

NLOS Mobile Terminal Position and Speed Estimation

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Abstract—Non-Line-of-Sight (NLOS) and multipath propagation conditions pose significant problems for most Mobile Terminal (MT) positioning approaches. This is because only path parameters of LOS paths are considered. When in fact all paths are considered, much more information for positioning becomes available, though proper consideration of the NLOS character of NLOS paths is required. On the other hand, channel parameters that have been used so far for positioning purposes concern a static channel snapshot. In the case of mobility, the path Doppler shifts provide information on the mobile terminal speed. In this paper we combine this information with traditional positioning information to jointly estimate the terminal's speed and position.

I. INTRODUCTION

The first attempts to estimate the location of a MT along with some very early results date back to the late sixties [1]. However, localization attracted a huge amount of interest only after the U.S. Federal Communications Commission (FCC) announced that it is mandatory for all wireless service providers to be able to provide location information to public safety services in case of an emergency [2], [3]. Undoubtedly that was just the initial motivation. During the attempt to meet with the FCC requirements in the predetermined time-interval, researchers envisioned new commercial services that could become feasible if the exact location of the mobile user is known to the provider. Specifically, location-sensitive billing, increased data rate due to optimum resource allocation, cellular phone fraud detection, cargo tracking, navigational and yellow-pages services can be introduced by wireless service providers, not only to attract new costumers but also to satisfy the demanding ones.

Amongst the numerous techniques that have been developed up to now, the most commonly used and accepted are the so-called geometrical ones. Geometrical techniques are primarily based on the estimation, usually in more than one Base Station (BS), of location-dependent parameters, such as the Angle of Arrival (AOA), the Time of Arrival (TOA), the time difference of Arrival (TDOA), a combination of two of the above (e.g. [4], [5]) or the estimation of the Received Signal Strength (RSS) [6], [7].

A main source of inaccuracies for geometrical techniques is multipath propagation [8]. If the receiver is unable to resolve the paths and determine which is the first-arriving one in order to identify it as the LOS path, the position error will

be very high. This problem has been considered in the past and solutions based on adding the spatial dimension [9], [10] have been found. Recently in [11], Qi et al considered a total different approach for positioning in multipath environments. Instead of estimating TOA, based solely on the LOS path, they investigated the enhancement in performance when multipath components are also being processed. They showed that the signal strength of those components and the variance of their delays play an important role in the enhancement.

The performance of geometrical techniques can also be seriously degraded by the complete lack of a LOS component. This is why the very first approaches were based on the assumption that a LOS path always exists. However in urban environments this condition is rarely met. The most common approach for localizing in these environments is to try to mitigate the NLOS error. This can be accomplished in various ways: Identifying the NLOS BSs so as to localize with just the remaining LOS ones [12], [13] is one way. Using all BSs but introducing either proper weighting to the measurements [14], [15] or a cost function that must be minimized [16], [17] in order to minimize the effect of the NLOS ones, is another. Both of these approaches require the reception of the signal in many BSs, some of which must necessarily be linked through a LOS path with the MT. A third and more appealing way, to mitigate the NLOS errors, is to introduce an appropriate NLOS channel model [18], [19] and use its propagation characteristics to derive new equations that must be satisfied by the MT position's coordinates.

The method proposed herein falls into the last category. It is based on the channel model introduced in [20], which enables us to express the coordinates of the MT as a function of the location-dependent parameters, mentioned above. It doesn't require the reception of the MT's transmitted signal in more than one BS and aims at providing a high location accuracy in strictly NLOS environments. Furthermore, in contrast to existing localization methods, we consider an environment that changes dynamically due to the movement of the MT, rather than a static one. By doing so, we introduce one more dimension to the localization procedure, namely the (variation in) time. This new dimension can be exploited to provide us with more information about the MT's position.

The problem of estimating the exact location of a MT can also be attacked from a different perspective. Continuously

estimating the coordinates is essentially equivalent to tracking the MT. Thus some interesting techniques based on Kalman filtering [21], [22] or the principles of Bayesian parameter estimation [23] have been introduced. We will show how the approach in [21] can be efficiently combined with our approach to mitigate errors introduced by the non-linearity of the movement of the MT on a larger time-scale.

The rest of the paper is organized as follows: In section II we present the channel model along with the assumptions we adopt. In section III we formulate the Maximum Likelihood joint estimation of the speed and the position for the general case when spatial and temporal correlation amongst the estimated parameters exist and can be computed. We then proceed with the derivation of the Cramer-Rao Bound in section IV. Identifiability concerns are briefly discussed in section V. A tracking procedure to enhance the performance of the proposed method is presented in section VI. Finally conclusions and suggestions for future work are given in section VII.

