Bounds On The Distortion Of Distributed Sensing Of Random Fields

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I. Abstract

Wireless sensor networks consist in a deployment over an area, of a cetain number of devices called sensors aimed to monitor their environment by cooperatively sensing information, processing it locally and then transmitting it in a wireless manner to a fusion center in which important data will be extracted and analysed. The range of applications of such networks has become very large: from environmental and habitat monitoring to military surveillance, civil and medical applications.

We restrict our work in a low rate application where the sensor network has to track a slowly time-varying physical random field . The latter depends on space coordinates, and we suppose that it could be written as

$$F(\mathbf{x}) = \sum_{i=1}^{N'} \sqrt{\lambda_i} U_i \phi_i(\mathbf{x})$$

where N' is the space dimension of the field, each λ_i is a constant representing the energy of the field in the i^{th} dimension, $\mathbf{U} = (U_1, \ldots, U_{N'})$ is a gaussian random vector with mean zero and identity covariance matrix and $\{\phi_1(\mathbf{x}), \ldots, \phi_{N'}(\mathbf{x})\}$ forms an orthonormal basis of space functions. We assume that the sensors are deployed in a random manner, but their coordinates are known to the fusion center. At the same time, each sensor senses a noisy value of the field at its coordinates, maps it into a signal with a specific signature and , in the end, sends it to the fusion center through a gaussian multiple access channel. The observation noises of the sensors are gaussian and an attenuation factor is taken into account between the fusion center and each sensor node. At the fusion center stage, it has to reconstruct an estimation value $\widehat{F}(\mathbf{x})$ of the field with a certain fidelity criterion that we seek to minimize. Here, this fidelity criterion is taken to be

$$D = \int_{\mathbf{x}\in\mathcal{A}} E\left[\left(F(\mathbf{x}) - \widehat{F}(\mathbf{x})\right)^2\right] d\mathbf{x}$$

where $\mathcal{A}$ is the area on witch the field exists. The only constraint that we have, is on the energy in the transmission stage: the sum of the energy of all the transmitted signals per field view is equal to E_T. For the mapping issue, we fix it by choosing a linear encoder leaving two variables to optimize in order to minimize the distortion D: the first one is the signals' signatures, the second is the distribution of the total energy on the sensors.

Most of the theoretical bounds that we found, are compared on a graph corresponding to a given sensor network model. In fact, we derive a lower bound on the distortion and an achievable distortion; this latter is compared not only to the lower bound but also to an achievable distortion found by Gaspard in [1] for a special case of our field, in particular, when the field is 'white' (all λ_i are equal). In a large domain of total energy, the result was a significant improvement in terms of achievability and the gap between our achievable distortion and the lower bound was small. Moreover, It is shown that in a perfect sensor network model, where observation noise doesn't exist, the achievable distortion goes to zero when the number of sensors goes to infinity. Another lower bound on the distortion has been found in a model where the sensors are constrained to transmit in a TDMA manner. It is shown that this lower bound doesn't depend on the number of sensors. In the end, in order to evaluate the performance of the linear encoder, a lower bound on the distortion over all possible encoders and decoders has been found ; this latter assumes that the sensors can exchange information between each other through noiseless links. It is shown that for a relatively large total energy, the linear coding scheme has good performance.

References

[1] M. Gastpar and M. Vetterly, "Power-bandwidth-distortion scaling laws for sensor networks," in *3rd international symposium on information processing in sensor networks (IPSN'04),* (Berkeley,CA), April 2004