x

c	Ψ
$c > 1$	$x^{1-c} \Gamma(c-1) / \Gamma(a) + o(x^{1-c})$
$c = 1$	$-[\Gamma(a)]^{-1} \log x + o(\log x)$
$c < 1$	$\Gamma(1-c) / \Gamma(a-c+1) + o(1)$

$$\kappa_l(\beta, t) = \begin{cases} \Gamma(n - m + 1) \frac{\Gamma(\beta - n + m - 1)}{\Gamma(\beta)} \prod_{k=2}^t \Gamma(k) \Gamma(n - m + k) \frac{\Gamma(\beta - n + m - 2k + 2) \Gamma(\beta - n + m - 2k + 1)}{\Gamma(\beta - k + 1) \Gamma(\beta - n + m - k + 1)} & t > 1, \\ \Gamma(n - m + 1) \frac{\Gamma(\beta - n + m - 1)}{\Gamma(\beta)} & t = 1, \\ 1 & t = 0. \end{cases} \quad (16)$$

$$\kappa_h(\beta, t) = \begin{cases} \prod_{k=1}^t \Gamma(k) \Gamma(n - m - \beta + k) & t > 0, \\ 1 & t = 0. \end{cases} \quad (17)$$

2. For $2/\eta \in (n+m-1, +\infty)$, termed *the low SCBR state*, the corresponding SNR coefficient is

$$c^*(\eta) = P_s M^{a_{ub}} \frac{\kappa_l(\frac{2}{\eta}, m)}{\prod_{k=1}^m \Gamma(n-k+1)\Gamma(m-k+1)}. \quad (34)$$

3. For $2/\eta \in [n-m+1, n+m-1]$, termed *the moderate SCBR state*, in the case where $2/\eta - (n-m-1)$ is not an even number, the corresponding SNR coefficient is

$$c^*(\rho, \eta) = P_s M^{a_{ub}} \frac{\kappa_l(\frac{2}{\eta}, l)\kappa_h(\frac{2}{\eta} - 2l, m-l)}{\prod_{k=1}^m \Gamma(n-k+1)\Gamma(m-k+1)}; \quad (35)$$

in the case where $2/\eta - (n-m-1)$ is an even number, the corresponding SNR coefficient is

$$c^*(\rho, \eta) = P_s M^{a_{ub}} \log\left(\frac{\rho}{M}\right) \frac{\kappa_l(\frac{2}{\eta}, l-1)\kappa_h(\frac{2}{\eta} - 2l, m-l)}{\prod_{k=1}^m \Gamma(n-k+1)\Gamma(m-k+1)}, \quad (36)$$

where $\frac{\frac{2}{\eta} - (n-m-1)}{2} < l \leq \frac{\frac{2}{\eta} - (n-m-1)}{2}$, $l \in \mathbb{Z}^+$,

Proof: When $2/\eta \in (0, n-m+1)$, $d_{ij} - \frac{2}{\eta} + 1 > 1$.

Following (25), using Table I, (15) and (27), we get

$$A_{ij}(\rho, \eta) = M^{\frac{2}{\eta}} \Gamma(d_{ij} - \frac{2}{\eta}) \quad (37)$$

and

$$a_{ij} = \frac{2}{\eta}. \quad (38)$$

By using Lemma 3, the coefficient in $\det \mathbf{G}(\rho, \eta)$

$$A(\rho, \eta) = M^{\frac{2m}{\eta}} \kappa_h\left(\frac{2}{\eta}, m\right). \quad (39)$$

In this case,

$$a_{ub}(\eta) = \frac{2m}{\eta}. \quad (40)$$

By Theorem 1,

$$c^*(\eta) = P_s M^{a_{ub}} \frac{\kappa_h(\frac{2}{\eta}, m)}{\prod_{k=1}^m \Gamma(n-k+1)\Gamma(m-k+1)}, \quad (41)$$

which monotonically decreases with n in view of Lemma 6, no matter whether n is the transmit antenna number or the receive antenna number. Therefore, Result 1. in Theorem 3 is proved.

