LINEAR DETECTORS FOR MULTI-USER MIMO SYSTEMS WITH CORRELATED SPATIAL DIVERSITY

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ABSTRACT

A multiuser CDMA systems with both the transmitting and the receiving sites equipped with multiple antenna elements is considered. The multiuser MIMO channel is correlated at the transmitting and the receiving sites. Multistage detectors achieving near-linear MMSE performance with a complexity order per bit linear in the number of users are proposed. The large system performance is analyzed in a general framework including any multiuser detector that admits a multistage representation. The performance of this large class of detectors is independent of the channel correlation at the transmitter. It depends on the direction of the channel gain vector of the user of interest if the channel gains are correlated.

1. INTRODUCTION

The seminal works in [1] and [2] on multiple antenna elements at the transmitter and the receiver show a huge increase in throughput of this point-to-point channel, referred to also as multiple input multiple output (MIMO) system. These promising results motivated the introduction of multiple antenna elements in the standardization of third generation systems based on code division multiple access (CDMA), e.g. UMTS.

The beneficial effects of spacial diversity, eventually obtained by multiple antenna elements at a single base station, on code division multiple access (CDMA) systems have been investigated in [3]. Modelling the spreading matrices as random matrices and focusing on linear minimum mean square error (MMSE) detectors, Hanly and Tse [3] found a very simple relation between the degrees of freedom introduced by spatial diversity (L receiving antennas) and the degree of freedom in frequency given by spread spectrum techniques (spreading factor N), when the channel gains are independent and identically distributed. The multi-antenna system behaves like a system with a single receive antenna but with spreading factor multiplied by the number of receiving antennas, and the received power of each user being the sum of the received powers at the individual antennas. This behaviour is known as *resource pooling* effect. It shows the possibility to trade bandwidth (spreading factor) with antennas and viceversa according to the peculiarity of the communication system.

The interchangeability between degrees of freedom in frequency and space suggests the idea of treating the two effects in the same way performing antenna array processing and multiuser detection jointly. Joint processing outperforms techniques that try to exploit separately the degrees of freedom in space and frequency significantly [4]. However, the optimal algorithms for this task are known for their prohibitive complexity. The linear MMSE detector has been proposed as a suboptimal approach able to attain good performance with a substantial reduction in complexity. However, when applied to large CDMA systems, i.e. systems with large spreading sequences and large number of users, its complexity is still very demanding for real-time implementations.

With the aim of finding a good trade-off between complexity and performance, also in the challenging scenario of large CDMA systems with random spreading, linear multistage detectors with universal weights have been proposed in [4, 5]. They are obtained as asymptotic approximation of the multistage Wiener filter (MSWF) [6]. These multistage detectors consist of a projector onto a Krylov subspace and a subsequent filter using universal weights as filter coefficients instead of tailored weights depending on the transmitted spreading sequences. The design of universal weights benefits from the asymptotic self averaging properties of random matrices and reduces the computationally demanding part of the detector into a computation of a polynomial depending on the statistical properties of random matrices via few essential system parameters. The assumption of independent channel gains underlies the design of universal weights in both works. Thanks to the additional feature of detecting jointly all K active users, the multistage detectors proposed in [4, 7] achieve near-LMMSE performance with the same complexity order per bit as the single user matched filter, also in the uplink. In fact, by processing jointly all users, most of projection computations becomes identical and the complexity drops by a factor of K. In [4] an asymptotic approximation of the polynomial expansion detectors [8] is also proposed.

In this work we generalize the results in [4] to a synchronous CDMA system with correlated spatial diversity and/or line of sight components. We refer to the asymptotic approximation of the MSWF detector as detector Type J-I to underline the joint projection of the received signal for all users and the asymptotic individual optimization of the filter coefficients for each user. The asymptotic approximation of the polynomial expansion detector is referred to as Type J-J detector to emphasize the joint optimization of the filter coefficients.

The design of the universal weights relies on (i) the convergence of the diagonal elements of the system correlation matrix $\mathbf{R}$ and of its positive powers when the system dimensions go to infinity with constant ratio, for detector Type J-I, (ii) the convergence of the traces of $\mathbf{R}$ and its powers for detector Type J-J.

