# Linear Multiuser Detection for CDMA Systems with Correlated Spatial Diversity

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*Abstract*— This contribution analyzes the behaviour of CDMA (code division multiple access) systems with correlated spatial diversity. The users transmit to one or more antenna arrays. The centralized receiver employs a linear multistage detector. We provide a unified framework for the performance analysis of any linear multiuser detector representable as a multistage Wiener filter in a large system with random spreading sequences and weak assumptions on the flat fading channel gains—the fading may be correlated and contain line-of-sight components. We show that, as the number of users and the spreading factor grow large with fixed ratio, the performance is independent of the correlation at the transmitting side but depends on the correlation at the receiver.

The mathematical tools provided in this work enable also the design of low complexity multistage detectors with universal weights following the same lines as in [7].

## I. INTRODUCTION

The pioneering works in [1] and [2] on multiple antenna elements at the transmitter and the receiver with independent and identically distributed (i.i.d.) channel gains promise huge increase in the throughput of wireless communication systems. In one hand this motivated the introduction of spatial diversity in the standardization of third generation wireless systems based on code division multiple access (CDMA), e.g. UMTS. In the other hand, it promoted the blossoming of studies on the capacity of such systems using more realistic models. In this stream are works that analyze the effect of channel correlation, line of sight components, multiple scattering, and key holes. Fading correlation and line of sight components where found to affect severely channel capacity. Therefore, it is important and of practical interest to consider their effects also in a CDMA system with spatial diversity.

Modelling the spreading matrices as random matrices, Hanly and Tse in [3] analyzed a synchronous CDMA system with independent spatial diversity both at the transmitters and the receivers as the number of active users in the system and the spreading factor tend to infinity with ratio converging to a constant. The assumption of independent channel gains and the use of linear minimum mean square error detectors at the receiver underlies this analysis.

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The large system performance[1] of other linear multiuser detectors has been investigated.

The reduced rank multistage Wiener filters (MSWF) for CDMA systems with spatial diversity and channel gains independent and identically distributed for each user and independent across all users was presented in [4]. For the same scenario, a unified framework for the analysis of any multiuser detector that admit a representation as multistage detector is proposed in [5]. In the same context, an efficient implementation of multistage detectors with asymptotic weighting coefficients has been proposed in [5]. The proposed multistage detectors achieve near-linear MMSE performance with the same complexity order per bit as the single user matched filter.

The large system performance of a linear MMSE detector for a general synchronous CDMA system eventually with correlated spatial diversity and line of sight components has been investigated in [6]. The results in [6] generalize and include the results in [3].

In this work we generalize the results in [5] to a synchronous CDMA system with correlated spatial diversity and/or line of sight components. We provide a unified framework for the large system performance analysis of any multiuser detector that can be represented as a multistage detector, e.g. the polynomial expansion detector, the linear MMSE detector, the MSWF, the parallel interference cancelling (PIC) detector. Those results rely on the convergence of the diagonal elements and of the trace of the system correlation matrix $\boldsymbol{R}$ and its positive powers as the system dimensions go to infinity with constant ratio. This convergence is also useful for the design of low complexity multistage detectors with universal weights (see [7]).

To compute the diagonal elements of $\boldsymbol{R}^m$, $m \in \mathbb{Z}^+$, we propose a recursive algorithm. Simplifications are possible in case of correlated fading channel. In fact, as already shown for the linear MMSE [6], for given correlated Rayleigh channels there exists a macro-diversity scenario with independent Rayleigh fading channels equivalent in terms of performance.

Surprisingly, the performance of linear multiuser detectors is independent of the correlation of the channel gains at the transmitters. Considering the sequence of the empirical joint distribution of the received amplitudes at each receiving antenna and assuming the convergence of this sequence to a deterministic joint distribution function, the performance

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[1]Throughout this work we refer to systems with number of active users and spreading factor that grow to infinity with ratio converging to a constant value as *large system* and the corresponding performance analysis as *large system analysis*.

is affected by the correlation at the receiver only through the correlation of the random variables defined by the limit joint distribution. The same considerations hold for the line of sight components.

