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Maximizing Multicell Capacity Using Distributed Power Allocation and Scheduling

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Abstract— **Joint optimization of transmit power and scheduling in wireless data networks promises significant system-wide capacity gains. However, this problem is known to be NP-hard and thus difficult to tackle in practice. We analyze this problem for the downlink of a multicell full reuse network with the goal of maximizing the overall network capacity. We propose a *distributed* power allocation and scheduling algorithm which provides significant capacity gain for any finite number of users. This distributed cell coordination scheme, in effect, achieves a form of *dynamic spectral reuse*, whereby the amount of reuse varies as a function of the underlying channel conditions and only limited inter-cell signaling is required.**

I. Introduction

System level performance of future wireless data networks like WiMAX, 3G/4G etc. are adversely affected by an intolerable level of interference in case of full reuse (in any dimension e.g. time or frequency slots, codes etc.) of the spectral resource. Fortunately, some form of coordination between the different cells occupying the same spectral resource can offer significant improvement. Optimal resource allocation requires complete information about the network in order to decide which users in which cells should transmit simultaneously with a given power, while incurring the least loss of capacity due to inter-cell interference. Some interesting results exist exploiting inter-cell coordination with goals such as maximizing system throughput [1]–[4], achieving a target carrier-to-interference ratio [5] or maintaining user queue stabilities [6]. All of these results however, rely on some form of centralized control to obtain gains at various layers of the communication stack. In a realistic network however, centralized multicell coordination is hard to realize in practice, especially in fast-fading environments.

In this paper we address the problem of *distributed inter-cell coordination* to maximize the system capacity. This means that cells know channel state information (CSI) of their own users but have no information on channel conditions of other cell users. The key idea here is to *switch off transmission in cells which do not contribute enough capacity to outweigh the interference degradation caused by them to the rest of the network*. We propose a distributed algorithm which allows a subset of the total number of cells to transmit simultaneously during a given scheduling period. Though other cells stay silent, they may be active during the next scheduling period. This approach can be considered as a distributed mechanism for *dynamic spectral reuse*. In contrast with traditional cellular networks, the reuse pattern obtained with this method is random, possibly highly irregular (Fig. 1) and varies from one scheduling period to the next as a function of the channel state information of the cell users. We show that the proposed power allocation and scheduling algorithm thus offers two types of gain:

- a dynamic spectral reuse gain thanks to the reduction of interference.
- a multi-user diversity gain through scheduling within each cell.

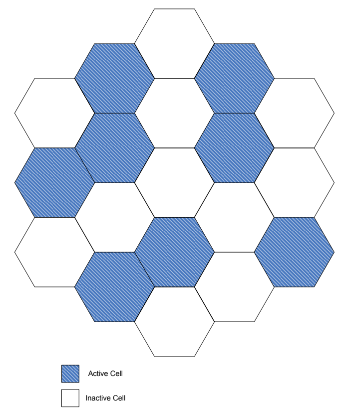

Fig. 1. Possible irregular reuse pattern at a given scheduling period due to dynamic spectral reuse.

We first build the framework for a single-carrier system, which is then extended to multi-carrier techniques like orthogonal frequency division multiple access (OFDMA), which has recently been adopted for WiMAX [7]. In this latter case, we obtain a novel distributed mechanism for *sub-carrier allocation* for multicell OFDMA networks, where frequency diversity gain can also be exploited. Numerical results under realistic wireless network settings are shown to exhibit significant capacity gains over traditional fixed spectral reuse schemes.

II. System Model

Consider the downlink of a multicell system, employing the *same spectral resource* in each cell giving rise to an interference-limited system. Power control is used in an effort to preserve power and to limit interference and fading effects.

We assume a peak power constraint, $P_{\max}$, at each access point (AP). Within each cell, we consider a multiple access scheme in which an orthogonally divided resource (time, frequency or codes) is used to separate transmissions to cell user terminals (UT). Each cell user is allocated a unique resource slot, but due to full reuse, the user "sees" interference from all neighboring co-channel cells that transmit.