To compute the diagonal elements of $\mathbf{R}^m$, $m \in \mathbb{Z}^+$ or the trace of $\mathbf{R}^m$ we propose a recursive algorithm for the general case and a simplified version for the correlated Rayleigh fading channels.

As shown in [7], the knowledge of the asymptotic diagonal elements of $\mathbf{R}^m$ enables the asymptotic analysis of any linear multiuser detector that can be expressed as linear multistage detector with projection in the same Krylov subspace. A part from the MSWFs and the polynomial expansion detectors, this class of detectors includes the parallel interference cancellers (PIC), the weighted PICs, asymptotically as the number of stages goes to infinity[1], the linear MMSE detectors, and the matched filter.

The large system analysis shows that the asymptotic performance of this large class of detectors is independent of the correlation of the channel gains at the transmitters. In contrast to the case of a system with a single receive antenna, the multiuser efficiency does not characterize univocally the system performance and varies from user to user according to the direction of the vector of the channel gains. This also has the following implication. While the MSWFs and the polynomial expansion detectors are equivalent for synchronous CDMA with single antennas [7] or multiple receiving antennas with independent and identically distributed channel gains, in case of perfect power control, the MSWFs outperform the polynomial expansion detectors also in case of perfect power control if the channel gains are correlated.

2. SYSTEM MODEL

We consider a CDMA system with spreading factor N and K' users. Each user employs a transmit antenna array with N_T elements sending independent data streams through each of the elements. Thus, we may speak of a system with $K = K'N_\text{T}$ virtual users. The signal is received by L receive antennas. These antennas can be part of an array or can be placed at different locations, but processed jointly.

The baseband discrete-time system model, as the channel is flat fading and the system is synchronous, is given by

$$\mathbf{y} = \mathbf{H}\mathbf{b} + \mathbf{n} \tag{1}$$

where $\mathbf{y}$ is the NL-dimensional vector of received signals, $\mathbf{b}$ is the K-dimensional vector of transmitted symbols, and $\mathbf{n}$ is discrete-time, circularly symmetric complex-valued additive white Gaussian noise with zero mean and variance σ^2. The influence of spreading and fading is described by the $NL \times K$ matrix

$$\mathbf{H} = \sum_{\ell=1}^{L} (\mathbf{S}\mathbf{D}\mathbf{\Lambda}_\ell) \otimes \mathbf{e}_\ell \tag{2}$$

where $\mathbf{S}$ is the $N \times K$ spreading matrix whose k^th column is the spreading sequence of the k^th virtual user. The diagonal square matrix $\mathbf{D} \in \mathbb{C}^{K\times K}$ contains the transmitted amplitudes of all virtual users such that its k^th diagonal element d_k is the amplitude of the signal transmitted by the virtual user indexed by k. The diagonal matrices $\mathbf{\Lambda}_1, \mathbf{\Lambda}_2, \ldots, \mathbf{\Lambda}_L \in \mathbb{C}^{K\times K}$ take into account the effect of the flat fading channel. The k-th diagonal element of $\mathbf{\Lambda}_\ell$ is the channel gain between the transmitting antenna element of the k^th virtual user and the ℓ^th receive antenna and will be denoted by $\lambda_{\ell k}$ in the following. The channel gains can be, in general, correlated and contain line of sight components as in Rice channels. $\mathbf{e}_\ell$ is the L-dimensional unit column vector whose elements are zero except the ℓ^th that equals 1, i.e. $\mathbf{e}_\ell = (\delta_{\ell j})_{j=1}^{L}$. In order to simplify notation, it will be helpful in the following to define the L-dimensional vectors $\mathbf{l}_k = d_k[\lambda_{1k}, \lambda_{2k}, \ldots, \lambda_{Lk}]^T$, $k = 1, \ldots, K$ and the diagonal square matrices $\mathbf{L}_\ell = \mathbf{D}\mathbf{\Lambda}_\ell$, $\ell = 1, \ldots, L$.

