

# Impact of Channel Side Information Availability at the Source Node on the Capacity of Multi-Hop MIMO Relay Networks

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**Abstract**—In this paper, we investigate the impact of channel side information (CSI) availability at the source node on the capacity of multi-hop fading relay channels where the nodes are all equipped with multiple antennas. We consider both Amplify-and-Forward (AF) and Decode-and-Forward (DF) relaying strategies. We study a system where the source node multiplexes the information into a number of parallel streams. When the source node has CSI of the source to relay links, we propose a transmission scheme where each stream is matched to a different relay node based on the conventional eigen-mode transmission. When no CSI is available at the source node, all cooperating relays are assumed to get involved in forwarding all data streams. We derive ergodic capacity expressions for all scenarios considered and observe that CSI availability at the source node provides significant gains over the case where no CSI is available. It should be noted that the CSI availability at the source node assumed in this paper is a limited one, in that, no CSI for the relay to destination links are assumed.

## I. INTRODUCTION

In the last decade, cooperative diversity has attracted significant attention thanks to its capability of providing spatial diversity gain in a *distributed* fashion. In cooperative diversity, the source node cooperates with a partner to transmit its information through destination node. This scheme effectively provides transmission through a virtual transmit antenna array, thereby providing more reliability in the transmission relative to transmitting alone [1], [2]. Cooperative diversity is also attractive since it requires no extra bandwidth for the transmission of information. To increase the channel capacity, several cooperation modes may be employed at the relay nodes [3], [4]. Among them, AF and DF are the two fundamental forwarding modes. In the AF mode, the relay node simply amplifies the received signal according to a power constraint and forwards this amplified version to the destination node. There is no demodulation, decoding and re-encoding processes in this mode; however, in the process the noise at the relay receiver is also amplified with the signal. In the DF mode on the other hand, the relay node fully decodes, re-encodes and retransmits the received signal. There is the possibility of decoding error propagation in this mode which results in a decrease in the system performance.

Up to now, there has been a significant body of work in the literature on cooperative diversity. Distributed space-time code design and information-theoretic performance limits for single antenna fading relay channels (with a finite number of nodes) have recently been studied in [5], [3], [6]. Capacity results for MIMO relay channels with a finite number of relays are investigated in [6]- [10].

In this paper, we examine fading relay networks where the source, the destination as well as the multiple relay nodes are all equipped with multiple antennas. We study capacity of multiple antenna, multi-hop relay networks where the source node has either no CSI available or has only limited information on the eigen-vector corresponding to the best eigen-value of source to relay node links. Each relay is assumed to have corresponding CSI from the source to relay and from relay to the destination links. When limited CSI is available at the source node, we propose a simple but efficient transmit best-eigen mode BF algorithm for both AF and DF relaying modes and derive the associated network capacity expressions. Specifically, we investigate the impact of channel side information availability and subsequently use of BF transmission at the source node on the MIMO multi-relay network capacity based on inter-user channel conditions as well as relaying modes. As conventional with BF, we assume that the source node directs its transmission power to the best eigen-channels.

This paper differentiates itself from previous works in a number of ways. First, a new, simple transmission strategy at the source node, MU-BF, is proposed for limited available CSI at the source. Second, with this proposed strategy, unlike the previous approaches, we release the restriction on the number of antennas at the relay nodes.

## II. MIMO MULTI-RELAY SYSTEM MODEL

We consider a wireless multi-hop (MH) relay network where a source node intends to communicate with a destination node using the cooperation of a multiple number of partnering relay nodes where all nodes are equipped with multiple antennas. We assume that there is no direct link between the source

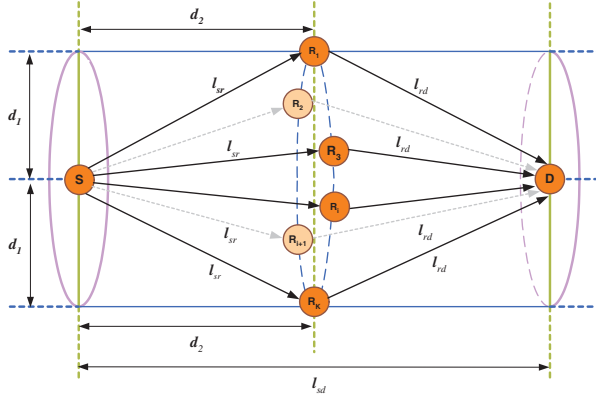


Fig. 1. Relay location setup: 3D Parallel Case.