Let us notice that, in contrast to the case of a system with a single receive antenna, the multiuser efficiency does not characterize univocally the system performance and varies according to the direction of the vector of the channel gains for each user.

## II. SYSTEM MODEL

We consider a CDMA system with spreading factor $N$ and $K'$ users. Each user employs a transmit antenna array with $N_{\mathrm{T}}$ elements sending independent data streams through each of the elements. Thus, we may speak of a system with $K = K' N_{\mathrm{T}}$ virtual users. The signal is received by $L$ receive antennas. These antennas can be part of an array or can be placed at different locations, but processed jointly.

The baseband discrete-time system model, as the channel is flat fading and the system is synchronous, is given by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{b} + \boldsymbol{n} \qquad (1)$$

where $\boldsymbol{y}$ is the $NL$-dimensional vector of received signals, $\boldsymbol{b}$ is the $K$-dimensional vector of transmitted symbols, and $\boldsymbol{n}$ is discrete-time, circularly symmetric complex-valued additive white Gaussian noise with zero mean and variance $\sigma^2$. The influence of spreading and fading is described by the $NL \times K$ matrix

$$\boldsymbol{H} = \sum_{l=1}^{L} (\boldsymbol{S}\boldsymbol{D}\boldsymbol{\Lambda}_l) \otimes \mathbf{e}_l \qquad (2)$$

where $\boldsymbol{S}$ is the $N \times K$ spreading matrix whose $k^{\mathrm{th}}$ column is the spreading sequence of the $k^{\mathrm{th}}$ virtual user. The diagonal square matrix $\boldsymbol{D} \in \mathbb{C}^{K \times K}$ contains the transmitted amplitudes of all virtual users such that its $k^{\mathrm{th}}$ diagonal element $d_k$ is the amplitude of the signal transmitted by the virtual user indexed by $k$. The diagonal matrices $\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_L \in \mathbb{C}^{K \times K}$ take into account the effect of the flat fading channel. The $k$-th diagonal element of $\boldsymbol{\Lambda}_l$ is the channel gain between the transmitting antenna element of the $k^{\mathrm{th}}$ virtual user and the $l^{\mathrm{th}}$ receive antenna and will be denoted by $\lambda_{lk}$ in the following. The channel gains can be, in general, correlated and contain line of sight components as in Rice channels. $\mathbf{e}_l$ is the $L$-dimensional unit column vector whose elements are zero except the $l^{\mathrm{th}}$ that equals 1, i.e. $\mathbf{e}_l = (\delta_{lj})_{j=1}^{L}$. In order to simplify notation, it will be helpful in the following to define the $L$-dimensional vectors $\boldsymbol{l}_k = d_k [\lambda_{1k}, \lambda_{2k}, \ldots, \lambda_{Lk}]^T$, $k = 1, \ldots, K$ and the diagonal square matrices $\boldsymbol{L}_\ell = \boldsymbol{D}\boldsymbol{\Lambda}_\ell$, $\ell = 1, \ldots, L$.

Let us consider the empirical joint distribution function of the random variables $(l_{1,k}, l_{2,k}, \ldots l_{L,k})$, $k = 1, \ldots, K$

$$F_{\boldsymbol{L}_1, \boldsymbol{L}_2, \ldots \boldsymbol{L}_L}^{(K)}(\boldsymbol{l}) = \frac{1}{K} \sum_{k=1}^{K} \mathbf{1}(\boldsymbol{l} - \boldsymbol{l}_k) \qquad (3)$$

where $\mathbf{1}(\cdot)$ is the $L$-dimensional indicator function. In the asymptotic design and analysis carried out in this work, we

assume that the sequence of the empirical joint distribution functions $\{F_{\boldsymbol{L}_1, \boldsymbol{L}_2, \ldots \boldsymbol{L}_L}(\boldsymbol{l})\}$ converges weakly with probability 1 to a limit distribution function $F_{\boldsymbol{L}_1, \boldsymbol{L}_2, \ldots \boldsymbol{L}_L}^{(K)}(\boldsymbol{l})$ with bounded support.

