MULTIVARIATE LP BASED MMSE-ZF EQUALIZER DESIGN CONSIDERATIONS AND APPLICATION TO MULTIMICROPHONE DEREVERBERATION

Mahdi Triki, Dirk T.M. Slock[*]

Eurecom Institute
2229 route des Crêtes, B.P. 193, 06904 Sophia Antipolis Cedex, FRANCE
Email: {triki,slock}@eurecom.fr

ABSTRACT

The linear prediction algorithm estimates a zero-forcing (ZF) equalizer from the SIMO channel output's second order statistics. Linear prediction can be easily extended in the presence of an additive white noise, since the white noise variance can be easily identified and compensated for in the reverberant signal covariance matrix. However, the presence of the additive noise has so far not been considered for the design of the ZF equalizer, and the resulting equalizer is not optimal. In this paper, we consider two issues in the design of the LP-based equalizer in the presence of additive white noise. First, we investigate the effect of relative subchannel delay compensation on the output SNR. We show that such relative delay can reduce considerably the output SNR. Then, we optimize the transformation of the multivariate prediction filter to a longer equalizer filter using the SNR criterion. The optimization corresponds to MMSE-ZF design, and the filter length increase allows for the introduction of some equalization delay, that can also be optimized.

Index Terms**—** Linear Prediction, ZF Equalization, Dereverberation, Time Delay compensation, MMSE-ZF.

1. INTRODUCTION

The problem of blind channel identification and equalization has motivated an intensive interest in communication and signal processing society. Most of the earlier approaches to blind identification are based on the use of higher order statistics, which known to suffer from many drawbacks. They usually require a large number of data samples and a heavy computational load, making them unattractive for practical applications. More recently, it has been shown that the second order statistics contain sufficient information for the identification and equalization of FIR channels. Blind multichannel equalization exploits the channel diversity introduced by sensor arrays and/or fractional sampling (single input/ multiple outputs). The multichannel diversity introduces a useful 'signal overdetermination' which can be exploited in terms of signal/noise subspaces decompositions.

Recently, the blind multi-channel equalization was suggested to the speech dereverberation problem. Blind dereverberation is the process of removing the effect of reverberation from an observed reverberant signal. Reducing the distortion caused by reverberation is a difficult blind deconvolution problem, due to the broadband nature of speech and the length of the equivalent impulse response from the speaker's mouth to the microphone. Let us consider a clean speech signal, $s(n)$, produced in a reverberant room. The reverberant speech signal observed on M distinct microphones can be written as:

$$\mathbf{y}(k) = \mathbf{H}(q)s(k) \tag{1}$$

where $\mathbf{y}(k) = [y_1(k) \cdots y_M(k)]^T$ is the reverberant speech signal, $\mathbf{H}(q) = [H_1(q) \cdots H_M(q)]^T = \sum_{i=0}^{L_h-1} \mathbf{h_i} q^{-i}$ is the SIMO channel transfer function, L_h is the channel length, and q^{-1} is the one sample time delay operator.

The blind dereverberation should face the channel/speech identifiability problem. In fact, for any scalar filter $\alpha(q)$, $(H(q)/\alpha(q)$, $\alpha(q)s(k))$ is also an acceptable solution for (1). In [1], the authors computes the multi-channel FIR equalizer using a subspace based method. The identifiability problem is solved using accurate information of the "source" (or "noise) subspace dimension. The validity of the technique hinges critically on the true channel impulse response being of strictly finite duration, and its successful identification requires knowledge of (at least a tide upper bound on) the channel length [2]. For acoustic case, the true channel impulse response length is generally unknown, or/and not defined. This is a major limitation to the practical applicability of the subspace based methods on speech dereverberation.

In contrast, the alternative Linear Prediction(LP) based technique (proposed and refined by Slock et al. [3, 4]) proved to be consistent in the presence of channel order error. This makes the LP equalizer one of most attractive solutions to the blind speech dereverberation. Compensating for the speech correlation is done via a prewhitening at the LP equalizer input[5, 6], or post-filtering of the LP residual[7, 8].

