Spatial Throughput Of Multi-hop Wireless Networks Under Different Retransmission Protocols^{*}

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Abstract

In this work, we are interested in characterizing the performance of decentralized multiple-access and retransmission schemes for multi-hop wireless networks. We assume that nodes are randomly distributed on the plane according to a homogeneous spatial Poisson process. We investigate two transmission strategies: one that maximizes the distance of a successful transmission for each hop towards the final destination; and the other that forwards packets to the closest node in range in the direction of the final destination where transmission is successful. This is useful in the context of multi-hop wireless networks as it is important to assess the tradeoff between spatial density of communication and the range of each transmission. The results of this work also show that coding and retransmissions provide means of reliable communication coupled with a completely decentralized multiple-access strategy.

1 Introduction

The study of wireless ad hoc networks has recently received significant attention. An ad hoc network is a collection of wireless nodes forming a network without the use of any existing network infrastructure or centralized coordination, where communication between any two nodes can either be direct or relayed through other nodes. This lack of any centralized control gives rise to many issues at the physical layer which make the analysis of such networks complex. In [1], Gupta and Kumar determined the capacity of wireless networks under certain assumptions and point out a basic behavior of current wireless networks. They showed that given n nodes in the unit disk and an uniform traffic pattern, the aggregate capacity is of $\Theta(\sqrt{n})$, thus resulting in a vanishing pernode capacity as the network population grows. In [2], the Gupta Kumar model was modified to take into account mobility and using only one-hop relaying, it was shown that an $\Theta(n)$ aggregate throughput can be obtained. These results provide expressions

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for the ad hoc network capacity and determine the scalability of such networks as the number of nodes increases to infinity.

In constrast to these macroscopic studies, in this work we are focus on a microscopic analysis of decentralized ad hoc wireless networks ruling out the possibility of coordination between nodes (e.g. TDMA-based exclusion techniques). In [3], a different approach illustrates the impact of an exponentially decaying traffic pattern and the relay load on the throughput in the context of a decentralized system with retransmission protocols. We jointly address the properties of the physical and the data link layer in the design of the media-access control (MAC) protocol. We provide a setting to characterize the performance of such networks. We assume that nodes access the channel at random and employ simple protocols to retransmit the erroneously received packets. We consider two possible retransmission protocols: the first is *Slotted Aloha* (using the wireless setting as described in [4]) where decoding considers only the most recent received block; the second is *Incremental Redundancy* where decoding takes into account all previously received signal blocks and performs soft combining until decoding is achieved successfully. We compare these strategies to the generalization of the collision channel without feedback or delay constraints [5], where the measure of success of a transmission will be an achievable ergodic throughput of this channel as it will be seen later.

For this analysis, the nodes are taken to be spatially distributed in the plane according to a homogeneous spatial Poisson process which leads to a new representation of interference and collisions between concurrent transmissions. To derive the throughput, we follow the analysis of Nelson and Kleinrock in [6] where they studied the spatial capacity of a slotted Aloha multi-hop network with capture. The spatial throughput is computed in terms of the product of the number of the simultaneously successful transmissions per unit area by the average jump (or expected forward progress) made by each transmission. We carry out its optimization with respect to the channel access probability p. The relationship between the spatial throughput and the Gupta-Kumar transport capacity is described in [7]. For the purpose of comparison, we consider two routing strategies, one that maximizes the expected forward progress, and the other that relays packets to the closest node in range at each hop towards the final destination for successful transmission.

The outline of the paper is as follows: In section 2, we describe the system model. Section 3 deals with the retransmission protocols. In Section 4, throughput expressions are derived and we show some numerical results. Finally, in Section 5 we draw some conclusions and point out future research directions.

