# EFFECTS OF TRANSMIT BEAMFORMING ON THE CAPACITY OF MULTI-HOP MIMO RELAY CHANNELS 

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#### Abstract

In this paper we investigate multi-hop fading relay channels where the source, the destination and the multiple relay nodes are all equipped with multiple antennas. We study the ergodic capacities of multiple relay networks based on Amplify-andForward $(A F)$ and Decode-and-Forward $(D F)$ relaying modes. We examine multi-user beamforming (MU-BF), where each data stream is assumed to be matched to a specified relay node, based on the conventional eigen-mode transmission for both modes, and derive ergodic capacity expressions. We also examine the impact of the number of selected relay nodes on the network capacity both for modes of relaying. We show that by using MU-BF at the source node and a maximum number of relay nodes selected for cooperation, the network gains from multiplexing, while beyond that number of cooperating relay nodes, it only gains from relay selection.


## I. Introduction

In the last decade, cooperative diversity has attracted significant attention thanks to its capability of providing spatial diversity gain in a distributed fashion. In cooperative diversity, the source node cooperates with a partner to transmit its information through destination node. This scheme effectively provides transmission through a virtual transmit antenna array, thereby providing more reliability in the transmission relative to transmitting alone $[1,2]$. Cooperative diversity is also attractive since it requires no extra bandwidth for the transmission of information. To increase the channel capacity, several cooperation modes may be employed at the relay nodes [3, 4]. Among them, AF and DF are the two fundamental forwarding modes. In the AF mode, the relay node simply amplifies the received signal according to a power constraint and forwards this amplified version to the destination node. There is no demodulation, decoding and re-encoding processes in this mode; however, in the process the noise at the relay receiver is also amplified with the signal. In the DF mode on the other hand, the relay node fully decodes, re-encodes and retransmits the received signal. There is the possibility of decoding error propagation in this mode which results in a decrease in the system performance.

Up to now, these has been a significant body of work in the literature on cooperative diversity. Distributed space-time code design and information-theoretic performance limits for single antenna fading relay channels (with a finite number of nodes) have recently been studied in $[5,3,6]$. Capacity results for MIMO relay channels with a finite number of relays are investigated in [6]- [10].

In this paper, we examine fading relay networks where the
source, the destination as well as the multiple relay nodes are all equipped with multiple antennas. We study the ergodic capacities of the AF and DF modes in this scenario. We study the capacity of multiple antenna, multi-hop relay networks when channel state information (CSI) is available at all transmit node(s). At the source node, we assume that only CSI of the source to relay nodes is available. At relay nodes, we assume that the CSI for source to relay as well as relay to destination channels are available. For such a system, we propose simple but efficient transmit best-eigen mode beamforming (BF) algorithms at the source node for AF and DF modes and derive the associated network capacity expressions. Specifically, we investigate the impact of BF transmission at the source node on the MIMO multi-relay network capacity based on inter-user channel conditions as well as relaying modes. As conventional with BF, we assume that the source node directs its transmission power to the best eigen-channels.

This paper differentiates itself from previous works in a number of ways. First, a new, simple transmission strategy at the source node, MU-BF, is proposed for limited available CSI at the source. Second, under the above strategy we study that the effects of multiple relay selection based on a simple metric.

The remainder of this paper is organized as follows. In section II we provide the system model for the MIMO multi-relay channel studied. In sections III and IV, we formulate the network capacity for this system when AF and DF relay modes are used, respectively. In both cases, a simple procedure is proposed to select a subset of the available relays for cooperation. Section V provides numerical results for a number of scenarios and finally conclusions are drawn in section VI.

