Power Allocation under Quality of Service Constraints for Uplink Multi-User Systems

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Abstract— **In this contribution, we derive the optimal power allocation policy in the case of an uplink multiuser system under quality of service constraints. Interestingly, using asymptotic results (in the number of users and dimensions) of random matrix theory and under the assumption of Minimum Mean Square Error Successive Interference Cancellation (MMSE-SIC) at the base station, the results show that the users requested rate can be satisfied based only on the knowledge of the statistical nature of the environment and not the channel realization as it is commonly assumed. Moreover, a framework is provided to determine the decoding order of users which minimizes the overall transmitted power. The results are then validated through simulations.**

I. Introduction

With the increasing importance of multiuser communication systems, one of the main issues is to satisfy the different users' rates under multi-user interference. In its full generality, this problem can be solved through proper power allocation (when the rate regions are achievable). However, the power allocation scheme depends on the channel realization, the type of receiver structure as well as the requested rates, as devised in contributions [1]–[7]. Moreover, the complexity of such a scheme increases with the number of users.

In this contribution, for a given target rate and using the Minimum Mean Square Error (MMSE) and Minimum Square Error Successive Interference Cancellation (MMSE-SIC) receivers, explicit expressions of the power allocated to different users are derived based on asymptotic random matrix theory results [8], [9]. Moreover, the decoding order for fixed requested rates in order to minimize the total required power is obtained. The results are applied for random systems (K users and a spreading length of N for CDMA type systems, or K users and N receiving antennas at the base station for multiuser MIMO systems) where the system entries are i.i.d. Gaussian variables.

In section II, the system model is presented. Section III addresses the general derivation of MMSE and MMSE-SIC receivers. Section IV deals with the power allocation formulation of these receivers, and in section V asymptotic expressions are provided based on random matrix theory. Finally in section VI numerical results are presented to validate the theoretical claims.

II. System Model

We consider a system composed of a base station with N transmit/receive dimensions and K users to be covered by the base station (N could be either the number of antennas in a MIMO system or the spreading length in the CDMA case). We are interested in the uplink scenario. Each user k is supposed to send a signal at a requested rate R_k. The input output relationship of the system is then given by:

$$\mathbf{y} = \mathbf{H}\mathbf{P}^{\frac{1}{2}}\mathbf{s} + \mathbf{n}, \quad (1)$$

where $\mathbf{y}$, $\mathbf{s}$, $\mathbf{n}$, $\mathbf{H}$ and $\mathbf{P}^{\frac{1}{2}}$ are respectively the received signal, transmitted signal, additive white Gaussian noise (AWGN) of variance σ^2, channel matrix, and diagonal matrix of transmitted powers. These terms are written out as: $\mathbf{y} = [y_1, y_2, \ldots, y_N]^T$, $\mathbf{s} = [s_1, s_2, \ldots, s_K]^T$, $\mathbf{n} = [n_1, n_2, \ldots, n_N]^T$,

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \ldots & h_{1K} \\ h_{21} & h_{22} & \ldots & h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1} & \ldots & \ldots & h_{NK} \end{bmatrix},$$

and

$$\mathbf{P}^{\frac{1}{2}} = \begin{bmatrix} p_1^{\frac{1}{2}} & 0 & 0 & \ldots & 0 \\ 0 & p_2^{\frac{1}{2}} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & p_K^{\frac{1}{2}} \end{bmatrix}.$$

We suppose that users can have different power allocations in order to satisfy their different (or equal) requested rates.

III. Multiuser receivers

A. MMSE receiver

The Minimum Mean Square Error (MMSE) receiver has several attributes that make it appealing for use. It is known to generate a soft decision output that maximizes the output Signal-to-Interference-plus-Noise Ratio (SINR) [10].