Let us consider the empirical joint distribution function of the random variables $(l_{1,k}, l_{2,k}, \ldots l_{L,k})$, $k = 1, \ldots, K$

$$F^{(K)}_{\mathbf{L}_1,\mathbf{L}_2,\ldots\mathbf{L}_L}(\mathbf{l}) = \frac{1}{K}\sum_{k=1}^{K} 1(\mathbf{l} - \mathbf{l}_k) \tag{3}$$

where $1(\cdot)$ is the L-dimensional indicator function. In the asymptotic design and analysis carried out in this work, we assume that the sequence of the empirical joint distribution functions $\{F^{(K)}_{\mathbf{L}_1,\mathbf{L}_2,\ldots\mathbf{L}_L}(\mathbf{l})\}$ converges weakly with probability 1 to a limit distribution function $F_{\mathbf{L}_1,\mathbf{L}_2,\ldots\mathbf{L}_L}(\mathbf{l})$ with bounded support.

In the following, the spreading matrix is modelled as a random matrix whose elements are independent and identically distributed (i.i.d.) with zero mean and variance $\frac{1}{N}$. Moreover, we assume the transmitted symbols to be uncorrelated random variables with zero mean and unit variance, i.e. $\mathrm{E}\{\mathbf{b}\mathbf{b}^H\} = \mathbf{I}_K$.

For clarity sake, we adopt the following notation:

- $\beta = \frac{K}{N}$ is the system load;
- $\mathbf{h}_k$ denotes the k^th column of $\mathbf{H}$;
- $\mathbf{T} = \mathbf{H}\mathbf{H}^H$;
- $\mathbf{R} = \mathbf{H}^H\mathbf{H}$.

3. MULTISTAGE DETECTION

The design of multistage detectors with universal weights for multiuser MIMO systems with correlated spatial diversity follows along the design of multistage detectors for synchronous CDMA systems with single antenna in [7]. The multistage detectors Type J-I perform the projection onto the Krylov subspace $\chi_{M,k}(\mathbf{H}) = \mathrm{span}(\mathbf{T}^m\mathbf{h}_k)|_{m=0}^{M-1}$ jointly for all users and the subsequent filtering individually for each user. The Type J-I detector for user k is defined as

$$\widehat{b}_k = \sum_{m=0}^{M-1} w_{k,m}\mathbf{h}_k^H\mathbf{T}^m\mathbf{y} \tag{4}$$

where $M < K$ is an integer and $w_{k,m}$ are the universal weights. The universal weights $w_{k,m}$ are obtained as

$$w_{k,m} = \lim_{\substack{N,K\to\infty \\ \frac{K}{N}\to\beta}} w_{k,m}(N)$$

where $w_{k,m}(N)$ are the tailored filter coefficients minimizing the mean square error (MSE) $\mathrm{E}\{\|b_k - \sum_{m=0}^{M-1} w_{k,m}(N)\mathbf{h}_k^H\mathbf{T}^m\mathbf{y}\|^2\}$. The tailored weight $w_{k,m}(N)$ is the $(m+1)^{st}$ element of the vector $\mathbf{w}_k(N)$ given by

$$\mathbf{w}_k(N) = \mathbf{\Xi}_k^{-1}(N)\boldsymbol{\xi}_k(N)$$

where $\mathbf{\Xi}_k(N) = \left((\mathbf{R}^{i+j})_{kk} + \sigma^2(\mathbf{R}^{i+j-1})_{kk}\right)_{i,j=1\ldots M}$ and $\boldsymbol{\xi}_k(N) = \left((\mathbf{R}^j)_{kk}\right)_{j=1\ldots M}$.

[1] Note that the convergence to the linear MMSE performance is very fast [9], exponential in the number of stages [10].

The matrix form of Type J-I detector for the joint projection is given by

$$\widehat{\mathbf{b}} = \sum_{m=0}^{M-1} \mathbf{W}_m \mathbf{R}^m \mathbf{H}^H \mathbf{y}$$

where $\mathbf{W}_m$ is a $K \times K$ diagonal matrix whose k^{th} diagonal element coincides with $w_{k,m}$.

Type J-J detectors perform the projection onto $\chi_{M,k}(\mathbf{H})$ and subsequently filter all projections with the same filter coefficients. They are defined as

$$\widehat{\mathbf{b}}_{\text{pe}} = \sum_{m=0}^{M-1} \overline{w}_m \mathbf{R}^m \mathbf{H}^H \mathbf{y}$$

where the scalar $\overline{w}_m$ are the universal weights of Type J-J detectors. The universal weights are obtained as

$$\overline{w}_m = \lim_{\substack{N,K\to\infty \\ \frac{K}{N}\to\beta}} \overline{w}_m(N)$$