2 System Model and Setting

2.1 Network and Propagation Model

We assume that nodes are distributed according to a Poisson point process on the plane with node density σ . This topology represents an instantaneous snapshot of a mobile network of nodes. Then, for any region **S** of area $A(\mathbf{S})$, the number of nodes in the region has a Poisson distribution with parameter $\sigma A(\mathbf{S})$, i.e.,

$$\Pr[k \text{ in } \mathbf{S}] = e^{-\sigma A(\mathbf{S})} \frac{(A(\mathbf{S}))^k}{k!}$$
(1)

The propagation model is described by two effects: the signal attenuation due to the distance r between the transmitter and the receiver, proportional to $r^{-\alpha}$, where α is

the power loss exponent (positive number) ; and Rayleigh fading that causes random power variations. The received power P_R from a mobile at distance r is expressed as $P_R = R_a^2 r^{-\alpha} P = \gamma r^{-\alpha} P$ where R_a is a Rayleigh distributed random variable (with unit power for simplicity), γ is an exponentially distributed random variable (γ has mean equal to 1) and P is the transmit power. This represents a narrowband channel with respect to the coherence bandwidth of the environment.

2.2 System Model and Setting

In the system we are considering, each node can transmit over a common wireless channel. Apart from the slotted transmission structure where nodes transmit packets within slots of defined duration, nodes are completely uncoordinated. This slotted transmission scheme requires some local frame synchronization method, for instance a form of distributed transmission of pilot signals. The signal model is given by:

$$y_{j,s} = \sum_{k} \sqrt{\gamma_{k,j,s} P r_{k,j}^{-\alpha}} x_{k,s} + n_{j,s}$$

$$\tag{2}$$

where the index s denotes the slot, $y_{j,s}$ the received signal at node j, $x_{k,s}$ the transmitted signal from node k, $n_{j,s}$ the background noise at node j. Moreover, for the purpose of our analysis, we make the following assumptions:

- An infinite number of packets is available for each source. A packet can be seen as a separate codeword for which transmission is stopped when an acknowledgment of successful decoding is returned by the receiver. Furthermore, we assume that the ACK/NACK feedback signaling channel is error-free and delay-free. The signaling overhead is insignificant with respect to the data channel.
- We suppose single-user decoding where each decoder treats the signals from other users as noise. The single-user decoder for each node has perfect knowledge of the channel gain and the total interference power (i.e. noise and interfering user traffic). This can be achieved in a real system by inserting some pilot symbols.
- We assume a block-fading channel model. The fading remains constant on the whole slot and is an i.i.d process across successive slots. In a real system, this can be achieved via slow frequency hopping across a large system bandwith.
- For each slot, each node transmits a packet with probability p and remains silent with probability 1 p such that transmit and receive nodes have spatial Poisson distributions with average node density $\sigma_t = \sigma p$ and $\sigma_r = \sigma(1-p)$ respectively. In a slow frequency-hopping system, 1/p could be the number of frequencies when nodes transmit only on a single frequency for any time-slot.
- Each node transmits with fixed power P.

This is a more general multiple-access scenario than the interference model considered in [1, 2] since we are at liberty to optimize the transmission probability p to randomize interference levels. Note that this creates a random exclusion area around each node.

3 Retransmission Protocols

3.1 Information Outage Probability

The instantaneous average mutual information for a (i, j) pair of nodes conditioned on the channel gain $\gamma_{i,j,s}$ and the interference power V is (in bit/dim):

$$I_{i,j,s} = I(X_{i,j,s}; Y_{j,s} | \gamma_{i,j,s}, V) = \log_2 \left(1 + \frac{\gamma_{i,j,s} P r_{i,j}^{-\alpha}}{N_0 + V} \right)$$
(3)

where N_0 is the background noise power, P is the transmit power and $r_{i,j} = |X_i - X_j|$ where X_j is the position of the receiver, V is defined as the summation of interference power contributions from all interfering transmitters: $V = \sum_{k \neq i} \gamma_{k,j,s} Pr_{k,j}^{-\alpha}$ (in the following we will drop the index s standing for the slot). $P_{out}(r_{i,j})$ is defined as the information outage probability of the channel, or the probability that the mutual information $I_{i,j}$ falls below some fixed spectral efficiency R. Expressions of the mutual information necessary for the outage probability evaluation are derived under the assumption that all user signals are Gaussian with flat power spectral density. The Gaussian assumption yields an upper-bound to the minimum achievable outage probability [8] [9]. $P_{out}(r_{i,j})$ is given by:

$$P_{out}(r_{i,j}) = \Pr\left(\log_2\left(1 + \frac{\gamma_{i,j}Pr_{i,j}^{-\alpha}}{N_0 + V}\right) \le R\right)$$

$$= 1 - e^{-\frac{(2^R - 1)N_0}{Pr_{i,j}^{-\alpha}}} \int e^{-v\frac{(2^R - 1)}{Pr_{i,j}^{-\alpha}}} f_V(v) dv$$
(4)

where $f_V(v)$ is the probability density function of the random variable V and v is the integration variable. Let us define $t = \frac{(2^R - 1)}{Pr_{i,j}^{-\alpha}}$, and $\int e^{-vt} f_V(v) dv = \phi_V(t)$ is the moment generating function MGF of V. To compute the latter, we follow the procedure in [10] (and references in it) where the problem is different since the fading was not considered. To compute the MGF of V, we restrict V to all nodes in a disk \mathbf{D}_b centered at the receiver and having radius b, then we let $b \to \infty$. Moreover, given k nodes in a region, V is a sum of independent random variables with uniform distribution and from [15], we obtain:

$$\phi_{V}(t) = \lim_{b \to \infty} \sum_{k=0}^{\infty} \Pr[k \text{ in } \mathbf{D}_{b}] E\left[e^{-tV} | k \text{ in } \mathbf{D}_{b}\right]$$
$$= \lim_{b \to \infty} \sum_{k=0}^{\infty} \Pr[k \text{ in } \mathbf{D}_{b}] E\left[e^{-tP\gamma r^{-\alpha}}\right]^{k}$$
(5)

Define $g(r) = r^{-\alpha}$ and $\beta = P\gamma$ as an exponential random variable with mean P, we obtain that $E\left[e^{-tP\gamma r^{-\alpha}}\right] = \int f_r(r)\phi_\beta(tr^{-\alpha})dr$, where $\phi_\beta(tr^{-\alpha}) = \frac{1}{1+Ptg(r)}$ is the MGF of β evaluated at tg(r), and $f_r(\cdot)$ is the probability density function of r. The probability density function of the distance between a transmitter and a receiver in a disk of radius b has the uniform distribution:

$$\begin{cases} f_r(r) = \frac{2r}{b^2} & \text{if } r \le b\\ 0 & \text{elsewhere} \end{cases}$$
(6)

Thus, (5) becomes (using the fact that all the interfering nodes form a Poisson process with density σ_t):

$$\phi_{V}(t) = \lim_{b \to \infty} \sum_{k=0}^{\infty} e^{-\sigma_{t}\pi b^{2}} \frac{(\sigma_{t}\pi b^{2})^{k}}{k!} \left(\int_{0}^{b} \frac{2r}{b^{2}} \frac{1}{1 + Ptg(r)} dr \right)^{k}$$

$$= \lim_{b \to \infty} e^{\sigma_{t}\pi b^{2} \left[\int_{0}^{b} \frac{2r}{b^{2}} \left(\frac{1}{1 + Ptg(r)} - 1 \right) dr \right]}$$

$$= \exp \left\{ -2\sigma_{t}\pi \int_{0}^{\infty} \frac{rPg(r)t}{1 + Ptg(r)} dr \right\}$$
(7)

Remember that $t = \frac{(2^R - 1)}{Pr_{i,j}^{-\alpha}}$ and with some manipulations, (4) becomes:

$$P_{out}(r_{i,j}) = 1 - e^{\left(-\frac{(2^R - 1)N_0}{Pr_{i,j}^{-\alpha}}\right)} e^{\frac{-2\sigma_t \pi \Gamma(2/\alpha)\Gamma(1 - 2/\alpha)r_{i,j}^2(2^R - 1)^{2/\alpha}}{\alpha}}$$
(8)

where $\Gamma(y) = \int_0^\infty x^{y-1} e^{-x} dx$ is the Gamma function.