When $2/\eta \in (n+m-1, +\infty)$, $d_{ij} - \frac{2}{\eta} + 1 < 1$. Thus,

$$A_{ij}(\rho, \eta) = M^{d_{ij}} \Gamma(d_{ij}) \frac{\Gamma\left(\frac{2}{\eta} - d_{ij}\right)}{\Gamma\left(\frac{2}{\eta}\right)}, \quad (42)$$

and

$$a_{ij} = i + j + n - m - 1. \quad (43)$$

By using Lemma 2 and 4,

$$A(\rho, \eta) = M^{mn} \kappa_l\left(\frac{2}{\eta}, m\right). \quad (44)$$

In this case,

$$a_{ub} = mn, \quad (45)$$

hence,

$$c^*(\eta) = P_s M^{a_{ub}} \frac{\kappa_l(\frac{2}{\eta}, m)}{\prod_{k=1}^m \Gamma(n-k+1)\Gamma(m-k+1)}. \quad (46)$$

Result 2. in Theorem 3 is thus proved.

When $2/\eta \in [n-m+1, n+m-1]$, define $l = \left\lceil \frac{\frac{2}{\eta} - (n-m-1)}{2} \right\rceil \in \mathbb{Z}^+$ and $\mathbf{G}(\rho, \eta)$ can be partitioned as in Lemma 5. In the case where $2/\eta - (n-m-1) \neq 2l$, using Lemma 3, 4 and 5, we get

$$c^*(\eta) = P_s M^{a_{ub}} \frac{\kappa_l(\frac{2}{\eta}, l)\kappa_h\left(\frac{2}{\eta} - 2l, m-l\right)}{\prod_{k=1}^m \Gamma(n-k+1)\Gamma(m-k+1)}. \quad (47)$$

In the case where $2/\eta - (n-m-1) = 2l$, in view of Table I,

$$g_{l,l}(\rho, \eta) = \left(\frac{\rho}{M}\right)^{-\frac{2}{\eta}} \log\left(\frac{\rho}{M}\right). \quad (48)$$

Therefore, when $\rho \rightarrow +\infty$,

$$\det \mathbf{A}(\rho, \eta) \sim \left(\frac{\rho}{M}\right)^{-l(l+n-m)} \log\left(\frac{\rho}{M}\right) \kappa_l\left(\frac{2}{\eta}, l-1\right) \quad (49)$$

where $\mathbf{A}(\rho, \eta)$ is the l -by- l submatrix of $\mathbf{G}(\rho, \eta)$ as in Lemma 5. In view of

$$\det \mathbf{C}(\rho, \eta) \sim \left(\frac{\rho}{M}\right)^{-\frac{2(m-l)}{\eta}} \kappa_h\left(\frac{2}{\eta} - 2l, m-l\right) \quad (50)$$

where $\mathbf{C}(\rho, \eta)$ is the $(m-l)$ -by- $(m-l)$ submatrix of $\mathbf{G}(\rho, \eta)$, using Lemma 5, we get

$$c^*(\rho, \eta) = P_s M^{a_{ub}} \log\left(\frac{\rho}{M}\right) \frac{\kappa_l(\frac{2}{\eta}, l-1)\kappa_h(\frac{2}{\eta} - 2l, m-l)}{\prod_{k=1}^m \Gamma(n-k+1)\Gamma(m-k+1)}. \quad (51)$$

In this case,

$$a_{ub}(\rho) = l(l+n-m) + \frac{2(m-l)}{\eta}. \quad (52)$$

So far, Result 3. of Theorem 3 is proved. \blacksquare

V. NUMERICAL ANALYSIS AND DISCUSSION

In Fig.1, we provide numerical and simulation results at high SCBR and high SNR. c^* denotes the SNR coefficient and D_{lb}^* denotes the approximate lower bound on distortion, $D_{lb}^* = c^* \rho^{-a_{ub}}$, which actually is the highest order term in the expression of D_{lb} . We also compare the approximate lower bound with Monte Carlo simulations; the latter are carried out by generating 10 000 realizations of \mathbf{H} and evaluating (23). As expected, the comparison shows an excellent agreement between analysis and simulation.

The plot demonstrates that at high SCBR and high SNR, when the number of antennas at one side is fixed and the number of antennas at the other side increases, the optimal distortion is monotonically decreasing. The monotonicity is due to behaviors of the SNR exponent a_{ub} and the SNR coefficient c^* . In the beginning, when $\min\{M, N\}$ increases,

D_{lb} is monotonically decreasing because a_{ub} is increasing and SNR is pretty high, even though c^* is increasing as well; then, when $\max\{M, N\}$ increases, D_{lb} is decreasing because the SNR coefficient c^* is decreasing whereas the SNR exponent a_{ub} keeps constant.

