Code Constructions for Non-coherent *On–off* Ultra-wideband Systems

Younes Souilmi, Raymond Knopp and Giuseppe Caire
Institut Eurécom
2229 route des Crêtes, 06904 Sophia Antipolis, France
Email:souilmi, knopp, caire@eurecom.fr

***Abstract*— We consider coding schemes for non-coherent *On–off* signaling over Ultra-wideband channels. Different code constructions are proposed and optimized using an exit chart based methodology. The performance of proposed codes is then measured using both an exit chart based analysis and Monte Carlo simulations. The optimized codes are shown to perform close to information-theoretic limits.**

I. Introduction

In considering signaling strategies for Ultra-Wideband (UWB) systems, [6], [7] Souilmi and Knopp evaluated the achievable rates for non-coherent detection, in the sense that the receiver has no channel side information of the underlying wideband channel process, of *On–off* UWB signaling. Although significant loss in information rates compared to AWGN channel can be expected due to the extreme bandwidth (even for the low spectral efficiency associated with proposed UWB regulatory constraints on bandwidth and power), losses with respect to coherent detection with incomplete side information (i.e. imperfect channel estimation) are small. The savings in terms of implementation complexity are thus justified from a practical standpoint.

The term *On–off* is defined as a signaling strategy having a two-mass input distribution with a mass point at the origin (i.e. zero energy symbol). Thus a *On–off* scheme consists of transmitting, at each symbol time, either a certain pulse $p(.)$ with probability η and not transmitting anything with probability $1-\eta$. When computing the attainable mutual information of this signaling scheme over non-coherent UWB channels, it is further shown that the optimal transmit probability η should vary with the system's average SNR.

In this work we consider suitable coding schemes for *On–off* with non-coherent detection over a UWB channel. Such a coding scheme needs to be a symmetric-input,asymmetric-output distribution binary code in order to correctly match the optimal input distribution for a given SNR level. Such coding schemes had been studied by Bennatan and Burshtein in [3], where the design of channel codes for non-uniform input distributions was considered for memoryless channels.

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An other alternative would consist on serially concatenating a symmetric output-distribution channel code with m-ary Pulse Position Modulation (PPM). The latter is a special case of *On–off* signaling corresponding to transmitting exactly one pulse during each block of m symbols (here $\eta = 1/m$). In [2], Peleg and Shamai, consided such a strategy for memoryless (rapidly-varying) Rayleigh fading channels .

In the following we make the choice of enforcing the considered code constructions to be of the form of binary symmetric-output code serially concatenated with an m-ary PPM modulator. The motivation for such a design choice is that designing binary codes with an asymmetric-output distribution is not a simple task. Furthermore, the use of binary symmetric-output distribution codes allows us to employ powerful optimization methods already developed in different contexts. The remainder of the paper is organized as follows. The main goal of this work is to present code constructions for m-PPM modulation and examine their ability to approach channel capacity over an UWB channel with no channel state information at receiver side. Section II deals with the underlying system model for transmission and reception as well as the channel model. In section III we derive and evaluate BICM constrained capacity over UWB channel. Section IV contains the description of the presented codes as well as their optimization methodology. Finally in section V we discuss the considered codes performance in terms of decoding convergence thresholds and bit error rates.

II. Channel Model

We consider Non-coherent m-PPM signaling for an Ultrawideband system as a special case of the previously introduced *On–off* signaling(1). Each m-PPM symbol, x_k, corresponds to choosing one out of m symbol times, constituting a PPM frame, in which to emit the transmit pulse $p(t)$. $x \in \{1,\ldots,m\}$ is simply the position within the PPM frame where the pulse is transmitted. We restrict our study to strictly time-limited memoryless real-valued signals, both at the transmitter and receiver. We consider a block fading channel model so that the channel impulse response is time-invariant in any interval of $[kT_c,(k+1)T_c)$, where T_c is the *coherence–time* of the channel. We denote the channel in any block by $h_k(t)$ which is assumed to be a zero-mean process. For simplicity in the analytical developments, we assume that the channel realization in every block is independent and identically distributed, so that $\mathrm{E}[h_k(t)h_l(u)] = R_h(t,u)\delta_{kl}$, where $R_h(t,u)$ is the auto-correlation function of the channel response in a particular interval. The received signal is

