BINARY POWER CONTROL FOR MULTI-CELL CAPACITY MAXIMIZATION

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ABSTRACT

We consider the problem of optimally allocating the base station transmit powers for a wireless multi-cellular (N-cell) system in order to maximize the total system throughput under interference and noise impairments, and short term (minimum and peak) power constraints. Employing dynamic reuse of spectral resources, we impose the power constraints at each base station and allow for coordination between the base stations. For the two-cell case, the capacity-optimal power allocation has been previously shown to be binary [1]. We now propose to perform binary power allocation, (by simply checking the corners of the domain resulting from the power constraints), also when $N > 2$, and we identify two scenarios in which the optimality of binary power control can be proven also for arbitrary N. Furthermore, in the general setting for $N > 2$, we demonstrate by simulations that a network performance with negligible loss, compared to the best non-binary scheme found by geometric programming, can be obtained.

1. INTRODUCTION

The need for higher spectrum efficiency motivates the search for system-wide optimization of the wireless resources. A key example of multi-cell resource allocation is that of power control, which serves as means for both battery savings at the mobile, and interference management. Traditional power control solutions are designed for voice-centric networks, hence aiming at guaranteeing a target signal-to-noise-and-interference ratio (SNIR) level to the users [2]. In modern wireless networks, link adaptation [3] is or will be implemented, and throughput maximization becomes a more relevant metric.

The simultaneous optimization of transmission rates and power with the aim of maximizing the multi-cell sum capacity is a difficult problem. Considering the problem of optimally allocating the transmit power for N concurrent communication links, a common approach is to use a high SNIR approximation to establish convexity in the sum-capacity objective function [4, 5]. However, this approximation by construction prohibits completely turning off the power of any base station at any time. This extra constraint may in fact cause the resulting power vector to steer away from the optimum solution. Indeed one of the major points made in this work is that the ability of shutting down one or more base stations can be instrumental in approaching maximum network capacity.

Restricting the scenario to interference limited systems, i.e., neglecting noise sources, in [5] the high SNIR assumption is used only for a set of active links, but although improving over the schemes presented in [4], the method "is still inferior to maximization of the actual aggregate throughput" [5]. Under a sum power constraint, [6] neglects noise sources and uses waterfilling to maximize the network capacity. But, due to the sum power constraint and neglection of noise, these results are not applicable to our analysis. We argue that for cellular networks, applying an individual power constraint at every base station is more realistic.

When modeling the transmission rate as a *linear* function of the received power, [7] shows that a base station when on should transmit at maximum power for optimality. This result has the merit of showing potential benefits of an on/off power control, but in general, the assumed linear relationship between rate and power is however unfortunately far from the truth since the rate is known to have a $\log(\cdot)$ behavior. The proof does not extend to arbitrarily increasing rate-power relations, and the results will not in general yield throughput-optimal power allocation. Nevertheless, here we show that when using a *low SNIR approximation*, the linear relation in [7] is indeed obtained, and thus the conclusions from that paper holds in this case, and can be extended to include a minimum power constraint at each base station.

In this paper we tackle the problem of power allocation in cellular networks without resorting to the restricting assumptions of high SNIR or interference-limited systems. The application we have in mind is a wireless data access network with best-effort type of quality-of-service, and where the total throughput across the network is the figure of merit. The system is assumed to be enabled with perfect link adaptation, so the user rate is adapted as a function of the user's SNIR as to always achieve Shannon capacity in any resource slot.

Our contributions are as follows: We consider the $N > 2$ cell case, and show that when either a geometric-arithmetic mean or a low-SNIR approximation is applicable binary power control[1] is still optimal (as always true for any SINR in the

[1] On/off power control and binary power control are equivalent if the minimum transmit power is zero.

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$N = 2$ case). In the general case for $N > 2$, we utilize the mathematical framework of geometric programming (GP) [8] in order to establish a sum capacity benchmark, and compare with our proposed binary power allocation through exhaustive simulations. Empirically, we demonstrate that the loss associated with restriction to binary power levels is negligible. On the other hand, discretizing the optimization space is highly beneficial: the feedback rate needed to communicate between network nodes is reduced, transmitter design is simplified, and finally, limiting the potential solutions to search over better facilitates distributed resource allocation [9].

The remainder of the present paper is organized as follows. We introduce the wireless system model under investigation in Section 2. In Section 3 we derive optimal power control schemes for sum throughput maximization. In Section 4 numerical results are presented, and finally conclusions are given in Section 5.

