

ON THE ACHIEVABLE RATES OF MULTIBAND UWB SYSTEMS

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Abstract

In this work we study the achievable rates of memoryless signaling strategies adapted to ultraWideBand (UWB) multipath fading channels. We focus on strategies which do not have explicit knowledge of the instantaneous channel realization, but may have knowledge of the channel statistics. We evaluate the average mutual information of the general binary flash-signaling rates as a function of channel statistics and derive random coding bounds for m -ary PPM using different non-coherent receivers as well as an imperfect coherent receiver. Then we extend the results to multi-band m -PPM signaling and show that for data rates on the order of 400 Mbits/s, at $4m$ distance between the transmitter and the receiver, can be achieved using simple non-coherent receivers.

I. INTRODUCTION

In this work, we consider achievable rates for transmission strategies suited to *Ultra-wideband (UWB)* systems and focus non-coherent receivers (i.e. those which do not perform channel estimation, but may have prior knowledge of the second-order channel statistics). Here we take a UWB system to be loosely defined as any wireless transmission scheme that occupies a bandwidth between 1 and 10 GHz and more than 25% of its carrier frequency in the case of a passband system.

The most common UWB transmission scheme is based on transmitting information through the use of short-term impulses, whose positions are modulated by a binary information source [1]. This can be seen as a special case of *flash signaling* coined by Verdu in [9]. Similar to direct-sequence spread-spectrum, the positions can further be modulated by an m -ary sequence (known as a *time-hopping sequence*) for mitigating inter-user interference in a multiuser setting [2]. This type of UWB modulation is a promising candidate for military imaging systems as well as other non-commercial sensor network applications because of its robustness to interference from signals (potentially from other non-UWB systems) occupying the same bandwidth. Based on recent documentation from the FCC it is also being considered for commercial adhoc networking applications based on peer-to-peer communications, with the goal be to provide low-cost high-bandwidth connections to the internet from small handheld terminals in both indoor and outdoor settings. Proposals for indoor wireless LAN/PAN systems in the 3-5 GHz band (802.15.3) are also considering this type of transmission scheme. These ideas can be extended to multiband setting. The current industrial trend is the use of multi-bands each of 500 MHz.

In this work, we focus on the case of non-coherent detection since it is well known [11] [12] that coherent detection is not required to achieve the *wideband* AWGN channel capacity, $C_\infty = \frac{P_R}{N_0 \ln 2}$ bits/s, where P_R is the received signal power in watts, and N_0 is the noise power spectral density. In [12] Telatar and Tse showed this to be the case for arbitrary channel statistics in the limit of infinite bandwidth. Their transmission model was based on frequency-shift keying (FSK) and it was shown that channel capacity is achieved using very high transmission duty cycle.

In [9] Verdu addresses the spectral efficiency of signaling strategies in the wideband regime under different assumptions regarding channel knowledge at the transmitter and receiver. The characterization is in terms of the minimum energy-per-bit to noise spectral density ratio $(E_b/N_0)_{\min}$ and the wideband slope S_0 . The latter quantity is measured in bits/s/Hz/3dB and represents growth of spectral efficiency at the origin as a function of E_b/N_0 . Verdu's work is fundamental to our problem since it shows that approaching C_∞ with non-coherent detection is impossible for practical data rates (>100 kbit/s) even for the vanishing spectral efficiency of UWB systems. This is due to the fact that S_0 is zero at the origin for non-coherent detection. To get an idea of the loss incurred, consider a system with a 2GHz bandwidth and data rate of 20 Mbit/s (this would correspond to a memoryless transmission strategy for channels with a 50ns delay-spread) yielding a spectral-efficiency of .01 bits/s/Hz. For Rayleigh statistics the loss in energy efficiency is on the order of 3dB, which translates into a factor 2 loss in data rate compared to a system with perfect channel state information at the receiver. The loss becomes less significant for lower data rates and/or higher bandwidths.

The main goal of this work is to examine under what conditions different non-coherent signaling strategies can approach the wideband channel capacity (i.e. with perfect channel knowledge at the receiver) subject to a large but finite bandwidth constraint and different propagation conditions.

