

# TRAINING SEQUENCE BASED MULTIUSER CHANNEL IDENTIFICATION FOR COCHANNEL INTERFERENCE CANCELLATION IN GSM

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## ABSTRACT

In this paper, we address the problem of the joint identification of the channel impulse responses of almost-synchronous cochannel users. This problem arises in a cellular system using the time division multiple access (TDMA) scheme where the base stations are assumed to be synchronized and the radius of the cells is limited to a few kilometers. After the partial equalization of the transmit pulse shape filter which compensates for the intersymbol interference (ISI) introduced by the GMSK modulation (used in GSM), the receiver should detect the active training sequences by a synchronization step and then estimates the corresponding channel impulse responses by the standard least squares (LS) method. The spatio-temporal receiver used in our simulations for demodulating the signal of interest modeling the interferers as Gaussian colored noise is the interference Cancelling Matched Filter (ICMF) which we introduced previously.

## 1. INTRODUCTION

The European digital cellular standard is the Global System for Mobile communications (GSM). In this system, the limited frequency spectrum is divided into narrow bands; each one is assigned to a cell. The carrier frequencies can be reused in distant cells but neighboring cells can not operate in the same band. Since the number of subscribers should be high, the tendency is to use small frequency reuse factors. However, the cochannel interference can limit seriously the performance of such a system. To further increase the capacity of the system, the Time Division Multiple Access (TDMA) enables some users (up to eight in GSM) to operate on the same carrier frequency [1]. Each user occupies one time slot per frame and the time slots are separated by a guard region to combat overlap between users of the same cell. We assume in the following that the base stations are synchronized and the radius of a cell is such that the maximum delay (Fig 1) of a cochannel interferer is a few symbol periods (three in our simulations).

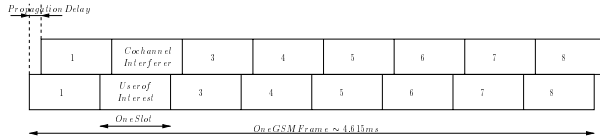


Figure 1: GSM Frame Structure.

If we know (i.e., correctly detect) the active training sequences,

we can estimate jointly all the channels over the intersection period of all the training sequences. As the number of parameters is the main limitation of this method, we propose a partial equalization of the transmit pulse shape [2]. Then, only a few coefficients (that correspond mainly to the propagation channel) per user need to be identified.

Consider the symbol sampled received signal

$$\mathbf{x}_k = \mathbf{Q}(q)d_k + \mathbf{n}_k \quad (1)$$

where  $\mathbf{x}_k$  is an  $m = 2pN_s$  by 1 vector,  $N_s$  is the number of sensors,  $p$  is the oversampling factor,  $d_k$  is the binary information sequence and  $\mathbf{Q}(q)$  is an  $m$  by 1 vector modeling the linear channel between the emitter and the receiver. The number 2 occurs for one dimensional constellation and accounts for the real and imaginary parts of the complex channel [3]. In simulations, we often assume  $\mathbf{n}_k$  to consist of temporally and spatially i.i.d. noise plus co-channel multi-user interference. If  $p \geq 2$  (i.e., Nyquist is satisfied), then the channel can be factored into  $\mathbf{Q}(q) = \mathbf{G}(q)\mathbf{C}(q)$  where  $\mathbf{G}(q)$  accounts for the known transmit pulse shape and  $\mathbf{C}(q)$  for the multipath propagation [4]. As it is pointed out in [2], the filter  $\mathbf{G}(q)$  can not be inverted by a linear equalizer and we try only to shorten its length by a filter  $\mathbf{F}(q)$  such that  $\mathbf{F}(q)\mathbf{G}(q) = q^{-\Delta}\mathbf{B}(q)$  where  $\Delta$  is a desirable delay. Filtering the received signal  $\mathbf{x}_k$  by  $\mathbf{F}(q)$ , the reduced-order model can be written as

$$\mathbf{y}_k = \sum_{i=0}^{N-1} \mathbf{h}_i d_{k-\Delta-i} + \mathbf{v}_k = \mathbf{H}_N D_N(k) + \mathbf{v}_k,$$

$$\mathbf{y}_k = \begin{bmatrix} y_{1,k} \\ \vdots \\ y_{m,k} \end{bmatrix}, \mathbf{v}_k = \begin{bmatrix} v_{1,k} \\ \vdots \\ v_{m,k} \end{bmatrix}, \mathbf{h}_k = \begin{bmatrix} h_{1,k} \\ \vdots \\ h_{m,k} \end{bmatrix}$$

$$\mathbf{H}_N = [\mathbf{h}_{N-1} \cdots \mathbf{h}_0], D_N(k) = [d_{k-\Delta-N+1}^H \cdots d_{k-\Delta}^H]^H \quad (2)$$

where the first subscript  $i$  denotes the  $i^{\text{th}}$  channel, and superscript  $H$  denotes Hermitian transpose. The term  $\mathbf{v}_k$  will be considered here to consist of both spatially and temporally correlated additive zero mean noise.