If the input signal is white, the output of the multichannel LP is

$$\mathbf{x}(k) = A(q)\mathbf{y}(k) = \mathbf{h_0} s(k) \tag{2}$$

where $A(q)$ is the multichannel linear predictor, and $\mathbf{h_0} = \mathbf{H}(+\infty)$ is the multichannel precursor coefficient. The LP equalizer is obtained by combining the column of the predictor. The LP approach can be easily extended to the presence of an additive white noise, since the white noise variance can be easily identified and compensated for in the reverberant signal covariance matrix. However, the predictor columns (each can regarded as a zero-forcing equalizer) have different behavior in the presence of observation noise[9]. This fact was not considered in the original LP algorithm, and then the computed equalizer is not optimal.

In this paper, we consider two important issues in the design of the LP-based equalizer in the presence of additive white noise. First, we

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investigate the effect of relative subchannel delay compensation on the output SNR (section 2). Then, we optimize the transformation of the multivariate prediction filter to a longer equalizer filter using the SNR criterion (section 3). The optimization corresponds to MMSE-ZF design, and the filter length increase allows for the introduction of some equalization delay, that can also be optimized.

2. TIME DELAY COMPENSATION FOR LP EQUALIZATION

Several authors point the lack of robustness of the LP equalizer in presence of additive noise. In particular, the algorithm overall performance rely on the particular realization of the multichannel precursor coefficient $\mathbf{h_0}$, yielding a prediction error signal with uncontrollable symbol-to-noise ratio[10]. In [11], Li et al. underline that some problems may arise when $\mathbf{h_0}$ have small entries.
In[10], Gesbert and Duhamel use several multistep prediction error signals to triangularize the multichannel system. In such a way, the proposed prediction scheme exploits the full channel structure. Thus, it provides more statistical efficiency in channel identification. In this section, we suggest alleviating this side effect by aligning the received signals on the various microphones (delay compensation for direct path). We demonstrate that it leads not only to an increase in the signal part energy $\left(\sigma_s^2 \|\mathbf{h_0}\|^2\right)$, but also to a decrease on the output MSE $= \left(\sigma_v^2 \frac{1}{2\pi j} \oint A(z)A^\dagger(z)\frac{dz}{z}\right)$.

<u>*Theorem 1:*</u> For a noisy SIMO dereverberation problem, the output SNR increases by relative subchannel delay compensation.

Proof: We consider the noisy SIMO system with M outputs:

$$\mathbf{x}(k) = \sum_{i=0}^{L_h-1} \mathbf{h}_i s(k-i) + \mathbf{v}(k) \tag{3}$$

where the channel noise $\mathbf{v}(k)$ is assumed zero mean white process. The input signal and the noise covariances are denoted respectively $E\left\{s^2(k)\right\} = \sigma_s^2$, and $E\left\{\mathbf{v}(k)\mathbf{v}^H(k)\right\} = \sigma_v^2 I_M$. If σ_v^2 is known, one can compensate for the noise covariance in the reverberant signal covariance matrix. And noise-free multichannel LP can be computed (using the cleaned covariance matrix). This linear predictor is then applied to the noisy signal $\mathbf{x}(k)$.
To describe the blind linear predictive algorithm, it is easier to first form the $(L_A M) \times (L_A + L_h - 1)$ block Toeplitz matrix

$$\mathcal{H} = \begin{bmatrix} \mathbf{h}_0\, \mathbf{h}_1 \, \cdots \, \mathbf{h}_{L_h-1} \, \mathbf{0} & \cdots & \mathbf{0} \\ 0 \quad \mathbf{h}_0\, \mathbf{h}_1 \, \cdots \, \mathbf{h}_{L_h-1} & \ddots & \vdots \\ \vdots \quad \ddots \quad \ddots & \ddots & \mathbf{0} \\ 0 \quad \cdots \quad \mathbf{0}\; \mathbf{h}_0\, \mathbf{h}_1 & \cdots & \mathbf{h}_{L_h-1} \end{bmatrix}$$