A. Signal Model

Consider N cells, and U_n users randomly distributed over each cell n. We denote the random channel gain between any arbitrary AP i and user u_n in cell n by $G_{u_n,i} \in \mathbb{R}^+$, and in what follows, assume that the coherence time of the channel is longer than the scheduling period. We also assume that perfect CSI is present at the receiver and transmitter. The received signal Y_{u_n} at the user is then given by

$$Y_{u_n} = \sqrt{G_{u_n,n}} X_{u_n} + \sum_{i \neq n}^{N} \sqrt{G_{u_n,i}} X_{u_i} + Z_{u_n},$$

where X_{u_n} is the signal from the serving AP, $\sum_{i \neq n}^{N} \sqrt{G_{u_n,i}} X_{u_i}$ is the sum of interfering signals from other cells and Z_{u_n} is additive white Gaussian noise.

III. Joint Power Allocation and Scheduling

To maximize the network throughput, power allocation should be jointly optimized with scheduling. In order to facilitate the problem formulation of the joint power allocation and scheduling problem, we state the following definitions:

Definition 1: A ***scheduling vector*** $\boldsymbol{U}$ contains the set of users simultaneously scheduled across all cells:

$$\boldsymbol{U} = [u_1 \, u_2 \, \cdots \, u_n \, \cdots \, u_N]$$

where $[\boldsymbol{U}]_n = u_n$. Noting that $1 \leq u_n \leq U_n$, the feasible set of scheduling vectors is given by $\Upsilon = \{\boldsymbol{U} \mid 1 \leq u_n \leq U_n \ \forall \ n = 1, \ldots, N\}$.

Definition 2: A ***transmit power vector*** $\boldsymbol{P}$ contains the transmit power values used by each AP to communicate with its respective user:

$$\boldsymbol{P} = [P_{u_1} \, P_{u_2} \, \cdots \, P_{u_n} \, \cdots \, P_{u_N}]$$

where $[\boldsymbol{P}]_n = P_{u_n}$. Due to the peak power constraint $0 \leq P_{u_n} \leq P_{\max}$, the feasible set of transmit power vectors is given by $\Omega = \{\boldsymbol{P} \mid 0 \leq P_{u_n} \leq P_{\max} \ \forall \ n = 1, \ldots, N\}$.

The joint power allocation and scheduling problem consists of finding the power allocation vector and scheduling vector that will maximize the chosen utility function: network capacity. The signal to interference-plus-noise ratio (SINR) of a user scheduled in cell n is given by

$$\Gamma([\boldsymbol{U}]_n, \boldsymbol{P}) = \frac{G_{u_n,n} P_{u_n}}{\sigma^2 + \sum_{i \neq n}^{N} G_{u_n,i} P_{u_i}},$$

where $P_{u_n} = \mathbb{E}|X_{u_n}|^2$, and we assume $\mathbb{E}|Z_{u_n}|^2 = \sigma^2$ for all n. Assuming an ideal link adaptation protocol, the per-cell network capacity at any given scheduling period can be expressed in bits/sec/Hz/cell, using the Shannon capacity, as

$$\mathcal{C}(\boldsymbol{U}, \boldsymbol{P}) \triangleq \frac{1}{N} \sum_{n=1}^{N} \log_2 \Big(1 + \Gamma([\boldsymbol{U}]_n, \boldsymbol{P})\Big). \qquad (1)$$

A. Optimal Power Allocation and Scheduling

Taking (1) as the objective function, the optimal power allocation and scheduling problem can be formulated as

$$(\boldsymbol{U}^*, \boldsymbol{P}^*) = \arg \max_{\substack{\boldsymbol{U} \in \Upsilon \\ \boldsymbol{P} \in \Omega}} \mathcal{C}(\boldsymbol{U}, \boldsymbol{P}), \qquad (2)$$

However, the solution is hard to realize due to the non-convexity of the problem.

IV. Distributed Power Allocation and Scheduling

A straightforward approach to problem (2) would be an exhaustive search over the sets Υ and Ω to find $\mathcal{C}^*$. But clearly, this approach entails a significant computational cost as well as feedback overhead. Moreover, due to the dependency of the capacity equation on global network knowledge, centralized processing would be required. We thus proceed to obtain a computationally simple and distributed, although sub-optimal, algorithm instead.