and the destination. The source,  $\mathcal{S}$ , transmits multiplexed data streams to the destination,  $\mathcal{D}$ , via the assistance of  $K$  relays, where the  $k$ 'th relay is  $\mathcal{R}_k$  for  $k \in \{1, 2, \dots, K\}$ . We assume that the available channel is divided into two *orthogonal* sub-channels in the time domain.  $\mathcal{S}$  communicates with the selected  $\mathcal{R}_k$ 's in the first time slot. In the second time slot,  $\mathcal{R}_k$ 's communicate with  $\mathcal{D}$ .

The channels  $\mathcal{S} \rightarrow \mathcal{D}$ ,  $\mathcal{S} \rightarrow \mathcal{R}_k$  and  $\mathcal{R}_k \rightarrow \mathcal{D}$  are assumed to experience Rayleigh block fading such that they remain constant for the duration of two consecutive time slots. We let the number of antennas at the source,  $k$ 'th relay and destination nodes be  $M$ ,  $N_{\mathcal{R}_k}$  and  $N$ , respectively. The channels among different nodes are assumed to be independent of one another. Similarly, independence is assumed between two consecutive time slot realizations of a channel. We let  $\mathbf{H}_k \in \mathbb{C}^{N_{\mathcal{R}_k} \times M}$  and  $\mathbf{G}_k \in \mathbb{C}^{N \times N_{\mathcal{R}_k}}$  represent the channel matrices for  $\mathcal{S} \rightarrow \mathcal{R}_k$  and  $\mathcal{R}_k \rightarrow \mathcal{D}$  links, respectively. For each link, the channel coefficients are assumed to include the path-loss exponent. That is, the channel coefficient from the  $m$ 'th transmitter antenna to  $n$ 'th receiver antenna follows the general form of

$$h(n, m) = l^{-\frac{\delta}{2}} \phi(t), \quad (1)$$

where  $\phi(t)$  is a complex Gaussian random variable with zero mean and unit variance and  $l$  is the distance between the transmitter and the receiver, and  $\delta$  is the path-loss exponent. In this paper, we assume  $\delta = 4$ . The system model considered in this paper is depicted in Figure 1. Noise terms at the front end of relays and the destination node are all assumed to be i.i.d. zero-mean circularly symmetric complex Gaussian with  $\sigma_r^2$  and  $\sigma_n^2$  variances, respectively. We assume that the total available transmit power for the source and the sum of all relay nodes to be  $P$  so if  $K$  relays are active in transmission, each relay is assumed to have  $P_r = P/K, \forall k$  as its power constraint. The total transmission powers are assumed to be independent of number of transmit antennas.

<sup>†</sup>Throughout the paper, the superscripts,  $T$ ,  $*$  and  $H$  stand for transposition, element-wise conjugate and conjugate transposition, respectively.  $\mathcal{E}[\cdot]$  denotes the expectation operator,  $\mathbf{I}_N$  is the  $N \times N$  identity matrix of appropriate dimensions.

### III. AF RELAYING MODE

#### A. No CSI at The Source Node

In this section, following [11], we investigate the MIMO multi-relay channel capacity when no CSI is available at the source node, full CSI is available at the destination node and only backward CSI is available at each relay node. We describe the received signal vectors at the relay nodes in the first time slot as  $\mathbf{r}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k$ ,  $k \in \{1, 2, \dots, K\}$ . These signals are subsequently multiplied by the gain matrix  $\mathbf{B}_k \in \mathbb{C}^{N_{\mathcal{R}_k} \times N_{\mathcal{R}_k}}$  and the resulting vectors,  $\mathbf{t}_k = \mathbf{B}_k \mathbf{r}_k$ , are transmitted by the relay nodes in the second time slot. In this case, after some manipulations, the network capacity can be derived as,