Equation (3) can be now written :

$$\mathbf{x}_{L_A}(k) = \mathcal{H}\,\mathbf{s}(k) + \mathbf{v}_{L_A}(k) \tag{4}$$

where: $\mathbf{x}_{L_A}(k) = \left[\ \mathbf{x}^T(k)\ \cdots\ \mathbf{x}^T(k-L_A+1)\ \right]^T$,
$\mathbf{s}(k) = \left[\ s(k)\ \cdots\ s(k-L_A-L_h+2)\ \right]^T$, and
$\mathbf{v}_{L_A}(k) = \left[\ \mathbf{v}^T(k)\ \cdots\ \mathbf{v}^T(k-L_A+1)\ \right]^T$.
With this notation, one can show that if $\mathbf{H}(q) = \sum_{i=0}^{L_h-1} \mathbf{h}_i q^{-i}$ has no zeros, the matrix $\mathcal{H}$ has full column rank. Then, the pseudoinverse $\mathcal{H}^{\#}$ exists, and the multichannel LP coefficients are given by

$$\left[\ A_{L_A,1}\ \cdots\ A_{L_A,L_A}\ \right] = -\left[\ \mathbf{h}_1 \cdots \mathbf{h}_{L_h-1}\ 0 \cdots 0\ \right]\mathcal{H}^{\#}$$

As the linear predictor is designed for the noise-free reverberant signal, the residual signal becomes

$$\widetilde{\mathbf{x}}(k) = \mathbf{x}(k) + \sum_{i=1}^{L_A} A_{L_A,i}\mathbf{x}(k-i) = \mathbf{h}_0 s(k) + \sum_{i=0}^{L_A} A_{L_A,i}\mathbf{v}(k-i)$$

The output MSE is given by

$$\begin{aligned} \text{MSE} &= \sigma_v^2 \text{tr}\left\{ I_M + \sum_{i=1}^{L_A} A_{L_A,i}A_{L_A,i}^H \right\} \\ &= \sigma_v^2 \text{tr}\left\{ I_M + \left[\ \mathbf{h}_1 \cdots \mathbf{h}_{L_h-1}\, \mathbf{0} \cdots \mathbf{0}\right] \left(\mathcal{H}^H\mathcal{H}\right)^{-1} \begin{bmatrix} \mathbf{h}_1^H \\ \vdots \\ \mathbf{h}_{L_h-1}^H \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \right\} \end{aligned}$$

where $tr\left\{.\right\}$ denotes the trace operator.
Note that the MSE depends only on the $(L_h-1)\times(L_h-1)$ upper block of the matrix $\left(\mathcal{H}^H\mathcal{H}\right)^{-1}$. On the other hand, taking into consideration the whiteness of reverberation and its decaying energy, one can show that this $(L_h-1)\times(L_h-1)$ upper block is almost diagonal, and that the MSE is given by

$$\begin{aligned} MSE &= \sigma_v^2 \left(M + \frac{||\mathbf{h}_1||^2}{||\mathbf{h}_0||^2} + \frac{||\mathbf{h}_2||^2}{||\mathbf{h}_0||^2 + ||\mathbf{h}_1||^2} + \cdots \right. \\ &+ \left. \cdots + \frac{||\mathbf{h}_{L_h-1}||^2}{||\mathbf{h}_0||^2 + \cdots + ||\mathbf{h}_{L_h-2}||^2} \right) \end{aligned} \tag{5}$$

The above equation shows how critical the energy of $\mathbf{h}_0$ is. In fact if $\|\mathbf{h}_0\|^2 \to 0$, not only the desired signal energy $(\sigma_s^2 \|\mathbf{h}_0\|^2) \to 0$, but also the MSE $\to \infty$.
On the other hand, one can show that

$$\frac{\partial \text{MSE}}{\partial ||\mathbf{h}_0||^2} < 0. \tag{6}$$

Equation (6) is not sufficient to prove that the relative compensation decreases the output MSE. In fact, by aligning the received data, we are not increasing the energy of $\mathbf{h}_0$ independently of $\mathbf{h}_i \quad i \neq 0$. We denote $\left\{\delta_i\right\}_{i\geq 0}$ the difference between the energy of the i^{th} channel tap before and after relative delay compensation. After time aligning, the output MSE becomes:

$$\begin{aligned} MSE &= \sigma_v^2 \left(M + \frac{||\mathbf{h}_1||^2 + \delta_1}{||\mathbf{h}_0||^2 + \delta_0} + \frac{||\mathbf{h}_2||^2 + \delta_2}{||\mathbf{h}_0||^2 + ||\mathbf{h}_1||^2 + \delta_0 + \delta_1} \right. \\ &+ \left. \cdots + \frac{||\mathbf{h}_{L_h-1}||^2 + \delta_{L_h-1}}{||\mathbf{h}_0||^2 + \cdots + ||\mathbf{h}_{L_h-2}||^2 + \delta_0 + \cdots + \delta_{L_h-2}} \right) \end{aligned}$$