$$r(t) = \sum_{k=0}^{N} \sqrt{mE_s}p\left(t - (k*m + x_k)T_s\right) * h_k(t) + z(t) \quad (1)$$

where k is the symbol index, mT_s the symbol duration, E_s the transmitted symbol energy, x_k is the transmitted symbol at time k, $p(t)$ is a unit-energy pulse of duration T_p, and $z(t)$ is white Gaussian noise with power spectral density N_0. A guard interval of length T_d is left at the end of each symbol (from our memoryless assumption) so that $T_s \geq T_p + T_d$, and the symbol interval $T_s \ll T_c$. The received signal bandwidth W is roughly $1/T_p$, in the sense that the majority of the signal energy is contained in this finite bandwidth.
Through a Karhunen-Loève expansion we rewrite the channel model in equation (1), for the nth slot (of duration T_s), as the equivalent set of parallel channels

$$r_{n,i} = \sqrt{mE_s\lambda_i}h_{n,i} + z_{n,i}; i = 1, ..., \infty \quad (2)$$

where $z_{n,i}$ is $\mathcal{N}(0, N_0)$ and $\{h_{n,i}\}$ are unit variance zero mean independent Gaussian variables. The $\{\lambda_i\}$ are the solution to

$$\lambda_i\phi_i(t) = \int_0^{T_d+T_p} R_o(t,u)\phi_i(u)du. \quad (3)$$

where ϕ_i and $R_o(t,u)$ are the eigenfunctions and the autocorrelation function of the composite channel $h_k(t)*p(t)$, respectively. Because of the band-limiting nature of the channels in this study, the channel will be characterized by a finite number, D, of significant eigenvalues which for rich environments will be close to $1 + WT_d$, in the sense that a certain proportion of the total channel energy will be contained in these D components. we will assume that the eigenvalues are ordered by decreasing amplitude. In the following we denote $\mathbf{R}_k$ the received signal corresponding to the kth transmitted PPM symbol

$$\begin{aligned} \underline{r}_n &= \{r_{n,1}, \ldots, r_{n,D}\} \\ \underline{R}_k &= \{\underline{r}_{m(k-1)+1}, \ldots, \underline{r}_{mk}\} \end{aligned} \quad (4)$$

Throughout the rest of the paper we assume that the channel is ergodic in the sense that it has independent and identically distributed realizations over any two different slots, so that $\mathrm{E}[h_n(t)h_l^*(u)] = R_h(t,u)\delta_{n,l}$, where $R_h(t,u)$ is the auto-correlation function of the channel response in a particular interval. Generally speaking, this channel model is useful only as a first approximation for short range communications. Nevertheless, for UWB signaling with non-coherent detection, this channel model is adequate thanks to the high diversity order D of UWB channels. Hence, the overall received energy over a typical UWB channel, conditioned on the transmitted symbol, is constant (see figure 1) irrespective of particular channel realizations. Thus, in a sense, the channel almost does not suffer any fading [1].

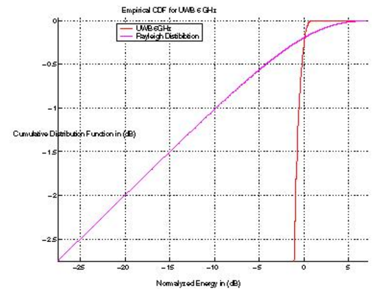

Fig. 1. CDF of total channel received energy

Therefore, system performance over the ergodic channel model is a significant measure of the performance of practical systems due to the fact that the probability of the information outage event is vanishing. The information outage event is defined as the probability of having the instentaneous mutual information, between the transmitted symbol and the received signal, less than the coding rate.