In the following, the spreading matrix is modelled as a random matrix whose elements are independent and identically distributed (i.i.d.) with zero mean and variance $\frac{1}{N}$. Moreover, we assume the transmitted symbols to be uncorrelated and identically distributed random variables with zero mean and unit variance, i.e. $\mathrm{E}\{\boldsymbol{b}\boldsymbol{b}^H\} = \boldsymbol{I}_K$.

For clarity sake, we adopt the following notation:

- $\beta = \frac{K}{N}$ is the system load;
- $\boldsymbol{h}_k$ denotes the $k^{\mathrm{th}}$ column of $\boldsymbol{H}$;
- $\boldsymbol{T} = \boldsymbol{H}\boldsymbol{H}^H$;
- $\boldsymbol{R} = \boldsymbol{H}^H\boldsymbol{H}$.

## III. MULTISTAGE DETECTION

We consider the large class of linear multiuser detectors that can be expressed as a multistage detector of rank $M \in \mathbb{Z}^+$ in the Krylov subspace

$$\chi_{M,k}(\boldsymbol{H}) = \mathrm{span}(\boldsymbol{T}^m \boldsymbol{h}_k)|_{m=0}^{M-1},$$

i.e.

$$\widehat{b}_k = \sum_{m=0}^{M-1} (\boldsymbol{w}_k)_m \boldsymbol{h}^H \boldsymbol{T}^m \boldsymbol{y}$$

where $\boldsymbol{w}_k$ is the $M$-dimensional vector of weight coefficients.

The multistage detector processing jointly all users is given by

$$\widehat{\boldsymbol{b}}_k = \sum_{m=0}^{M-1} \boldsymbol{W}_m \boldsymbol{R}^m \boldsymbol{H}^H \boldsymbol{y}$$

where $\boldsymbol{W}_m$, $m = 0, \ldots, M-1$ are the $K \times K$ diagonal matrices of weight coefficients such that $(\boldsymbol{W}_m)_{kk} = (\boldsymbol{w}_k)_m$.

This class of detectors includes the most popular linear multiuser detectors:

- If the weight matrices are proportional to the identity matrix, i.e. $\boldsymbol{W}_m = w_m \boldsymbol{I}$, with $w_m$ fixed coefficients the multistage detector coincides with the the Parallel Interference Cancelling detector, eventually weighted.
- When the weight matrices are designed according to a Minimum Mean Square Error criterion, so that $\mathrm{E}\{\|\boldsymbol{b} - \widehat{\boldsymbol{b}}\|^2\}$ is minimized, the multistage detector is equivalent to a multistage Wiener filter of rank $M$ in terms of performance.
- Assuming the matrix of weights proportional to the identity matrix and enforcing the again the MMSE criterion we obtain the polynomial expansion detector of rank $M$.
- A polynomial expansion detector or a multistage Wiener filter of rank $M = K$ coincide with a linear MMSE detector.

The weights of a polynomial expansion detector of rank $M$ are given by

$$\boldsymbol{w} = \boldsymbol{\Phi}^{-1}\boldsymbol{\varphi}$$

where $\boldsymbol{\Phi}$ is an $M \times M$ matrix with $(i,j)$ element $\boldsymbol{\Phi}_{ij} = \text{trace}\boldsymbol{R}^{i+j} + \sigma^2\text{trace}\boldsymbol{R}^{i+j-1}$, $i,j = 1,\ldots,M$. $\boldsymbol{\varphi}$ is an $M$-dimensional column vector with $i^{\text{th}}$ element $\varphi_i = \text{trace}\boldsymbol{R}^i$. The weights of a multistage detector equivalent to the MSWF can be obtained by