## 2. SLOT SYNCHRONIZATION

In GSM, each user can be synchronized within its proper cell by the synchronization burst [5]. However, in the absence of a prior knowledge, the cochannel users can not be synchronized with the user of interest even if the base stations are synchronous. At this

point we should emphasize that we deal with slot synchronization and not with frame synchronization since the behavior of the cochannel users is unpredictable. Since in GSM only eight training sequences are possible and if the cellular design is performed properly, one can assume that the training sequences are different between the user of interest and the six cochannel interferers on the first tier. In the following, we discuss two simple methods to achieve slot synchronization.

### 2.1. Least-Squares method

Assume that we receive a burst of data

$$\mathbf{Y}_M(M) = \mathcal{T}_M(\mathbf{H})D_{M+N-1}(M) + \mathbf{V}_M(M) \quad (3)$$

or  $\mathbf{Y} = \mathcal{T}(\mathbf{H})D + \mathbf{V}$ , where  $\mathbf{Y} = [\mathbf{y}_1^H \cdots \mathbf{y}_M^H]^H$  and similarly for  $\mathbf{V}$ , and  $\mathcal{T}(\mathbf{H})$  is a block Toeplitz matrix with  $M$  block rows and  $[\mathbf{h}_{N-1} \cdots \mathbf{h}_0 \ 0_{m \times (M-1)}]$  as first block row. Without loss of generality we assume that the first user is the user of interest and that the receiver is synchronized to this user. The LS technique also used in [6] for the single user can be extended to the multi-user case to estimate the beginning  $k$  of a training sequence  $\{a^{(i)}\}$ , of a cochannel interferer, of length  $L$  by

$$\begin{aligned} [\hat{k}, \hat{i}] &= \arg \min_{k \in \mathcal{W}, i, h_i} \|\mathbf{Y}_{L-N+1}(k+L-1) - \mathcal{A}_i h_i\|^2 \\ &= \arg \min_{k \in \mathcal{W}, i} \left\| P_{\mathcal{A}_i}^\perp \mathbf{Y}_{L-N+1}(k+L-1) \right\|^2 \end{aligned} \quad (4)$$

where  $\mathcal{A}_i = A_i \otimes I_m$ ,  $I_m$  is the  $m$  by  $m$  identity matrix,

$$A_i = \begin{bmatrix} a_N^{(i)} & \cdots & a_1^{(i)} \\ \vdots & & \vdots \\ a_L^{(i)} & \cdots & a_{L-N+1}^{(i)} \end{bmatrix}, \quad h_i = \begin{bmatrix} \mathbf{h}_0^{(i)} \\ \vdots \\ \mathbf{h}_{N-1}^{(i)} \end{bmatrix} \quad (5)$$

$\mathcal{W}$  is a search window and  $P_{\mathcal{A}_i}^\perp$  is the projection matrix onto the orthogonal complement of the space spanned by the columns of  $\mathcal{A}_i$ .

For this approach to work, we assume that the search window  $\mathcal{W}$  is small. The bounds for  $k$  can be predicted, for the worst case, in the case of synchronous base stations where the propagation delay is the only source of asynchronism.

### 2.2. Correlation method

This method is essentially the same as the previous one except that we use the correlation structure of the training sequences (the training sequences can be considered as ‘‘deterministic white noise’’). At each sampling instant  $k \in \mathcal{W}$  we calculate a windowed normalized correlation coefficient defined as

$$C_k^{(i)} = \frac{\frac{1}{N} \sum_{n=1}^N \left( \frac{1}{16} \sum_{j=6}^{21} a_{-j}^{(i)*} \mathbf{y}_{k+L+n-j}^H \right) \left( \frac{1}{16} \sum_{l=6}^{21} a_{-l}^{(i)} \mathbf{y}_{k+L+n-l} \right)}{\sqrt{\left( \frac{1}{16} \sum_{j=6}^{21} |a_{-j}^{(i)}|^2 \right) \left( \frac{1}{N} \sum_{n=1}^N \frac{1}{16} \sum_{j=6}^{21} \mathbf{y}_{k+L+n-j}^H \mathbf{y}_{k+L+n-j} \right)}} \quad (6)$$

where  $a_{-j}^{(i)} = a_{27-j}^{(i)}$  is the  $(27-j)^{th}$  symbol corresponding to the  $i^{th}$  training sequence.