$$C_{AF}^{NC/SL} = \frac{1}{2} \mathcal{E}_{\mathcal{H}} \left[ \log \det \left( \mathbf{I}_N + \frac{P}{M\sigma_n^2} (\mathbf{W}\mathcal{H})(\mathbf{W}\mathcal{H})^H \right) \right], \quad (2)$$

where  $\mathbf{W} \in \mathbb{C}^{N \times N}$  is the whitening matrix for the equivalent noise seen at the destination,  $\mathbf{n}_{eq} = \sum_{k=1}^K \mathbf{G}_k \mathbf{B}_k \mathbf{n}_k + \mathbf{n}_d$  and the overall equivalent channel matrix is given by

$$\mathcal{H} = \sum_{k=1}^K \mathbf{G}_k \mathbf{B}_k \mathbf{H}_k. \quad (3)$$

#### B. Transmission With MU-BF Method At The Source Node

We now derive the capacity of the multiple antenna multi-relay network when the source has  $\mathcal{S} \rightarrow \mathcal{R}_k$  CSI availability and using this information employs a MU-BF scheme. The source multiplexes the data to be transmitted into different data streams, each intended for different directions and each corresponding to one of the selected cooperating relays. Each relay assumes non-intended data streams as interference. Using the  $\mathcal{S} \rightarrow \mathcal{R}_k$  CSI knowledge, the source performs best eigenmode MU-BF which is conducted on the directions of the eigenvectors corresponding to the maximum eigenvalues of each of the  $\mathcal{S} \rightarrow \mathcal{R}_k$  links. We equally distribute transmit power among data streams. Each relay is assumed to use a Matched-Filter (MF) to capture the portion of the signal intended for itself.

We define the transmitted signal vector as

$$\mathbf{x} = \sum_{k=1}^K \mathbf{w}_k s_k \quad (4)$$

where  $\mathbf{w}_k$ 's are the eigenvectors corresponding to the maximum eigenvalues of the matrix  $\mathbf{H}_k^H \mathbf{H}_k$ .

In the first time slot, the source node transmits while the relays listen and use MF to capture the signals intended for them. In the second time slot, the cooperating relays, using the available relay-destination CSI, beam their amplified signals via the best-eigen mode that corresponds to the channel between the relay and the destination node. The destination node is assumed to conduct multi-dimensional decoding.

In the first time slot, the signal at the  $k$ 'th relay is given by

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k = \mathbf{H}_k \mathbf{w}_k s_k + \sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{H}_k \mathbf{w}_i s_i + \mathbf{n}_k, \quad (5)$$

where  $\mathbf{n}_k \in \mathcal{C}^{N_{\mathcal{R}_k} \times 1}$  is temporally and spatially white with  $\mathcal{CN}(\mathbf{0}, \sigma_r^2 \mathbf{I}_{N_{\mathcal{R}_k}})$ . Then the signal at the output of the MF at the  $k$ 'th relay is

$$\begin{aligned} u_k &= (\mathbf{H}_k \mathbf{w}_k)^H \mathbf{r}_k \\ &= \|\mathbf{H}_k \mathbf{w}_k\|^2 s_k + \mathbf{w}_k^H \mathbf{H}_k^H \mathbf{H}_k \sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{w}_i s_i + (\mathbf{H}_k \mathbf{w}_k)^H \mathbf{n}_k. \end{aligned} \quad (6)$$

and therefore, the SINR at the output of the MF of the  $k$ 'th relay is

$$\Psi_k = \frac{\|\mathbf{H}_k \mathbf{w}_k\|^4 P/K}{\sum_{\substack{i=1 \\ i \neq k}}^K |\mathbf{w}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{w}_i|^2 \frac{P}{K} + \|\mathbf{H}_k \mathbf{w}_k\|^2 \sigma_r^2}. \quad (7)$$