$$\boldsymbol{w}_k = \boldsymbol{\Phi}_k^{-1}\boldsymbol{\varphi}_k \qquad k = 1,2\ldots K$$

where $\boldsymbol{\Phi}_k$ is an $M \times M$ matrix with $(i,j)$ element $(\boldsymbol{\Phi}_k)_{ij} = (\boldsymbol{R}^{i+j})_k + \sigma^2(\boldsymbol{R}^{i+j-1})_k$, $i,j = 1,\ldots,M$ and $\boldsymbol{\varphi}_k$ is an $M$-dimensional column vector with elements $(\boldsymbol{\varphi}_k)_i = (\boldsymbol{R}^i)_k$, $i = 1,\ldots M$, and $(\boldsymbol{R}^\ell)_k$ denotes the $k^{\text{th}}$ diagonal element of the matrix $\boldsymbol{R}^\ell$.

The SINR of user $k$ at the output of a multistage detector with weighting vector $\boldsymbol{w}_k$ is given by

$$\text{SINR}_k = \frac{\boldsymbol{w}_k^H \boldsymbol{\varphi}_k \boldsymbol{\varphi}_k^T \boldsymbol{w}_k}{\boldsymbol{w}_k^H \left(\boldsymbol{\Phi}_k - \boldsymbol{\varphi}_k \boldsymbol{\varphi}_k^T\right) \boldsymbol{w}_k}.$$

It specializes for the polynomial expansion detector to

$$\text{SINR}_{pe,k} = \frac{1}{\dfrac{(\boldsymbol{\varphi})^T(\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}_k(\boldsymbol{\Phi})^{-1}\boldsymbol{\varphi}}{\left(\boldsymbol{\varphi}(\boldsymbol{\Phi})^{-1}\boldsymbol{\varphi}\right)^2} - 1}$$

and for the MSWF to

$$\text{SINR}_{MSWF,k} = \frac{\boldsymbol{\varphi}_k^T \boldsymbol{\Phi}_k^{-1}\boldsymbol{\varphi}_k}{1 - \boldsymbol{\varphi}_k^T \boldsymbol{\Phi}_k^{-1}\boldsymbol{\varphi}_k}$$

The asymptotic analysis of linear multistage detectors that admit a representation as multistage detectors reduces to determine the asymptotic values of $(\boldsymbol{R}^\ell)_k$ (and $\text{trace}\boldsymbol{R}^\ell$ for the polynomial expansion detectors). Additionally, following the same line as in [7], these asymptotic values can be utilized for the design of multiuser detectors with the same complexity order per bit as the matched filter.

The following theorem shows that $(\boldsymbol{R}^m)_k$ converges almost surely to a deterministic value conditionally on $\boldsymbol{l}_k$. A recursive algorithm to compute such a limiting value is provided.

*Theorem 1:* Let $\boldsymbol{S}$ be an $N \times K$ complex matrix with random i.i.d. zero mean entries with variance $\text{E}\{|s_{ij}|^2\} = \frac{1}{N}$, and $\lim_{N\to\infty} E\{N^3|s_{ij}|^6\} < +\infty$. Let $\boldsymbol{l}_k$ be the vector of the received amplitudes of the virtual user $k$. Let us assume that, almost surely, the empirical joint distribution of $\boldsymbol{l}_1, \boldsymbol{l}_2, \ldots \boldsymbol{l}_K$ converges to some limiting joint distribution $F_{\boldsymbol{l}}(\ell_1, \ell_2, \ldots, \ell_L)$ with bounded support as $K \to \infty$. $\boldsymbol{L}_\ell$, $\ell = 1,\ldots,L$, is a $K \times K$ diagonal matrix whose $k^{\text{th}}$ element coincides with the $\ell^{\text{th}}$ component of $\boldsymbol{l}_k$, i.e. $(\boldsymbol{L})_{kk} = (\boldsymbol{l}_\ell)_k$. Define $\boldsymbol{H} = \sum_{\ell=1}^{L}\boldsymbol{S}\boldsymbol{L}_\ell \otimes \boldsymbol{e}_\ell$ and assume that the spectral radius of the matrix $\boldsymbol{R} = \boldsymbol{H}^H\boldsymbol{H}$ is upper bounded. Then, as $N, K \to \infty$ with $\frac{K}{N} \to \beta$ and $L$ fixed, the diagonal elements of the matrix $\boldsymbol{R}^m$ corresponding to the virtual user $k$, with given fading amplitude $\boldsymbol{l}_k$, converges with probability 1 to the deterministic value