## II. MIMO Multi-Relay System Model

We consider a wireless multi-hop (MH) relay network where a source node intends to communicate with a destination node using the cooperation of a multiple number of partnering relay nodes where all nodes are equipped with multiple antennas. We assume that there is no direct link between the source and the destination. The source, $\mathcal{S}$, transmits multiplexed data streams to the destination, $\mathcal{D}$, via the assistance of $K$ relays, where the $k$ 'th relay is $\mathcal{R}_{k}$ for $k \in\{1,2, \ldots, K\}$. We assume that the available channel is divided into two orthogonal sub-channels in the time domain. $\mathcal{S}$ communicates with the selected $\mathcal{R}_{k}$ 's in the first time slot. In the second time slot, $\mathcal{R}_{k}$ 's communicate with $\mathcal{D}$.
The channels $\mathcal{S} \rightarrow \mathcal{D}, \mathcal{S} \rightarrow \mathcal{R}_{k}$ and $\mathcal{R}_{k} \rightarrow \mathcal{D}$ are assumed to experience Rayleigh block fading such that they remain constant for the duration of two consecutive time slots. We let the


Figure 1: Relay location setup: 3D Parallel Case.
number of antennas at the source, $k$ 'th relay and destination nodes be $M, N_{\mathcal{R}_{k}}$ and $N$, respectively. The channels among different nodes are assumed to be independent of one another. Similarly, independence is assumed between two consecutive time slot realizations of a channel. We let $\mathbf{H}_{k} \in C^{N_{\mathcal{R}_{k}} \times M}$ and $\mathbf{G}_{k} \in C^{N \times N_{\mathcal{R}_{k}}}$ represent the channel matrices for $\mathcal{S} \rightarrow \mathcal{R}_{k}$ and $\mathcal{R}_{k} \rightarrow \mathcal{D}$ links, respectively. For each link, the channel coefficients are assumed to include the path-loss exponent. That is, the channel coefficient from the $m$ 'th transmitter antenna to $n$ 'th receiver antenna follows the general form of

$$
\begin{equation*}
h(n, m)=l^{-\frac{\delta}{2}} \phi(t), \tag{1}
\end{equation*}
$$

where $\phi(t)$ is a complex Gaussian random variable with zero mean and unit variance and $l$ is the distance between the transmitter and the receiver, and $\delta$ is the path-loss exponent. In this paper, we assume $\delta=4$. The system model considered in this paper is depicted in Figure 1. Noise terms at the front end of relays and the destination node are all assumed to be i.i.d. zeromean circularly symmetric complex Gaussian with $\sigma_{r}^{2}$ and $\sigma_{n}^{2}$ variances, respectively.

We assume that the total available transmit power for the source and the sum of all relay nodes to be $P$ so if $K$ relays are active in transmission, each relay is assumed to have $P_{r}=$ $P / K, \forall k$ as its power constraint. The total transmission powers are assumed to be independent of number of transmit antennas.

## III. AF Relaying Mode with MU-BF and Relay Selection

In this section, we derive the capacity of the multiple antenna multi-relay network when MU-BF is employed at the source node using only the $\mathcal{S} \rightarrow \mathcal{R}_{k}$ CSI and the cooperating relays use the AF protocol. In the AF mode, each relay simply scales the received signal subject to its transmit power limitation and re-transmits it. No demodulation, decoding/re-encoding takes place at the relay nodes.

We investigate conventional MU-BF at the source such that the node multiplexes different data streams, each intended for different directions and each corresponding to one of the selected cooperating relays. Each relay assumes non-intended signals as interference. According to $\mathcal{S} \rightarrow \mathcal{R}_{k}$ CSI at the source, we perform best eigen-mode MU-BF which is conducted on the directions of the eigenvectors corresponding to the maximum eigenvalues of each of the $\mathcal{S} \rightarrow \mathcal{R}_{k}$ links. We
equally distribute transmit power among the channel directions (best eigen-mode beamformers), $\mathcal{S} \rightarrow \mathcal{R}_{k}$ links. Furthermore, we assume that each relay uses matched-filter (MF) to capture the portion of the signal intended for itself.

Let us define the transmitted signal vector as

$$
\begin{equation*}
\mathbf{x}=\sum_{k=1}^{K} \mathbf{w}_{k} s_{k} \tag{2}
\end{equation*}
$$

where $\mathbf{w}_{k}$ 's are the eigenvectors corresponding to the maximum eigenvalues of the matrix ${ }^{1} \mathbf{H}_{k}^{H} \mathbf{H}_{k}$. Note that all matrices are Hermitian matrices, so they are unitarily diagonalizable. We define the generalized transmit covariance matrix as $\mathbf{Q}_{x}=\mathcal{E}\left[\mathbf{x x}^{H}\right]$.