III. Coding Schemes

A. BICM

Our reference coding scheme, will be a standard convolutional code used in Bit Interleaved Coded Modulation (BICM) construction. The encoder is obtained by the serial concatenation of a convolutional code and m-ary PPM modulation, through a bit interleaver figure (2) (the accumulator we can see on the figure will be added later). Here the interleaver is assumed to be an ideal one (i.e. of infinite depth). The incoming information bits are first encoded with the convolutional code and passed through a bit interleaver. The coded bits are then grouped into sequences of m bits each and finally mapped onto corresponding m-PPM symbols and transmitted over the channel. The bit interleaver can be seen as a one-to-one correspondence $\pi : k \longrightarrow (k', i)$, where k denotes the time ordering of the coded bits c_k, k' denotes the time ordering of the signals $x_{k'}$, and i indicates the position of the bit c_k in the label of $x_{k'}$.
The channel ergodicity assumption implies that a sufficiently large number of channel realizations span the codeword length. This can be achieved by first interleaving the transmitted symbols, using an infinite depth interleaver, before sending them over the channel. Thus, we can assume that the deinterleaved symbols at the receiver face independent channel realizations.

[1]From the perspective of a non-coherent detector which captures the received energy over the channel.

1) Capacity: We compute the constrained[2] capacity of BICM construction over the considered channel (equation II). Note that here the capacity by allowing the convolutional code in figure (2) to be replaced by any possible binary code. In the following we drop the time index k in equation (4) for a better clarity of mathematical developments. Letting $P(\underline{R}|z)$ denote the transition probability of the transmission channel, the capacity of the considered system, in bits per second, can be written as follows [8]

$$\hat{C} = \frac{1}{mT_s}\left(m - \sum_{i=1}^{m} \underset{b,\underline{R}}{E}\left[\log_2\left(\frac{\sum_{z\in\mathcal{X}} P(\underline{R}|z)}{\sum_{z\in\mathcal{X}_b^i} P(\underline{R}|z)}\right)\right]\right) \tag{5}$$

where $\mathcal{X}_b^i$ denotes the set of codewords x whose ith label position is equal to b.Due to the symmetry of m-PPM modulation, $\hat{C}$ is not sensitive to particular choices of the labeling function (that maps bit sequences onto m-PPM symbols). Thus $\hat{C}$ can be rewritten as follows

$$\begin{aligned}\hat{C} &= \frac{1}{T_s}\left(1 - \underset{b,\underline{R}}{E}\left[\log_2\left(\frac{\sum_{z\in\mathcal{X}} P(\underline{R}|z)}{\sum_{z\in\mathcal{X}_b^1} P(\underline{R}|z)}\right)\right]\right)\\ &= \frac{1}{T_s}\left(1 - \frac{1}{2m}\sum_{x=1}^{m} \underset{\underline{R}|x}{E}\left[\log_2\left(1+\frac{\sum_{z\in\mathcal{X}_0^1} P(\underline{R}|z)}{\sum_{z\in\mathcal{X}_1^1} P(\underline{R}|z)}\right)\right]\right.\\ &+ \left.\underset{\underline{R}|x}{E}\left[\log_2\left(1+\frac{\sum_{z\in\mathcal{X}_1^1} P(\underline{R}|z)}{\sum_{z\in\mathcal{X}_0^1} P(\underline{R}|z)}\right)\right]\right)\end{aligned} \tag{6}$$

the channel transition probability is given by

$$\begin{aligned}P(\underline{R}|z) &= \prod_{i=1}^{D} \frac{1}{mE_s\lambda_i + N_0} e^{-\frac{|r_{z,i}|^2}{mE_s\lambda_i + N_0}}\\ &\prod_{\substack{j=1\\ j\neq z}}^{m}\prod_{i=1}^{D} \frac{1}{N_0} e^{-\frac{|r_{j,i}|^2}{N_0}}\end{aligned} \tag{7}$$