$$R^m(\boldsymbol{l}_k) \stackrel{a.s.}{=} \lim_{K=\beta N\to\infty}(\boldsymbol{R}^m)_k$$

with $R^m(\boldsymbol{l})$ determined by the following recursion

$$R^m(\boldsymbol{l}_k) = \sum_{s=0}^{m-1} g(\boldsymbol{T}^{m-s-1},\boldsymbol{l})R^s(\boldsymbol{l})$$

$$\boldsymbol{T}^m = \sum_{s=0}^{m-1}\beta\text{E}\{R^{m-s-1}(\boldsymbol{l})\boldsymbol{l}\boldsymbol{l}^H\}\boldsymbol{T}^s$$

$$g(\boldsymbol{T}^{m-1},\boldsymbol{l}) = \boldsymbol{l}^H\boldsymbol{T}^s\boldsymbol{l}.$$

The recursion is initialized by $R^0(\boldsymbol{l}) = 1$ and $\boldsymbol{T}^0 = \boldsymbol{I}_L$. The proof is in [8]. This theorem yields the following corollary to compute $m_{\boldsymbol{R}}^m = \lim_{K=\beta N\to\infty}\frac{1}{N}\text{trace}\boldsymbol{R}^m$, $m \in \mathbb{Z}^+$, the asymptotic eigenvalue moments of the matrix $\boldsymbol{R}^m$.

*Corollary 1:* Let $\boldsymbol{S}$, $\boldsymbol{H}$, $\boldsymbol{R}$, and $\boldsymbol{l}_k$ be defined as in Theorem 1. Let the assumptions of Theorem 1 be satisfied. Then, the asymptotic eigenvalue moments of the matrix $\boldsymbol{R}$ are given by

$$m_{\boldsymbol{R}}^m = \text{E}\{R^m(\boldsymbol{l})\}$$

where $R^m(\boldsymbol{l})$ is obtained by the recursion in Theorem 1 and the expectation is taken over the limiting joint distribution $F_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L)$ defined in Theorem 1.
Making use of the following correspondences

$$\begin{array}{ccc} R^m(\boldsymbol{l}) & \to & \rho_m(\boldsymbol{l}) \\ \boldsymbol{T}^m & \to & \mu_m \\ g(\boldsymbol{T}^m,\boldsymbol{l}) & \to & u_m \\ R^m\boldsymbol{l}\boldsymbol{l}^H & \to & \boldsymbol{v}_m \end{array}$$

Theorem 1 and Corollary 1 yield a simple algorithm for the computation of $R^m(\boldsymbol{l})$ and $m_{\boldsymbol{R}}^m$, $m \in \mathbb{Z}^+$.

*Algorithm 1:*
$1^{\text{st}}$ step Let $\rho_0(\boldsymbol{l}) = 1$ and $\boldsymbol{\mu}_0 = \boldsymbol{I}$.
$\ell^{\text{th}}$ step
- Define $u_{\ell-1}(\boldsymbol{l}) = \boldsymbol{l}^H\boldsymbol{\mu}_{\ell-1}\boldsymbol{l}$.
- Define $\boldsymbol{v}_{\ell-1}(\boldsymbol{l}) = \rho_{\ell-1}(\boldsymbol{l})\boldsymbol{l}\boldsymbol{l}^H$ and write it as a polynomial in the monomials $l_1^{r_1}\ldots l_L^{r_L}, \bar{l}_1^{s_1}\ldots\bar{l}_L^{s_L}$.
- Define $m_{\boldsymbol{l}}^{(r_1,\ldots,r_L,s_1,\ldots,s_L)} = \text{E}\{\prod_{\ell=1}^{L} l_\ell^{r_\ell}\bar{l}_\ell^{s_\ell}\}$ and replace all monomials $\prod_{\ell=1}^{L} l_\ell^{r_\ell}\bar{l}_\ell^{s_\ell}$ in $\boldsymbol{v}_{\ell-1}(\boldsymbol{l})$ by the corresponding $m_{\boldsymbol{l}}^{(r_1,\ldots,r_L,s_1,\ldots,s_L)}$. Assign the result to $\boldsymbol{V}_{\ell-1}$.
- Set