2. SYSTEM MODEL

We consider a system in which N neighboring base stations communicate with mobile terminals over a coverage area. In each cell, we consider an orthogonal multiple access scheme such that in any given *spectral resource slot* (where resource slots can be time or frequency slots in TDMA/FDMA, or codes in orthogonal CDMA) a single user is supported. The spectral resource slots are shared by all cells, leading in general to an interference and noise impaired system. Scheduling policies can in principle also be incorporated, by suitably modifying the users' channel statistics. However, in order to focus solely on power control, we do not explicitly consider scheduling here. The communication links are considered to be downlink, but the results can also be generalized to an uplink scenario. Furthermore, we emphasize that our analysis is valid for any geometry, even for non-cellular systems such as ad-hoc networks, as long as the sum of link capacities is an relevant performance metric.

The data destined for user u_n is transmitted with power P_n. Each base station is in general assumed to operate under both minimum and peak power constraints,

$$P_{\text{min}} \leq P_n \leq P_{\text{max}}, \quad n = 1, 2, \ldots, N. \tag{1}$$

Denote by $G_{n,i}(m)$ the channel power gain to the selected mobile user $u_n(m)$ in cell n from the cell i base station in slot m. We will suppress the slot index from now on, concentrating on one arbitrary slot. The channel gains are assumed to be constant over each such resource slot, i.e., we have a block fading scenario. Note that the gains $G_{n,n}$ correspond to the desired communication links, whereas the $G_{n,i}$, for $n \neq i$, correspond to the unwanted interference links. Assuming the transmitted symbols to be independent random variables with zero mean and unit variance, the signal to noise-plus-interference ratio (SNIR) for each user is given by

$$\text{SNIR}_{u_n} = \frac{P_n G_{n,n}}{\sigma^2_{Z_n} + \sum_{j \neq n} P_j G_{n,j}}, \tag{2}$$

where $\sigma^2_{Z_n}$ is the variance of the independent zero-mean additive white gaussian noise (AWGN) in cell n.

Under the assumption that capacity-achieving codes for AWGN channels can be employed, the achievable rate (in information bits/s/Hz) of user u_n is given by

$$R_{u_n} = \log_2(1 + \text{SNIR}_{u_n}). \tag{3}$$

From (2), (3) the total achievable throughput (sum rate) $R = \sum_{n=1}^{N} R_{u_n}$ is then found as

$$R = \sum_{n=1}^{N} \log_2\big(1 + \frac{P_n G_{n,n}}{\sigma^2_{Z_n} + \sum_{j \neq n} P_j G_{n,j}}\big). \tag{4}$$

Finally, we note that our system model with (possibly different) noise levels $\{\sigma^2_{Z_n}\}_{n=1}^{N}$ also accommodates the modeling of additional interfering sources disturbing the users differently, contrary to previous works. One important application of this is when, for complexity reduction, the joint multi-cell power allocation is undertaken for a subnet (cluster) of neighboring cells only. In this case $\sigma^2_{Z_i}$ represents the combined effect of noise and interference received from out-of-cluster cells by the i^{th} user.

3. TRANSMIT POWER ANALYSIS

This section presents the general optimal power allocation scheme $\mathbf{P}^* = (P_1^*, \ldots, P_N^*)$, which has as inputs the channel gains $\{G_{n,i} > 0\}$, and the AWGN variances $\{\sigma^2_{Z_n} > 0\}$. We search for the optimal power allocation by approaching the following optimization problem,

$$\mathbf{P}^* = \arg \max_{\mathbf{P} \in \Omega^N} R, \tag{5}$$

where $\Omega^N = \{\mathbf{P} | \, P_{\text{min}} \leq P_n \leq P_{\text{max}}, n = 1, \ldots, N\}$ is the feasible set and R is given in (4). Since Ω^N is a closed and bounded set and $R : \Omega^N \to \mathbb{R}$ is continuous, (5) has a solution. Note that for $P_{\text{min}} = 0$, Ω^N admits solutions where some base stations shut down the power completely, which from a cellular engineering point of view can be interpreted as a form of dynamic channel reuse.

3.1. The 2-Cell Case

For the two-cell case ($N = 2$) it is possible to find a closed form solution by an analytical derivation [1][2]. Defining $\Delta\Omega^2 = \{(P_{\text{max}}, P_{\text{min}}), (P_{\text{min}}, P_{\text{max}}), (P_{\text{max}}, P_{\text{max}})\}$, the optimal power allocation can then be stated as

$$(P_1^*, P_2^*) = \arg \max_{(P_1, P_2) \in \Delta\Omega^2} R(P_1, P_2), \tag{6}$$

Hence, for a two-cell system the optimal power control is *binary*, depending on the noise and channel gains, transmit at full power only at base station 1, and minimum power at base station 2, vice versa, or both at full power.