where $\overline{w}_m(N)$ are the tailored filter coefficients minimizing the MSE $\mathrm{E}\{\|\mathbf{b} - \sum_{m=0}^{M-1} \overline{w}_m(N)\mathbf{R}^m\mathbf{H}^H\mathbf{y}\|^2\}$. The tailored weight $\overline{w}_m(N)$ is the $(m+1)^{st}$ element of the vector $\overline{\mathbf{w}}(N)$ given by

$$\overline{\mathbf{w}}_k(N) = \mathbf{\Xi}^{-1}(N)\boldsymbol{\xi}(N)$$

with $\mathbf{\Xi}(N) = \left(\text{trace}(\mathbf{R}^{i+j}) + \sigma^2 \text{trace}(\mathbf{R}^{i+j-1})\right)_{i,j=1\ldots M}$ and $\boldsymbol{\xi}(N) = \left(\text{trace}(\mathbf{R}^j)\right)_{j=1\ldots M}$.

The design of universal weights reduces to the computation of the asymptotic values $R_{kk,\infty}^m = \lim_{K=\beta N\to\infty}(\mathbf{R}^m)_{kk}$ for Type J-I detectors and to the computation of $m_{\mathbf{R}}^m = \lim_{K=\beta N\to\infty} \frac{1}{K}\text{trace}(\mathrm{R}^m)$, the asymptotic eigenvalues moments of $\mathbf{R}$, for Type J-J detectors.

The following theorem shows that $(\mathbf{R}^m)_{kk}$ converges almost surely to a deterministic value conditionally on $\mathbf{l}_k$.

Theorem 1 *Let $\mathbf{S}$ be an $N \times K$ complex matrix with random i.i.d. zero mean entries with variance $\mathrm{E}\{|s_{ij}|^2\} = \frac{1}{N}$, and $\lim_{N\to\infty} E\{N^3|s_{ij}|^6\} < +\infty$. Let $\mathbf{l}_k$ be the vector of the received amplitudes of the virtual user k. Let us assume that, almost surely, the empirical joint distribution of $\mathbf{l}_1, \mathbf{l}_2, \ldots \mathbf{l}_K$ converges to some limiting joint distribution $F_{\mathbf{l}}(\ell_1, \ell_2, \ldots, \ell_L)$ with bounded support as $K \to \infty$. $\mathbf{L}_\ell$, $\ell = 1, \ldots, L$, is a $K \times K$ diagonal matrix whose k^{th} element coincides with the ℓ^{th} component of $\mathbf{l}_k$, i.e. $(\mathbf{L}_\ell)_{kk} = (\mathbf{l}_\ell)_k$. Define $\mathbf{H} = \sum_{\ell=1}^{L} \mathbf{S}\mathbf{L}_\ell \otimes \mathbf{e}_\ell$ and assume that the spectral radius of the matrix $\mathbf{R} = \mathbf{H}^H\mathbf{H}$ is upper bounded. Then, as $N, K \to \infty$ with $\frac{K}{N} \to \beta$ and L fixed, the diagonal elements of the matrix $\mathbf{R}^m$ corresponding to the virtual user k, with given fading amplitude $\mathbf{l}_k$, converges with probability 1 to the deterministic value*

$$R_m(\mathbf{l}_k) \stackrel{a.s.}{=} \lim_{K=\beta N\to\infty} (\mathbf{R}^m)_{kk}$$

with $R_m(\mathbf{l})$ determined by the following recursion

$$R_m(\mathbf{l}_k) = \sum_{s=0}^{m-1} g(\mathscr{T}^{m-s-1}, \mathbf{l}) R_s(\mathbf{l})$$

$$\mathscr{T}_m = \sum_{s=0}^{m-1} \beta \mathrm{E}\{R_{m-s-1}(\mathbf{l})\mathbf{l}\mathbf{l}^H\} \mathscr{T}_s$$

$$g(\mathscr{T}_{m-1}, \mathbf{l}) = \mathbf{l}^H \mathscr{T}_s \mathbf{l}.$$

The recursion is initialized by $R_0(\mathbf{l}) = 1$ and $\mathscr{T}_0 = \mathbf{I}_L$.

The proof is in [11].

This theorem yields the following corollary to compute $m_{\mathbf{R}}^m$.