For slowly fading channels, the denominator is constant and then  $C_k^{(i)}$  is proportional to the norm of the channel impulse response. As before, one should try with all the training sequences,

find the peaks of cross-correlation indicating the location of the training sequences. Then,

$$[\hat{k}, \hat{i}] = \arg \max_{k \in \mathcal{W}, i} C_k^{(i)}. \quad (7)$$

### 2.3. Averaging correlations over more than one burst

A simple (but a realistic) model for the propagation channel is the frequency selective model which is parameterized with the delays of the principle paths (6 or 12 in GSM [7]) and their complex amplitudes. From burst to burst, the variation of the channel impulse responses (CIRs) is due to the variations of the phases; the delays and the magnitudes can be assumed to be constant. Thus, one can average the squared cross-correlation coefficients over a small number of bursts (10 in our simulations). If more than one time slot per frame is used (for the purpose of increasing the data rate as it is the case for EDGE [8]) then an averaging time of 10ms (two frames) should be sufficient to detect correctly the training sequence of the interferer. For high signal to interference ratios (SIRs), we can improve this result by removing from the received data the contribution of the user of interest over its training sequence since the quality of the channel estimate for the user of interest is good. In practice, we shall estimate the signal to interference plus noise ratio (SINR) and decide w.r.t. a threshold to achieve the synchronization over the received signal or after removing the contribution of the user of interest.

## 3. SINGLE USER RECEPTION WITH INTERFERENCE CANCELLATION

Let  $u_k$  be the zero-mean temporally and spatially white noise part

in  $\mathbf{n}_k$  (eq. 1) with variance  $\sigma_u^2 = \frac{\sigma_a^2}{2\pi j} \oint \frac{dz}{z} \mathbf{Q}^\dagger(z) \mathbf{Q}(z)$  where  $\mathbf{Q}^\dagger(z) = \mathbf{Q}^H(1/z^*)$ . After the channel shortening step, the contribution of this noise in  $\mathbf{v}_k$  (eq. 2) becomes  $w_k = \mathbf{F}(q)u_k$  with a power spectrum density (psd)  $S_{ww}(z) = \sigma_u^2 \mathbf{F}(z) \mathbf{F}^H(z)$ .

### 3.1. Joint estimation of the CIRs over the overlapped time of the training sequences

Let  $\delta$  be the delay that can occur between the slot of the user of interest and that of an interferer,  $\theta = [h^H h_i^H]^H$  the vector of the CIRs of the user of interest and the interferer and  $\mathcal{A} = [\mathcal{A}_1 \mathcal{A}_i]$ .  $\mathcal{A}_1$  is formed by the last lines of  $\mathcal{A}_1$  after removing the first  $m\delta$  lines, and  $\mathcal{A}_i$  is formed by the first lines of  $\mathcal{A}_i$ , the last  $m\delta$  lines being removed. Then, the CIRs are the solution of the problem

$$\hat{\theta} = \arg \min_{\theta} \|\mathbf{Y}_{L-N+1-\delta}(k^\circ + L - 1) - \mathcal{A}\theta\|_{R_{ww}^{-1}}^2, \quad (8)$$

where  $k^\circ$  is the starting instant of the training sequence of the user of interest and  $R_{ww} = E(WW^H)$  with  $W = W_{L-N+1-\delta}(k)$ . Theoretically  $R_{ww} = \sigma_u^2 \mathcal{T}(\mathbf{F}) \mathcal{T}(\mathbf{F})^H$  where the value of  $\sigma_u^2$  is of no interest for the previous LS problem.