[2]In [1] this result was derived for the case of $P_{\text{min}} = 0$, but the result is extended here to $P_{\text{min}} > 0$. Letting $P_{\text{min}} > 0$ might be necessary in some scenarios to ensure that all users receive a minimum transmission, such as control information or pilot symbols.

3.2. Binary Power Control in the N-cell Case

For $N > 2$, analytical treatment of the optimization problem (5) proves to be very challenging, because of the lack of convexity and the fact that the above analysis from the two-cell case does not generalize to N cells. However, motivated both by the optimality of binary power allocation for the two-cell case and its potential in complexity reduction and finding distributed solutions, we will investigate the properties of binary power control also in the N-cell case.

Binary power control for N cells is done by evaluating $R(\mathbf{P})$ at the corners of Ω^N, and picking the maximum value. Mathematically formulated,

$$\mathbf{P}_{\text{bin}} = \arg \max_{\mathbf{P} \in \Delta\Omega^N} R(\mathbf{P}), \tag{7}$$

where $\Delta\Omega^N$ is the set of $2^N - 1$ corner points of Ω^N, excluding the all-P_{min} point. We now proceed by considering binary power control for N cells in three cases, 1) approximation by the arithmetic-geometric means inequality, 2) the low-SNIR regime, and 3) the general case.

3.2.1. *Arithmetic mean-geometric mean approximation*

Writing (4) as the log of products, we can apply the arithmetic-geometric means inequality to obtain

$$\begin{aligned} R(\mathbf{P}) &= \log_2 \left(\prod_{n=1}^{N} 1 + \frac{P_n G_{n,n}}{\sigma_{Z_n}^2 + \sum_{j \neq n} P_j G_{n,j}} \right) \\ &\leq N \log_2 \left(1 + \frac{1}{N} \sum_{n=1}^{N} \frac{P_n G_{n,n}}{\sigma_{Z_n}^2 + \sum_{j \neq n} P_j G_{n,j}} \right). \end{aligned} \tag{8}$$

Now, in scenarios where the right hand side of the above inequality can be used as an *approximation* of $R(\mathbf{P})$, i.e.,

$$R(\mathbf{P}) \approx N \log_2 \left(1 + \frac{1}{N} \sum_{n=1}^{N} \frac{P_n G_{n,n}}{\sigma_{Z_n}^2 + \sum_{j \neq n} P_j G_{n,j}} \right), \tag{9}$$

the optimization problem (5) simplifies, and we can analytically find a closed form solution. As always true in the two-cell case, the optimal power control is binary when (9) holds.

Theorem 1. *The sum throughput maximizing power control is binary for any N when* (9) *holds.*

Proof. Due to the monotonicity of the log-function, we establish the result by showing that the argument of the logarithm on the right hand side of (9) is convex in each variable P_k.

$$\begin{aligned} &\frac{\partial^2}{\partial P_k^2} \left(1 + \frac{1}{N} \sum_{n=1}^{N} \frac{P_n G_{n,n}}{\sigma_{Z_n}^2 + \sum_{j \neq n} P_j G_{n,j}} \right) = \\ &\frac{1}{N} \sum_{n \neq k} \frac{2 P_n G_{n,n} G_{n,k}^2}{(\sigma_{Z_n}^2 + \sum_{j \neq n} P_j G_{n,j})^3} \geq 0. \end{aligned} \tag{10}$$

Now, for any $\mathbf{P}$ where at least one of its components is not an endpoint of its interval, there is another point $\mathbf{P}'$ with $R(\mathbf{P}') \geq R(\mathbf{P})$ such that one more component is at an endpoint of its interval. □

For more details on the arithmetic-geometric mean approximation and its applications to power control, see [10].

3.2.2. *Low-SNIR regime*

In the low-SNIR regime we can apply a first order Taylor approximation of the achievable rate, thus simplifying the problem, specifically for low SNIR: $\log_2(1 + \text{SNIR}) \approx \frac{\text{SNIR}}{\ln 2}$. Thus, we have

$$R(\mathbf{P}) \approx \frac{1}{\ln 2} \sum_{n=1}^{N} \frac{P_n G_{n,n}}{\sigma_{Z_n}^2 + \sum_{j \neq n} P_j G_{n,j}}, \tag{11}$$

and again find that binary power control is optimal, which is easily seen from the proof of Theorem 1[3]. In fact, the objective function obtained by both the low-SNIR approximation and the arithmetic-geometric means approximation is maximized by the same binary power values.