Corollary 1 *Let $\mathbf{S}$, $\mathbf{H}$, $\mathbf{R}$, and $\mathbf{l}_k$ be defined as in Theorem 1. Let the assumptions of Theorem 1 be satisfied. Then, the asymptotic eigenvalue moments of the matrix $\mathbf{R}$ are given by*

$$m_{\mathbf{R}}^m = \mathrm{E}\{R^m(\mathbf{l})\}$$

where $R^m(\mathbf{l})$ is obtained by the recursion in Theorem 1 and the expectation is taken over the limiting joint distribution $F_{\mathbf{l}}(l_1, l_2, \ldots, l_L)$ defined in Theorem 1.

Theorem 1 and Corollary 1 yield a simple algorithm for the computation of $R_m(\mathbf{l})$ and $m_{\mathbf{R}}^m$, $m \in \mathbb{Z}^+$.

Algorithm 1

1$^{\text{st}}$ step *Let $\rho_0(\mathbf{l}) = 1$ and $\boldsymbol{\mu}_0 = \mathbf{I}$.*

ℓ^{th} step • *Define $u_{\ell-1}(\mathbf{l}) = \mathbf{l}^H \boldsymbol{\mu}_{\ell-1} \mathbf{l}$.*

- *Define $\mathbf{v}_{\ell-1}(\mathbf{l}) = \rho_{\ell-1}(\mathbf{l})\mathbf{l}\mathbf{l}^H$ and write it as a polynomial in the monomials $l_1^{r_1} \ldots l_L^{r_L}, \bar{l}_1^{s_1} \ldots \bar{l}_L^{s_L}$.*
- *Define $m_{\mathbf{l}}^{(r_1,\ldots,r_L,s_1,\ldots,s_L)} = \mathrm{E}\{\prod_{\ell=1}^{L} l_\ell^{r_\ell} \bar{l}_\ell^{s_\ell}\}$ and replace all monomials $\prod_{\ell=1}^{L} l_\ell^{r_\ell} \bar{l}_\ell^{s_\ell}$ in $\mathbf{v}_{\ell-1}(\mathbf{l})$ by the corresponding $m_{\mathbf{l}}^{(r_1,\ldots,r_L,s_1,\ldots,s_L)}$. Assign the result to $\mathbf{V}_{\ell-1}$.*
- *Set*

$$\rho_\ell(\mathbf{l}) = \sum_{s=0}^{\ell-1} u_{\ell-s-1}(\mathbf{l}) \rho_s(\mathbf{l})$$

$$\boldsymbol{\mu}_\ell = \sum_{s=0}^{\ell-1} \beta \mathbf{V}_{\ell-s-1} \boldsymbol{\mu}_s.$$

- *Assign $\rho_\ell(\mathbf{l})$ to $R^\ell(\mathbf{l})$.*
- *Write $\rho_\ell(\mathbf{l})$ as a polynomial in $l_1, \ldots l_L, \bar{l}_1, \ldots \bar{l}_L$ and replace all monomials $\prod_{\ell=1}^{L} l_\ell^{r_\ell} \bar{l}_\ell^{s_\ell}$ in $\rho_\ell(\mathbf{l})$ by the correspondent moments $m_{\mathbf{l}}^{(r_1,\ldots,r_L,s_1,\ldots,s_L)}$ and assign the result to $m_{\mathbf{R}}^\ell$.*

If the channels at the receiving site are independent, the previous algorithm simplifies since the matrix $\mathscr{T}^s$, $s \in \mathbb{Z}^+$, is diagonal. If the coefficients are asymptotically independent and identically distributed as in the micro-diversity scenario analyzed in [3] the limiting diagonal elements of the matrix $\mathbf{R}_\ell$ and the eigenvalue moments $m_{\mathbf{R}}^\ell$ can be derived from Algorithm 1 in [7] for synchronous single receiving antenna systems by replacing

(i) β with $\beta' = \frac{K}{LN}$;
(ii) The received energy of user k at a single antenna, by the total received energy of user k at all antennas, $\mathbf{l}^H\mathbf{l}$;
(iii) The moments of the received energy at a single antenna by the moments of the total received energy at all antennas $\mathrm{E}\{|\mathbf{l}^H\mathbf{l}|^s\}$.