At first, we consider the delay compensation of only one subchannel. We denote by $\tau_d \neq 0$ the relative delay of this subchannel. Energy conservation leads to:

$$\sum_{s=0}^{\infty} \delta_{k+s\tau_d} = 0 \quad k \in [0, (\tau_d - 1)]$$

If we assume the channel energy to be decreasing with time lag and/or the relative delay τ_d is large enough, we have:

$$\begin{aligned} \delta_i > 0\ ,\ \ \forall i < \tau_d \\ \delta_i < 0\ ,\ \ \forall i \geq \tau_d \end{aligned}$$

Now, we consider the relative time compensation of the whole multichannel. $\tau_d \neq 0$ will denote the minimum non-zero relative delay on different subchannels. If M increases, and for acoustic subchannels, we have:

$$\frac{\delta_i}{\delta_0+\cdots+\delta_{i-1}} \approx \frac{||\mathbf{h}_i||^2}{||\mathbf{h}_0||^2+\cdots+||\mathbf{h}_{i-1}||^2} \quad \forall i < \tau_d$$
$$\frac{\delta_i}{\delta_0+\cdots+\delta_{i-1}} < \frac{||\mathbf{h}_i||^2}{||\mathbf{h}_0||^2+\cdots+||\mathbf{h}_{i-1}||^2} \quad \forall i \geq \tau_d$$

Therefore, the output MSE decreases after relative delay compensation. On the other hand, the desired signal energy $(\sigma_s^2\,||\mathbf{h}_0||^2)$ increases.

To illustrate the effect of the data alignment on the SNR of the LP output, we consider a rectangular room with dimensions $L_x = 8m$, $L_y = 10m$, and $L_z = 4m$, and with wall reflection coefficients $\rho_x = \rho_y = \rho_z = 0.9$ ($T_{60} \approx 250ms$). A white noise is used as the source signal (sampled at 8 Khz). The reverberant speech signal is observed on $M = 16$ distinct microphones. A computer implementation (graciously provided by Geert Rombouts from K.U. Leuven) of the image method as described in [13] is used to generate synthetic room impulse response between the source and the microphones.

The Matched Filter Bound (MFB) is defined as MFB $= \frac{\sigma_s^2}{\sigma_v^2}\,\|H\|^2$. The MFB can be also called "channel SNR"[9]. The MFB can be interpreted as the SNR of the Maximum Likelihood estimation of the symbol $s(k)$ assuming that all other symbols $s(n)\quad n \neq k$ are known[14]. It is clear that the MFB constitute an upper bound on the output SNR. Furthermore, we consider the evaluation criterion:

$$\frac{MFB}{SNR_{out}} \geq 1 \tag{7}$$

Note that for any zero-forcing equalizer, this criterion do not depend on $\frac{\sigma_s^2}{\sigma_v^2}$. Figure 1 compares the performance of the LP algorithms with and without relative time-delay compensation (averaged over 100 Monte Carlo runs). One can remark that the alignment of the received signals provides better results; and that it is crucial when the number of sub-channels increases.

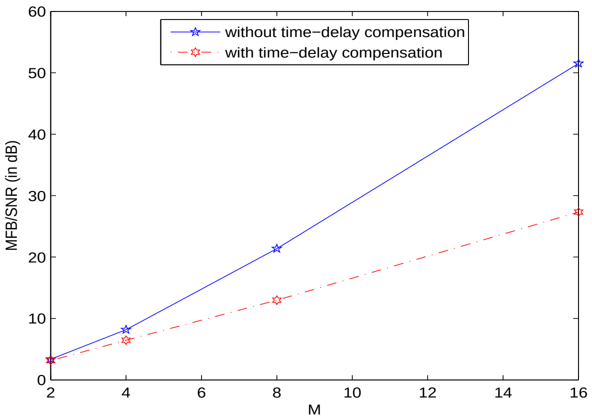

Fig. 1. $\frac{MFB}{SNR_{out}}$ with and without relative time-delay compensation