3.2 Slotted Aloha

The Slotted Aloha protocol can provide random multiple access to a common channel with minimal coordination between the channel users. This is a simple scheme where the transmitter sends a codeword to the receiver and waits for an acknowledgment from the receiver. A positive acknowledgment (ACK) means the codeword is received successfully, whereas a negative acknowledgment (NACK) means that errors are detected by the receiver. When the transmitter gets a NACK, it will resend the previous codeword to the receiver until it gets an ACK from the receiver. A data packet collision occurs whenever two or more users transmit at about the same time. When packets from different nodes collide, it may still be possible to successfully decode the packet with the strongest received signal power, which is known as the "capture effect" [11]. Following the analysis of [4], we define the throughput as:

$$\eta = \frac{R}{\tau} \tag{9}$$

where τ is the mean delay measured in slots for the transmission of an information message and R is the spectral efficiency in bit/dim. In Aloha, the receiver has no memory of the past signals, and the probability of successful decoding after l transmitted slots is given by (in the following, we drop the indices i, j standing for the positions of transmitter receiver for simplicity, we keep only the slot index s in the mutual information):

$$\Pr(I_1 < R, I_2 < R, \cdots, I_l > R) = P_{out}^{l-1}(1 - P_{out})$$
(10)

and the mean delay is given by:

$$\tau = \frac{(1 - P_{out})\sum_{l=1}^{\infty} lP_{out}^{l-1}}{p} = \frac{1}{p(1 - P_{out})}$$
(11)

Combining (9) and (11) we obtain:

$$\eta = Rp(1 - P_{out}) \tag{12}$$

3.3 Incremental Redundancy

The basic idea behind incremental redundancy is that the code rate is adjusted by incrementally transmitting redundancy information until decoding is successful. Indeed, if the receiver fails to successfully decode a packet, a NACK is sent to the transmitter. This latter will send additional new redundancy bits which are accumulated and processed by the receiver. As explained in [12], incremental redundancy can be achieved by using rate compatible punctured convolutional codes (RCPC). Transmission starts with the highest rate code of the RCPC code family and additional redundancy bits are sent whenever needed. To study the achievable rate for incremental redundancy, we consider that node k encodes its message information of b bits each independently of other nodes by using a channel code with code book $\mathcal{C}_k \subset \mathbb{C}^{LN}$ where N is the slot length and L is the accumulate number of slots. For the sake of computing information theoretic quantities, we let $L \to \infty$ $N \to \infty$. Codewords are divided into L sub-blocks of length N, and we let $\mathcal{C}_{k,l}$ for $l = 1, \dots, L$ denote the punctured code of length lNobtained from \mathcal{C}_k by deleting the last L - l sub-blocks. If successful decoding occurs at the *l*-th transmission, the effective coding rate for the current codeword is R/l bit/dim where R = b/N. In incremental redundancy, the receiver has memory of the past signals since it accumulates mutual information. Since $\Pr(I_m^{ir} < R) \leq \Pr(I_n^{ir} < R)$ for $m \leq n$, where $\Pr(I_l^{ir} < R) = \Pr(\sum_{s=1}^l I_s < R)$ (the index s stands for the slot sequence), the probability of successful decoding after l transmitted slots is given by:

$$\Pr(I_1^{ir} < R, I_2^{ir} < R, \cdots, I_l^{ir} > R) = \Pr(I_{l-1}^{ir} < R) - \Pr(I_l^{ir} < R)$$

and the mean delay is given by:

$$\tau = \frac{\sum_{l=0}^{\infty} \Pr(I_l^{ir} < R)}{p} \tag{13}$$

The throughput is then:

$$\eta = \frac{Rp}{\sum_{l=0}^{\infty} \Pr(\sum_{s=1}^{l} I_s < R)}$$
(14)

One can notice that $\Pr(\sum_{s=1}^{l} I_s < R)$ is the cumulative density function of the sum of l i.i.d random variables distributed as I_s and evaluated in R. This can be computed numerically by using the characteristic function and discrete Fourier transforms as we have already computed the cumulative density function of I_s in closed form (8).