The relays amplify the streams prior to their transmission. Due to power constraints at the relays, the filtered signals are scaled with scaling factors,  $f_k$ , calculated as,

$$f_k = \sqrt{\frac{P_r}{\|\mathbf{H}_k \mathbf{w}_k\|^4 \frac{P}{K} + \sum_{\substack{i=1 \\ i \neq k}}^K |\mathbf{w}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{w}_i|^2 \frac{P}{K} + \|\mathbf{H}_k \mathbf{w}_k\|^2 \sigma_r^2}}. \quad (8)$$

In the second time slot, each of the scaled signals is transmitted via BF to the destination. We assume each relay has forward CSI to form the beam [10]. Best-eigen mode BF is conducted where the BF vector is selected according to the best corresponding eigenvalue. Therefore the BF vector for the  $k$ 'th relay node,  $\mathbf{v}_k$  for  $k \in \{1, 2, \dots, K\}$ , corresponds to best eigen-mode of the channel  $\mathcal{R}_k \rightarrow \mathcal{D}$ .

Assuming that the BF vectors at the relay nodes all have unit norms, the transmit signals at the relay nodes can be expressed as  $\mathbf{t}_k = f_k u_k \mathbf{v}_k$ . Then the received signal vector at the destination node, assuming relay nodes transmit synchronously, can be expressed as

$$\begin{aligned} \mathbf{y} &= \sum_{k=1}^K \left[ \left( \sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{O}_i \right) \mathbf{w}_k + \mathbf{G}_k \mathbf{v}_k f_k \|\mathbf{H}_k \mathbf{w}_k\|^2 \right] s_k \\ &\quad + \sum_{k=1}^K \mathbf{G}_k \mathbf{v}_k f_k (\mathbf{H}_k \mathbf{w}_k)^H \mathbf{n}_k + \mathbf{n}_d \end{aligned} \quad (9)$$

where  $\mathbf{O}_k = \mathbf{G}_k \mathbf{v}_k f_k \mathbf{w}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{w}_k$ ,  $k \in \{1, 2, \dots, K\}$ . If we represent the total noise term as  $\mathbf{z}$ ,  $\mathbf{y}$  can be rewritten compactly as

$$\mathbf{y} = \mathbf{A} \mathbf{s} + \mathbf{z} \quad (10)$$

where  $\mathbf{s} = [s_1 \dots s_K]^T$  is the transmit signal vector and  $\mathbf{A} \in \mathcal{C}^{N \times K}$  is the channel matrix for the second time slot and is written as

$$\mathbf{A} = \left[ \left[ \left( \sum_{\substack{i=2 \\ i=2}}^K \mathbf{O}_i \right) \mathbf{w}_1 + \mathbf{G}_1 \mathbf{v}_1 f_1 \|\mathbf{H}_1 \mathbf{w}_1\|^2 \right] \dots \left[ \left( \sum_{i=1}^{K-1} \mathbf{O}_i \right) \mathbf{w}_K + \mathbf{G}_K \mathbf{v}_K f_K \|\mathbf{H}_K \mathbf{w}_K\|^2 \right] \right]. \quad (11)$$

In  $\mathbf{y}$ , the noise term  $\mathbf{z}$  is no longer white. Thus, a whitening process is necessary at the receiver of the destination node [11]. Let the whitening matrix be  $\mathbf{W} \in \mathcal{C}^{N \times N}$ . Then the overall compound channel equation can be written as

$$\tilde{\mathbf{y}} = \mathbf{W} \mathbf{y} = \mathbf{W} \mathbf{A} \mathbf{s} + \tilde{\mathbf{z}} \quad (12)$$

where  $\tilde{\mathbf{z}} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_N)$  corresponds to the whitened noise term. Then, the corresponding ergodic network capacity expression for this system can be calculated as

$$C_{AF}^{BF} = \frac{1}{2} \mathcal{E}_{\{\mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K} \left[ \log_2 \det \left( \mathbf{I}_N + \frac{P}{K \sigma_n^2} \mathbf{W} \mathbf{A} (\mathbf{W} \mathbf{A})^H \right) \right]. \quad (13)$$

#### IV. DF RELAYING MODE

In this section, we determine the capacity of multiple antenna equipped multi-relay network under investigation when the relays use the DF strategy. In the DF mode, the signal vector received by each relay node at the first time slot is demodulated, decoded and re-encoded before transmission.