Notation: Throughout the paper, upper case and lower case boldface symbols will represent matrices and column vectors respectively. $(\cdot)^t$ will denote the transpose of any vector or matrix. In order to make the notation more compact, any random vector \mathbf{a} with elements a_{ij}, \dots, a_{kl} will be denoted as $\mathbf{a}_{ij:kl}$.

II. CHANNEL MODEL

In the following analysis, we consider the single bounce model, slightly different versions of which have been introduced and employed in localization techniques by Miao et al [20], [19] and Jazzar and Caffery [18]. The single bounce model describes accurately numerous scenarios, despite the fact that it is very simple. Its wide applicability stems from the fact that in a physical propagation environment, the more bounces, the larger the attenuation will be, not only because the scatterer absorbs some of the signal's energy but also because more bounces usually implies a longer path length. Thus if a limited number of NLOS signal components with non-negligible energy arrive at the receiver, it is reasonable to assume that they have bounced only once.

The aforementioned approaches consider a static propagation environment, i.e. they assume that the MT is not moving. We, on the other hand, are particularly interested in dynamically changing environments. It is well known that the shift in frequency due to the movement of a terminal (Doppler effect) depends strongly on the magnitude and the direction of its speed vector. Therefore estimating this shift can provide us with valuable information for the localization procedure, as will be shown below.

Let ϕ , ψ and d denote the AoA, AoD and length of the NLOS path respectively¹. Based on the single bounce model we can express these location-dependent parameters explicitly as a function of the MT coordinates, x and y , its speed components (projection to the same axes), v_x and v_y and the

¹The length of the path is just the product of the speed of light times the corresponding estimated delay.

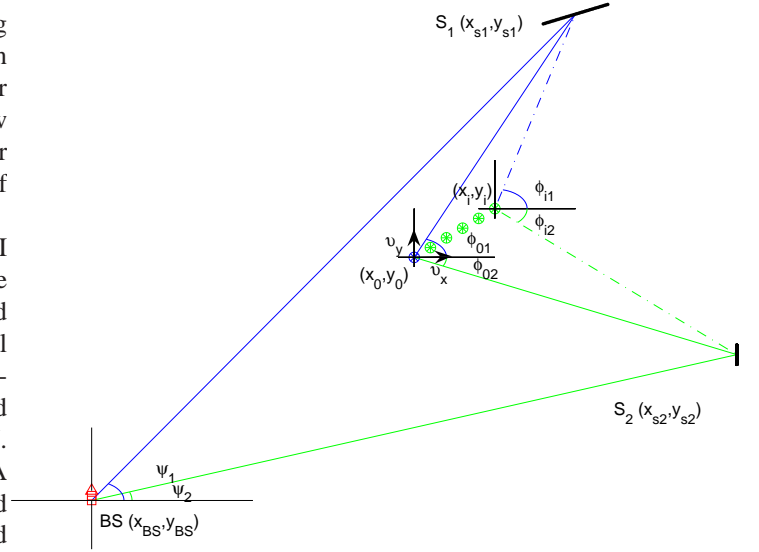


Fig. 1. Single Bounce model

coordinates of the scatterers. With respect to figure 1 and using subscript ij for the parameters at time instant t_i , $0 \leq i < N_t$ and corresponding to path (or scatterer²) j , $1 \leq j \leq N_s$, the channel's parameters, whose value has been estimated a-priori, are given by:

$$\phi_{ij} = \begin{cases} \tan^{-1} \frac{y_{sj} - y_i}{x_{sj} - x_i}, & \frac{y_{sj} - y_i}{x_{sj} - x_i} > 0 \\ \pi + \tan^{-1} \frac{y_{sj} - y_i}{x_{sj} - x_i}, & \frac{y_{sj} - y_i}{x_{sj} - x_i} < 0 \end{cases} \quad (1)$$

$$\psi_{ij} = \psi_j = \begin{cases} \tan^{-1} \frac{y_{sj} - y_{BS}}{x_{sj} - x_{BS}}, & \frac{y_{sj} - y_{BS}}{x_{sj} - x_{BS}} > 0 \\ \pi + \tan^{-1} \frac{y_{sj} - y_{BS}}{x_{sj} - x_{BS}}, & \frac{y_{sj} - y_{BS}}{x_{sj} - x_{BS}} < 0 \end{cases} \quad (2)$$

$$d_{ij} = \sqrt{(y_{sj} - y_i)^2 + (x_{sj} - x_i)^2} + \sqrt{(y_{sj} - y_{BS})^2 + (x_{sj} - x_{BS})^2} \quad (3)$$