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II. SYSTEM MODEL

We restrict our study to strictly time-limited memoryless real-valued signals, both at the transmitter and receiver. We consider a block fading channel model so that the channel impulse response is time-invariant in any interval of $[kT_c, (k+1)T_c)$, where T_c is the *coherence-time* of the channel. We denote the channel in any block by $h_k(t)$ which is assumed to be a zero-mean process, motivated by the fact that scattering off objects causes 180 degree phase reversals in the impinging components of the wavefront. For simplicity in the analytical developments, we assume that the channel realization in every block is independent and identically distributed, so that $E[h_k(t)h_l(u)] = R_h(t, u)\delta_{kl}$, where $R_h(t, u)$ is the auto-correlation function of the channel response in a particular interval. The received signal is

$$r(t) = \sum_{k=0}^N s(u_k) \sqrt{E_s} p(t - kT_s) * h_k(t) + z(t) \quad (1)$$

where k is the symbol index, T_s the symbol duration, $E_s = PT_s$ the transmitted symbol energy, u_k is the transmitted symbol at time k , $p(t)$ and $s(u_k)$ are the assigned pulse and amplitude for symbol u_k , and $z(t)$ is white Gaussian noise with power spectral density $N_0/2$. For all k , $p(t)$ is a unit-energy pulse of duration T_p . This signaling model encompasses modulation schemes such as flash signaling, m -ary PPM, amplitude, and differential modulation. A guard interval of length T_d is left at the end of each symbol (from our memoryless assumption) so that $T_s \geq T_p + T_d$, and the symbol interval $T_s \ll T_c$. The received signal bandwidth W is roughly $1/T_p$, in the sense that the majority of the signal energy is contained in this finite bandwidth.

The previous scheme can be generalized through the use of multi-band signaling, where parallel independent signal streams are transmitted on the F sub-carriers. The spacing between two adjacent sub-carriers (in order to insure their orthogonality for rectangular $p(t)$) is taken to be $\Delta F = \frac{1}{T_p}$. In this case, $F = \lfloor BT_p \rfloor$ where B is the total system bandwidth. This memoryless transmission strategy resembles OFDM signaling, where the guard-interval plays the role of the cyclic prefix. Within each sub-band, however, impulsive signaling is still used.

The large bandwidths considered here (≥ 500 MHz) provide a high temporal resolution and enable the receiver to resolve a large number of paths of the impinging wavefront. Providing that the channel has a high diversity order (i.e. in rich multipath environments), the total channel gain is slowly varying compared to its constituent components. It has been shown [5], [7], [8] through measurements that in indoor environments, the UWB channel can contain several hundreds of paths of significant strength. We may assume, therefore, that for all practical purposes, the total received energy should remain constant at its average path strength, irrespective of the particular channel realization. Variations in the received signal power will typically be caused by shadowing rather than fast fading.

Through a Karhunen-Loève expansion we rewrite the channel model in (1), for each symbol k , as the equivalent set of parallel channels

$$\begin{aligned} r_{k,i} &= \sqrt{\mu_1} h_i \sqrt{E_s \lambda_i} s(u_k) + z_i, i = 1, \dots, \infty \\ R_k &= \{r_{k,1}, r_{k,2}, \dots\} \end{aligned} \quad (2)$$

where z_i is $\mathcal{N}(0, N_0/2)$ and $\{h_i\}$ are unit variance zero mean independent gaussian variables. The $\{\lambda_i\}$ are the solution to

$$\lambda_i \phi_i(t) = \int_0^{T_d+T_p} R_o(t, u) \phi_i(u) du. \quad (3)$$

where ϕ_i and $R_o(t, u)$ are the eigenfunctions and the autocorrelation function of the composite channel $h_k(t) * p(t)$, respectively. Because of the bandlimiting nature of the channels in this study, the channel will be characterized by a finite number, D , of significant eigenvalues which for rich environments will be close to $1 + 2WT_d$, in the sense that a certain proportion of the total channel energy will be contained in these D components. Based on measurement campaigns [3] we see that the number of significant eigenvalues can be large but significantly less than the approximate dimension of the signal-space $1 + 2WT_d$ [4, Chapter 8]. This is due to insufficient scattering in short-range indoor environments. For notational convenience, we will assume that the eigenvalues are ordered by decreasing amplitude.

III. NON-COHERENT DETECTION

In this section we consider non-coherent receivers that may or may not have access to the second-order channel statistics. The motivation for such a study is to derive receivers that are very low-cost from an implementation standpoint. We assume that the transmitter does not have any side information about the channel and that it is constrained to use flash-like signaling.

A. Average Mutual Information

In the case of vanishing signal-to-noise ratio, the simplest form of flash-signaling is optimal [9], [10]. Using the notation from the previous section, we express the flash signaling scheme as

$$u_k = \begin{cases} 1 & \text{with probability } \eta \\ 0 & \text{with probability } (1 - \eta) \end{cases} \quad (4)$$