As described in the previous section, in the first time slot, $\mathcal{S}$ transmits meanwhile the relays listen and use MF to capture the signals intended for them. In the second time slot, the cooperating relays, using the available relay-destionation CSI, scale their signal subject to the specified power constraint and transmit this scaled signal via the best-eigen mode that corresponds to the channel between the relay in question and $\mathcal{D}$. We assume that the destination node performs multi-dimensional decoding. However, it is also possible to use simple decoding if we use a Zero-Forcing (ZF) filter at the destination node.
In the first time slot, the signal at the $k$ 'th relay is given by

$$
\begin{equation*}
\mathbf{r}_{k}=\mathbf{H}_{k} \mathbf{x}+\mathbf{n}_{k}=\mathbf{H}_{k} \mathbf{w}_{k} s_{k}+\sum_{\substack{i=1 \\ i \neq k}}^{K} \mathbf{H}_{k} \mathbf{w}_{i} s_{i}+\mathbf{n}_{k} \tag{3}
\end{equation*}
$$

where $\mathbf{n}_{k} \in \mathcal{C}^{N_{\mathcal{R}_{k}} \times 1}$ is temporally and spatially white with $\mathcal{C N}\left(\mathbf{0}, \sigma_{r}^{2} \mathbf{I}_{N_{\mathcal{R}_{k}}}\right)$. Then the signal at the output of the MF at the $k$ 'th relay is

$$
\begin{align*}
u_{k} & =\left(\mathbf{H}_{k} \mathbf{w}_{k}\right)^{H} \mathbf{r}_{k} \\
& =\left\|\mathbf{H}_{k} \mathbf{w}_{k}\right\|^{2} s_{k}+\mathbf{w}_{k}^{H} \mathbf{H}_{k}^{H} \mathbf{H}_{k} \sum_{\substack{i=1 \\
i \neq k}}^{K} \mathbf{w}_{i} s_{i}+\left(\mathbf{H}_{k} \mathbf{w}_{k}\right)^{H} \mathbf{n}_{k}, \tag{4}
\end{align*}
$$

Therefore, the SINR at the output of the MF of the $k$ 'th relay is given as

$$
\begin{equation*}
\Psi_{k}=\frac{\left\|\mathbf{H}_{k} \mathbf{w}_{k}\right\|^{4} P / K}{\sum_{\substack{i=1 \\ i \neq k}}^{K}\left|\mathbf{w}_{k}^{H} \mathbf{H}_{k}^{H} \mathbf{H}_{k} \mathbf{w}_{i}\right|^{2} \frac{P}{K}+\left\|\mathbf{H}_{k} \mathbf{w}_{k}\right\|^{2} \sigma_{r}^{2}} \tag{5}
\end{equation*}
$$

Due to power constraints at the relays, the received filtered signals are scaled to meet the power constraints of these nodes. Scaling factors, $f_{k}$, at the relays are calculated as

$$
\begin{equation*}
f_{k}=\sqrt{\frac{P_{r}}{\left\|\mathbf{H}_{k} \mathbf{w}_{k}\right\|^{4} \frac{P}{K}+\sum_{\substack{i=1 \\ i \neq k}}^{K}\left|\mathbf{w}_{k}^{H} \mathbf{H}_{k}^{H} \mathbf{H}_{k} \mathbf{w}_{i}\right|^{2} \frac{P}{K}+\left\|\mathbf{H}_{k} \mathbf{w}_{k}\right\|^{2} \sigma_{r}^{2}}} \tag{6}
\end{equation*}
$$

${ }^{1 T}$ and ${ }^{H}$ stand for transposition and conjugate transposition. Bold uppercase letters refers to matrix while bold lowercase letters refer to column vector of appropriate dimensions. $\operatorname{tr}(\mathbf{A})$ and $\operatorname{det}(\mathbf{A})$ stand for trace and determinant of the matrix $\mathbf{A}$.