Furthermore, considering the movement of the MT, which is depicted in the figure 1 as consecutive points starting at (x_0, y_0) and passing through (x_i, y_i) , we can express the Doppler shifts $f_{d,ij}$ as a function of the same parameters:

$$f_{d,ij} = \frac{f_c}{c} v \cos(\phi_{ij} - \alpha_v) = \frac{f_c}{c} \frac{v_x(x_{sj} - x_i) + v_y(y_{sj} - y_i)}{\sqrt{(y_{sj} - y_i)^2 + (x_{sj} - x_i)^2}} \quad (4)$$

where f_c is the carrier frequency, c is the speed of light, $v = \sqrt{v_x^2 + v_y^2}$ is the magnitude and $\alpha_v = \tan^{-1}(v_y/v_x)$ is the direction of the speed. Assuming that $(N_t - 1) \times \bar{d}t$ is small (e.g. fraction of a second), where N_t represents the number of times we repeated the observations in time and $\bar{d}t$

²For the sake of simplicity we assume that the scatterers do not move. The extension to the more general case is beyond the scope of this paper.

is the average time between subsequent observations³, we can approximate the movement of the MT with a linear one of constant speed, i.e.

$$x_i = x_0 + v_x dt_{i0}, y_i = y_0 + v_y dt_{i0}, 1 \leq i < N_t \quad (5)$$

where $dt_{i0} = t_i - t_0$. Substituting (5) in (1), (3) and (4) we get the AoAs, the path lengths and the Doppler Shifts as a function of only the initial position and the speed:

$$\phi_{ij} = \begin{cases} \tan^{-1} \frac{y_{sj} - (y_0 + v_y dt_{i0})}{x_{sj} - (x_0 + v_x dt_{i0})}, & \frac{y_{sj} - (y_0 + v_y dt_{i0})}{x_{sj} - (x_0 + v_x dt_{i0})} > 0 \\ \pi + \tan^{-1} \frac{y_{sj} - (y_0 + v_y dt_{i0})}{x_{sj} - (x_0 + v_x dt_{i0})}, & \frac{y_{sj} - (y_0 + v_y dt_{i0})}{x_{sj} - (x_0 + v_x dt_{i0})} < 0 \end{cases} \quad (6)$$

$$d_{ij} = \sqrt{(y_{sj} - (y_0 + v_y dt_{i0}))^2 + (x_{sj} - (x_0 + v_x dt_{i0}))^2} + \sqrt{(y_{sj} - y_{BS})^2 + (x_{sj} - x_{BS})^2} \quad (7)$$

$$f_{d,ij} = \frac{f_c v_x (x_{sj} - (x_0 + v_x dt_{i0})) + v_y (y_{sj} - (y_0 + v_y dt_{i0}))}{c \sqrt{(y_{sj} - (y_0 + v_y dt_{i0}))^2 + (x_{sj} - (x_0 + v_x dt_{i0}))^2}} \quad (8)$$

III. JOINT ESTIMATION OF SPEED AND INITIAL POSITION

We are interested in estimating jointly the MT's coordinates at time 0, namely x_0 and y_0 and its speed components v_x and v_y , which, as mentioned above, remain constant during the short period of the estimation procedure. These two pairs of parameters (parameters of interest) compose a vector which we denote as $\mathbf{p}_{int} = [x_0, y_0, v_x, v_y]^t$. The rest of the unknown parameters on the right hand side of equations (2),(6)-(8), which are the coordinates of the scatterers are just nuisance parameters and they compose the vector $\mathbf{p}_{nuis} = [x_{s1}, y_{s1}, \dots, x_{sN_s}, y_{sN_s}]^t$. The set of all of the above $2N_s + 4$ parameters compose the vector:

$$\mathbf{p} = [\mathbf{p}_{int}^t, \mathbf{p}_{nuis}^t]^t \quad (9)$$

Let $\hat{\boldsymbol{\phi}} \triangleq \hat{\boldsymbol{\phi}}_{01:(N_t-1)N_s}$, $\hat{\boldsymbol{\psi}} \triangleq \hat{\boldsymbol{\psi}}_{1:N_s}$, $\hat{\mathbf{d}} \triangleq \hat{\mathbf{d}}_{01:(N_t-1)N_s}$, $\hat{\mathbf{f}}_d \triangleq \hat{\mathbf{f}}_{d,01:(N_t-1)N_s}$ be the random vectors containing the estimated channel-dependent parameters and $\boldsymbol{\phi} \triangleq \boldsymbol{\phi}_{01:(N_t-1)N_s}$, $\boldsymbol{\psi} \triangleq \boldsymbol{\psi}_{1:N_s}$, $\mathbf{d} \triangleq \mathbf{d}_{01:(N_t-1)N_s}$, $\mathbf{f}_d \triangleq \mathbf{f}_{d,01:(N_t-1)N_s}$ be the vectors containing the true value of the entries of the above vectors. Define the following vectors of size $N = (3N_t + 1)N_s$:

$$\boldsymbol{\theta} = [\boldsymbol{\phi}^t, \boldsymbol{\psi}^t, \mathbf{d}^t, \mathbf{f}_d^t]^t \quad (10)$$

$$\hat{\boldsymbol{\theta}} = [\hat{\boldsymbol{\phi}}^t, \hat{\boldsymbol{\psi}}^t, \hat{\mathbf{d}}^t, \hat{\mathbf{f}}_d^t]^t \quad (11)$$

$$\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta} \quad (12)$$

The vector $\boldsymbol{\theta}$ is deterministic and contains the true value of all channel-dependent parameters. The vector $\hat{\boldsymbol{\theta}}$ is a Gaussian random vector with mean value $\boldsymbol{\theta}$ and covariance matrix $\mathbf{C} \triangleq \mathbf{C}_{\hat{\boldsymbol{\theta}}}$. The mean value vector $\boldsymbol{\theta}$ depends on the entries of \mathbf{p} . It is

³By considering the average, we overcome the restriction of uniformly spaced measurement times.

likely that the covariance matrix \mathbf{C} also depends on the same parameters, however exploring such relationship is beyond the scope of this paper, so we will limit our analysis to the case where \mathbf{C} does not depend on the entries of \mathbf{p} . On the other hand, to make our analysis more general, we will not assume that \mathbf{C} is necessarily diagonal, i.e. we do not require that all the entries of $\hat{\boldsymbol{\theta}}$ are independent. For example the entries of any of the vectors $\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\psi}}, \hat{\mathbf{d}}, \hat{\mathbf{f}}_d$ with the same time index i could be correlated. The p.d.f of $\boldsymbol{\theta}$ conditioned on \mathbf{p} is given by:

$$f(\hat{\boldsymbol{\theta}}|\mathbf{p}) = \frac{1}{(2\pi)^{\frac{1}{2}N}(\det \mathbf{C})^{1/2}} e^{-\frac{1}{2}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}(\mathbf{p}))^t \mathbf{C}^{-1}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}(\mathbf{p}))} \quad (13)$$

To obtain a Maximum Likelihood (ML) estimate of our parameters of interest, we need to maximize $f(\hat{\boldsymbol{\theta}}|\mathbf{p})$ -or equivalently maximize or minimize a corresponding likelihood- with respect to both the parameters of interest and the nuisance parameters. Define a log-likelihood obtained by taking the natural logarithm of $f(\hat{\boldsymbol{\theta}}|\mathbf{p})$ and ignoring the constant terms as:

$$\mathcal{L} \triangleq \mathcal{L}(\boldsymbol{\theta}(\mathbf{p})) = \frac{1}{2}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}(\mathbf{p}))^t \mathbf{C}^{-1}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}(\mathbf{p})) \quad (14)$$

Then the ML estimate denoted as $\hat{\mathbf{p}}$ is given by:

$$\hat{\mathbf{p}} = \underset{\mathbf{p}}{\operatorname{argmin}}\{\mathcal{L}\} \quad (15)$$

IV. CRAMER-RAO BOUND

According to the Cramer-Rao Bound (CRB) for an unbiased estimator $\hat{\mathbf{p}}$ of \mathbf{p} , the correlation matrix of the parameter estimation errors $\hat{\mathbf{p}}$ is bounded below by the inverse of the Fisher Information Matrix (FIM) as shown below⁴:

$$R_{\hat{\mathbf{p}}\hat{\mathbf{p}}} = E\{(\hat{\mathbf{p}} - \mathbf{p})(\hat{\mathbf{p}} - \mathbf{p})^t\} \geq \mathbf{J}^{-1} \quad (16)$$

where the FIM is given by:

$$\mathbf{J} = E\left\{\left(\frac{\partial \mathcal{L}}{\partial \mathbf{p}}\right)\left(\frac{\partial \mathcal{L}}{\partial \mathbf{p}}\right)^t\right\} = \frac{\partial \boldsymbol{\theta}^t}{\partial \mathbf{p}} \mathbf{C}^{-1} \frac{\partial \boldsymbol{\theta}}{\partial \mathbf{p}^t} \quad (17)$$