In the low-SNIR case the binary power allocation is also optimal for a weighted sum rate criterion, $R_{\text{w}} = \sum_{n=1}^{N} w_n R_n$, $w_n \geq 0$, which we state as a corollary.

Corollary 1. *In the low-SNIR regime, for a weighted sum rate criterion, the sum throughput maximizing power control is binary.*

Proof. The result follows by the rules of differentiation. □

3.2.3. *General case*

In the general case, when none of the above approximations hold, we have not found mathematical relations establishing the performance of binary power control, and hence we resort to exhaustive numerical simulations, trying to cover typical settings for cellular networks. To evaluate the performance of binary power control against a non-binary benchmark we capitalize on geometric programming [8], as discussed next.

3.3. Geometric Programming Power Control

As the yardstick for binary power control performance we use power control by geometric programming. First, we provide a brief background on geometric programming [8]. A *monomial* is a function $f : \mathbf{R}_{++}^n \rightarrow \mathbf{R}$: $g(\mathbf{P}) = c P_1^{a^{(1)}} \cdots P_n^{a^{(n)}}$, where $\mathbf{R}_{++}^n$ is the strictly positive quadrant of $\mathbf{R}^n$, $c > 0$ is

[3]For the case of $P_{\text{min}} = 0$, this was independently reported also by the authors in [11]. Further, in [7], also with $P_{\text{min}} = 0$, the optimization problem (5) with R as a linear function of the received power, similar to (11), is considered and an alternative proof for on/off power control is given.

a constant, and $a^{(i)} \in \mathbf{R}$, $i = 1, \ldots, n$. A sum of monomials is called a *posynomial*:

$$f(\mathbf{P}) = \sum_{k=1}^{K} c_k P_1^{a_k^{(1)}} P_2^{a_k^{(2)}} \cdots P_n^{a_k^{(n)}}. \tag{12}$$

Then, a geometric program in standard form is written as:

$$\begin{aligned} \text{minimize} \quad & f_0(\mathbf{P}), \\ \text{subject to} \quad & f_i(\mathbf{P}) \le 1, \; i = 1, \ldots, I \\ & g_m(\mathbf{P}) = 1, \; m = 1, \ldots, M, \end{aligned} \tag{13}$$

where $f_i,\ i = 0, \ldots, I$ are posynomials and $g_m,\ m = 1 \ldots M$ are monomials. Now, using the results in [8], the optimization problem in (5) can formulated as follows

$$\begin{aligned} \text{minimize} \quad & \prod_{n=1}^{N} \frac{1}{1 + \mathrm{SNIR}_{u_n}}, \\ \text{subject to} \quad & \frac{P_n}{P_{\max}} \le 1, \; n = 1, \ldots, N, \\ & \frac{P_{\min}}{P_n} \le 1, \; n = 1, \ldots, N. \end{aligned} \tag{14}$$

Inspecting (14), we see that the constraints are monomials (and hence posynomials), but the objective function is a *ratio* of posynomials, as shown by

$$\prod_{n=1}^{N} \frac{1}{1 + \mathrm{SNIR}_{u_n}} = \prod_{n=1}^{N} \frac{\sigma_{Z_n}^2 + \sum_{j \neq n} G_{n,j} P_j}{\sigma_{Z_n}^2 + \sum_{j=1}^{N} G_{n,j} P_j}, \tag{15}$$

and the fact that posynomials are closed under multiplication. Hence, (14) is not a GP in standard form, but a *signomial* programming (SP) problem [8]. SP problems can be solved by solving a series of GPs [8]. To transform it to standard form we follow [8], approximating the denominator posynomial of (15) by a monomial. Specifically, denote the denominator posynomial of (15) as $g(\mathbf{P})$. Since a posynomial is a sum of monomials, write $g(\mathbf{P}) = \sum_i u_i(\mathbf{P})$ where $u_i(\mathbf{P})$ is a monomial. Then, $g(\mathbf{P})$ can be well approximated with a monomial $\tilde{g}(\mathbf{P})$ as follows

$$g(\mathbf{P}) \ge \tilde{g}(\mathbf{P}) = \prod_i \Big(\frac{u_i(\mathbf{P})}{\alpha_i} \Big)^{\alpha_i}, \tag{16}$$

where $\alpha_i = u_i(\mathbf{P})/g(\mathbf{P})$. By using (16), (15) is now a ratio of a posynomial and a monomial. This ratio is again a posynomial, and hence (14) is approximated and transformed to standard form, and can be solved using GP techniques.