3. MMSE-ZF LP COMBINATION FOR BLIND MULTICHANNEL EQUALIZATION

The output of the multichannel linear predictor is

$$\mathbf{x}(k) = \mathbf{h_0} s(k) + A(q)\mathbf{v}(k) \tag{8}$$

In original LP equalizer, the columns of the predictor $A(q) = I + \sum_{i=1} A_i q^{-i}$ are combined using the weighing vector $\mathbf{h}_0^H$, i.e.,

$$F_{LP}(q) = \mathbf{h}_0^H A(q) \tag{9}$$

This choice maximizes the power of the signal part but not necessarily the output SNR. In[9], Gazzah computes the weighing vector by maximizing the output SNR, i.e.,

$$\mathbf{w} = \arg\max_{\mathbf{w}} \frac{\sigma_s^2}{\sigma_v^2}\frac{\|\mathbf{w}\|^2}{\mathbf{w}AA^H\mathbf{w}^H} \tag{10}$$

The proposed equalizer is:

$$F_{MLP}(q) = \mathbf{h}_0^H (AA^H)^{-1} A(q) \tag{11}$$

where $A = [IA_1 \cdots A_L]$. The author shows that the proposed equalizer output not only outperforms the original LP equalizer, but also attains the lowest achievable (by any no-delay ZF equalizer) MSE.

In this section, we generalize the previous approach by considering a weighting filters to combines the columns of the $A(q)$. This will allow the design of non-zero-delay ZF equalizer. For a given length filter L_w, and an equalization delay $d \leq (L_w - 1)$ The weighting filter are optimized by maximizing the output SNR, under the d-delay zero-forcing constraint, .i.e.

$$\begin{cases} \mathbf{w} = \arg\max_{\mathbf{w}} \frac{\sigma_s^2}{\sigma_v^2} \dfrac{1}{\frac{1}{2\pi j}\oint \mathbf{w}(q)A(q)A^\dagger(q)\mathbf{w}^\dagger(q)\frac{dz}{z}} \\ \mathbf{w}(q).\mathbf{h_0} = q^{-d} \end{cases} \tag{12}$$

where $A^\dagger(q)$ denotes the $A(q)$ matched filter.
To Solve the optimization problem, it is easier to first form the $(L_w.M)\times((L_w+L-1).M)$ and $(L_w.M)\times L_w$ block Toeplitz matrix

$$\mathcal{A} = \begin{bmatrix} I & A_1 & \cdots & A_L & 0 & \cdots & 0 \\ 0 & I & A_1 & \cdots & A_L & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & \\ 0 & \cdots & 0 & I & A_1 & \cdots & A_L \end{bmatrix} \quad H_0 = \begin{bmatrix} \mathbf{h_0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{h_0} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h_0} \end{bmatrix}$$

The optimisation in (12) becomes

$$\begin{cases} W_{L_w,d} = \arg\min_{W} WR_wW^H \\ W_{L_w,d}H_0 = e_d \end{cases} \tag{13}$$

where $W = [\mathbf{w}_1 \cdots \mathbf{w}_{L_w}]$ is a $(M.L_w)$ vector characterizing the weighting filter coefficients $\left(\mathbf{w}(q) = \sum_{i=1}^{L_w} \mathbf{w}_i q^{-i}\right)$, $R_w = \sigma_v^2 \mathcal{A}\mathcal{A}^H$ represents the output noise covariance matrix, and $e_d = [0...0\,1\,0...0]$ is the $(d+1)^{th}$ vector of the $\mathbb{R}^{L_w}$ canonical basis. Using Lagrange optimization, on can show that the optimal weighting filter is given by

$$W_{L_w,d} = e_d\left(H_0^H R_w^{-1} H_0\right)^{-1} H_0^H R_w^{-1} \tag{14}$$

The achieved output MSE is

$$\text{MSE} = \sigma_v^2 e_d\left(H_0^H R_w^{-1} H_0\right)^{-1} e_d^H \tag{15}$$