This result can be obtained directly from Theorem 1 in [3] as proposed in [4] or, alternatively, from Algorithm 1 noting that $\mathbf{V}_s$ is proportional to the identity matrix and $R^\ell(\mathbf{l})$ is a function of $\mathbf{l}^H\mathbf{l}$.

In practice, fading amplitudes are often complex Gaussian distributed and correlated[2] and their limiting joint distribution is given as

$$f_{\mathrm{l}}(\mathbf{l}) = \frac{1}{\pi^L \det \mathbf{C}_{\mathrm{l}}} \exp\left(-\mathbf{l}^H \mathbf{C}_{\mathrm{l}}^{-1} \mathbf{l}\right). \tag{5}$$

In the absence of power control, i.e. $\mathbf{D} = \mathbf{I}_K$, $\mathbf{C}_{\mathrm{l}}$ is the correlation matrix of the fading at the receiving side with entries $r_{ij} = \mathrm{E}\left\{\lambda_i \lambda_j^*\right\}$. Consider the eigenvalue decomposition $\mathbf{C}_{\mathrm{l}} = \mathbf{M}\boldsymbol{\Psi}\mathbf{M}^H$ with $\boldsymbol{\Psi} = \mathrm{diag}(\psi_1, \ldots, \psi_L)$ and the change of variables $\mathbf{g} = \mathbf{M}^H \mathbf{l}$ and $\mathbf{g}_k = [g_{1k}, \ldots, g_{Lk}]^T = \mathbf{M}^H \mathbf{l}_k$ creating statistically independent components in the random vector $\mathbf{g}$. Then, substituting $\mathbf{l} = \mathbf{M}\mathbf{g}$, $\mathbf{l}_k = \mathbf{M}\mathbf{g}_k$ and taking into account that $g_1, g_2 \ldots g_L$, the components of $\mathbf{g}$, are independent complex Gaussian variables with variances $\psi_1, \psi_2, \ldots \psi_L$, Algorithm 1 can be simplified as follows:

Algorithm 2

1st step *Let* $\rho_0(\mathbf{g}) = 1$ *and* $\mu_{0,\ell} = 1$, *for* $\ell = 1, \ldots L$.

ℓth step
- *Define* $u_{n-1,\ell}(\mathbf{g}) = \sum_{\ell=1}^{L} \mu_{n-1,\ell} |g_\ell|^2$.
- *Define* $v_{n-1,\ell}(\mathbf{g}) = \rho_{n-1}(\mathbf{g}) |g_\ell|^2$, $\ell = 1, \ldots L$ *and write them as polynomials in the monomials* $\prod_{\ell=1}^{L} |g_\ell|^{2r_\ell}$.
- *Define* $m_{\mathbf{g}}^{(r_1,\ldots,r_L)} = \prod_{\ell=1}^{L} \mathrm{E}\{|g_\ell|^{2r_\ell}\}$ *and replace all monomials* $\prod_{\ell=1}^{L} |g_\ell|^{2r_\ell}$ *in* $v_{n-1,\ell}(\mathbf{g})$, $\ell = 1, \ldots L$ *by the corresponding* $m_{\mathbf{g}}^{(r_1,\ldots,r_L)}$. *Assign the result to* $V_{n-1,\ell}$, $\ell = 1, \ldots L$, *respectively.*
- *Set*

$$\rho_n(\mathbf{g}) = \sum_{s=0}^{n-1} u_{n-s-1}(\mathbf{g}) \rho_s(\mathbf{g})$$

$$\mu_{n,\ell} = \sum_{s=0}^{n-1} \beta V_{n-s-1,\ell} \mu_{s,\ell}.$$

- *Assign* $\rho_n(\mathbf{M}^H \mathbf{l})$ *to* $R^n(\mathbf{l})$.
- *Write* $\rho_n(\mathbf{g})$ *as a polynomial in the monomials* $\prod_{\ell=1}^{L} |g_\ell|^{2r_\ell}$ *and replace all monomials* $\prod_{\ell=1}^{L} |g_\ell|^{2r_\ell}$ *in* $\rho_n(\mathbf{g})$ *by the correspondent moments* $m_{\mathbf{g}}^{(r_1,\ldots,r_L)}$ *and assign the result to* $m_{\mathbf{R}}^n$.