In the second time slot, each of the scaled signals is transmitted via BF to the destination. We assume each relay has forward CSI to form the beam [10]. Best-eigen mode BF is conducted where the BF vector is selected according to the best corresponding eigenvalue. Therefore the BF vector for the $k$ 'th relay node, $\mathbf{v}_{k}$ for $k \in\{1,2, \ldots, K\}$, corresponds to best eigen-mode of the channel $\mathcal{R}_{k} \rightarrow \mathcal{D}$.

Assuming that the BF vectors at the relay nodes all have unit norms, the transmit signals at the relay nodes can be expressed as $\mathbf{t}_{k}=f_{k} u_{k} \mathbf{v}_{k}$. Then the received signal vector at the destination node, assuming relay nodes transmit synchronously, can be expressed as

$$
\begin{align*}
\mathbf{y}= & \sum_{k=1}^{K}\left[\left(\sum_{\substack{i=1 \\
i \neq k}}^{K} \mathbf{O}_{i}\right) \mathbf{w}_{k}+\mathbf{G}_{k} \mathbf{v}_{k} f_{k}\left\|\mathbf{H}_{k} \mathbf{w}_{k}\right\|^{2}\right] s_{k}  \tag{7}\\
& +\sum_{k=1}^{K} \mathbf{G}_{k} \mathbf{v}_{k} f_{k}\left(\mathbf{H}_{k} \mathbf{w}_{k}\right)^{H} \mathbf{n}_{k}+\mathbf{n}_{d}
\end{align*}
$$

where $\mathbf{O}_{k}=\mathbf{G}_{k} \mathbf{v}_{k} f_{k} \mathbf{w}_{k}^{H} \mathbf{H}_{k}^{H} \mathbf{H}_{k}, k \in\{1,2, \ldots, K\}$. If we represent the total noise term as $\mathbf{z}, \mathbf{y}$ can be rewritten compactly as

$$
\begin{equation*}
\mathbf{y}=\mathbf{A s}+\mathbf{z} \tag{8}
\end{equation*}
$$

where $\mathbf{s}=\left[s_{1} \cdots s_{K}\right]^{T}$ is the transmit signal vector and $\mathbf{A} \in$ $\mathbf{C}^{N \times K}$ is the channel matrix for the second time slot and is written as

$$
\begin{align*}
\mathbf{A}= & {\left[\left(\sum_{i=2}^{K} \mathbf{O}_{i}\right) \mathbf{w}_{1}+\mathbf{G}_{1} \mathbf{v}_{1} f_{1}\left\|\mathbf{H}_{1} \mathbf{w}_{1}\right\|^{2}\right) \cdots } \\
& \left.\left(\left(\sum_{i=1}^{K-1} \mathbf{O}_{i}\right) \mathbf{w}_{K}+\mathbf{G}_{K} \mathbf{v}_{K} f_{K}\left\|\mathbf{H}_{K} \mathbf{w}_{K}\right\|^{2}\right)\right] \tag{9}
\end{align*}
$$

In $\mathbf{y}$, the noise term $\mathbf{z}$ is no longer white. Thus, a whitening process is necessary at the receiver of the destination node [11]. Let the whitening matrix be $\mathbf{W} \in \mathbf{C}^{N \times N}$. Then the overall compound channel equation can be written as

$$
\begin{equation*}
\widetilde{\mathbf{y}}=\mathbf{W y}=\mathbf{W A s}+\widetilde{\mathbf{z}} \tag{10}
\end{equation*}
$$

where $\widetilde{\mathbf{z}} \sim \mathcal{C N}\left(\mathbf{0}, \sigma_{n}^{2} \mathbf{I}_{N}\right)$ corresponds to the whitened noise term.

Then, the corresponding ergodic network capacity expression for this system can be calculated as

$$
\begin{equation*}
C_{A F}=\frac{1}{2} \mathcal{E}_{\mathbf{H}_{k}, \mathbf{G}_{k}}\left[\log \operatorname{det}\left(\mathbf{I}_{N}+\frac{P}{K \sigma_{n}^{2}} \mathbf{W} \mathbf{A}(\mathbf{W A})^{H}\right)\right] . \tag{11}
\end{equation*}
$$

## A. Relay Subset Selection for AF Mode

We now investigate the effects of selecting different number of relay nodes for cooperation. The source node multiplexes as many data streams as the selected number of relays. In this subsection, we conduct calculations to assess the achieved gains as the number of cooperating relay nodes is incremented. Note that, we assume that the transmit power available for each relay is given by $P_{r}=P / K$ when $K$ relays are selected to cooperate. It is worthwhile to emphasize once again that we assume the source node does not have $\mathcal{R}_{k} \rightarrow \mathcal{D}$ CSI.