$s(0) = 0$, $s(1) = \sqrt{\frac{E_s}{\eta}}$, and $T_s = T_d + T_p$. We assume that the h_i are Gaussian ergodic sequences, which implies that the system's temporal resolution is not fine enough to resolve all the degrees of freedom of the considered channel and that the projection of $h(t)$, on each of the kernel's directions, is the combination of a relatively large number of independent multipath components. Frequency-domain measurements of UWB channels [5] have shown that channel components, at a particular frequency, can be considered to fade according to Rayleigh statistics, indicating that this assumption is quite reasonable. Then R is a zero-mean Gaussian vector with covariance matrix $E[RR^T] = \text{diag}(s(u_k)E_s\lambda_i + \frac{N_0}{2})$. We show in Appendix I that the average mutual information as a function of the received symbol energy is given by

$$I(u_k; R_k)[E_s] = -\frac{1}{T_s} \frac{E}{Y} \left[\eta \log \left(\eta + (1-\eta) \sqrt{\prod_{i=1}^D 1 + \frac{2E_s\lambda_i}{\eta N_0}} e^{(-\frac{1}{2} Y (\text{diag}(\frac{N_0\eta}{2E_s\lambda_i}))^{-1} Y^T)} \right) \right. \\ \left. + (1-\eta) \log \left((1-\eta) + \frac{\eta}{\sqrt{\prod_{i=1}^D 1 + \frac{2E_s\lambda_i}{\eta N_0}}} e^{(\frac{1}{2} Y (\text{diag}(\frac{N_0\eta}{2E_s\lambda_i} + 1))^{-1} Y^T)} \right) \right] \text{ bits/s} \quad (5)$$

with Y a zero-mean gaussian random vector with unit-variance independent components. This expression is easily computed numerically.

In the case of multi-band signalling, the total available power to the transmitter is split over the F parallel sub-bands. Denoting the symbol vector $U_k = (u_{k,1}, u_{k,2}, \dots, u_{k,F})$, we have that the total average mutual information between the set of transmitted symbols U_k and the set of received signals on the F sub-bands is given by

$$I(U_k; R_k^1, R_k^2, \dots, R_k^F)[E_s] = \sum_{i=1}^F I(u_{k,i}; R_k^i)[E_s/F] \\ = FI(u_k; R_k)[E_s/F] \quad (6)$$

where R_k^f is the observation vector on the f th sub-band during the k th time-interval. We should note that throughout all the paper, we assume that different sub-bands are statistically independent.

B. m -ary PPM with Energy Detection

In this section we consider random coding bounds for flash-signaling implemented with m -PPM modulation both the optimal non-coherent detector, for known second order channel statistics, and a suboptimal mismatched energy detector. m -PPM can be seen as a specially-designed channel code for flash-signaling. Each m -PPM symbol corresponds to choosing one out of m symbol times, constituting a PPM frame, in which to emit the transmit pulse $p(t)$, which is a special case of flash signaling with $\eta = 1/m$ and exactly one pulse transmitted per frame. The data is encoded using a randomly generated codebook $\mathcal{C} = \{C_1, C_2, \dots, C_M\}$ of cardinality M and codeword length N . Each codeword C_l is a sequence $C_l = (c_{1,l}, c_{2,l}, \dots, c_{N,l})$ corresponding to the emission timeslot indices within each of the N frames used for its transmission, where $c_{i,j} \in \{0, m-1\}$. Let C_w be the transmitted codeword, using the notation of model (1) we have for $k \in [nm, (n+1)m-1]$ that $u_k = \mathcal{I}(c_{n,w} = (k \bmod m))$ and $s(u_k) = u_k/\eta$. \mathcal{I} stands for the indicator function.

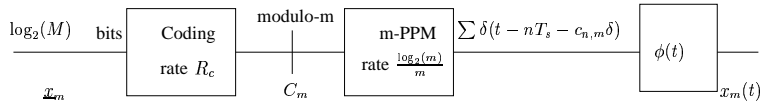


Fig. 1. Transmitter block diagram.

For all $n \in [1, N]$ and $k \in [1, m]$ let $R_{n,k} = R_{nm+k}$, $\underline{R}_N = \{R_{n,k} | n = 1 \dots N, k = 1 \dots m\}$ and w denotes the index of the transmitted codeword.

1) *Channel-matched Non-Coherent Detector*: The Maximum likelihood non-coherent detector can be written (see Appendix II) as

$$\hat{k} = \underset{k}{\operatorname{argmax}} \Pr(\underline{R}_N/w = k) = \underset{k}{\operatorname{argmax}} \frac{1}{N} \sum_{n=1}^N q_{n,k} \quad (7)$$

with $q_{n,k} = R_{n,k} Q^{-1} R_{n,k}^T$ and $Q = \text{diag}(\frac{N_0}{2} (1 + \frac{N_0}{2E_s\lambda_i}))^{-1}$.

For the derivation of random coding bounds we use the following sub-optimal receiver. The decoder forms the decision variables

$$q_k = \frac{1}{N} \sum_{n=1}^N q_{n,k} \quad (8)$$

¹this detector is equivalent to the classical estimator-correlator [13]

and uses the following threshold rule to decide on a message: if q_k exceeds a certain threshold ρ for exactly one value of k , say \hat{k} , then it will declare that \hat{k} was transmitted. Otherwise, it will declare a decoding error. This is the same sub-optimal decoding scheme considered in [12].