A. Distributed Iterative Approach in the Interference Limited Regime

Let $\mathscr{N}$ be the set of indices of all presently active cells. A cell should be deactivated if this action results in an increase in network capacity. Denoting the cell which is to be potentially turned off by m, the network capacity with and without cell m turned off is given by the LHS and the RHS of (3) respectively, and after simple manipulations (5). Assuming high SINR regime in all "on" cells, and an interference-limited system, we can simplify the condition (5) as

$$\frac{G_{m,m} P_m}{\sum_{\substack{i \neq m \\ i \in \mathscr{N}}} G_{m,i} P_i} < \frac{\prod_{\substack{n \in \mathscr{N} \\ n \neq m}} \sum_{\substack{i \neq n \\ i \in \mathscr{N}}} G_{n,i} P_i}{\prod_{\substack{n \in \mathscr{N} \\ n \neq m}} \sum_{\substack{i \neq n,m \\ i \in \mathscr{N}}} G_{n,i} P_i} \qquad (6)$$

Evaluating (6) still requires global channel state knowledge as well as searching over the sets Υ and Ω. We therefore exploit the following results which will allow us to further simplify the problem in the case of large network size (N).

1) Interference Modeling: In order to obtain a distributed algorithm dependent only on locally available information, we use the *interference-ideal* model [8]. Fortunately, full reuse networks lend themselves to simpler modeling of the total interference experienced by the user, due mostly to the large number of interference sources averaging themselves out at the receiver. This allows us to simplify modeling of the interference in large full-reuse networks by stating that the total interference at a receiver is only weakly dependent on

$$\sum_{n\in\mathcal{N}} \log_2\left(1+\frac{G_{n,n}P_n}{\sigma^2+\sum\limits_{\substack{i\neq n\\ i\in\mathcal{N}}} G_{n,i}P_i}\right) < \sum_{\substack{n\in\mathcal{N}\\ n\neq m}} \log_2\left(1+\frac{G_{n,n}P_n}{\sigma^2+\sum\limits_{\substack{i\neq n,m\\ i\in\mathcal{N}}} G_{n,i}P_i}\right) \tag{3}$$

$$\Downarrow$$

$$\log_2\left(1+\frac{G_{m,m}P_m}{\sigma^2+\sum\limits_{\substack{i\neq m\\ i\in\mathcal{N}}} G_{m,i}P_i}\right)+\sum_{\substack{n\in\mathcal{N}\\ n\neq m}} \log_2\left(1+\frac{G_{n,n}P_n}{\sigma^2+\sum\limits_{\substack{i\neq n\\ i\in\mathcal{N}}} G_{n,i}P_i}\right) < \sum_{\substack{n\in\mathcal{N}\\ n\neq m}} \log_2\left(1+\frac{G_{n,n}P_n}{\sigma^2+\sum\limits_{\substack{i\neq n,m\\ i\in\mathcal{N}}} G_{n,i}P_i}\right) \tag{4}$$

$$\Downarrow$$

$$\left(1+\frac{G_{m,m}P_m}{\sigma^2+\sum\limits_{\substack{i\neq m\\ i\in\mathcal{N}}} G_{m,i}P_i}\right)\prod_{\substack{n\in\mathcal{N}\\ n\neq m}} \left(1+\frac{G_{n,n}P_n}{\sigma^2+\sum\limits_{\substack{i\neq n\\ i\in\mathcal{N}}} G_{n,i}P_i}\right) < \prod_{\substack{n\in\mathcal{N}\\ n\neq m}} \left(1+\frac{G_{n,n}P_n}{\sigma^2+\sum\limits_{\substack{i\neq n,m\\ i\in\mathcal{N}}} G_{n,i}P_i}\right) \tag{5}$$

its position in the cell when there are a large number of interferers, i.e. a dense network. This can be formalized as

$$\sum_{i\neq n}^{N} G_{u_n,i}P_i \approx G\sum_{i\neq n}^{N} P_i$$

where G is a constant which does not depend on the location of u_n, but depends on pathloss and link budget parameters. One of the key ideas in our approach is that G (average interference gain) need NOT be estimated.

2) Binary Power Allocation: An interesting result pertaining to problem (2) for the two-cell case is presented in [9]. It is shown that the optimal power allocation, for any scheduling vector, lies in the binary feasible set

$$\Omega^B = \{\boldsymbol{P} \mid P_{u_n}=0 \text{ or } P_{u_n}=P_{\max}]\}.$$

Moreover, numerical results suggest that with a greater number of cells this binary allocation, although not strictly globally capacity-optimal in the Shannon sense, is close to the optimal power allocation [1], [9]. This motivates restricting the search for levels to Ω^B also for an arbitrary number of cells.