##### A. No CSI at The Source Node

Here, we derive the capacity expressions for the system when DF is employed and no BF at the source node is conducted. We assume each relay node as well as the destination node use Zero-Forcing (ZF) algorithm to capture the signal and subsequently conduct parallel decoding. In this case, the source broadcasts  $M$  multiplexed data streams in the first time-slot and each relay tries to decode all data streams *perfectly*. We assume that the source broadcast is conducted at a data rate that enables all relays to decode the streams perfectly. Thus, there is some rate loss in the broadcast channel. However, multiple relays may be exploited in this scheme by selecting the best capacity achieving subset of relays providing relay selection diversity. Note that It is necessary to have  $M \leq N_{\mathcal{R}_k} \leq N$ , for  $k \in \{1, 2, \dots, K\}$  so that all of the relays may decode all of the data streams.

In the first time slot, the source transmits  $\mathbf{x} \in \mathcal{C}^{M \times 1}$  with the covariance matrix  $\mathbf{Q}_x = (P/M) \mathbf{I}_M$ . Then, the received signal vectors at the relays are passed through ZF filters, independently decoded and re-encoded, and in the second time slot, the re-encoded signal vectors,  $\mathbf{t} \in \mathcal{C}^{M \times 1}$  with the covariance matrices  $\mathbf{Q}_t = (P_r/M) \mathbf{I}_M$ , are transmitted such that  $P_r = P/K$ . Here, we utilize only  $M$  randomly selected antennas at each relay node. The corresponding channel matrices from  $\mathcal{R}_k \rightarrow \mathcal{D}$  are denoted as  $\tilde{\mathbf{G}}_k \in \mathcal{C}^{N \times M}$ . Assuming  $K$  relays participating in the transmission, the received signal at the destination is given by,

$$\mathbf{y} = \sum_{k=1}^K \tilde{\mathbf{G}}_k \mathbf{t} + \mathbf{n}_d = \tilde{\mathbf{G}} \mathbf{t} + \mathbf{n}_d \quad (14)$$

where  $\tilde{\mathbf{G}} = \sum_{k=1}^K \tilde{\mathbf{G}}_k$  is the equivalent  $N \times M$  channel matrix. The corresponding ZF matrices at each relay and the destination node are given, respectively, as

$$\begin{aligned} \mathbf{H}_k^\dagger &= (\mathbf{H}_k^H \mathbf{H}_k)^{-1} \mathbf{H}_k^H, \\ \tilde{\mathbf{G}}^\dagger &= (\tilde{\mathbf{G}}^H \tilde{\mathbf{G}})^{-1} \tilde{\mathbf{G}}^H. \end{aligned} \quad (15)$$

Then, the streams at the outputs of the relays and the destination node are given by

$$\begin{aligned} \mathbf{r}'_k &= \mathbf{H}_k^\dagger \mathbf{r}_k = \mathbf{s} + \mathbf{H}_k^\dagger \mathbf{n}_k \\ \mathbf{y}' &= \tilde{\mathbf{G}}^\dagger \mathbf{y} = \mathbf{t} + \tilde{\mathbf{G}}^\dagger \mathbf{n}_d = \sqrt{\frac{P_r}{P}} \mathbf{s} + \tilde{\mathbf{G}}^\dagger \mathbf{n}_d, \end{aligned} \quad (16)$$

respectively. Thus, the SNR terms for relay nodes and the destination node for  $j$ th data stream can be expressed as

$$\Gamma_j^k = \frac{P/M}{\sigma_r^2 \|\mathbf{H}_k^\dagger\|_j^2}, \quad j = 1, 2, \dots, M, \quad (17)$$

$$\Gamma_j^{des} = \frac{P_r/M}{\sigma_n^2 \|\tilde{\mathbf{G}}^\dagger\|_j^2}, \quad j = 1, 2, \dots, M, \quad (18)$$

where  $\Gamma_j^k$  and  $\Gamma_j^{des}$  are the SNR values observed at  $k$ th relay node and the SNR seen at the destination node for the  $j$ th data stream, respectively. Here  $\|\tilde{\mathbf{G}}^\dagger\|_j^2$  is the norm of the  $j$ th row of matrix  $\tilde{\mathbf{G}}^\dagger$ .