We propose a simple best subset of relay selection mechanism in this paper, where the SINR values are used as the selection metric. Assume that $L$ relay nodes are to be selected out of $K$ relays. We use an exhaustive search to find the best $L$-relays based on this metric. Assume $\pi_{b}=\left[\pi_{b}(1), \pi_{b}(2), \ldots, \pi_{b}(L)\right]$, $b \in\{1,2, \ldots, B\}$, is one of the possible combinations for the relay selection where each element corresponds to a selected relay. Assuming the source uses MU-BF for each multiplexed data steam, the transmit signal can be written as

$$
\begin{equation*}
\mathbf{x}_{b}=\sum_{i=1}^{L} \mathbf{w}_{\pi_{b}(i)} s_{i} \tag{12}
\end{equation*}
$$

where $\mathbf{w}_{\pi_{b}(i)}$ is the best eigen-value corresponding eigenvector for the link $\mathcal{S} \rightarrow \mathcal{R}_{\pi_{b}(i)}$. Following the procedure described in the previous section, the SINR at the MF output of the $\pi_{b}(k)$ 'th relay is calculated as

$$
\begin{equation*}
\Psi_{\pi_{b}(k)}=\frac{\left\|\mathbf{H}_{\pi_{b}(k)} \mathbf{w}_{\pi_{b}(k)}\right\|^{4} P / L}{\sum_{\substack{i=1 \\ i \neq k}}^{L}\left|\mathbf{w}_{\pi_{b}(k)}^{H} \mathbf{H}_{\pi_{b}(k)}^{H} \mathbf{H}_{\pi(k)} \mathbf{w}_{\pi_{b}(i)}\right|^{2} \frac{P}{L}+\Delta_{k}} \tag{13}
\end{equation*}
$$

where $\Delta_{k}=\left\|\mathbf{H}_{\pi_{b}(k)} \mathbf{w}_{\pi_{b}(k)}\right\|^{2} \sigma_{r}^{2}$ for $k \in\{1,2, \ldots, L\}$. We select the relay combination, $\pi_{b^{*}}$, such that the maximum sum of SINR's over all selected relays is achieved. In other words, we select the group of relays, $\pi_{b^{*}}$, such that

$$
\begin{equation*}
\pi_{b^{*}}=\arg \max _{\pi_{b}: b \in\{1,2, \ldots, B\}}\left\{\sum_{k=1}^{L} \Psi_{\pi_{b}(k)}\right\} \tag{14}
\end{equation*}
$$

## IV. DF Relaying Mode with MU-BF and Relay Selection

We now determine the capacity of multiple antenna equipped multi-relay networks employing MU-BF at the source node and the DF protocol at the selected cooperating relays. We assume that the destination node employs ZF decoder and therefore a maximum of $N$ relays may be selected. In this mode, we select a subset of the relays such that each selected relay can perfectly decode its intended signal. As in the previous case, the best capacity achieving subset of $L$-relays is selected for cooperation. The remaining relays remain silent throughout the communication period.

As in AF mode, we propose to use the MU-BF scheme where we assign a single data stream for each of the selected relay nodes under the assumption that the selected number of relays, $L$, is less then number of antennas at the destination, $N$. We assume that the source has instantaneous CSI of $\mathcal{S} \rightarrow \mathcal{R}_{k}$, $k=1,2, \ldots, K$ and that relays have forward CSI and they beamform on the best eigen-mode corresponding to their link to the destination node. Thus, the transmit signal model and the corresponding BF vectors at the source node are identical to the AF case. Similarly, we assume that the BF vector from the $k$ 'th relay, $\mathbf{v}_{k}$ for $k \in\{1,2, \ldots, K\}$, corresponds to best eigen-mode of the link $\mathcal{R}_{k} \rightarrow \mathcal{D}$. Assuming that the BF vectors at relays have unit norms, the transmit signals at relays can
be expressed as