Armed with these results and simplifications we now proceed to obtain a distributed algorithm. Using the interference-ideal model on the RHS of (6), for cell m to be deactivated (all other cells being static) we require

$$\frac{G_{m,m}P_m}{\sum\limits_{\substack{i\neq m\\ i\in\mathcal{N}}} G_{m,i}P_i} < \frac{\prod\limits_{\substack{n\in\mathcal{N}\\ n\neq m}} G\sum\limits_{\substack{i\neq n\\ i\in\mathcal{N}}} P_i}{\prod\limits_{\substack{n\in\mathcal{N}\\ n\neq m}} G\sum\limits_{\substack{i\neq n\neq m\\ i\in\mathcal{N}}} P_i}.$$

As all "on" cells transmit with $P_{\max}$ and denoting $|\mathcal{N}|=\tilde{N}$, cell m will be active if

$$\frac{G_{m,m}}{\sum\limits_{\substack{i\neq m\\ i\in\mathcal{N}}} G_{m,i}} > \left(\frac{\tilde{N}-1}{\tilde{N}-2}\right)^{(\tilde{N}-1)}. \tag{7a}$$

This requires knowledge of the number of active cells, which can be easily determined by measuring the number of received pilot signals. Additionally, we see that as the size of the network increases,

$$\lim_{N\to\infty}\left(\frac{\tilde{N}-1}{\tilde{N}-2}\right)^{(\tilde{N}-1)} = e.$$

Thus, for a large network size, a cell m will be active if the user signal-to-interference ratio of the scheduled user is more than e,

$$\text{SIR}([\boldsymbol{U}]_m) = \frac{G_{m,m}}{\sum\limits_{\substack{i\neq m\\ i\in\mathcal{N}}} G_{m,i}} > e. \tag{7b}$$

Notice that evaluating (7b) requires knowledge of only the cell user SIR, which can be easily measured and communicated back to the AP. We thus obtain a surprisingly simple, yet powerful condition allowing an AP to determine in a distributed manner, whether it should be active or inactive. Moreover, for each cell to fulfill the condition (7b) and thus contribute to the system capacity, the user with the best SINR for a given power allocation should be scheduled. Depending on the size of the network either (7a) or (7b) could be used. In what follows, we use (7b) as the activity condition in order to demonstrate its robustness for realistic network sizes.

Distributed Algorithm: An iterative approach is adopted to obtain a fully distributed algorithm for power allocation and user scheduling. Starting with a full power allocation vector, each cell simultaneously measures the SIR of the best user and based on (7b) remains active or inactive during the next iteration. Similarly, at every iteration, inequality (7b) is evaluated for the user with the best SIR based on the power allocation resulting from the previous iteration, and the power allocation is updated. The algorithm is run until the cell capacity stabilizes or for a given number of iterations. The pseudo-code for this approach is given in Algorithm 1.

B. Extension to Multicell OFDMA Networks

With the same goal of system capacity maximization, the proposed algorithm can be easily extended to multicell multi-carrier systems. Consider a full reuse multicell OFDMA

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Algorithm 1 A Distributed Iterative Power Allocation and Scheduling Algorithm

$[\boldsymbol{P}^{(1)}]_n = P_{\max} \ \forall \ n$
for $t = 1 : IT_{\max}$ **do**
  $[\boldsymbol{U}^{(t)}]_n = \arg\max_{u_n} \Gamma(u_n, \boldsymbol{P}^{(t)})$
  if $\Gamma([\boldsymbol{U}^{(t)}]_n, \boldsymbol{P}^{(t)}) > e$ **then**
    $[\boldsymbol{P}^{(t+1)}]_n = P_{\max}$
  else
    $[\boldsymbol{P}^{(t+1)}]_n = 0$
  end if
end for

network in which the available frequency band is divided into a number of intra-cell orthogonal sub-carriers. The advantage of OFDMA lies in frequency-selective channels where a user experiencing fading on one sub-carrier, can be scheduled on another where it sees a better channel. For OFDMA, the proposed algorithm is simply run independently over all sub-carriers in parallel. In this case, the algorithm will jointly schedule the user and power for *each sub-carrier* in the same way as described in the single-carrier case. If a cell cannot schedule a user which contributes enough capacity to the system to outweigh the interference produced, it will remain silent on that specific sub-carrier. As we focus on system capacity maximization, no user to sub-carrier allocation fairness constraint is imposed, and at a given scheduling instant a user may be allocated a number of sub-carriers or none at all. The result of this algorithm on OFDMA systems is illustrated in fig. 2, where we show a possible sub-carrier reuse pattern.