Assuming all of the relays can decode *perfectly*, the achievable data rates for each multiplexed data stream are given by

$$C_j = \min \left\{ \log_2(1 + \Gamma_j^1), \dots, \log_2(1 + \Gamma_j^K), \log_2(1 + \Gamma_j^{des}) \right\} \quad (19)$$

which is only possible if the source is able to use variable rate coding depending on the instantaneous capacity feedbacks for each data stream from the relays and the destination. The overall instantaneous network capacity in this case can be expressed as

$$C_{DF}^{NC SI} = \frac{1}{2} \mathcal{E}_{\{\mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K} \left[ \sum_{j=1}^M C_j \right]. \quad (20)$$

### B. Transmission With MU-BF Method At The Source Node

We now determine the capacity of multiple antenna equipped multi-relay networks employing MU-BF at the source node when the relays employ DF. Since the destination node is assumed to use ZF, only  $L \leq N$  relays out of  $K$  relay nodes can be used in the transmission. If  $K \geq N$ , it is possible to select the best subset of the available relay nodes. Here, for simplicity, we assume that there are  $K$  relay nodes in system with  $K \leq M \leq N$ .

As in the AF mode, we propose to use the MU-BF scheme where only one data stream is assigned to each relay node. The relay nodes beamform on the best eigen-mode corresponding to their link to the destination node. The source node transmitted signal and the corresponding BF vectors at the source node are identical to the AF case. Similarly, the BF vector from the  $k$ 'th relay,  $\mathbf{v}_k$  for  $k \in \{1, 2, \dots, K\}$ , corresponding to best eigen-mode of the link  $\mathcal{R}_k \rightarrow \mathcal{D}$  is unchanged. Assuming that the BF vectors at relays have unit norms, the transmitted signals at the relays can be expressed as

$$\mathbf{t}_k = \mathbf{v}_k s_k, \quad k \in \{1, 2, \dots, K\} \quad (21)$$

where we assume that  $s_k$  is the signal intended for relay  $k$  with  $\mathcal{E}[|s_k|^2] = P/K = P_r$ .

In the first time slot, different data streams are transmitted by the source node to different relays. Since, ZF is assumed to be employed at the destination node, it is possible to evaluate the individual capacities of the individual data streams from the relay nodes to the destination node independently resulting in a less complex decoding process. The composite received signal vector at the destination can be written as

$$\begin{aligned} \mathbf{y} &= \sum_{k=1}^K \mathbf{G}_k \mathbf{t}_k + \mathbf{n}_d = \sum_{k=1}^K \mathbf{G}_k \mathbf{v}_k s_k + \mathbf{n}_d \\ &= [\mathbf{G}_1 \mathbf{v}_1 \dots \mathbf{G}_K \mathbf{v}_K] \mathbf{s} + \mathbf{n}_d \\ &= \mathbf{G} \mathbf{s} + \mathbf{n}_d \end{aligned} \quad (22)$$

where  $\mathbf{s} = [s_1, s_2, \dots, s_L]^T$  is the multiplexed transmit signal vector at the source node, and  $\mathbf{G} \in \mathbb{C}^{N \times K}$  is the compound channel. The corresponding ZF matrix at the destination node is given by

$$\mathbf{G}^\dagger = (\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}^H \quad (23)$$

resulting in an output sequence of

$$\mathbf{y}' = \mathbf{G}^\dagger \mathbf{y} = \mathbf{s} + \mathbf{G}^\dagger \mathbf{n}_d. \quad (24)$$

Then, the SNR for  $k$ th data stream can be expressed as

$$\Gamma_k = \frac{P/K}{\sigma_n^2 \|\mathbf{G}^\dagger\|_k^2} \quad (25)$$

Therefore, the network capacity can be calculated as

$$C_{DF}^{BF} = \frac{1}{2} \mathcal{E}_{\{\mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K} \left[ \sum_{k=1}^K \min \left\{ \log_2(1 + \Psi_k), \log_2(1 + \Gamma_k) \right\} \right] \quad (26)$$

where  $\Psi_k$  is the SINR observed at the  $k$ 'th relay node given in (7) and  $\Gamma_k$  is the SNR at the destination node for  $k$ 'th data stream.