Thus, exploiting symmetry of the channel transition probability and making the assumption that m is an even number, we re-write $\hat{C}$ as follows

$$\begin{aligned}\hat{C} &= \frac{1}{T_s}\left(1 - \frac{1}{2}\underset{\underline{R}|x=1}{E}\left[\log_2\left(1+\frac{\sum_{z=1}^{m/2} P(\underline{R}|z)}{\sum_{z=(m/2)+1}^{m} P(\underline{R}|z)}\right)\right]\right.\\ &+ \left.\frac{1}{2}\underset{\underline{R}|x=1}{E}\left[\log_2\left(1+\frac{\sum_{z=(m/2)+1}^{m} P(\underline{R}|z)}{\sum_{z=1}^{m/2} P(\underline{R}|z)}\right)\right]\right)\end{aligned} \tag{8}$$

Using equations (8) and (7) we can numerically evaluate $\hat{C}$. On the other hand the capacity of non-coherent UWB channel constrained to the use of m-PPM (Coded modulation capacity) is given by

$$I(x;\underline{R}) = \frac{1}{mT_s}\left(\log_2(m) - \underset{\underline{R}|x=1}{E}\left[1 + \sum_{j=2}^{m}\frac{P(\underline{R}|x=j)}{P(\underline{R}|x=1)}\right]\right) \tag{9}$$

[2]Constrained to the use of m-PPM modulation

B. Convolutional Code+Binary Accumulator

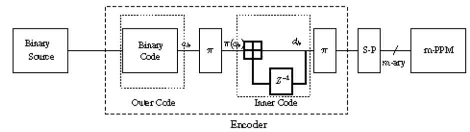

Fig. 2. Transmitter block diagram.

In order to obtain a more powerful coding scheme, we explore in this section a new construction figure (2). The construction is obtained by serial concatenation of the previous encoder and a unit-memory binary accumulator followed by a bit interleaver. The accumulator sums the incoming bit $\pi(c_k)$ with the previous output bit, d_{k-1}, in order to produce the new output bit d_k. The accumulator is rate one code, thus the overall coding rate of the proposed scheme is equal to the coding rate of the convolutional code R_c.

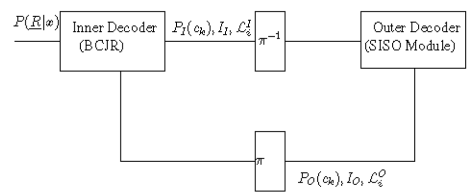

Fig. 3. Decoder block diagram.

1) Decoding: Decoding is performed in an iterative manner. At each iteration the two decoder blocks (see figure 3) exchange extrinsic information an recompute soft outputs on the coded bits. The decoding schedule at each decoding iteration is a two-step process: i) first, the inner decoder uses the likelihoods $P_O(c_k)$, obtained from the outer decoder at the previous iteration, as a priori probabilities on the coded bits c_k in order to marginalize, using the BCJR algorithm, the likelihoods on the transmitted symbols $P_{CH}(\underline{R}_n|x_n)$; obtained through the transmission channel and compute new likelihoods on the coded bits c_k. ii) Second, the outer decoder uses the new likelihoods, $P_I(c_k)$, computed by the inner decoder in order to produce at its turn new likelihoods on the coded bits c_k. For the first iteration $P_O(c_k)$ are initialized with equiprobabilities. At the end of the decoding process, the outer code makes hard decisions on the information bits.

C. m-Ary Accumulator

We now replace the bit accumulator (and the bit interleaver following it) in the previous scheme by a weighted unit-memory symbol-level accumulator (figure 4. The incoming symbol u_i is added to the previously transmitted symbol x_{i-1} multiplied by a factor f. Note that u_i, x_i, f, as well as the sum and product operations are defined over $GF(q)$. Throughout the paper q will be chosen to be equal to m. Again the overall coding rate is equal to the code rate of the convolutional code, since the symbol accumulator is a rate one code.