C. Fairness Issues

As we focus in this work on capacity maximization schemes, it is expected that fairness issues will arise with regard to some cells that might experience long periods of silence due to prolonged detrimental fading conditions or a poor user distribution. However, we draw the reader's attention to the fact that solutions akin to the single-cell scheduling scenario, giving various levels of fairness-capacity trade-off, can be used also in the multicell context, e.g. use of proportional-fair type measures [10]. Hence, we may alternatively use a capacity measure for each cell that is normalized by the throughput of the cell. Moreover, when multiple orthogonal units are employed, a cell that is inactive for one code, frequency, or time slot may be active on another. Investigations of the fairness-capacity trade-off are however, left for future work.

V. Numerical Results & Discussion

Monte-Carlo simulations to measure performance of the distributed algorithm are carried out for a single-carrier system only, as the gains will evidently hold for OFDMA systems as well. We consider an operating frequency of 1.8 GHz and two system layouts: a hexagonal cellular system with

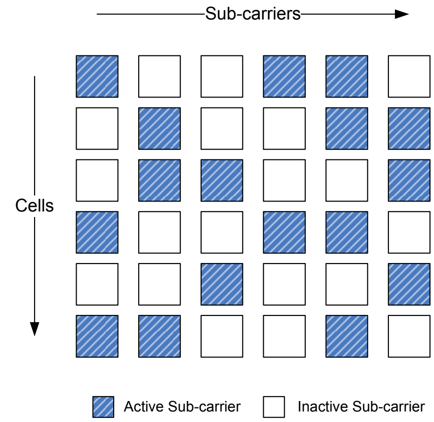

Fig. 2. Snapshot of a full reuse multicell OFDMA network. Possible sub-carrier reuse pattern at a given scheduling period due to dynamic sub-carrier allocation.

cells of radius 200 meters and a square grid with each cell side measuring 500 meters. In our simulations we assume for simplicity that the number of users in each cell is the same, although this is not a restriction of the algorithm. Gains for *all inter-cell and intra-cell* AP-UT links are based on the COST-231 [11] path loss model including lognormal shadowing with standard deviation of 10 dB, as well as fast fading which is assumed i.i.d. with distribution $\mathcal{CN}(0, 1)$. The peak power constraint is given by $P_{\max} = 1$ watt and the maximum number of iterations by $IT_{\max} = 5$.

We compare the distributed approach with full reuse, as well as with traditional fixed reuse patterns under a max-SINR scheduling policy i.e. the user with the best SINR is scheduled .

A. Comparison with Exhaustive Search

We first compare the distributed algorithm with an exhaustive search approach in a 7 cell hexagonal system. The exhaustive search algorithm considers all possible combinations of binary power allocation vectors $\boldsymbol{P} \in \Omega^B$ and schedules the user with the maximum SINR based on the chosen $\boldsymbol{P}$. This will thus serve as an optimal solution for problem (2) if $\boldsymbol{P}$ is restricted to Ω^B instead of Ω, and will demonstrate just how much gain may theoretically be exploited through joint binary power allocation and scheduling. We consider for this case only a 7 cell hexagonal network, as Monte-Carlo simulations of the exhaustive search approach prove cumbersome even for a small network (e.g. if $N = 7$ and $U = 8$, then the number of combinations are $(2^N - 1)(U^N) = 1.27 \times 10^9$). For one user there is no multi-user diversity gain, and the distributed algorithm is able to exploit approximately 50% of the available dynamic spectral reuse gain as compared to Full Reuse (Fig. 3). As the number of users increases, all the algorithms tend towards to keeping all cells on. Figure 4 shows however that with exhaustive search, fewer cells are active for a given number of users than with the proposed distributed algorithm.