## V. SIMULATION SCENARIOS AND NUMERICAL RESULTS

As shown in Figure 1, we consider a network geometry where all relay nodes are equidistant from the source node and also equidistant from the destination node. In Figure 1, we let  $d_1 = 0.5$  and assume the distance between the source and the destination node is normalized to unity,  $l_{sd} = 1$ .

In Figure 2 and Figure-3, the network ergodic capacities of the system employing AF and DF relaying modes are plotted, respectively, for  $M = N_{\mathcal{R}_k} = N = 8$  and  $P/\sigma_n^2 = 10$ dB for  $K = 2, 4, 8$  relay nodes, as a function of the projected distance between the source node and the relay nodes,  $d_2$ .

In Figure 2, when relay nodes are closer to the source node, transmission without BF provides higher performance. But if relays move towards the destination node, the gain provided by MU-BF is observed. As all of the partnering nodes are assumed to have 8 antennas, the system provides full multiplexing gain when only one relay node is used in the scheme for the case of no CSI at the source. On the other hand, as can be seen from Figure 2 if there are not enough

relay nodes, full multiplexing gain can not be achieved with MU-BF. It should be noted that if all of the relay nodes have single antenna then MU-BF provides significant gains over non-BF case.

For the DF mode, we observe in Figure 3 that for the case of no CSI availability at the source node, utilizing more relay nodes degrades the system performance. This is due to the fact that all of the partnering relay nodes are required to be able to decode *perfectly*. It is observed that the system performance improves when compared to the non-BF transmission scheme when the relay nodes are equidistant from the source and the destination nodes. It can be also said that with MU-BF, the system performance is more stable as a function of  $d_2$ , which may be a desired situation for some wireless applications.

## VI. CONCLUSIONS

This paper is concerned with the derivation of network capacities for a system employing MU-BF at the source node when multiple antennas are available at all nodes. Both AF and DF relaying strategies are considered in the paper. A TDMA-based cooperation protocol is assumed. A limited CSI is assumed for the source node where the eigen-vectors corresponding to the best eigen-value of the  $\mathcal{S} \rightarrow \mathcal{R}_k$  links are considered to be available. It is assumed that the relays have forward and backward CSI availability. The proposed MU-BF at the source node uses multiplexed data streams, each indented for a different relay node. When the relays use the AF strategy, we assume they employ MF and transmit the received signal after scaling. In this case, we observe that using a larger number of relay nodes builds up the channel rank and increases the overall capacity. When the AF strategy is employed, we observe that the MU-BF scheme provides significant gains over the non-BF case the when relay nodes are closer to the destination node. When the relay nodes employ the DF strategy, we observe that using more relay nodes results in a performance degradation for the non-BF case. On the other hand, when the MU-BF scheme is employed, the system performance gets better as the the number of relay nodes increases. The MU-BF scheme provides a better capacity performance when compared to the non-BF case when the relay nodes are placed in the midpoint between the source and the destination nodes. In general, the MU-BF scheme causes the system capacity to be less dependent to the location of the relay nodes when the DF relaying strategy is employed.

## REFERENCES

- [1] A. Sendonaris, E. Erkip and B. Aazhang, "User Cooperation Diversity Part-I: System Description," *IEEE Transactions on Communications*, November 2003.
- [2] J.N. Laneman, D.N.C. Tse and G.W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Transactions on Information Theory*, December 2004.
- [3] J.N. Laneman, D.N.C. Tse and G.W. Wornell, "Distributed Space-Time-Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks," *IEEE Transactions on Information Theory*, October 2003.
- [4] M. Janani, A. Hedayat, T.E. Hunter, and A. Nosratinia, "Coded Cooperation in Wireless Communications: Space-Time Transmission and Iterative Decoding," *IEEE Transactions on Signal Processing*, February 2004.