$$
\begin{equation*}
\mathbf{t}_{k}=\sqrt{\frac{L P_{r}}{P}} \mathbf{v}_{k} s_{k} \tag{15}
\end{equation*}
$$

where we assume that $s_{k}$ is the signal intended for relay $k$.
In the first part of transmission, we different data streams are transmitted to different relays. Since a ZF filter is assumed to be employed at the destination node, it is possible to evaluate the individual capacities of each of the data streams from the selected relay nodes to the destination node independently resulting in a less complex decoding process. Assuming $L \leq N$ relays participate in the communication between the source and the destination nodes and $\pi \in \mathbf{R}^{L \times 1}$ holds the selected relay indexes, the composite received signal vector, $\mathbf{y}$ at the destination node can be written as

$$
\begin{align*}
\mathbf{y} & =\sum_{k=1}^{L} \mathbf{G}_{\pi(k)} \mathbf{t}_{\pi(k)}+\mathbf{n}_{d}=\sum_{k=1}^{L} \sqrt{\frac{L P_{r}}{P}} \mathbf{G}_{\pi(k)} \mathbf{v}_{\pi(k)} s_{k}+\mathbf{n}_{d} \\
& =\sqrt{\frac{L P_{r}}{P}}\left[\begin{array}{lll}
\mathbf{G}_{\pi(1)} \mathbf{v}_{\pi(1)} & \ldots & \left.\mathbf{G}_{\pi(L)} \mathbf{v}_{\pi(L)}\right] \mathbf{s}+\mathbf{n}_{d} \\
& =\mathbf{G}_{\pi} \mathbf{s}+\mathbf{n}_{d}
\end{array}\right.
\end{align*}
$$

where $\mathbf{s}=\left[s_{1}, s_{2}, \ldots, s_{L}\right]^{T}$ is the multiplexed transmit signal vector at the source node, and $\mathbf{G}_{\pi} \in C^{N \times L}$ is the compound channel. Then the corresponding ZF matrix at the destination node is given by

$$
\begin{equation*}
\mathbf{G}_{\pi}^{\dagger}=\left(\mathbf{G}_{\pi}^{H} \mathbf{G}_{\pi}\right)^{-1} \mathbf{G}_{\pi}^{H} \tag{17}
\end{equation*}
$$

resulting in an output sequence of

$$
\begin{equation*}
\mathbf{y}^{\prime}=\mathbf{G}_{\pi}^{\dagger} \mathbf{y}=\mathbf{s}+\mathbf{G}_{\pi}^{\dagger} \mathbf{n}_{d} . \tag{18}
\end{equation*}
$$

Then, the SNR for $k$ th data stream can be expressed as

$$
\begin{equation*}
\Gamma_{k}=\frac{P / L}{\sigma_{n}^{2}\left\|\mathbf{G}_{\pi}^{\dagger}\right\|_{k}^{2}} \tag{19}
\end{equation*}
$$

where $\left\|\mathbf{G}_{\pi}^{\dagger}\right\|_{k}^{2}$ is the norm of $k$ th row of matrix $\mathbf{G}^{\dagger}$. Therefore, the network capacity can be calculated as

$$
\begin{equation*}
C_{D F}=\frac{1}{2} \mathcal{E}_{\mathbf{H}_{k}, \mathbf{G}_{k}}\left[\sum_{k=1}^{L} \min \left\{\log _{2}\left(1+\Psi_{k}\right), \log _{2}\left(1+\Gamma_{k}\right)\right\}\right] \tag{20}
\end{equation*}
$$

where $\Psi_{k}$ is the SINR observed at the $k$ 'th relay node and $\Gamma_{k}$ is the SNR at the destination node for $k$ 'th data stream for $k=$ $1,2, \ldots, L$.

## A. Relay Subset Selection for DF Mode

Since the signal models are identical at the relay nodes, the relay subset selection for DF mode is the same as the AF mode. In the following section we will observe that once a sufficient number relays is selected such that the maximum achievable multiplexing gain is obtained for the system under investigation, selecting additional relays do not provide any significant gains on the network capacity.