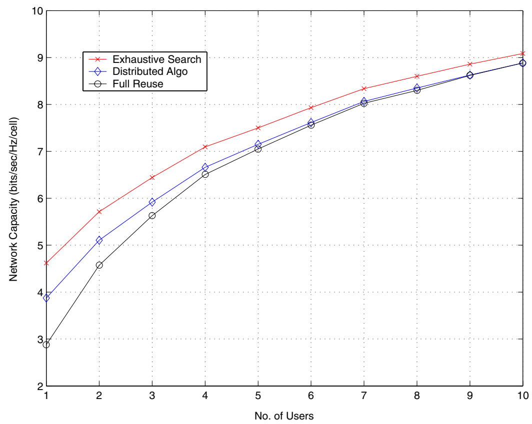

Fig. 3. Network capacity vs. number of users for hexagonal cellular system with 7 cells. Distributed approach lies between the optimal exhaustive search approach and the MAX-SINR-ON algorithm. Convergence to Full Reuse occurs as the number of users increases.

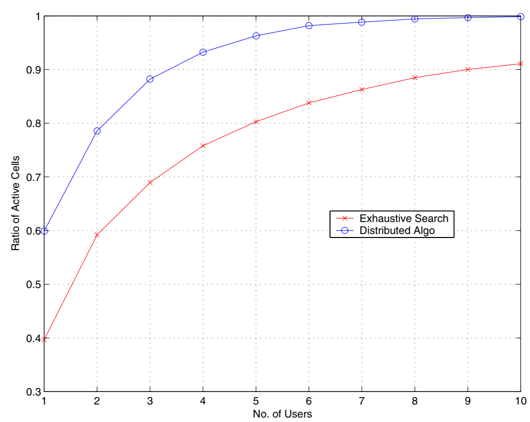

Fig. 4. Ratio of active cells vs. number of users for hexagonal cellular system with 7 cells.

B. Comparison with Static Schemes

In this section we compare our novel distributed algorithm against traditional fixed reuse pattern schemes employing maximum SINR scheduling. Comparison is done with a 19 hexagonal cell system with reuse cluster sizes 3 and 4. As the number of users increase, network capacity for all schemes improves due to the multi-user diversity gain (Fig. 5). Full reuse outperforms traditional frequency reuse schemes as greater spectral reuse maximizes the average system capacity. The distributed algorithm outperforms all other schemes due to dynamic spectral reuse which adapts the reuse pattern to the channel conditions as opposed to the static schemes. The results for just one user are of particular interest. In this case there is no multi-user diversity gain, and therefore this demonstrates the performance of dynamic binary power allocation alone with a round-robin type scheduling policy. The gain of the distributed approach over full reuse is almost 50% (Fig. 5), which demonstrates the merit of inter-cell coordination through dynamic spectral reuse.

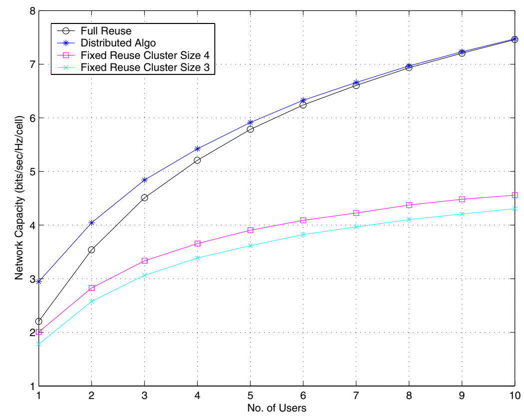

Fig. 5. Network capacity vs. number of users for hexagonal cellular system with 19 cells. Distributed approach provides gain for small number of users and converges to Full Reuse. Dynamic resource allocation outperforms fixed spectral reuse schemes.

VI. Conclusion

We presented in this work a novel distributed algorithm for power allocation and scheduling for capacity maximization in full reuse multicell networks. The key idea is to combine intra-cell multi-user diversity gain with dynamic spectral reuse gain through inter-cell coordination to maximize the overall system capacity. Relying on local cell information, cells which do not offer enough capacity to outweigh interference caused to the network are deactivated. The approach can be applied to OFDMA networks as well, where an added frequency diversity gain can be exploited by scheduling users over sub-carriers. Comparisons with traditional fixed reuse schemes in a realistic network demonstrated significant capacity gains.

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