As far as the MMSE SINR is concerned and considering Eq.(1), the output of the MMSE detector, denoted by $\hat{\mathbf{s}} =$

$[\hat{s}_1, \ldots, \hat{s}_K]^T$, is given by

$$\begin{aligned}\hat{\mathbf{s}} &= E\left(\mathbf{s}\mathbf{y}^H\right)\ \left[E(\mathbf{y}\mathbf{y}^H)\right]^{-1}\mathbf{y} \\ &= \mathbf{P}^{\frac{1}{2}}\mathbf{H}^H\left(\mathbf{H}\mathbf{P}\mathbf{H}^H + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{y} \\ &= \mathbf{P}^{\frac{1}{2}}\mathbf{H}^H\mathbf{A}^{-1}\mathbf{y},\end{aligned}$$

with $\mathbf{A} = \mathbf{H}\mathbf{P}\mathbf{H}^H + \sigma^2\mathbf{I}_N$. Each component $\hat{s}_k$ of $\hat{\mathbf{s}}$ is corrupted by the effect of both thermal noise and "multi-user interference" due to the contributions of the other symbols $\{s_l\}_{l\neq k}$. Let us now derive the expression of the SINR at one of the K outputs of the MMSE detector. Let $\mathbf{h}_k$ be the column of $\mathbf{H}$ associated to element s_k, and $\mathbf{U}$ the $N \times (K-1)$ matrix that remains after extracting $\mathbf{h}_k$ from $\mathbf{H}$. The component $\hat{s}_k$ after MMSE equalization has the following form:

$$\hat{s}_k = \eta_{\mathbf{h}_k} s_k + \tau_k,$$

where

$$\eta_{\mathbf{h}_k} = p_k^{\frac{1}{2}}\mathbf{h}_k^H\mathbf{A}^{-1}p_k^{\frac{1}{2}}\mathbf{h}_k, \qquad (2)$$

$$\tau_k = p_k^{\frac{1}{2}}\mathbf{h}_k^H\mathbf{A}^{-1}\mathbf{H}P^{\frac{1}{2}}[s_1, \ldots, s_{k-1}, 0, s_{k+1}, \ldots, s_K]^T \qquad (3)$$

$$+ p_k^{\frac{1}{2}}\mathbf{h}_k^H\mathbf{A}^{-1}\mathbf{n}. \qquad (4)$$

The SINR_k at the output k of the MMSE detector can be shown to be expressed as:

$$\begin{aligned}\text{SINR}_k &= \frac{E[|\eta_{\mathbf{h}_k} x_k|^2 \mid \mathbf{H}]}{E[|\tau_k|^2 \mid \mathbf{H}]} \\ &= \frac{(\eta_{\mathbf{h}_k})^2}{\eta_{\mathbf{h}_k}(1-\eta_{\mathbf{h}_k})} \\ &= \frac{\eta_{\mathbf{h}_k}}{1-\eta_{\mathbf{h}_k}}.\end{aligned}$$

Writing $\mathbf{H}\mathbf{H}^H = \mathbf{U}\mathbf{U}^H + \mathbf{h}_k\mathbf{h}_k^H$ and invoking the matrix inversion lemma[1], we get after some simple algebra another useful expression for this SINR (see e.g. [11]):

$$\text{SINR}_k = p_k\mathbf{h}_k^H\left(\mathbf{U}\mathbf{P}_k\mathbf{U}^H + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{h}_k,$$

where $\mathbf{P}_k$ is the power matrix, from which the k-th column and row have been removed. Note that in practical coding schemes, after applying the MMSE receiver, one will minimize $|\hat{s}_k - \eta_{\mathbf{h}_k}s_k|^2$ with respect to the alphabet in use s_k.

B. MMSE SIC receiver

The MMSE receiver has the advantage of a very low complexity implementation. This feature (due in part to its linearity) has triggered the search for other MMSE based receivers such as the MMSE Successive Interference Cancellation (MMSE-SIC) [12], [13], which is at the heart of very famous schemes such as BLAST [14].

The algorithm relies on a sequential detection of the received block [15]. Recall that $\mathbf{y} = \mathbf{H}\mathbf{P}^{\frac{1}{2}}\mathbf{s} + \mathbf{n}$. At the first step of the method, an MMSE equalization of matrix $\mathbf{T}_{N,K} = \mathbf{H}$ is performed by a multiplication of $\mathbf{y}$ by matrix

$$\mathbf{F}_1 = \mathbf{P}^{\frac{1}{2}}\mathbf{T}_{N,K}^H(\mathbf{T}_{N,K}\mathbf{P}\mathbf{T}_{N,K}^H + \sigma^2\mathbf{I})^{-1}.$$