An upper bound of the decoding error probability is then given by the following theorem

Theorem 1: The probability of codeword error is upper bounded by

$$\Pr[\text{error}] \leq M \min_{t>0} \exp -N \left[t\rho - \ln \left((1-\eta) \prod_{i=1}^D \frac{1}{\sqrt{1 - \frac{2t}{1 + \frac{N_0}{2E_s\lambda_i}}}} + \eta \prod_{i=1}^D \frac{1}{\sqrt{1 - \frac{4E_s\lambda_i t}{N_0}}} \right) \right] \quad (9)$$

with $\rho = 2(1-\epsilon)E_s/N_0$, and $\eta = 1/m$.

Proof: see Appendix III ■

The decoding error probability in equation (9) decays to zero exponentially in N as long as the transmission rate R satisfies

$$R = \frac{1}{mNT_s} \log(M) \leq \max_{t>0} \frac{1}{T_s} \left(t\rho - \ln \left((1-p) \prod_{i=1}^D \left(1 - \frac{2t}{1 + \frac{N_0}{2E_s\lambda_i}} \right)^{-\frac{1}{2}} + p \prod_{i=1}^D \left(1 - \frac{4E_s\lambda_i t}{N_0} \right)^{-\frac{1}{2}} \right) \right) \quad (10)$$

Due to the finite cardinality of the symbol alphabet our information rate is bounded by

$$R \leq \frac{\log 2(m)}{mT_s} \text{bits/s} \quad (11)$$

2) *Mismatched non-coherent detector:* We now consider the case where the receiver does not have access to channel statistics and/or is constrained to use a time-invariant front-end filter because of implementation considerations. The received signal is first filtered by the time-limited unit-energy filter $f(t)$ of duration T_f , this filtering aims to reduce the amount of receiver noise while capturing the majority of its information bearing part.

$$\begin{aligned} r_f(t) = (r * f)(t) &= ((s + z) * f)(t) \\ &= s_f(t) + z_f(t) \end{aligned} \quad (12)$$

then for each potential emission position $t_{n,k} = ((n-1)m + k)T_s$ we capture the received energy on the interval from $t_{n,k}$ to $t_{n,k} + T_s$

$$q_{n,k} = \int_{t_{n,k}}^{t_{n,k} + T_s} r_f^2(t) dt \quad (13)$$

$$q_{n,k} = \begin{cases} \sum_{i=1}^D \lambda_i r_{f,i}^2 & c_{n,w} = k \\ \sum_{i=1}^D \mu_i r_{f,i}^2 & c_{n,w} \neq k \end{cases} \quad (14)$$

with λ_i and μ_i being the solutions to

$$\lambda_i \phi_i(t) = \int_0^{T_d+T_p} \mathbf{R}_{s_f+z_f}(t,u) \phi_i(u) du \quad (15)$$

$$\mu_i \theta_i(t) = \int_0^{T_d+T_p} \mathbf{R}_{z_f}(t,u) \theta_i(u) du \quad (16)$$

where $\mathbf{R}_{s_f+z_f}(t,u)$ and $\mathbf{R}_{z_f}(t,u)$ are the autocorrelation functions of the filtered received signal respectively with and without the presence of a transmitted pulse and $\{\phi_{n,1}, \dots, \phi_{n,D}\}$ and $\{\theta_{n,1}, \dots, \theta_{n,D}\}$ are the resulting basis functions in the Karhunen-Loève decompositions. The $r_{f,i}$ are zero mean unit variance random variables resulting from the projection of the received signal on the basis functions.

Using the same decoding rule as in the previous section, we derive an upper bound on the decoding error probability

Theorem 2: The probability of codeword error is upper bounded by

$$\Pr[\text{error}] \leq M \min_{t>0} \exp -N \left[t\rho - \ln \left((1-p) \prod_{i=1}^D \frac{1}{\sqrt{1 - \mu_i t}} + p \prod_{i=1}^D \frac{1}{\sqrt{1 - \lambda_i t}} \right) \right] \quad (17)$$

with $\rho = (1-\epsilon) \sum_{i=1}^D \lambda_i$, and $\eta = 1/m$.

The proof is identical to the previous case and is omitted.

A question of significant importance is whether coherent detection schemes (i.e. RAKE receivers) are robust to channel estimation imperfections in UWB systems. In this section we address the problem of characterizing the degradation of coherent detection performance due to an additive channel estimation noise. This will be achieved through the derivation of an upper bound on the decoding error probability.