4 Throughput expressions

The spatial throughput is expressed as a function of the product of the number of the simultaneously successful transmissions per unit space by the average jump made by each transmission, a result that we maximize with respect to the channel access probability p. To calculate the spatial throughput, we introduce the expected forward progress as defined in [13], and in [6] where it is assumed that a packet is randomly relayed to one of the neighboring terminals within a circle of defined radius (constrained range) in a capture environment. The expected forward progress of a packet in the direction of its final destination F, is the distance Z between the transmitter and the receiver (an intermediate node) projected onto a line towards the final destination and the transmission to that receiver is successful (notice that to make the calculations simple, the forward progress

is assumed to be the same for any node on the line perpendicular to the direction of the destination, assumption that is reasonable since the distance r in (fig. 1) is much smaller than the source destination distance). In the following we present two routing strategies:

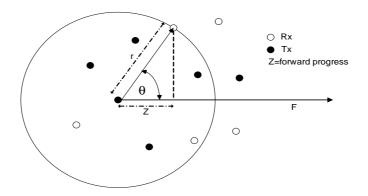


Figure 1: The forward progress.

one that maximizes the expected forward progress by moving the packet to the node most forward towards the final destination; and a strategy that moves the packet to the closest node in range. Concerning the closest node in range, similar strategy is considered in [14] in the context of mobile infostations networks, and in [2] where the transmission is spread to a large number of intermediate mobile relay nodes, and whenever they get close to the final destination, they hand the packets off to it, this leads to a transmit range on the order of $O\left(\frac{1}{\sqrt{n}}\right)$, n being the number of nodes in an unit area. In our analysis, we assess the trade-off between the spatial transmission concurrency and the spectral efficiency of the connections. This is explained by realizing that when we decrease the hop-distance, there are less simultaneous transmissions in a given area but we decrease the mutual interference. This leads to an increase in the achievable rate of each pair and consequently of the spectral efficiency of each link, but the potential spatial transmission concurrency is not fully utilized and moreover the number of hops to reach the destination increases. We define the spatial throughput C as the product of the mean total distance traversed in one hop by all transmissions initialized in an unit area $\sigma p E[Z_u]$ (where $E[Z_u]$ is the expected forward progress for strategy u) by the bit rate R, mainly:

$$C = R\sigma_t E[Z_u] \tag{15}$$

4.1 Maximal Expected Forward Progress

As stated before, the forward progress (the distance traversed in one hop for a successful transmission) is $Z = h(r) = r \cos(\theta)\psi(r)$ (see (fig.1), where $\psi(r)$ is a measure of the success of a transmission, and $\psi(r) = 1 - P_{out}$ for slotted Aloha and $\psi(r) = \frac{1}{\sum_{l=0}^{\infty} \Pr(\sum_{s=1}^{l} I_s < R)}$ for incremental redundancy. To derive the expected maximal forward progress, we compute: (Ω is the set of all receivers, and since we are looking for a receiver that maximizes the forward progress, we consider a sender-centric transmission model and we restrict to all receivers in a half disk \mathbf{D}_a of radius a, the half disk in the direction of the destination, then we let $a \to \infty$, moreover notice that we are using σ_r the density of receivers):

$$\Pr\left(Z_1 = \max_{j \in \Omega} h(r_j) \le z\right) = \lim_{a \to \infty} \sum_{k=0}^{\infty} \Pr\left(h(r_j) \le z\right)^k e^{-\sigma_r \pi a^2/2} \frac{(\sigma_r \pi a^2/2)^k}{k!}$$
$$= \lim_{a \to \infty} \exp\left\{-\sigma_r \pi a^2/2 \Pr\left(h(r_j) > z\right)\right\}$$
(16)

Pr $(h(r_j) > z) = \frac{4}{\pi a^2} \int_{\{r \ge 0: z/r\psi(r) < 1\}}^a r \arccos(z/r\psi(r)) dr$ and θ is uniformly distributed over $[-\pi/2, \pi/2]$. By combining the latter to (16), we obtain:

$$E[Z_1] = \int_0^1 1 - e^{-2\sigma_r \left[\int_{\{r \ge 0: z/r\psi(r) < 1\}} r \arccos(z/r\psi(r)) dr \right]} dz$$
(17)