### 3.2. Step1: demodulation of the user of interest

Once the quality of the channel estimates is improved w.r.t. the least squares method for a single user, we can replace the Wiener

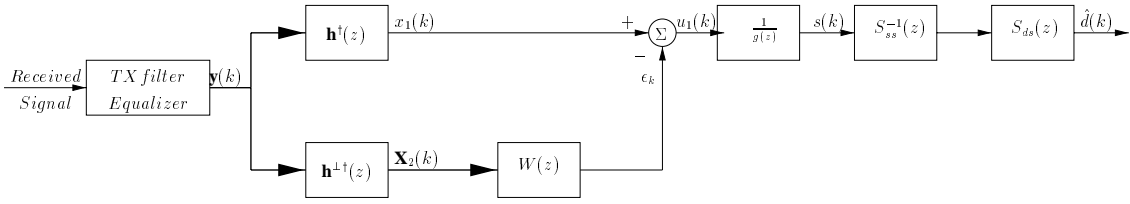


Figure 2: Optimal receiver for one user in the presence of colored noise.

filter  $W(z)$  of the ICMF [9] by its theoretical expression (parameterized with the CIRs and the variance  $\sigma_u^2$ ). In the simulations below, the output of the ICMF (Fig 2) feeds a scalar minimum mean-square-error linear-equalizer (MMSE-LE). Alternatively, the Viterbi algorithm (VA) could be used.

To find  $\hat{\sigma}_u^2$ , we estimate a sample covariance matrix  $\hat{R}_{yy}$  of the signal  $\mathbf{Y}$  such that the signal part is rank-deficient and we calculate the correlation matrix  $R_{ww}$  of the filter  $\mathbf{F}(z)$ . It follows that

$$\hat{\sigma}_u^2 = \lambda_{\min}(\mathcal{L}^{-1} \hat{R}_{yy} \mathcal{L}^{-H}) = \lambda_{\min}(\hat{R}_{yy}, R_{ww}) \quad (9)$$

is the minimal eigenvalue of the matrix  $\mathcal{L}^{-1} \hat{R}_{yy} \mathcal{L}^{-H}$  where  $\mathcal{L}$  is the lower triangular square-root factor of  $R_{ww} = \mathcal{L} \mathcal{L}^H$ .  $\hat{\sigma}_u^2$  is also the minimal generalized eigenvalue of the matrices  $\hat{R}_{yy}$  and  $R_{ww}$ .

### 3.3. Step2: iterative solution

After the first step, all the symbols of the user of interest are known (estimated). If the bit error rate (BER) is not very bad, the whole burst can act as a training sequence. So, we can improve the quality of the channel estimates and subsequently the BER for the information sequence of the user of interest. We assume in the following that the distribution of the symbols is Gaussian with the known symbols as the mean. For the channel estimates we can introduce the Gaussian Maximum Likelihood (GML) method or reduce ourselves to the optimally weighted LS method ([10] and references therein). To that end we rewrite the received burst as  $\mathbf{Y} = \mathcal{T}D + \mathcal{T}_i D_i + W$  and we decompose the contribution of the interferer as  $\mathcal{T}_i D_i = \mathcal{T}_{i,k} D_{i,k} + \mathcal{T}_{i,u} D_{i,u}$  where  $D_{i,k}$  is the interferer's training sequence and  $D_{i,u}$  is its unknown information sequence. With our model, we get for the covariance matrix  $C_{YY} = \sigma_u^2 \mathcal{T}_{i,u} \mathcal{T}_{i,u}^H + R_{WW}$ . Then, the CIRs are the solution of the problem

$$\hat{\theta} = \arg \min_{\theta} \left\| \mathbf{Y} - \mathcal{T}\hat{D} - \mathcal{T}_{i,k} D_{i,k} \right\|_{C_{YY}^{-1}}^2, \quad (10)$$

$\hat{D}$  contains the symbols of the user of interest estimated at the first step and its known training sequence. With these estimates we detect the symbols of the user of interest, as in the first step, after an optimal interference cancellation part (carried out by the ICMF) and a scalar (MMSE) equalizer. This step can be reiterated but we reach rapidly a local minima. In general, one iteration suffices since we can not improve significantly the BER at the second iteration.

## 4. SIMULATIONS

We consider the typical urban (TU) propagation environment. For this environment, we consider the 6-tap ( $L_c = 6$ ) statistical channel impulse responses as specified in the ETSI standard [7] and

this for both the user of interest and the interferer. A random delay, taken uniformly over one symbol period, is introduced in the (multi-)channel for each user. We consider in our simulations four channels: one sensor, an oversampling factor of  $p = 2$  and we exploit the one dimensional character of the constellation. We show respectively in Fig 3 and in Fig 4, the probability of a successful detection of the training sequence of the interferer and the probability of a successful synchronization with its time slot as a function of SIR for a fixed SNR = 20dB. These curves are averaged over 1000 realizations of the symbols and the channel impulse responses of the user of interest and the interferer, according to the statistical channel model, with the channel response of the interferer being rescaled to have a desired SIR. The random delay between the users and the propagation delay  $\delta$  (between the time slots of the two users) are taken constant every 10 consecutive bursts. For the synchronization step, we make a decision every 10 bursts so we actually average estimates over 100 realizations of the uniformly-distributed delays. The threshold considered for the estimated SINR is 5dB. If  $\widehat{\text{SINR}} > 5\text{dB}$  then the contribution of the signal of interest is removed from the received data before the synchronization step.