Although Theorem 2 implies that the commutation between numbers of transmit antennas and receive antennas has no effect on a_{ub} , it can affect the optimal distortion by affecting the coefficient c^* as shown by the expressions in Theorem 3. As Fig.1(b) shows, between a couple of commutative antenna allocation schemes ($M \neq N$), the scheme whose number of transmit antennas is the minimum between the two antenna numbers suffers less distortion than the other. The fundamental reason is that under certain total transmit power constraint, average transmit power per transmit antenna is higher for the scheme allocated less transmit antennas.

In the case of moderate or low SCBR, as Theorem 2 implies, a_{ub} is monotonically increasing with both antenna numbers. The optimal distortion is thus monotonically decreasing with antenna numbers in the high SNR regime regardless of c^* 's tendency.

In the case of transmission over a frequency-selective channel, following Lemma 3 in [13], when the length of block approaches to infinity, $T \rightarrow \infty$,

$$D_{lb}(\rho) = P_s \mathbb{E}[\det(\mathbf{I} + \frac{\rho}{M} \bar{\mathbf{H}}\bar{\mathbf{H}}^\dagger)^{-\frac{2}{\eta}}] \quad (53)$$

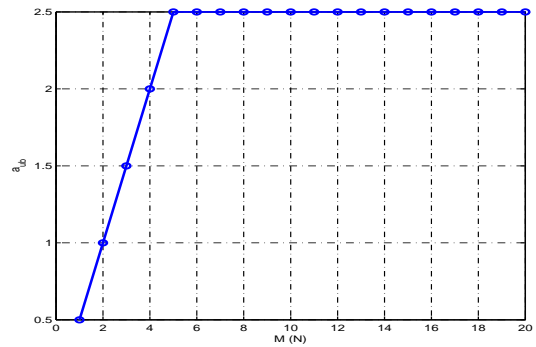
where $\bar{\mathbf{H}}$ can be regarded as a frequency-flat channel matrix of size $L \max\{M, N\} \times \min\{M, N\}$ where L is the number of channel taps. Therefore, regarding the foregoing analysis, frequency selectivity always benefits the optimal end-to-end distortion.

VI. CONCLUSION

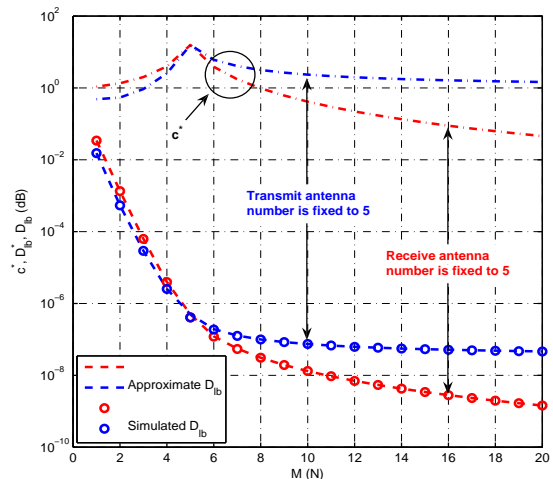
In this paper, We have derived the compact analytic expression of the lower bound on the optimal end-to-end mean quadratic distortion of MIMO links for a continuous white thermal noise source. Stemming from it, we have derived the upper bound on the optimal distortion SNR exponent and the SNR coefficient for any SCBR. We have presented that at the asymptotically high SNR, in the high SCBR state, although the SNR exponent is not affected by $\max\{M, N\}$, the coefficient is monotonically decreasing. It implies the goodness of increasing $\max\{M, N\}$ for a MIMO system in the high SCBR state. We also have presented that the commutation between the transmit antenna number and the receive antenna number can affect the achievable optimal distortion, which is due to the difference between essences of transmission and receiving, and the frequency selectivity can benefit the end-to-end distortion.

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(a) Upper bound on distortion SNR exponent



(b) Coefficients and lower bounds on optimal distortion

Fig. 1. The number of antennas at the other side is fixed to 5, the SCBR η is 4, and the SNR per receive antenna ρ is 30dB.

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