$$\rho_\ell(\boldsymbol{l}) = \sum_{s=0}^{\ell-1} u_{\ell-s-1}(\boldsymbol{l})\rho_s(\boldsymbol{l})$$

$$\boldsymbol{\mu}_\ell = \sum_{s=0}^{\ell-1}\beta\boldsymbol{V}_{\ell-s-1}\boldsymbol{\mu}_s.$$

- Assign $\rho_\ell(\boldsymbol{l})$ to $R^\ell(\boldsymbol{l})$.
- Write $\rho_\ell(\boldsymbol{l})$ as a polynomial in $l_1,\ldots l_L, \bar{l}_1,\ldots\bar{l}_L$ and replace all monomials $\prod_{\ell=1}^{L} l_\ell^{r_\ell}\bar{l}_\ell^{s_\ell}$ in $\rho_\ell(\boldsymbol{l})$ by the correspondent moments $m_{\boldsymbol{l}}^{(r_1,\ldots,r_L,s_1,\ldots,s_L)}$ and assign the result to $m_{\boldsymbol{R}}^\ell$.

If the channels at the receiving site are independent, the previous algorithm simplifies since the matrix $\boldsymbol{T}^s$, $s \in \mathbb{Z}^+$,

is diagonal. If the coefficients are asymptotically independent and identically distributed as in the micro-diversity scenario analyzed in [3] the limiting diagonal elements of the matrix $\boldsymbol{R}^\ell$ and the eigenvalue moments $m_{\boldsymbol{R}}^\ell$ can be derived from Algorithm 1 in [7] for synchronous single receiving antenna systems by replacing

(i) $\beta$ with $\beta' = \frac{K}{LN}$;
(ii) The received energy of user $k$ at a single antenna, by the total received energy of user $k$ at all antennas, $\boldsymbol{l}^H\boldsymbol{l}$;
(iii) The moments of the received energy at a single antenna by the moments of the total received energy at all antennas $\mathrm{E}\{|\boldsymbol{l}^H\boldsymbol{l}|^s\}$ .

This result can be obtained directly from Theorem 1 in [3] as proposed in [5] or, alternatively, from Algorithm 1 noting that $\boldsymbol{V}_s$ is proportional to the identity matrix and $R^\ell(\boldsymbol{l})$ is a function of $\boldsymbol{l}^H\boldsymbol{l}$.

In practice, fading amplitudes are often complex Gaussian distributed and correlated. Rayleigh fading also violates the demand for a distribution with bounded support in Theorem 1. However, it can be approximated arbitrary closely by a distribution with bounded support. Thus, from an engineering perspective, we need not worry about that fact. Assume that the limiting joint distribution is given as

$$f_l(\boldsymbol{l}) = \frac{1}{\pi^L \det \boldsymbol{C_l}} \exp\left(-\boldsymbol{l}^H \boldsymbol{C_l}^{-1} \boldsymbol{l}\right). \tag{4}$$