4. NUMERICAL RESULTS

Based on the system model described in Section 2, we will now through Monte Carlo simulations evaluate a cellular system, assuming that the users are uniformly distributed in each cell, following the spatial channel models developed in [12].

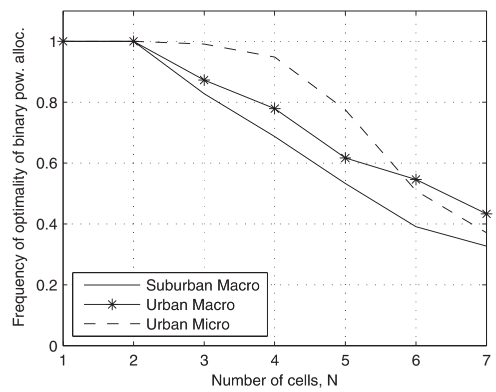

Fig. 1. Frequency of optimality of binary power allocation, relative to optimal (GP) power allocation.

Specifically, the following environments are considered: the suburban macrocell, the urban macrocell, and the urban microcell line of sight (LOS). In the macrocell environments the maximum transmit power $P_{\max} = 10$ W, while for the microcell setting $P_{\max} = 1$ W. All three scenarios are simulated with $P_{\min} = 0$ W. Depending on the model, BS-to-user distances should exceed $20 - 35$ m, thus we exclude users from being located in a circular disk of radius $20 - 35$ m around each base station.

4.1. Network Capacity Statistics

For the three simulation settings, Table 1 depicts the average per-cell capacity, defined as $\frac{R}{N}$, obtained by using GP and binary power control, and as a reference full power in all cells. It is clear that introducing power control improves the throughput performance, in particular for the urban microcell environment. However, note the only marginal improvement in going from binary power control to optimal GP power control based on geometric programming. As seen from the table, the average per cell capacity decreases as the number of cells increase. This is to be expected since all cells share the same spectral resources. As an example of how instrumental it is to be able to operate some cells at minimum power, we see that the *system capacity* in the urban microcell environment is less for two cells than for one cell using full power. However, using binary and GP power control, we observe an *increase* in system capacity, due to better management of interference.

In Fig. 1 we have plotted the frequency of optimality of binary power control, i.e., how often binary power control is optimal. It is seen that for one and two cells, binary power control is indeed always optimal, while for more than two cells it is no longer so. However, as shown in Table 1, the gap between the optimal (GP) power control and the suboptimal binary power control is still marginal. This demonstrated near-optimality of binary power control has several implications in the design and analysis of wireless networks. First, the complexity transmitter design is reduced since only a two

Table 1. Network capacity statistics

	Average pr. cell capacity (bits/s/Hz) shown in (GP, Binary, Full) triplets		
Number of cells (N)	Suburban Macro	Urban Macro	Urban Micro
1	$(6.02, 6.02, 6.02)$	$(5.13, 5.13, 5.13)$	$(11.96, 11.96, 11.96)$
2	$(4.93, 4.93, 4.74)$	$(4.40, 4.40, 4.27)$	$(6.64, 6.64, 4.54)$
3	$(4.41, 4.40, 4.02)$	$(4.03, 4.03, 3.75)$	$(6.03, 6.03, 3.39)$
4	$(4.03, 4.01, 3.53)$	$(3.70, 3.69, 3.33)$	$(4.66, 4.65, 2.91)$
5	$(3.98, 3.95, 3.45)$	$(3.68, 3.67, 3.28)$	$(3.88, 3.85, 2.75)$
6	$(3.81, 3.78, 3.25)$	$(3.54, 3.53, 3.11)$	$(3.41, 3.36, 2.58)$
7	$(3.67, 3.64, 3.08)$	$(3.42, 3.41, 2.97)$	$(3.06, 3.00, 2.40)$

level power control is required. Second, when searching for distributed power control algorithms, binary power control provides a key simplification of the problem by enabling *distributed* control of the power allocation [9].

5. CONCLUSIONS

We have analyzed transmit power allocation for an N-cell wireless system under a sum-capacity maximization criterion and minimum and peak power constraints at each base station. Assuming perfect channel gain information to be available, we investigated the system capacity without power control, with binary power control, and with (optimal) GP-based power control. We show that the optimal power control is binary for two cells, as well as when the network throughput can be approximated either by low-SNIR assumption or by a geometric-arithmetic means inequality. In the general case when $N > 2$, it was demonstrated by computer simulations that when restricting ourselves to binary power levels only a negligible loss occurs. For practical systems, these results are of importance since the transmitter design is simplified and the search for distributed algorithms becomes more manageable.

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