- [5] R.U. Nabar, H. Bölcskei, F.W. Kneubuhler, "Fading Relay Channels: Performance Limits and Space-Time Signal Design," *IEEE Journal on Selected Areas in Communications*, August 2004.
- [6] G. Kramer, M. Gastpar, and P. Gupta, Cooperative Strategies and Capacity Theorems for Relay Networks," *IEEE Transactions on Information Theory*, February 2004.
- [7] B. Wang, J. Zhang, A. Host-Madsen, "On the Capacity of MIMO Relay Channels," *IEEE Transactions on Information Theory*, January 2005.
- [8] D.P. Palomar, A. Agustin, O. Munoz and J. Vidal, "Decode-and-Forward Protocol for Cooperative Diversity in Multi-Antenna Wireless Networks," *Proceedings of IEEE CISS*, March 2004.
- [9] Y. Fan and J.S. Thompson, "MIMO Configurations for Relay Channels: Theory and Practice," *to be published in IEEE Transactions on Wireless Communications*, 2007.
- [10] H. Bölcskei, R.U. Nabar, Ö Oyman and A.J. Paulraj, "Capacity Scaling Laws in MIMO Relay Networks," *IEEE Transactions on Wireless Communications*, June 2006.
- [11] A. Wittneben A. and B. Rankov, "Impact of Cooperative Relays on the Capacity of Rank-Deficient MIMO Channels," *Proceedings of the 12th IST Summit*, June 2003.

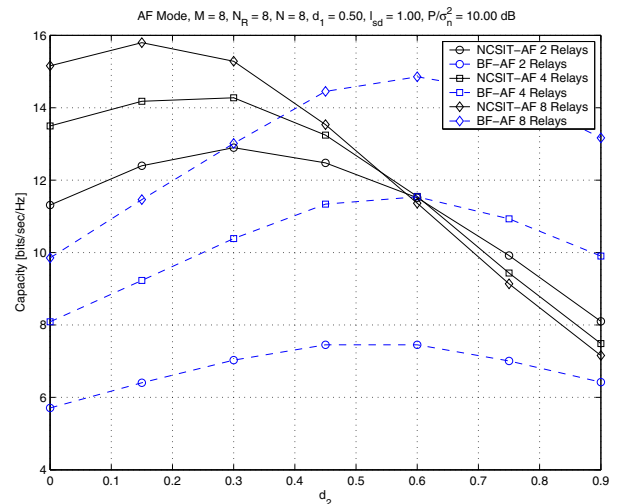


Fig. 2. Network capacities for NCSI AF and MU-BF AF modes are given. For 2, 4 and 8 relay nodes and  $M = N = N_{\mathcal{R}_k} = 8$ ,  $P/\sigma_n^2 = 10\text{dB}$ .

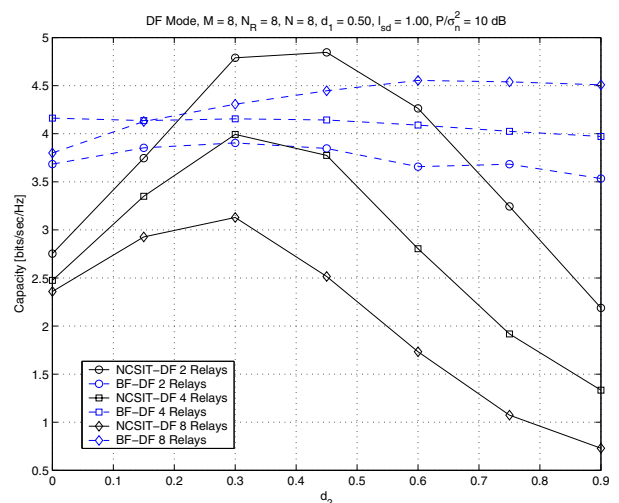


Fig. 3. Network capacities for NCSI DF and MU-BF DF modes are given. For 2, 4 and 8 relay nodes and  $M = N = N_{\mathcal{R}_k} = 8$ ,  $P/\sigma_n^2 = 10\text{dB}$ .