Suppose that the algorithm starts by decoding symbol s_K. The estimated symbol goes through a turbo-decoder chain in order to improve the reliability of the detection process. Assuming a perfect decision (this is possible if the information s_K has been encoded at a rate of $\log_2(1 + \text{SINR}_K)$), the resulting estimated symbol $\hat{s}_K$ is subtracted from the vector of received samples in the following manner:

$$\mathbf{r}_2 = \mathbf{r}_1 - p_K^{\frac{1}{2}}\hat{s}_K\mathbf{t}_K,$$

where $\mathbf{t}_i$ represents the i^{th} column of $\mathbf{T}_{N,K}$ and vector $\mathbf{r}_1 = \mathbf{y}$. This introduces one degree of freedom for the next cancelling vector choice which enables to reduce the noise plus interference influence and yields an increase in the decision process reliability.

The second step can be virtually represented by a completely new system of $K-1$ symbols $(s_1, \ldots, s_{K-1})$ transmitted with powers $(p_1, \ldots, p_{K-1})$ by an $N \times (K-1)$ matrix $\mathbf{T}_{N,K-1}$ on the same flat frequency fading channel. Equalizing with matrix

$$\mathbf{F}_2 = \mathbf{P}_{K-1}^{\frac{1}{2}}\mathbf{T}_{N,K-1}^H(\mathbf{T}_{N,K-1}\mathbf{P}_{K-1}\mathbf{T}_{N,K-1}^H + \sigma^2\mathbf{I}_N)^{-1},$$

one can retrieve symbol s_{K-1} which has been encoded at a rate of $\log_2(1+\text{SINR}_{K-1})$. The same process described at the beginning can be re-iterated. The advantage of such a scheme is that

$$\text{SINR}_{(K-1)}^{\text{SIC}} \geq \text{SINR}_{(K-1)}^{\text{MMSE}},$$

which means that one is able to convey more information on the second symbol (since the SINR increases) than with MMSE equalization.

The complete detection algorithm can thus be summarized in table I, where $\mathbf{T}_{N,K-i}$ denotes the matrix obtained by suppressing columns $(K, \ldots, K-i+1)$ of $\mathbf{T}_{N,K}$ and $\mathbf{f}_i^{(i)}$ is the i^{th} row of $\mathbf{F}_i$ (the MMSE filtering matrix) at step i. Once the symbol is detected (Eq. (5)), a decision modeled by operator $\widehat{D}$ is made (Eq. (6)). In our case, $\widehat{D}$ is the decision operator made after equalization. When coding is applied, $\widehat{D}$ will simply represent the soft decision operator.

IV. Power Allocation Formulation

In general, the users achievable rates depend on their channel energy as well as their decoding orders. In many applications, users request a target rate whatever the channel conditions may be. In this case, the base station has to allocate the adequate power to the users to satisfy their requirements as well as to make the system decodable. In its full generality, the problem is still an open issue and has not been solved [1]–[7]. For a fixed number of users and in the case where one would like to minimize the total power, the decoding ordering

[1]The matrix inversion lemma states that for any invertible matrix $\mathbf{F}$ and $\mathbf{E}$: $(\mathbf{D}^{-1} + \mathbf{F}\mathbf{E}^{-1}\mathbf{F}^H)^{-1} = \mathbf{D} - \mathbf{D}\mathbf{F}(\mathbf{E} + \mathbf{F}^H\mathbf{D}\mathbf{F})^{-1}\mathbf{F}^H\mathbf{D}^H$.

- Initialization:

$i \leftarrow 1;$
$\mathbf{r}_1 = \mathbf{y};$
$\mathbf{F}_1 = \mathbf{P}^{\frac{1}{2}} \mathbf{T}_{N,K}^H (\mathbf{T}_{N,K} \mathbf{P} \mathbf{T}_{N,K}^H + \sigma^2 \mathbf{I})^{-1};$

- Recursion:

$\tilde{s}_i = \mathbf{F}_i^{(i)} \mathbf{r}_i; \quad (5)$
$\hat{s}_i = \widehat{D}(\tilde{s}_i); \quad (6)$
$\mathbf{r}_{i+1} = \mathbf{r}_i - \hat{s}_i \mathbf{m}_i;$
$\mathbf{F}_{i+1} = \mathbf{P}_{K-i}^{\frac{1}{2}} \mathbf{T}_{N,K-i}^{\ \ H} (\mathbf{T}_{N,K-i} \mathbf{P}_{K-i} \mathbf{T}_{N,K-i}^H + \sigma^2 \mathbf{I})^{-1};$
$i \leftarrow i + 1;$

TABLE I
DETECTION ALGORITHM FOR MMSE-SIC

of the users is a NP complete problem. As a typical example, suppose in the following that the K users request target rates R_k for which the target SINR γ_k is thus given by (supposing Gaussian at the output of the receiver): $\gamma_k = 2^{R_k} - 1$.

A. MMSE receiver

In this case the SINR at the output of the receiver for user k is given by

$$p_k = \frac{\gamma_k}{\mathbf{h}_k^H \left(\sum_{l=1, l \neq k}^K p_l \mathbf{h}_l \mathbf{h}_l^H + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{h}_k}$$

For a given power budget $P = \sum p_k$, the system may not have a solution, depending on the targeted rates of the users.

B. MMSE-SIC receiver

Let us now derive for the MMSE SIC receiver the SINR expression for a given decoding order. Suppose that all the $K - 1$ users have been successively decoded using the SIC approach (for the moment, no decoding order is specified for the users, which is certainly an important issue). In this case, at the last iteration, we have:

$$\mathbf{r}_K = \mathbf{h}_K p_K^{\frac{1}{2}} s_K + \mathbf{n}.$$

The SINR at the output of the MMSE filter for user K is given by:

$$\text{SINR}_K = \gamma_K = \frac{\mathbf{h}_K^H \mathbf{h}_K p_K}{\sigma^2}.$$

Therefore,

$$p_K = \frac{\gamma_K \sigma^2}{\mathbf{h}_K^H \mathbf{h}_K}.$$

This analysis can be extended to iteration i obtaining the corresponding SINR as

$$\text{SINR}_i = \gamma_i = p_i \mathbf{h}_i^H \left(\sum_{l=i+1}^K p_l \mathbf{h}_l \mathbf{h}_l^H + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{h}_i,$$

and the power

$$p_i = \frac{\gamma}{\mathbf{h}_i^H \left(\sum_{l=i+1}^K p_l \mathbf{h}_l \mathbf{h}_l^H + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{h}_i}.$$

As one can observe, the same problems as in the MMSE case arise, additionally having to choose an adequate decoding order to minimize the total required power.

V. ASYMPTOTIC ANALYSIS

In order to provide a suitable power allocation scheme, we suppose the number of users K and the number of dimensions N very high, such as $N, K \to \infty$ but the ratio $\frac{K}{N}$ tends to a fixed constant α. Moreover, we suppose the entries h_{ik} of the matrix $\mathbf{H}$ to be i.i.d zero mean gaussians. In this case, using result of random matrix theory and based on the results of [16] and [17], one can show that:

Result: For a high number of users the SINR γ^k at the output of

- the MMSE receiver is given by:

$$\gamma^k = p_k \frac{1}{\sigma^2 + \frac{1}{N} \sum_{l=1}^K \frac{p_l}{1+\gamma^l}}.$$

- the MMSE-SIC receiver is given by:

$$\gamma^k = p_k \frac{1}{\sigma^2 + \frac{1}{N} \sum_{l=k+1}^K \frac{p_l}{1+\gamma^l}}.$$

Interestingly, irrespective of the channel realization and based only on the target rates, one can determine the optimal power allocation. From a practical stand point, this result is very interesting as it reduces the computational complexity since the powers assigned to the different users need only to be computed with respect to the statistical environment and not the channel realization.