The derivation of \mathbf{J} is given in the Appendix. Each of the four first diagonal entries of \mathbf{J}^{-1} is just the lower bound for the variance of the estimation error of each one of our parameters of interest, namely the coordinates and the speed components of the MT. These bounds are plotted in figures 2 and 3 as a function of the standard deviation of the entries of $\hat{\mathbf{f}}_d$, assuming these entries have the same standard deviation⁵. In the first figure we show that ML estimation can potentially achieve better performance as more information from different paths becomes available, since the CRB is dramatically decreased as N_s is increased from 1 to 3. In the second figure we show that the same effect can happen with an increase in the number of measurements ($N_t = 10, 20, 50$), in a predetermined time interval.

⁴For matrices \mathbf{A} and \mathbf{B} , $\mathbf{A} \geq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is non-negative definite.

⁵This assumption is valid if the power of the noise remains constant during the short observation time.

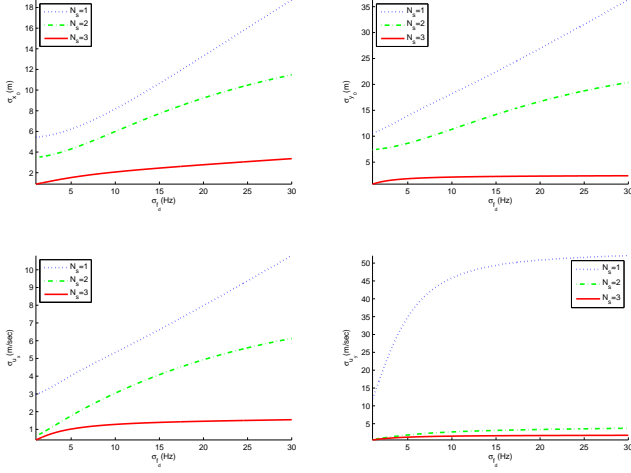


Fig. 2. Standard Deviation of \mathbf{p} versus $\sigma_{\hat{f}_d}$ (various N_s)

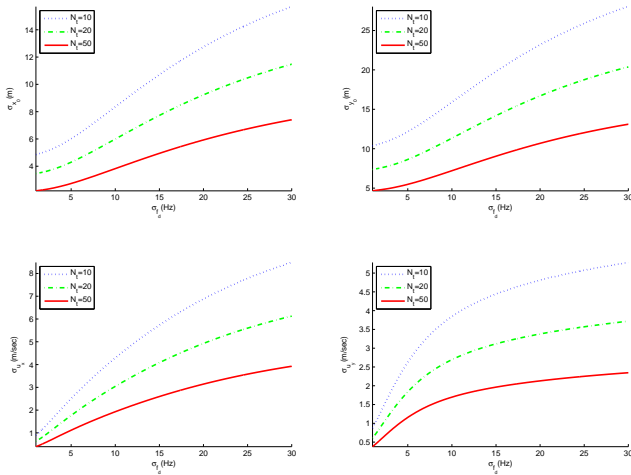


Fig. 3. Standard Deviation of \mathbf{p} versus $\sigma_{\hat{f}_d}$ (various N_t)

The measurements are uniformly spaced and dt is such that $N_t dt = 1\text{sec}$. The carrier frequency is $1.9 \times 10^9\text{Hz}$. The standard deviation of each of the entries of ϕ, ψ and \mathbf{d} are $2^\circ, 2^\circ$ and 5m , respectively. The cross-correlation is assumed to be negligible. Finally the true values of the entries of \mathbf{p} are given in table I. The values for the coordinates are typical values for picocells and the values for the speed components correspond to average walking speed.

V. IDENTIFIABILITY CONCERNS

One major advantage of taking into account information about the speed in addition to the information about the location is the ability to perform Maximum Likelihood estimation with as few as one NLOS signal component. This can be proven to be extremely useful for cases when the resolution of the channel impulse response as a function of time and

TABLE I
COORDINATES AND SPEED COMPONENTS CONSIDERED FOR COMPUTING THE CRB

(x_{BS}, y_{BS})	(x_0, y_0)	(v_x, v_y)	(x_{s1}, y_{s1})	(x_{s2}, y_{s2})	(x_{s3}, y_{s3})
(0,0)	(30,20)	(2,-1.5)	(20,20)	(35,15)	(25,5)

delay is low, leading to the estimation of the parameters of just one separable path. It is easy to show that identifiability in an error-free scenario can be achieved if the speed of the MT and the parameters of a single path are known. The straight-line equation derived in [20]

$$y_i = a_i x_i + b_i \quad (18)$$

with the constant terms depending on the estimated AOA, AODs and path lengths according to

$$a_i = \frac{\cos \phi_i + \cos \psi}{\sin \phi_i + \sin \psi} \quad (19)$$

$$b_i = -a_i(x_{BS} - d_i \sin \psi) + y_{BS} - d_i \cos \psi \quad (20)$$

and eq. (5), provide a set of equations from which the coordinates of the MT (x_i, y_i) can be computed.