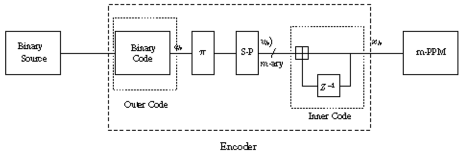

Fig. 4. Transmitter block diagram:m-ary Accumulator .

1) Exit Chart Analysis: Given the code construction, presented in this section, one can still optimize the performance of the code by making adequate choice of the convolutional code component. In order to perform this optimization over the set of all non-degenerated convolutional codes, we analyze the behavior of the concatenated code in the limit of infinite block lengths. The analysis is performed using exit charts of the code components taken separately. The exit chart of a block code is defined as transfer function T that gives, for a given extrinsic input mutual information I_{in}, the corresponding output mutual information I_{out}. I_{in} (respectively I_{out}) is the mutual information between the likelihood received (respectively emitted) through the extrinsic channel and its corresponding coded bit. In the following we note I_{In} (respectively I_O) such a quantity over the directional extrinsic channel from the inner decoder toward the outer decoder (respectively from the outer decoder toward the inner decoder)

$$\begin{aligned} I_{In} &= I\left(c_i, \mathcal{L}_i^{In}\right) \\ I_O &= I\left(c_i, \mathcal{L}_i^{O}\right) \end{aligned}$$

We associate to each decoding block an exit function as follows

$$\text{Inner decoder:} \qquad I_O = f\left(I_{In}, \mathcal{L}_{CH}\right) \tag{10}$$

$$\text{Outer decoder:} \qquad I_{In} = g\left(I_O\right) \tag{11}$$

Thus the iterative decoding process converges (i.e. achieves error free decoding) if and only if

$$x < g\left(f\left(x, L_{CH}\right)\right) \quad \forall x \in [0,1) \tag{12}$$

This condition prevents from having any fixed point, other than $x = 1$, for the function $x \longmapsto f\left(g\left(x\right), L_{CH}\right)$. Note that the existence for such a fixed point x_0 would mean that, if the decoder is initiated at a point lower than x_0, the decoding will stick at this point and thus do not achieve $I_o = 1$(i.e. do not achieve error free decoding). For the seek of feasibility of the estimation of functions $f(.)$ and $g(.)$, we make the assumption that the extrinsic channel is a Gaussian symmetric channel. Which implies that $P^O(c_i) = \frac{1}{2}(-1)^{c_i}\mathcal{L}_i^O$ and $P^{In}(c_i) = \frac{1}{2}(-1)^{c_i}\mathcal{L}_i^{In}$ where $\mathcal{L}_i^O$ and $\mathcal{L}_i^{In}$ are Gaussian distributed variables with mean respectively μ^O and μ^{In}, and variance respectively $2\mu^O$ and $2\mu^{In}$(from the symmetry assumption).

I_O (respectively I_{In}) is linked to μ_O (respectively μ_{In}) through the following bijection relationship $I_O = \mathcal{J}\left(\mu_O\right)$ (respectively $I_{In} = \mathcal{J}\left(\mu_{In}\right)$). Where $\mathcal{J}$ is an invertible function defined as in [9]. We compute the exit functions through Monte Carlo simulation. For a given input mutual information I_{in} we generate iid input log-likelihood ratios $\mathcal{L}_k^{int}$ according to its corresponding symmetric Gaussian distribution. Then for each of them we compute the output log-likelihood ratio $\mathcal{L}_k^{out}$ using the BCJR algorithm and obtain the output mutual information as

$$\begin{aligned} I_{out} &= 1 - \underset{\mathcal{L}^{out}}{E}\left[h\left(\frac{1}{1+e^{\mathcal{L}^{out}}}\right)\right] \\ &= 1 + \frac{1}{\log(2)}\underset{\mathcal{L}^{out}}{E}\left[\frac{\mathcal{L}^{out}e^{\mathcal{L}^{out}}}{1+e^{\mathcal{L}^{out}}} - \log\left(1+e^{\mathcal{L}^{out}}\right)\right] \end{aligned} \tag{13}$$

The code optimization procedure consist on picking, among all rate $1/2$ convolutional code generators, the one that achieves, the lowest, necessary transmitted SNR per bit for error free decoding.