In Fig 5 we denote by MF the curve for a matched filter receiver ( $W(z) \equiv 0$ ), by ICMF the curve for an ICMF receiver where the CIR is estimated jointly with that of the cochannel interferer over the overlapped part of the training sequences and the theoretical expression of the Wiener filter is used. In a second step, all the estimated symbols of the user of interest act as a training sequence. The channel estimates used in the ICMF are the solution of (eq. 10) and we call this receiver 2-step ICMF. The Hybrid receiver is the two-stage CCI/ISI reduction of [11] and the VA (assuming the noise and the cochannel users as a white noise as it is the case for the current GSM) are used as a benchmark to the proposed method. We should emphasize on the fact that these two receivers act directly on the received signal without equalization of the transmit pulse shape.

## 5. CONCLUSIONS

We have extended the correlation method (to achieve a slot synchronization) to the case of (strong) cochannel interferers by shortening the common part (pulse transmit filter) of the CIRs. Then, we demodulate the signal of interest by the ICMF followed by a (MMSE) scalar equalizer. The performance of this receiver can be improved by 3dB in a second step. Clearly, the proposed method outperforms the existing algorithms for the strong cochannel interference case. However, the performance of all the simulated algorithms are far from the (well-approximated) theoretical bound of the VA [12]:  $P_e = \frac{1}{2} \text{erfc}(\sqrt{\text{MFB}})$  where  $\text{erfc}$  is the complementary error function and MFB is the matched filter bound.

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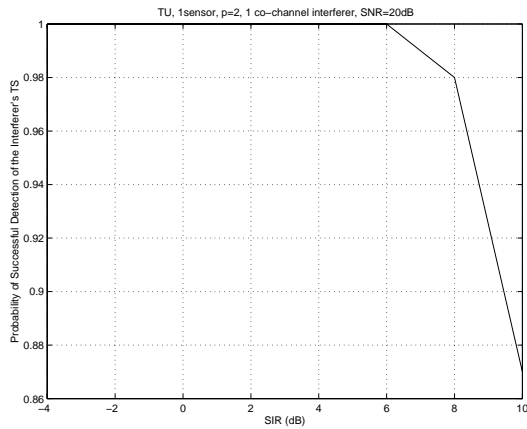


Figure 3: Probability of Successful Detection of the Interferer's training sequence

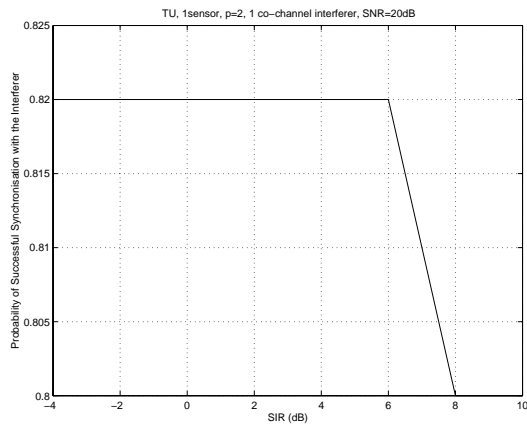


Figure 4: Probability of Successful Synchronization with the Interferer

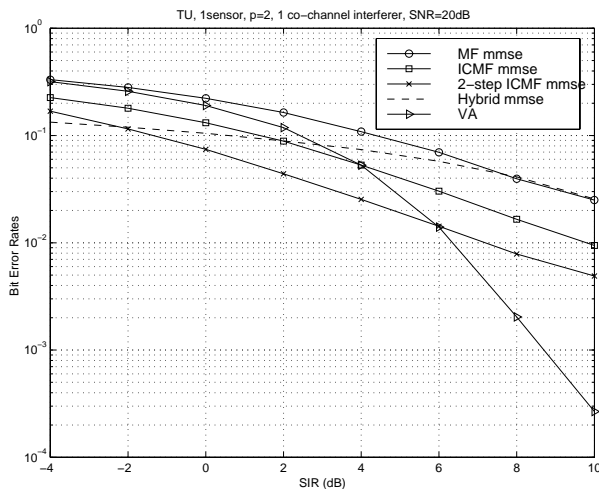


Figure 5: Averaged Bit Error Rate Curves