4.2 Expected Forward Progress for the Closest Node in Range Strategy

We need first to derive the probability density function of the minimum distance between the transmitter and the receiver among all the receive node distances (in a half disk of radius a). By using order statistics (see for example [15]), we have:

$$f_{r}(r) = \lim_{a \to \infty} \sum_{k=0}^{\infty} \frac{2kr}{a^{2}} \left(1 - \left(\frac{r}{a}\right)^{2} \right)^{k-1} e^{-\sigma_{r}\pi \frac{a^{2}}{2}} \frac{(\sigma_{r}\pi \frac{a^{2}}{2})^{k}}{k!}$$
$$= \sigma_{r}\pi r e^{-\sigma_{r}\pi \frac{r^{2}}{2}}$$
(18)

Moreover, the average distance of a node to its closest neighboring node is given by (using again the Gamma function $\Gamma(y) = \int_0^\infty x^{y-1} e^{-x} dx$):

$$E[r] = \int_0^\infty \sigma_r \pi r^2 e^{-\sigma_r \pi \frac{r^2}{2}}$$
(19)

$$= \frac{1}{\sqrt{2\sigma_r}} \tag{20}$$

Remember that the node density is the number of nodes per area, and for an unit area, the average nearest neighbor distance is on the order of $O\left(\frac{1}{\sqrt{n}}\right)$, which is similar to the strategy used in [2] as explained above. The expected forward progress becomes:

$$E[Z_2] = \int_0^1 \int_{\{r \ge 0: z/r\psi(r) < 1\}} \frac{2 \arccos(\frac{z}{r\psi(r)})}{\pi} f_r(r) \mathrm{d}r \mathrm{d}z \tag{21}$$

4.3 Numerical Results

The throughput is expressed as a function of different system parameters: the transmit SNR $\frac{P}{N_0}$, the target information rate R, the transmit probability p and the node density σ . The optimal throughput is derived by maximizing over the transmit probability p (where (17) (21) are solved by using numerical integration). In Slotted Aloha, since the throughput for very high target information rate R goes to zero (actually one can say that the mean delay τ is growing faster than R which from (9) leads to a zero throughput) and the throughput is zero for (R = 0), there exists an optimal target information rate R given a node density, transmit probability and transmit SNR as shown in Fig.2.

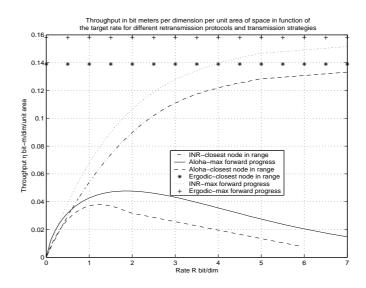


Figure 2: The Spatial throughput (in bit-meter per dimension per unit area for different retransmission protocols and transmissions strategies. Transmit SNR = 5dB, node density $\sigma = 1$, power loss exponent $\alpha = 4$.

Incremental redundancy is capacity achieving since it benefits from the accumulation of information (this process permits some averaging of the fading and interference affecting the useful signal). *Ergodic* in Fig.2 stands for the case where we replace $\psi(r)$ by $E[I_s]$ in (17) (21) where $E[I_s]$ is the ergodic capacity, i.e. the maximum achievable throughput on this channel, without feedback or delay constraints (I_s is defined in (3)).

The closest node in range strategy (in a microscopic analysis) is performing worst than the maximal forward progress strategy, this is in contrast to the results stemming from the Gupta Kumar model where communication is limited to nearest neighbors. The maximal expected forward strategy permits the computation of the optimal hop (or relay) distance.

5 Conclusions and Future Work

We derived formulas for the spatial throughput for simple retransmission protocols and transmission strategies for random networks described by a spatial Poisson point process. It is shown that coding and retransmission protocols are a viable and simple solution for providing fully decentralized multiple-access communications in ad hoc wireless networks despite harsh propagation characteristics (interference from nearby competing nodes). Random exclusion and a decentralized protocol allow for the mitigation of the interference coming from other nodes. A routing protocol allow for the mitigation of the interference progress is shown to significantly outperform nearest-neighbour based schemes. Future work will focus on more advanced strategies for cooperation, the analysis of practical coding strategies and distributed synchronization methods.

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