## V. Simulation Scenarios and Numerical Results

As shown in Figure 1, we consider a network geometry where all relay nodes are equidistant from the source node and also equidistant from the destination node. In Figure 1, we let $d_{1}=0.5$ and assume the distance between the source and the destination node is normalized to unity, $l_{s d}=1$.

In Figures 2 and 3, the network ergodic capacities of the system when AF and DF relaying modes are employed are plotted, respectively, for $M=N_{\mathcal{R}_{k}}=N=8$ and $P / \sigma_{n}^{2}=10 \mathrm{~dB}$, as a function of the projected distance between the source node and the relay nodes, $d_{2}$. For comparison, the ergodic capacity results for AF and DF modes when no CSI is available at the source node (NCSIT) is included in the plots. The plots include curves for different number of selected cooperating relay nodes. In Figure 2, we observe that selecting more nodes, and thus multiplexing the transmitted signal into more parallel streams, a capacity increase is observed. However, the amount of increase gets smaller for each additional selected relay. We observe that using a subset of the available relays as opposed to using the complete set, results in an inferior performance since the total available multiplexing gain is not fully utilized in this case. For the DF mode, we observe in Figure 3 that when no CSI is available at the source node, (NCSIT-DF) and random relay selection is employed, selecting only one relay results in the best performance compared to other cases. When limited CSI is available and BF is employed, (BF-DF), we can observe that capacity increase is achieved until a maximum of two relays are selected for cooperation. Using more relays results in a reduction in the capacity. This is due to the presence of the total power constraint for the relay nodes imposed in the system model, and the fact that the gain provided by BF in the DF mode is mostly dominated by power sharing.

In Figure 4, network ergodic capacities of the AF and DF relaying modes are plotted for $M=N_{\mathcal{R}_{k}}=N=8$ and $P / \sigma_{n}^{2}=10 \mathrm{~dB}$ as a function of the number of selected relay nodes. The solid and dotted curves provide results for the AF and DF modes, respectively. It is observed that incrementally selecting more relays builds up the equivalent channel rank of the system and thus multiplexing gains are observed in the AF mode; but after one exceeds the highest possible system rank, it is possible to obtain only a slight gain by adding more relays for cooperation. For DF mode on the other hand, selecting two relays provides us the highest capacity. Further increases in the employed number of relays reduce the overall capacity since the total power limitation in the relays cause reductions in the capacity of the individual streams from the relays.

## VI. Conclusions

In this paper we study the network capacities for a system employing MU-BF at the source node and AF and DF modes at the cooperating relay nodes when multiple antennas are available at all nodes. A TDMA-based cooperation protocol is assumed. The source has only $\mathcal{S} \rightarrow \mathcal{R}_{k}$ CSI knowledge and relays have both forward and backward CSI. We investigate the effect of relay selection based on a simple best sum SINR value for both AF and DF modes. The proposed BF at the source node pro-
vides multiplexed data streams, each indented for a different cooperating relay node. When the relays use AF, they employ MF and transmit the received signal after scaling. In this case, we observe that using more relay nodes builds up the channel rank and increases the overall capacity. However, the capacity increase is smaller as more relays are included in the communication. On the other hand, when the relays employ DF, utilization of the complete set of relays is not desirable when a total power limitation is invoked on the relays. For the system under investigation, we observe that selecting two relays yields the highest capacity.

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Figure 2: Network capacities for NCSI-AF and incremental relay selection BF-AF modes are given. 8 Relay nodes and $M=N=N_{\mathcal{R}_{k}}=8, P / \sigma_{n}^{2}=10 \mathrm{~dB}$.


Figure 3: Network capacities for 8 Relays and $M=N=$ $N_{\mathcal{R}_{k}}=8$ for DF Upper Bound, NCSIT-DF and BF-DF relaying modes. $P / \sigma_{n}^{2}=10 \mathrm{~dB}$.


Figure 4: Network capacities for incremental relay selection BF-AF modes are given. 8 Relay nodes and $M=N=N_{\mathcal{R}_{k}}=$ $8, P / \sigma_{n}^{2}=10 \mathrm{~dB}$.