Consider the case where a noisy estimate of the channel $\tilde{h}(t) = h(t) + n(t)$ is available at the receiver side, where $n(t)$ is a white gaussian zero-mean random process with variance σ_h^2 . We use the same system described in (III-B.1) Projecting $r(t)$ and the noisy channel $\tilde{h}(t) * p(t) = \tilde{h}_p(t)$ over the same Karhunen-Loève basis functions as in (2) we obtain correlate the received signal with the noisy channel estimate (i.e. a RAKE receiver), $\tilde{h}_{p,k,i}$ for all the possible pulse emission positions at each signal frame. An upper bound of the decoding error probability is given by the following theorem

Theorem 3: The probability of codeword error is upper bounded by

$$\Pr[\text{error}] \leq M \min_{t>0} \exp -N \left[t\rho - \ln \left((1-\eta) \prod_{i=1}^D \frac{1}{\sqrt{1 - \frac{N_0}{2} (E_s \lambda_i + \sigma_h^2) t^2}} + \eta \prod_{i=1}^D \frac{1}{\sqrt{1 - (\frac{N_0}{2} \sigma_h^2 + E_s \lambda_i \sigma_h^2 + \frac{N_0}{2} E_s \lambda_i) t^2 - 2E_s \lambda_i t}} \right) \right] \quad (18)$$

with $\rho = (1 - \epsilon)E_s$, and $\eta = 1/m$.

The proof is again identical to the first case. Note that for the following numerical evaluations, the filter $f(t)$ is assumed to be a prolate spheroidal wave function ([14]), which implies that the filtered noise eigenvalues distribution $\{\mu_i\}$ has a flat profile, $\mu_i \approx N_0/2D$ over its D most significant values².

V. DISCUSSION

We show in Fig. 2,3 the numerical evaluation of the bounds in the previous sections for a single 1 GHz band and delay-spread of 25ns for two different eigenvalue distributions. We show the achievable information rates as a function of the received SNR. The symbol alphabet size m is optimized numerically for different values of the SNR. For the case of flash signaling the emission probability η is similarly optimized numerically. The composite channel eigenvalues distribution, $\{\lambda_i\}$, is taken to have an exponential profile $\lambda_i = \alpha e^{\beta i}$. This model seems to accurately model actually measured UWB channel eigenvalues distributions ([3]). For a rapidly decaying exponential eigenvalue distribution we see that the mismatched energy detector suffers a penalty on the order of a factor of 2 with respect to the receiver matched to the second order statistics. An m -PPM based system can, however, approach the information rates of general flash signaling. For slowly decaying eigenvalue distributions (i.e. many very significant eigenvalues) there is little difference between matched and mismatched receivers. It also turns out that the optimization of the modulation size, as a function of the system operating SNR, leads to a constant received peak SNR and an outer code rate on the order of 1/2 irrespective of average received SNR.

The imperfect coherent detector, even for the case of low estimation noise variance, does not outperform the simple non-coherent receivers. This is due to the large bandwidth (number of degrees of freedom) of UWB channels. Nevertheless, all the considered receivers suffer a degradation on the order of a factor of 2 with respect to the ideal channel capacity. It is reasonable to assume, therefore, that if very accurate channel estimates can be obtained for the case of slowly time-varying systems and at the expense of increased receiver complexity, we can approach the ideal channel capacity.

In Fig. 4 we show the achievable rates for multiband signaling with a total system bandwidth of 7.5 GHz and a delay spread of 25ns, now as a function of the distance between the transmitter and receiver assuming a path loss model, $P_R = Kd^{-3.1}$ and an attenuation of 80dB at 10m. This is based on reported measurements in [6]. The eigenvalue distribution is a rapidly decaying exponential. Here we see that high information rates (400 MBit/s at 4m) are attainable even with extremely simple receiver structures.

APPENDIX I MUTUAL INFORMATION

$$\begin{aligned} I(u_k; R_k)[E_s] &= \eta \int_{R_k} P(R_k|u_k = 1) \log \left(\frac{P(R_k|u_k = 1)}{\eta P(R_k|u_k = 1) + (1-\eta)P(R_k|u_k = 0)} \right) \\ &+ (1-\eta) \int_{R_k} P(R_k|u_k = 0) \log \left(\frac{P(R_k|u_k = 0)}{\eta P(R_k|u_k = 1) + (1-\eta)P(R_k|u_k = 0)} \right) \end{aligned} \quad (19)$$

²Strictly speaking, the eigenvalues $\{\mu_i\}$ are all distinct.