In the absence of power control, i.e. $\boldsymbol{D} = \boldsymbol{I}_K$, this implies that $\boldsymbol{C_l}$ is the correlation matrix of the fading at the receiving side with entries

$$r_{ij} = \mathrm{E}\left\{\lambda_i \lambda_j^*\right\}. \tag{5}$$

Consider the eigenvalue decomposition

$$\boldsymbol{C_l} = \boldsymbol{M}\boldsymbol{\Psi}\boldsymbol{M}^H \tag{6}$$

with $\boldsymbol{\Psi} = \mathrm{diag}(\psi_1, \ldots, \psi_L)$ and the change of variables

$$\boldsymbol{g} = \boldsymbol{M}^H\boldsymbol{l} \tag{7}$$
$$\boldsymbol{g}_k = [g_{1k}, \ldots, g_{Lk}]^T = \boldsymbol{M}^H\boldsymbol{l}_k \tag{8}$$

creating statistically independent components in the random vector $\boldsymbol{g}$. Then, substituting $\boldsymbol{l} = \boldsymbol{M}\boldsymbol{g}$, $\boldsymbol{l}_k = \boldsymbol{M}\boldsymbol{g}_k$ and taking into account that $g_1, g_2 \ldots g_L$, the components of $\boldsymbol{g}$, are independent complex Gaussian variables with variances $\psi_1, \psi_2, \ldots \psi_L$, Algorithm 1 can be simplified as follows:

*Algorithm 2:*

$1^{\mathrm{st}}$ step Let $\rho_0(\boldsymbol{g}) = 1$ and $\mu_{0,\ell} = 1$, for $\ell = 1, \ldots L$.

$\ell^{\mathrm{th}}$ step
- Define $u_{n-1,\ell}(\boldsymbol{g}) = \sum_{\ell=1}^L \mu_{n-1,\ell}|g_\ell|^2$.
- Define $v_{n-1,\ell}(\boldsymbol{g}) = \rho_{n-1}(\boldsymbol{g})|g_\ell|^2$, $\ell = 1, \ldots L$ and write them as polynomials in the monomials $\prod_{\ell=1}^L |g_\ell|^{2r_\ell}$.
- Define $m_{\boldsymbol{g}}^{(r_1,\ldots,r_L)} = \prod_{\ell=1}^L \mathrm{E}\{|g_\ell|^{2r_\ell}\}$ and replace all monomials $\prod_{\ell=1}^L |g_\ell|^{2r_\ell}$ in $v_{n-1,\ell}(\boldsymbol{g})$, $\ell = 1, \ldots L$ by the corresponding $m_{\boldsymbol{g}}^{(r_1,\ldots,r_L)}$. Assign the result to $V_{n-1,\ell}$, $\ell = 1, \ldots L$, respectively.

- Set

$$\rho_n(\boldsymbol{g}) = \sum_{s=0}^{n-1} u_{n-s-1}(\boldsymbol{g})\rho_s(\boldsymbol{g})$$

$$\mu_{n,\ell} = \sum_{s=0}^{n-1} \beta V_{n-s-1,\ell}\mu_{s,\ell}.$$

- Assign $\rho_n(\boldsymbol{M}^H\boldsymbol{l})$ to $R^n(\boldsymbol{l})$.
- Write $\rho_n(\boldsymbol{g})$ as a polynomial in the monomials $\prod_{\ell=1}^L |g_\ell|^{2r_\ell}$ and replace all monomials $\prod_{\ell=1}^L |g_\ell|^{2r_\ell}$ in $\rho_n(\boldsymbol{g})$ by the correspondent moments $m_{\boldsymbol{g}}^{(r_1,\ldots,r_L)}$ and assign the result to $m_{\boldsymbol{R}}^n$.

## IV. CONCLUSIONS

In this contribution we determined the large system performance of any linear multiuser detector that admits a multistage representation in CDMA systems with random spreading and spatial diversity in the general case as the channel gains are correlated and with line of sight components. This result includes as special cases the results in [5] derived there under the constraints of independence of the channel gains and uniformly distributed phases.

It is shown that the correlation at the transmitting sides does not affect the performance of the large class of linear multiuser detectors under investigation, while the system is sensitive to the correlation at the receiving site.

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