4. PERFORMANCE ANALYSIS

The asymptotic signal-to-interference and noise ratio (SINR) of user k at the output of a multistage detector with weighting vector $\widetilde{\mathbf{w}}_k$ is given by

$$\mathrm{SINR}_k = \frac{\widetilde{\mathbf{w}}_k^H \boldsymbol{\xi}_k \boldsymbol{\xi}_k^T \widetilde{\mathbf{w}}_k}{\widetilde{\mathbf{w}}_k^H \left(\boldsymbol{\Xi}_k - \boldsymbol{\xi}_k \boldsymbol{\xi}_k^T\right) \widetilde{\mathbf{w}}_k}$$

where $\boldsymbol{\Xi}_k = \lim_{K=\beta N \to \infty} \boldsymbol{\Xi}_k(N)$ and $\boldsymbol{\xi}_k = \lim_{K=\beta N \to \infty} \boldsymbol{\xi}_k(N)$. It specializes for the polynomial expansion detector to

$$\mathrm{SINR}_{\mathrm{pe},k} = \frac{1}{\dfrac{\boldsymbol{\xi}^T \boldsymbol{\Xi}^{-1} \boldsymbol{\Xi}_k \boldsymbol{\Xi}^{-1} \boldsymbol{\xi}}{\left(\boldsymbol{\xi} \boldsymbol{\Xi}^{-1} \boldsymbol{\xi}\right)^2} - 1}$$

[2]Rayleigh fading violates the demand for a distribution with bounded support in Theorem 1. However, it can be approximated arbitrary closely by a distribution with bounded support. Thus, from an engineering perspective, we need not worry about that fact.

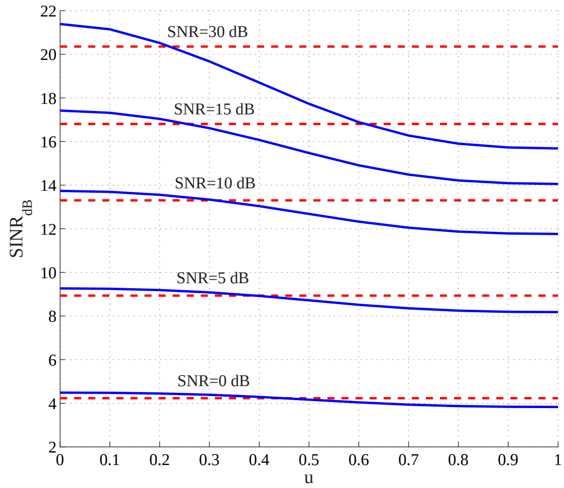

Figure 1: Output SINR in decibel of a Type J-I detector with $M = 4$ versus the coefficient u of the linear combination $\mathbf{v} = u\mathbf{v}_{\mathrm{MAX}} + (1-u)\mathbf{v}_{\mathrm{MIN}}$ for several values of the input SNR and correlated Rayleigh fading (solid lines) or independent and identically distributed fading (dashed lines).

with $\boldsymbol{\Xi} = \lim_{K=\beta N \to \infty} \boldsymbol{\Xi}(N)$ and $\boldsymbol{\xi} = \lim_{K=\beta N \to \infty} \boldsymbol{\xi}(N)$.

The asymptotic SINR at the output of a MSWF is given by

$$\mathrm{SINR}_{\mathrm{MSWF},k} = \frac{\boldsymbol{\xi}_k^T \boldsymbol{\Xi}_k^{-1} \boldsymbol{\xi}_k}{1 - \boldsymbol{\xi}_k^T \boldsymbol{\Xi}_k^{-1} \boldsymbol{\xi}_k}.$$

5. NUMERICAL RESULTS

The numerical results presented in this work were obtained using $L = 3$ receiving antennas at the base station and assuming a system load $\beta = \frac{1}{2}$. The channel was flat Rayleigh fading with limiting joint distribution (5) and correlation matrix

$$\mathbf{C}_{\mathrm{l}} = \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.5 & 1 & 0.5 \\ 0.3 & 0.5 & 1 \end{bmatrix}.$$