D. Extension:IRA Codes With a symbol Accumulator

In this section we introduce an extension to the previous scheme (figure III-C) through the replacement of the convolutional code by an irregular non-systematic repetition code. This modification aims to allow more degrees of freedom to the code optimization for a potentially better matching to the used modulation and channel statistics. The irregular repetition code is characterized, from its Tanner graph representation (figure 5), by its information bits edge degree distribution $\{\lambda_i\}$ and grouping factor a. Where λ_i is defined as the fraction of graph edges connected to a bit node of degree d_i equal to i. We denote d the maximum edge degree. Thus $\sum_{i=2}^{d}\lambda_i = 1$.

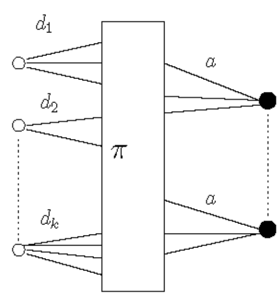

Fig. 5. Irregular repetition Tanner graph.

The overall coding rate, of the concatenated code, is equal to coding rate of the irregular repetition code and is given by

$$R_c = a\sum_{i=2}^{d}\lambda_i/i \tag{14}$$

1) Code Optimization: The degrees of freedom of the considered coding scheme are the information bits degree distribution $\{\lambda_i\}$ and the grouping factor a. Thus, code optimization consist on finding the combination of $\{\lambda_i\}$ and a that maximizes the code rate for a given SNR under the condition that the iterative decoding converges and is error free. We use the code optimization methodology introduced in [4], [5]. The exit function of the accumulator $f(.)$ is obtained using the same method and assumption as in (section III-C.1). Given the relative simplicity of the graph of a repetition code, $g(.)$ can be analytically derived, using the same method as in [9], and shown to be written as follows

$$g(x) = \sum_{i=1}^{d} \lambda_i \mathcal{J}\left((i-1)\mathcal{J}^{-1}\left(1 - \mathcal{J}\left((a-1)\mathcal{J}^{-1}(1-x)\right)\right)\right) \tag{15}$$

We solve the linear programming problem

$$\begin{cases} \text{maximize} \quad R_c = a\sum_{i=2}^{d}\lambda_i/i \quad \text{subject to} \\ \sum_{i=2}^{d}\lambda_i = 1, \quad \lambda_i \geq 0 \quad \forall i \\ x < g\left(f\left(x, L_{CH}\right)\right) \quad \forall x \in [0,1) \end{cases} \tag{16}$$

IV. Optimization Results and Simulations

All simulation and code optimization results were obtained for a pulse duration $T_p = 1e-9s$ and channel delay spread $T_d = 25e-9s$. Table (IV) shows code optimization results for the IRA type of codes with an m-ary accumulator for m-PPM modulation sizes equal to 4, 8, and 16. The maximum bit degree d was taken to be equal to 100. The optimized codes achieves convergence thresholds as close as 0.37 dB from the capacity limit. We note that the coding rates, corresponding to the distribution with lowest convergence threshold, have values around .5 which is in-line with the result, on optimal coding rate, from the capacity analysis of m-PPM. In figure (IV) we see a comparison of convergence threshold of the considered coding schemes, for different modulation size values. We can see that the use of the m-ary accumulator, instead of the binary one, reduced the distance to the capacity limit by about 0.5 dB. Figure (IV) contains bit error rates of the considered code constructions, obtained by simulations for block codes of 10000 bits and using randomly generated interleavers. We notice a gap, on the order of 1 dB, between the convergence thresholds obtained by the exit chart analysis and those obtained by simulation. This means that randomly generated interleavers are suboptimal (for this block size) and thus need to be optimized. Note also that the use of the Gaussian approximation of the extrinsic channel, usually lead to slightly too optimistic results [4].