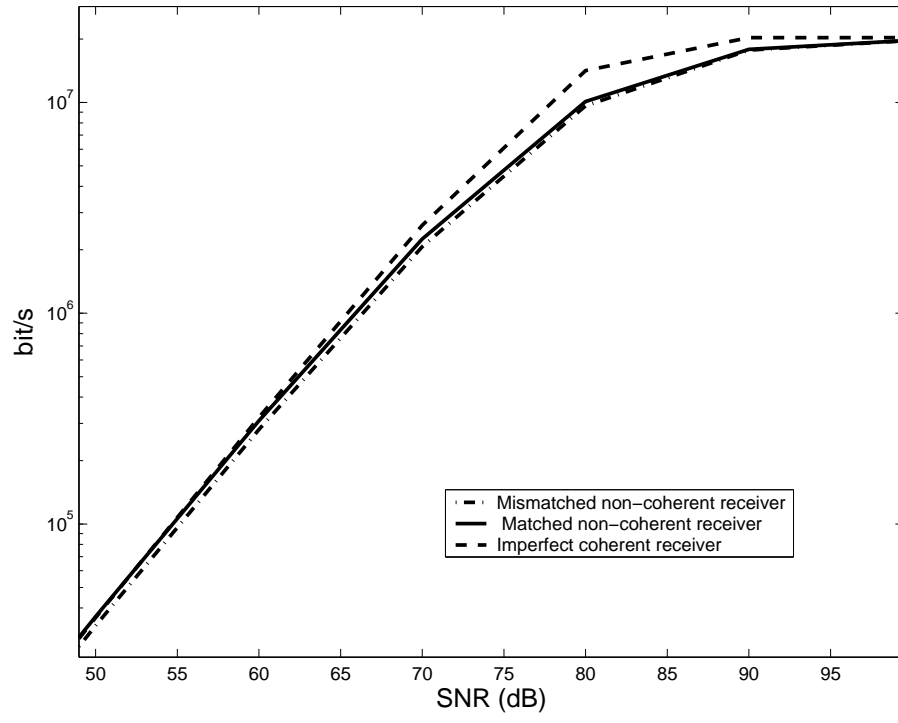


Fig. 2. Achievable rates of the considered receivers with a flat channel eigen values distribution: $T_d=25$ ns, $W=1$ GHz

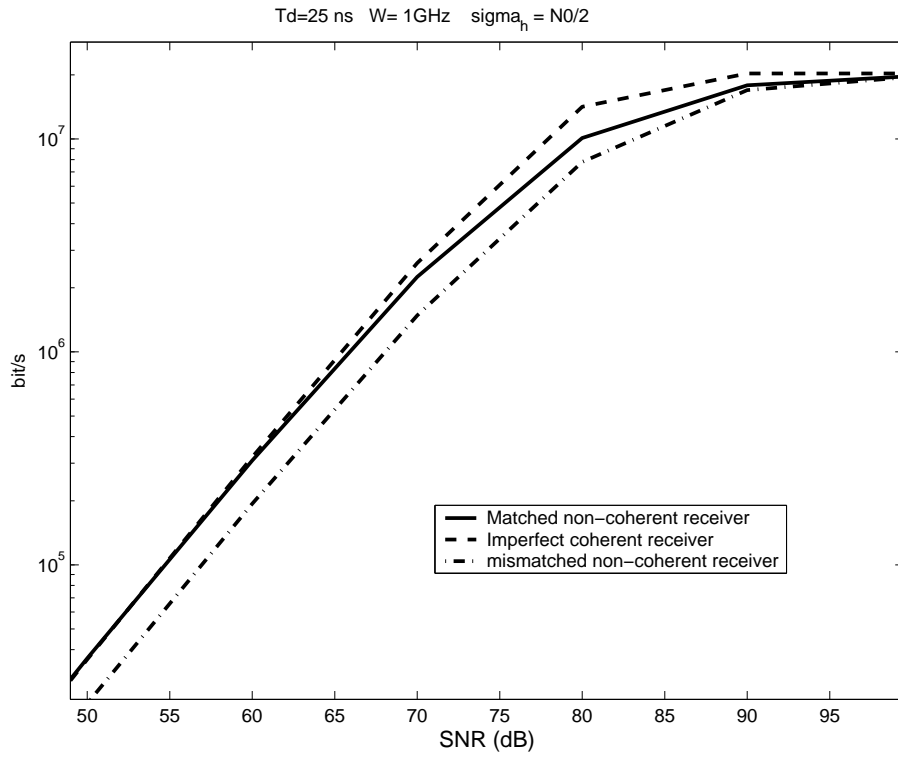


Fig. 3. Achievable data rates of the considered receivers with a rapidly decaying channel eigen values distribution: $T_d=25$ ns, $W=1$ GHz