VI. ENHANCEMENT THROUGH KALMAN FILTERING ON A LARGER TIME-SCALE

The method proposed in section III can only be applied on a finer time-scale since it is based on the assumption that the movement of a MT is linear. In practice that is true only for very small time intervals, during which the curve that the MT might be possibly moving on can be approximated by a straight line and any non-zero acceleration can be neglected. However we are interested in tracking the MT for time intervals much larger than $N_t \overline{dt}$. On a much larger time-scale, for which the considered time instances k are multiples of $N_t \overline{dt}$ our parameters of interest \mathbf{p}_{int} can be tracked using a standard mobility model:

$$\mathbf{p}_{int}(t_{k+1}) = \mathbf{S} \mathbf{p}_{int}(t_k) + \boldsymbol{\alpha} \quad (21)$$

where $\boldsymbol{\alpha} = [0, 0, \alpha_x(t_k) N_t \overline{dt}, \alpha_y(t_k) N_t \overline{dt}]^t$ with its non-zero entries being Gaussian random variables representing the unknown acceleration and

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & N_t \overline{dt} & 0 \\ 0 & 1 & 0 & N_t \overline{dt} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (22)$$

However, the exact values of the entries of $\mathbf{p}_{int}(t_k)$ are not available. Their estimated value derived by ML estimation might contain small errors, thus:

$$\widehat{\mathbf{p}}_{int}(t_k) = \mathbf{p}_{int}(t_k) + \mathbf{n} \quad (23)$$

where $\mathbf{n} = [\tilde{x}_0, \tilde{y}_0, \tilde{v}_x, \tilde{v}_y]$ contains the speed and position estimation errors and its entries are all Gaussian random

variables. Let $\widehat{\mathbf{p}}_{int}(t_{0:k}) = [\widehat{\mathbf{p}}_{int}^t(t_0), \dots, \widehat{\mathbf{p}}_{int}^t(t_k)]^t$ denote the vector containing all previously estimated values. Then, since all the errors are Gaussian distributed, the optimal ML/MMSE estimate for time k is given by:

$$\widehat{\mathbf{p}}_{int}(t_k) = E\{\mathbf{p}_{int}(t_k) | \widehat{\mathbf{p}}_{int}(t_{0:k})\} \quad (24)$$

As pointed out in [21], the above optimal estimate can be computed recursively with the use of a Kalman filter. The derivation of the algorithm is straightforward and can be found in their paper. To initiate the recursive algorithm, $\widehat{\mathbf{p}}_{int}(t_0)$ can be used. An initial value for the conditional covariance matrix $E\{(\mathbf{p}_{int}(t_k) - \widehat{\mathbf{p}}_{int}(t_k))(\mathbf{p}_{int}(t_k) - \widehat{\mathbf{p}}_{int}(t_k))^t | \widehat{\mathbf{p}}_{int}(t_{0:k})\}$ is also needed. The most reasonable choice for an initial covariance matrix is a scaled identity matrix. However an inference on the cross correlations of the position's and speed's components could be based on the first estimated values, obtained by the finer time-scale ML estimation (e.g. using the CRB in (16)).

VII. CONCLUSIONS

In this paper, we investigated how information about the movement of a mobile terminal can be integrated in traditional geometrical localization techniques to improve their accuracy. Such information is available through the Doppler frequency shifts. Instead of considering a static channel snapshot, the proposed method considers a set of adjacent snapshots, separated by a very small amount of time. This enables us to assume that consecutive positions of the MT satisfy a linear equation, thus although the aforementioned amount of information can be utilized in the estimation procedure, only two extra parameters, namely the projections of the speed vector along the two axes need to be jointly estimated. Disregarding the information about the movement and considering static snapshots would lead to suboptimal solutions for location estimation. A ML solution was formulated for the case when estimates about the AoAs, AoDs, delays and frequency shifts are available. The Cramer Rao Bound was derived to serve as an indication on the attainable performance of the proposed problem formulation. Moreover, identifiability was shown to be possible with as few as one NLOS component received at only one BS. Finally, we suggested Kalman filtering on a larger time-scale as an efficient method not only to enhance the performance of the ML estimation but also to combat the errors possibly introduced by an acceleration of the MT.