In case of correlated fading channel, the output SINR of a linear MMSE depends on the direction of the channel gain vector of the user of interest [12]. For correlated *Rayleigh* fading the performance is maximum or minimum when the channel gain vector is parallel to some of the eigenvectors of the correlation matrix $\mathbf{C}_{\mathrm{l}}$ [12]. The same property holds also for Type J-I and Type J-J detectors, as verified numerically. Let us denote by $\mathbf{l}_{\mathrm{MAX}}$ and $\mathbf{l}_{\mathrm{MIN}}$ the eigenvectors corresponding to the maximum and minimum eigenvalues, respectively. Figure 1 shows the performance of a Type J-I detector with $M = 4$ as the channel gain vector $\widetilde{\mathbf{l}}$ span the subspace $\{\mathbf{l}_{\mathrm{MIN}}, \mathbf{l}_{\mathrm{MAX}}\}$, i.e. it is a linear combination $\widetilde{\mathbf{l}} = u\mathbf{l}_{\mathrm{MAX}} + (1-u)\mathbf{l}_{\mathrm{MIN}}$. The solid lines plot the output SINR as a function of u, the coefficient of the linear combination $\widetilde{\mathbf{l}}$, for different values of the input SNR. The performance is maximum when the channel gain vector is parallel to $\mathbf{l}_{\mathrm{MIN}}$ and minimum when the the channel gain vector is parallel to $\mathbf{l}_{\mathrm{MAX}}$. The gap between maximum and minimum SINR increases as the input SNR increases. The dashed lines illustrate the performance of the same detector for a multiuser MIMO system with independent Rayleigh fading for the sake

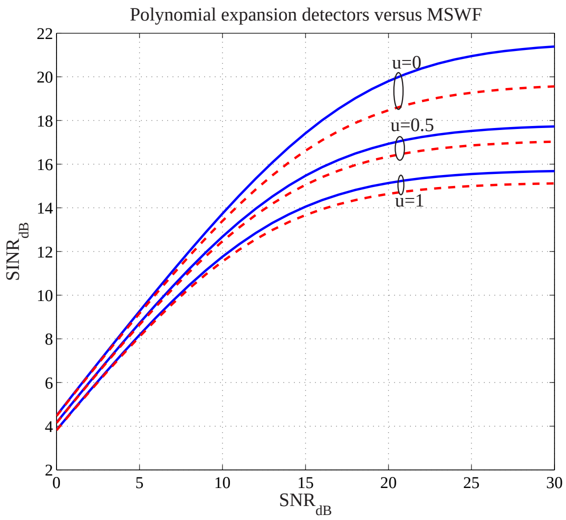

Figure 2: Asymptotic output SINR in decibel of polynomial expansion detectors (dashed lines) and MSWF (solid lines) with $M = 4$ versus SNR for several coefficients u of the linear combination $\mathbf{v} = u\mathbf{v}_{\text{MAX}} + (1-u)\mathbf{v}_{\text{MIN}}$.

of comparison. For independent Rayleigh fading, the SINR does not depend on the direction of the channel gain vectors and it has an intermediate value between the maximum and the minimum SINR for a correlated fading channel.

In Figure 2 the asymptotic output SINR of polynomial expansion detectors or Type J-J detectors (dashed lines) and of MSWF or Type J-I detectors (solid lines) is plotted as a function of the input SNR for three different channel gain vectors $\widetilde{1}$ ($u = 0, 0.5, 1$) and perfect power control. In case of single receive antenna or multiple antennas with independent and identically distributed fading gains, the MSWF and the polynomial expansion detectors are equivalent if perfect power control is performed [4, 7]. On the contrary, for correlated fading channels, even in case of perfect power control, the MSWF outperforms the correspondent polynomial expansion detector. The gap between the performance of the two detectors increases as the input SNR increases and/or u decreases.

6. CONCLUSIONS

In this contribution we propose two multistage detectors for CDMA systems with random spreading and spatial diversity in the general case as the channel gains are correlated and with line of sight components. Type J-I detector achieves near linear MMSE performance with the same complexity order per bit as the matched filter. The results presented include as special cases the results in [4] derived there under the constraints of independence of the channel gains and uniformly distributed phases.

A framework for the asymptotic performance analysis of any multistage detector with projection onto the Krylov subspace $\chi_{M,k}(\mathbf{H})$ is also provided. It is shown that the correlation at the transmitting sides does not affect the performance of the large class of linear multiuser detectors under investigation, while the system is sensitive to the correlation at the receiving site. The performance depends on the direction of the channel gain vector of the user of interest. In this scenario, the MSWF outperforms the correspondent polynomial expansion detector also in case of perfect power control.

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