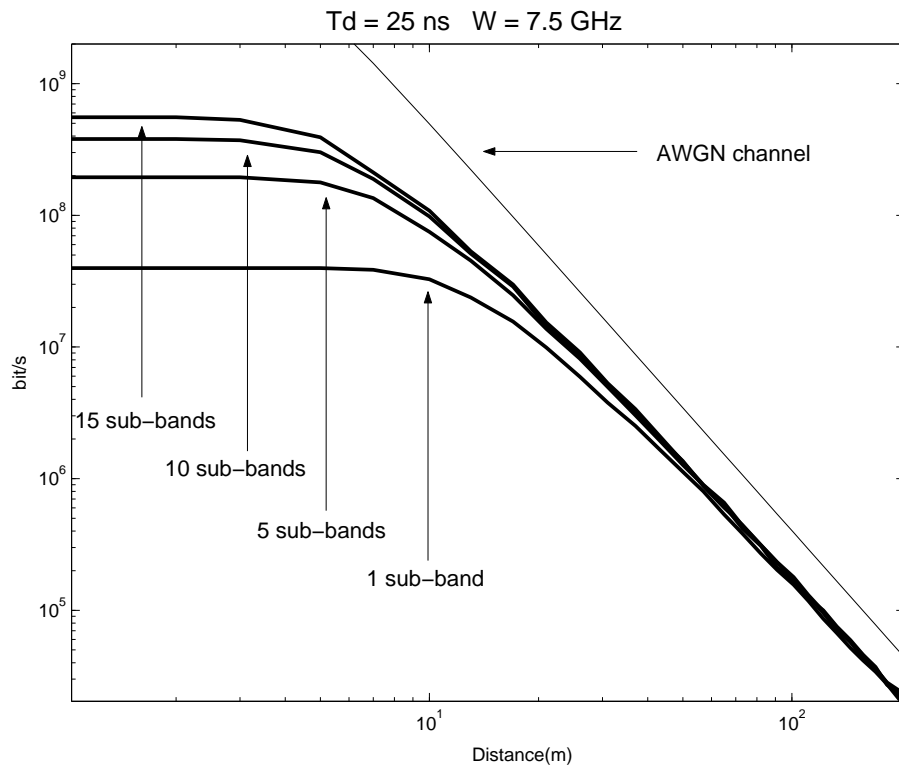


Fig. 4. Achievable data rates vs. distance of non-coherent multiband flash-signaling: $T_d=25\text{ns}$, $W=7.5\text{GHz}$

with

$$\begin{aligned}
 P(R_k | u_k = 0) &= \frac{1}{\text{sqr}t(\pi N_0)^N} \exp\left(-\frac{1}{2} R_k \left(\frac{N_0}{2} \mathbf{I}\right)^{-1} R_k^T\right) \\
 P(R_k | u_k = 1) &= \frac{1}{\sqrt{(2\pi)^N \prod_{i=1}^N \left(\frac{N_0}{2} + \frac{\lambda_i E_s}{\eta}\right)}} \exp\left(-\frac{1}{2} R_k \left(\text{diag}\left(\frac{N_0}{2} + \frac{\lambda_i E_s}{\eta}\right)\right)^{-1} R_k^T\right)
 \end{aligned} \quad (20)$$

then performing respectively the variable changes $Y = \sqrt{\frac{2}{N_0}} R_k$ and $Y = R_k \text{diag}\left(\sqrt{\frac{N_0}{2} + \frac{\lambda_i E_s}{\eta}}\right)^{-1}$ in the first and second integral of the right-hand side of equation 19 we obtain the desired result.

APPENDIX II

DERIVATION OF THE MAXIMUM LIKELIHOOD RECEIVER

The conditional probability in (7) is written as

$$\begin{aligned}
 \Pr(\underline{R}_N / w = k) &\stackrel{(a)}{=} \prod_{n=1}^N \prod_{j=1}^m \Pr(R_{n,j} / w = k) \\
 &= \prod_{n=1}^N \left(\prod_{j \neq c_{n,k}} \left(\prod_{i=1}^D \left(\frac{1}{\sqrt{\pi N_0}} e^{-\frac{r_{(n-1)m+j,i}^2}{N_0}} \right) \right) \prod_{i=1}^D \left(\frac{1}{\sqrt{2\pi (E_s \lambda_i + \frac{N_0}{2})}} e^{-\frac{r_{(n-1)m+c_{n,k},i}^2}{2(E_s \lambda_i + \frac{N_0}{2})}} \right) \right)
 \end{aligned}$$

in (a) we use the fact that, conditioned on the transmitted codeword, the observation vectors within one particular frame $R_{n,j} j = 0, \dots, m-1$ are statistically independent. The decision rule can thus be written equivalently as

$$\begin{aligned}
\hat{k} &= \underset{k}{\operatorname{argmin}} \sum_{n=1}^N \left(\sum_{\substack{j=1 \\ j \neq c_{n,k}}}^m \sum_{i=1}^D \frac{r_{(n-1)m+j,i}^2}{N_0} + \sum_{i=1}^D \frac{r_{(n-1)m+c_{n,k},i}^2}{2(E_s \lambda_i + \frac{N_0}{2})} \right) \\
&= \underset{k}{\operatorname{argmin}} \sum_{n=1}^N \left(\sum_{i=1}^D \frac{r_{(n-1)m+c_{n,k},i}^2}{2(E_s \lambda_i + \frac{N_0}{2})} - \sum_{i=1}^D \frac{r_{(n-1)m+c_{n,k},i}^2}{N_0} \right) \\
&= \underset{k}{\operatorname{argmax}} \sum_{n=1}^N R_{n,k} Q^{-1} R_{n,k}^T
\end{aligned} \tag{21}$$