Note that the delay $d \geq 0$ can be easily optimized by minimizing the output MSE.
For $L_w = 1,\ d = 0$, we recover the solution proposed in [9], i.e.,

$$W_{1,0} \propto \mathbf{h_0}\left(AA^H\right)^{-1} \tag{16}$$

If $L_w \to \infty$, and for an appropriate choice of the delay d_∞, one can show that

$$\mathbf{w}_{\infty,d_\infty}(q) = \left(h_0^H A^{-\dagger}(q) A^{-1}(q) h_0\right)^{-1} h_0^H A^{-\dagger}(q) A^{-1} \quad (17)$$

Exploiting the fact that $A^{-1}(q)h_0 = H(q)$, one can show that the obtained ZF equalizer corresponds to the MMSE-ZF equalizer:

$$\begin{aligned} F_{\infty,d_\infty}(q) &= \mathbf{w}_{\infty,d_\infty}(q)A(q) \quad (18) \\ &= \left(H^\dagger(q)H(q)\right)^{-1} H^\dagger(q) \end{aligned}$$

We illustrate the behavior of the proposed scheme, and we provide a comparison with the scheme proposed in[9] and the MMSE-ZF equalizer. Monte-Carlo simulations are constructed using the reverberation scenario described in section 2.

Figure 2 compares the performance of the different ZF equalizer (averaged over 10 Monte Carlo runs). We verify that if we consider zero delay equalization, increasing the order of weighting filter do not increase the performance; which is coherent with the results reported in[9]. On the other hand, despite achieving the MMSE-ZF equalization performance requires long filters and large delays (due to the acoustic channel length); considerable gains can be achieved by allowing even small delays (7.5 dB using 9 taps weighting filters (M=8)).

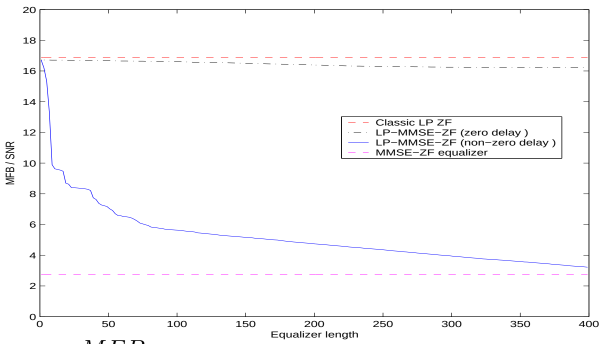

Fig. 2. $\dfrac{MFB}{SNR_{out}}$ for different ZF equalization scheme(averaged over 20 Monte Carlo runs).

Then, we investigate the performance of the proposed scheme function of the number of sub-channels M (figure 3). Curves show that the gain, due to the use of non-zero delay equalization, increases with M. The reason is: the more sub-channel we have, the more freedom degrees (in the weighting filters) we can optimize, and the better output SNR we achieve.

4. CONCLUSIONS

In this paper, we consider two issues in the design of the LP-based equalizer in the presence of additive white noise. First, we investigate the effect of relative subchannel delay compensation on the output SNR. We show that such relative delay compensation can increase considerably the output SNR. Then, we optimize the transformation of the multivariate prediction filter to a longer equalizer filter using the SNR criterion. The optimization corresponds to MMSE-ZF design, and the filter length increase allows for the introduction of some equalization delay, that can also be optimized. Simulations show that considerable gains can be achieved by allowing even small equalization delays.

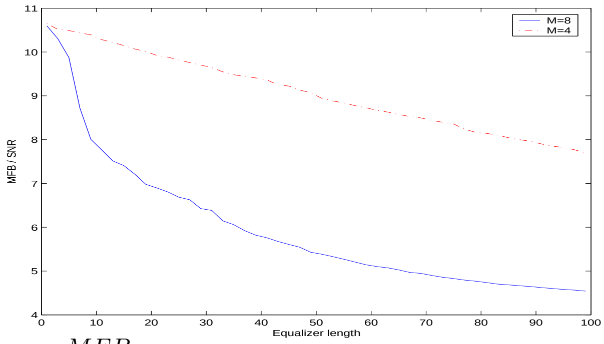

Fig. 3. $\dfrac{MFB}{SNR_{out}}$ for non-zero delay ZF equalization function of the number of sub-channels.

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