Although the framework provided herein is as general as possible, there are still some problems that need to be tackled and some special cases that would broaden the applicability of the method. It has been assumed throughout the whole paper that the AoAs and the AoDs are measured with respect to the same axes system. However if the AoAs are measured at the MT (e.g with the use of multiple antenna elements), this is not the case. A new unknown nuisance parameter will have to be introduced in the model then, namely the orientation of the MT. Some possible extensions of our framework would be the cases when a LOS path is also present, or more than one

BS are employed. The latter case would possibly require the introduction of delay offsets as nuisance parameters, due to lack of synchronization. MT orientation and BS delay offsets issues along with the extension to include a LOS component will be treated in future work.

APPENDIX

In the following analysis we derive a general expression for all the entries of the FIM. Let's start by defining the differences in the coordinates of the positions of the MT and between the coordinates of the scatterers and the BS:

$$\Delta_{yij} = (y_{sj} - y_0 - v_y dt_{i0}) \quad (25)$$

$$\Delta_{xij} = (x_{sj} - x_0 - v_x dt_{i0}) \quad (26)$$

$$\Delta_{yj} = (y_{sj} - y_{BS}) \quad (27)$$

$$\Delta_{xj} = (x_{sj} - x_{BS}) \quad (28)$$

The partial derivatives of all different kinds of entries of $\boldsymbol{\theta}$ - which could be an AoA ϕ , an AoD ψ , a path length d or a doppler shift f_d - with respect to all different kinds of entries of \mathbf{p} - which could be the coordinates or the speed components of the MT x_0, y_0, v_x, v_y or the coordinates of the scatterers x_{sj}, y_{sj} - can be expressed as a function of the above differences and the speed components. These expressions are given below.

If the entry of $\boldsymbol{\theta}$ is an AoA:

$$\frac{\partial \phi_{ij}}{\partial y_{sj}} = -\frac{\partial \phi_{ij}}{\partial y_0} = \frac{\Delta_{xij}}{\Delta_{xij}^2 + \Delta_{yij}^2} \quad (29)$$

$$\frac{\partial \phi_{ij}}{\partial v_y} = \frac{-dt_{i0} \Delta_{xij}}{\Delta_{xij}^2 + \Delta_{yij}^2} \quad (30)$$

$$\frac{\partial \phi_{ij}}{\partial x_{sj}} = -\frac{\partial \phi_{ij}}{\partial x_0} = \frac{-\Delta_{yij}}{\Delta_{xij}^2 + \Delta_{yij}^2} \quad (31)$$

$$\frac{\partial \phi_{ij}}{\partial v_x} = \frac{dt_{i0} \Delta_{yij}}{\Delta_{xij}^2 + \Delta_{yij}^2} \quad (32)$$

If the entry of $\boldsymbol{\theta}$ is an AoD:

$$\frac{\partial \psi_{ij}}{\partial y_{sj}} = \frac{\Delta_{xj}}{\Delta_{xj}^2 + \Delta_{yj}^2} \quad (33)$$

$$\frac{\partial \psi_{ij}}{\partial x_{sj}} = \frac{-\Delta_{yj}}{\Delta_{xj}^2 + \Delta_{yj}^2} \quad (34)$$

$$\frac{\partial \psi_{ij}}{\partial y_0} = \frac{\partial \psi_{ij}}{\partial v_y} = 0 \quad (35)$$

$$\frac{\partial \psi_{ij}}{\partial x_0} = \frac{\partial \psi_{ij}}{\partial v_x} = 0 \quad (36)$$

If the entry of $\boldsymbol{\theta}$ is a path length:

$$\frac{\partial d_{ij}}{\partial y_{sj}} = \frac{\Delta_{yij}}{\sqrt{\Delta_{xij}^2 + \Delta_{yij}^2}} + \frac{\Delta_{yj}}{\sqrt{\Delta_{xj}^2 + \Delta_{yj}^2}} \quad (37)$$

$$\frac{\partial d_{ij}}{\partial y_0} = \frac{\Delta_{yij}}{\sqrt{\Delta_{xij}^2 + \Delta_{yij}^2}} \quad (38)$$

$$\frac{\partial d_{ij}}{\partial v_y} = \frac{dt_{i0} \Delta_{yij}}{\sqrt{\Delta_{xij}^2 + \Delta_{yij}^2}} \quad (39)$$