APPENDIX III PROOF OF THEOREM 1

The decision variable for the transmitted codeword $C_w \in \mathcal{C}$ is given by

$$\frac{1}{N} \sum_{n=1}^N R_{n,w} Q^{-1} R_{n,w}^T \tag{22}$$

by the ergodicity of the noise process, this time-average will exceed the threshold with probability arbitrarily close to 1 for any $\epsilon > 0$ as N gets large. For all $k \neq w$ We bound the probability $\Pr[q_k \geq \rho]$ using a Chernoff bound

$$\begin{aligned}
\Pr[q_k \geq \rho] &= \Pr[Nq_k \geq N\rho] \\
&= \mathbb{E}_{\mathcal{C}}[\Pr[Nq_k \geq N\rho/\mathcal{C}]] \\
&\stackrel{(b)}{\leq} \mathbb{E}_{\mathcal{C}} \left[\min_{t>0} e^{-tN\rho} \prod_{n=1}^N \mathbb{E}[e^{tq_{n,k}}/\mathcal{C}] \right]
\end{aligned} \tag{23}$$

in (b), $q_{n,k}$ are statistically independent from our block fading and random coding assumptions.

We have that for all $c_{n,k} = c_{n,w}$

$$\mathbb{E}[e^{tq_{n,k}}] = \prod_{i=1}^D \left(1 - \frac{4E_s \lambda_i t}{N_0} \right)^{-\frac{1}{2}} \tag{24}$$

and for all $c_{n,k} \neq c_{n,w}$

$$\mathbb{E}[e^{tq_{n,k}}] = \prod_{i=1}^D \left(1 - \frac{2t}{1 + \frac{N_0}{2E_s \lambda_i}} \right)^{-\frac{1}{2}} \tag{25}$$

Let l be the number of collisions between codewords C_w and C_k , $l = \operatorname{card}(n = 1 \dots N/c_{n,w} = c_{n,k})$, then we have that

$$\begin{aligned}
\Pr[q_k \geq \rho/\mathcal{C}] &= \Pr[q_k \geq \rho/l] \\
&\leq \min_{t>0} \left[e^{-Nt\rho} \left(\prod_{i=1}^D \left(1 - \frac{4E_s \lambda_i t}{N_0} \right)^{-\frac{1}{2}} \right)^l \left(\prod_{i=1}^D \left(1 - \frac{2t}{1 + \frac{N_0}{2E_s \lambda_i}} \right)^{-\frac{1}{2}} \right)^{N-l} \right]
\end{aligned} \tag{26}$$

Averaging over all the realizations of the randomly generated codebook we obtain

$$\begin{aligned}
\mathbb{E}_C [\Pr [q_k \geq \rho/C]] &\leq \mathbb{E}_l \left[\min_{t>0} e^{-Nt\rho} \prod_{i=1}^D \frac{1}{\left(1 - \frac{4E_s \lambda_i t}{N_0}\right)^{\frac{l}{2}} \left(1 - \frac{2t}{1 + \frac{N_0}{2E_s \lambda_i}}\right)^{\frac{N-l}{2}}} \right] \\
&\stackrel{(c)}{\leq} \min_{t>0} \mathbb{E}_l \left[e^{-Nt\rho} \prod_{i=1}^D \frac{1}{\left(1 - \frac{2t}{1 + \frac{N_0}{2E_s \lambda_i}}\right)^{\frac{(N-l)}{2}} \left(1 - \frac{4E_s \lambda_i t}{N_0}\right)^{\frac{l}{2}}} \right] \\
&= \min_{t>0} e^{-Nt\rho} \left(\sum_{l=0}^N \binom{l}{N} p^l (1-p)^{N-l} \prod_{i=1}^D \left(1 - \frac{2t}{1 + \frac{N_0}{2E_s \lambda_i}}\right)^{-\frac{N-l}{2}} \left(1 - \frac{4E_s \lambda_i t}{N_0}\right)^{-\frac{l}{2}} \right) \\
&= \min_{t>0} e^{-Nt\rho} \left((1-p) \prod_{i=1}^D \frac{1}{\sqrt{1 - \frac{2t}{1 + \frac{N_0}{2E_s \lambda_i}}}} + p \prod_{i=1}^D \frac{1}{\sqrt{1 - \frac{4E_s \lambda_i t}{N_0}}} \right)^N \tag{27}
\end{aligned}$$

note that in (c) we perform a looser minimization operation for the sake of feasibility of the analytical developments. Using a union bound we obtain the desired result.

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