1) MMSE receiver:

a) General rate requirements: In this section each user is supposed to have a (possibly) different rate requirement. Each user will allocate the power such as:

$$p_k = \gamma^k \xi, \quad (7)$$

where

$$\xi = \sigma^2 + \frac{1}{N} \sum_{l=1}^K \frac{\xi \gamma^l}{(1 + \gamma^l)}, \quad (8)$$

As a consequence, ξ is solution of

$$\xi = \frac{\sigma^2}{(1 - \frac{1}{N} \sum_{l=1}^K \frac{\gamma^l}{1+\gamma^l})}. \quad (9)$$

Hence, the rate requirements are always satisfied if $K - \sum_{l=1}^K \frac{l}{1+\gamma^l} < N$ and in particular if $K \leq N$. For different rate requirements (linked to the different SINR γ^l requirements) one can determine the different powers using Eq. (7) and

Eq. (8)).In the case of equal rates requirements, $\gamma^l = \gamma$ for all l, the previous expression particularizes to $\xi = \sigma^2 + \alpha\frac{\xi\gamma}{1+\gamma}$ which yields $\xi = \frac{\sigma^2(1+\gamma)}{1+\gamma(1-\alpha)}$. Therefore,

$$p_k = \frac{\sigma^2\gamma(1+\gamma)}{1+\gamma(1-\alpha)}$$

is a constant depending only on the same required rate for all the users, the system load and noise variance.

b) High rate requirements: For high rate requirements, $\gamma^l >> 1$, we have, $\xi = \sigma^2 + \alpha\xi$ which yields

$$\xi = \frac{\sigma^2}{1-\alpha}.$$

This equation has a solution only for $\alpha \leq 1$. Interestingly, if the rates are high, each user can allocate his power depending only on his requested rate and not the requested rates of all the other users.

2) MMSE-SIC receiver: One of the main issues of the MMSE-SIC receiver is to find the decoding order that minimizes the total required power for a given set of requested rates.

a) General rate requirements:

Result: Let us assume, without loss of generality that we have ordered the users according to increasing requested rates $\gamma^1 \leq \gamma^2 \leq ... \leq \gamma^K$. Then, for the MMSE-SIC receiver, the users should be decoded in precisely that order and the assigned power to each of them is given by

$$p_k = \gamma^k\sigma^2 \prod_{i=k+1}^{K} [1 + \frac{1}{N}\frac{\gamma^i}{1+\gamma^i}]. \quad (10)$$

Proof: We can write $p_k = \alpha_k\gamma^k$, where

$$\alpha_k = \sigma^2 + \frac{1}{N}\sum_{l=k+1}^{K}\frac{p_l}{1+\gamma^l}$$

hence,

$$\alpha_{k-1} = \alpha_k(1 + \frac{1}{N}\frac{\gamma^k}{1+\gamma^k}),$$

$$\alpha_j = \alpha_k \prod_{i=j+1}^{k}(1 + \frac{1}{N}\frac{\gamma^i}{1+\gamma^i}). \quad (11)$$

It can be seen that if we exchange the decoding order of two consecutive users, the powers assigned to all the remaining ones will be kept constant. Without loss of generality, let us consider users decoded in arbitrary consecutive positions j and $j-1$. In this case the required powers to be allocated to them is:

$$p_j = \alpha_j\gamma^j \qquad \text{and} \qquad p_{j-1} = \alpha_j(1 + \frac{1}{N}\frac{\gamma^j}{1+\gamma^j})\gamma^{j-1},$$

whereas if the order of these two users is exchanged

$$p_j^* = \alpha_j\gamma^{j-1} \qquad \text{and} \qquad p_{j-1}^* = \alpha_j(1 + \frac{1}{N}\frac{\gamma^{j-1}}{1+\gamma^{j-1}})\gamma^j,$$

resulting in a difference in needed power

$$\Delta = p_j + p_{j-1} - (p_j^* + p_{j-1}^*) = \frac{\alpha_j}{N}(\frac{\gamma^j\gamma^{j-1}}{1+\gamma^j} - \frac{\gamma^j\gamma^{j-1}}{1+\gamma^{j-1}}),$$

which is positive whenever $\gamma^{j-1} > \gamma^j$. As a consequence, by exchanging the decoding order of consecutive users in order of increasing required SINR, we reduce the needed power. Since this does not affect the remaining ones, the same happens for the total power. The procedure can be repeated starting from any given order and therefore it has been shown that the ranking is indeed optimal. The expression for the power allocation derives directly from Eq. (11) and the fact that $\alpha_K = \sigma^2$

b) High rate requirements: At high rate requirements, $\gamma^l >> 1$, we have,

$$\gamma^K = \frac{p_K}{\sigma^2},$$

$$\gamma^{K-1} = \frac{p_{K-1}}{\sigma^2 + 1/N\sigma^2}.$$

Taking into account that the asymptotically high number of dimension, higher negative powers of N can be neglected, obtaining

$$\gamma^k = \frac{p_k}{\sigma^2 + \frac{K-k}{N}\sigma^2}.$$

Interestingly, the powers of each user can be determined without the need of the base station but only depending on the rank of decoding, which as seen previously would be in order of increasing SINR requirements to minimize the total power required. The power needed for each user can be obtained by particularizing the result of Eq. (10).