$$\frac{\partial d_{ij}}{\partial x_{sj}} = \frac{\Delta_{xij}}{\sqrt{\Delta_{xij}^2 + \Delta_{yij}^2}} + \frac{\Delta_{xj}}{\sqrt{\Delta_{xj}^2 + \Delta_{yj}^2}} \quad (40)$$

$$\frac{\partial d_{ij}}{\partial x_0} = \frac{\Delta_{xij}}{\sqrt{\Delta_{xij}^2 + \Delta_{yij}^2}} \quad (41)$$

$$\frac{\partial d_{ij}}{\partial v_x} = \frac{dt_{i0}\Delta_{xij}}{\sqrt{\Delta_{xij}^2 + \Delta_{yij}^2}} \quad (42)$$

Finally, if the entry of θ is a Doppler Shift:

$$\frac{\partial f_{d,ij}}{\partial y_{sj}} = -\frac{\partial f_{d,ij}}{\partial y_0} = \frac{f_c v_y \Delta_{xij}^2 - v_x \Delta_{xij} \Delta_{yij}}{c (\Delta_{xij}^2 + \Delta_{yij}^2)^{3/2}} \quad (43)$$

$$\frac{\partial f_{d,ij}}{\partial v_y} = \frac{f_c \Delta_{yij} + v_y dt_{i0} (\Delta_{yij}^2 - 1) + v_x dt_{i0} \Delta_{xij} \Delta_{yij}}{c (\Delta_{xij}^2 + \Delta_{yij}^2)^{3/2}} \quad (44)$$

$$\frac{\partial f_{d,ij}}{\partial x_{sj}} = -\frac{\partial f_{d,ij}}{\partial x_0} = \frac{f_c v_x \Delta_{yij}^2 - v_y \Delta_{xij} \Delta_{yij}}{c (\Delta_{xij}^2 + \Delta_{yij}^2)^{3/2}} \quad (45)$$

$$\frac{\partial f_{d,ij}}{\partial v_x} = \frac{f_c \Delta_{xij} + v_x dt_{i0} (\Delta_{xij}^2 - 1) + v_y dt_{i0} \Delta_{xij} \Delta_{yij}}{c (\Delta_{xij}^2 + \Delta_{yij}^2)^{3/2}} \quad (46)$$

Define the subscripts $1 \leq m, n \leq N_s + 4$. Taking the partial derivative of the log-likelihood \mathcal{L} with respect to any entry p_m of \mathbf{p} we obtain:

$$\frac{\partial \mathcal{L}}{\partial p_m} = \frac{1}{2} \frac{\partial \tilde{\theta}^t}{\partial p_m} \mathbf{C}^{-1} \tilde{\theta} + \frac{1}{2} \tilde{\theta}^t \mathbf{C}^{-1} \frac{\partial \tilde{\theta}}{\partial p_m} = -\frac{\partial \theta^t}{\partial p_m} \mathbf{C}^{-1} \tilde{\theta} \quad (47)$$

By multiplying two partial derivatives of \mathcal{L} with respect to two entries p_m and p_n of \mathbf{p} we get the following:

$$\frac{\partial \mathcal{L}}{\partial p_m} \frac{\partial \mathcal{L}}{\partial p_n} = \frac{\partial \theta^t}{\partial p_m} \mathbf{C}^{-1} \tilde{\theta} \tilde{\theta}^t \mathbf{C}^{-1} \frac{\partial \theta}{\partial p_n} \quad (48)$$

Then by taking the expected value of the product with respect to $\tilde{\theta}$ we obtain the general expression for any entry J_{mn} of the FIM:

$$J_{mn} = E_{\tilde{\theta}} \left\{ \frac{\partial \mathcal{L}}{\partial p_m} \frac{\partial \mathcal{L}}{\partial p_n} \right\} = \frac{\partial \theta^t}{\partial p_m} \mathbf{C}^{-1} \frac{\partial \theta}{\partial p_n} \quad (49)$$

where the row vector $\frac{\partial \theta^t}{\partial p_m}$ is a concatenation of four vectors as follows:

$$\frac{\partial \theta^t}{\partial p_m} = \left[\frac{\partial \phi^t}{\partial p_m}, \frac{\partial \psi^t}{\partial p_m}, \frac{\partial \mathbf{d}^t}{\partial p_m}, \frac{\partial \mathbf{f}_d^t}{\partial p_m} \right] \quad (50)$$

and the column vector $\frac{\partial \theta}{\partial p_n}$ is defined similarly. The final step to obtain any entry J_{mn} of the FIM is just to replace the entries of these two vectors with the appropriate expressions from the right hand side of equations (29)-(46).

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