	m=4		m=8		m=16	
	i	λ_i	i	λ_i	i	λ_i
	3	0.1194	3	0.0837	5	0.1370
	4	0.5260	4	0.1132	3	0.1662
	9	0.2098	6	0.4681	5	0.6013
	10	0.1448	7	0.3349	10	0.0955
a	2		3		6	
Rate	0.4182		0.5462		0.5746	
Eb/N_0	9.76		8.06		7.29	
$(Eb/N_0)_{gap}$	0.44		0.37		0.41	

Fig. 6. Decoding Thresholds for IRA with an m-ary Accumulator

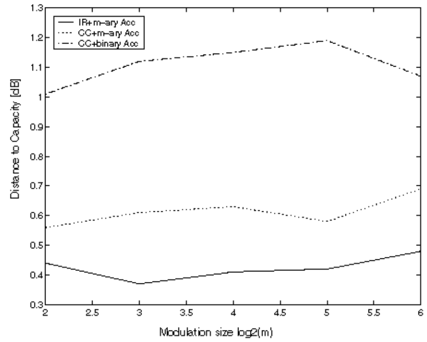

Fig. 7. Distance to Capacity

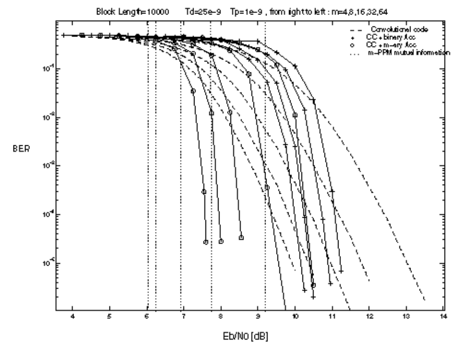

Fig. 8. Decoding simulations

References

[1] Verdu, S. ," Spectral efficiency in the wideband regime " *IEEE Transactions on Information Theory,* Vol. 48 Issue: 6 , June 2002 pp. 1319 -1343.

[2] Peleg, M.; Shamai, S.; "Efficient communication over memoryless Rayleigh fading channels with turbo coding/decoding" *Communication Theory Mini-Conference,* , 6-10 June 1999 PP. 83 - 88.

[3] Bennatan, A.; Burshtein, D.; "On the application of LDPC codes to arbitrary discrete-memoryless channels" *IEEE Transactions on Information Theory,*Vol. 50 , Issue: 3 , March 2004, PP. 417 - 438.

[4] A. Roumy; S. Guemghar; G. Caire; S. Verdu,"Design Methods for Irregular Repeat-Accumulate Codes" *IEEE Transactions on Information Theory,* Vol. 50, Issue:8, pp. 1711-1727, August 2004.

[5] S. ten Brink and G. Kramer, "Turbo Processing for Scalar and Vector Channels" in *Proc. 3rd Int. Symp. Turbo Codes and Related Topics,* Brest, France, Sept. 2003, pp. 23-30.

[6] Souilmi Y.; Knopp R. ; "On the Achievable Rates of Multiband Ultra-Wideband Systems: Ultra-Wideband Systems" *International Journal of Wireless Information Networks,* October 2003, vol. 10, iss. 4.

[7] Souilmi, Younes;Knopp, Raymond, "Challenges in UWB signaling for ad-hoc networking" *DIMACS Series in Discrete Mathematics and Theoretical Computer Science,* Volume 62, November 1, 2003, pp 271-284.

[8] G. Caire; G. Taricco; E. Biglieri,"Bit-Interleaved Coded Modulation" *IEEE Transactions on Information Theory,* Vol. 44, Issue:3, pp. 927-946, May 1998.

[9] Sae-Young Chung; Richardson, T.J.; Urbanke, R.L., "Analysis of sum-product decoding of low-density parity-check codes using a Gaussian approximation" *IEEE Transactions on Information Theory* Vol. 47 , Issue: 2 , pp. 657 - 670, Feb 2001.