VI. Simulations

In this section, numerical results are given to illustrate the theoretical claims. Fig. 1 and Fig. 2 show the comparison between the requested rates (achieved with the asymptotic assumption), with the ones achieved with 40 instantaneous realizations of the channel for a system with N=16 and K=6 users and N=128 and K=48. It can be seen that the performance is quite close to theoretical values, and it is acceptable even for rather small systems. Of course, as expected, the average error reduces with increasing number of users and dimensions. In Fig.3, comparative results for MMSE and MMSE-SIC receivers are shown, with curves plotting the required power as a function of the requested rate, equal for all users. Different cases in terms of the ratio $\alpha = \frac{K}{N}$ are considered. The gain in performance obtained by the use of the MMSE-SIC algorithm becomes significant when the system is sufficiently loaded, i. e., when α approaches 1.

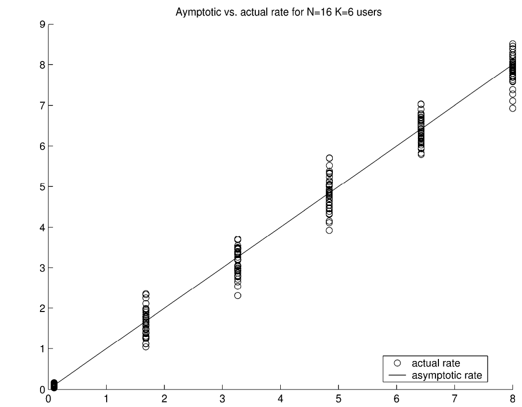

Fig. 1. Comparison between the asymptotic and real rate for a system with N=16 and K=6

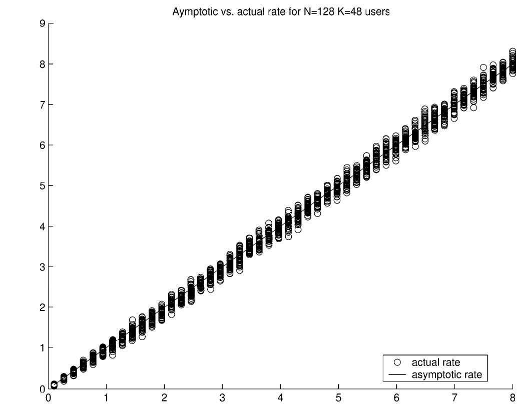

Fig. 2. Comparison between the asymptotic and real rate for a system with N=128 and K=48

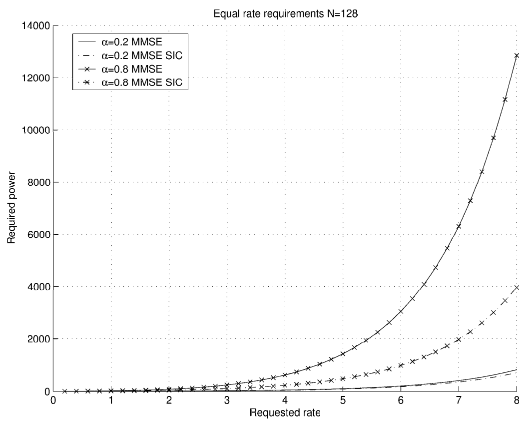

Fig. 3. Needed powers for MMSE and MMSE-SIC for different values of $\alpha = N/K$ and fixed number of dimensions N=128

VII. Conclusion

In this paper, the optimal decoding order for a MMSE-SIC receiver structure is obtained under different rate requirements by the users. Interestingly, the optimal power allocation depends only on the rate requirements, variance of the noise and load of the system and not the particular channel realizations. Extensions of these results to the case of non i.i.d. channels are being considered.

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