

# LINEAR PRECODING FOR MIMO TRANSMISSION WITH PARTIAL CSIT

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## ABSTRACT

We have previously introduced linear precoding schemes for spatial multiplexing, based on spatial spreading and delay diversity. These schemes were designed for the case of absence of Channel State Information at the Transmitter (CSIT) (whereas at the receiver the coherent case of full CSIR is assumed). These spatiotemporal spreading schemes have been shown to allow to attain the optimal rate-diversity trade-off at high SNR. On the other hand, the capacity achieving solution for the case of full CSIT is based on spatial prefiltering with water filling on the streams. Extensions of this scheme have been proposed recently for the case of partial CSIT, but like the full CSIT schemes, they do not take advantage of maximal diversity. In this paper a combination of the solutions described above for no and full CSIT is proposed for the general case of partial CSIT, exploiting diversity sources in all cases of CSIT.

## 1. INTRODUCTION

The growing demand for multimedia services in wireless communications has steadily increased the demand for capacity. On the other hand, spectral efficiency is necessary due to a limited radio spectrum. Multiple-Input Multiple-Output (MIMO) channels have shown good spectral efficiencies over the past years, growing approximately linearly with the number of antennas under ideal propagation [1]. Multiple data streams can be transmitted over the multiple paths between a transmit-receive antenna pair, which are generally considered independent. The MIMO wireless channel is usually described by a matrix  $\mathbf{H}$  with random complex entries and dimension  $N_R \times N_T$ , being  $N_T$  and  $N_R$  the number of transmit and receive antennas, respectively. At the receiver, the multiple data streams sent over the channel can be linearly recovered if  $\text{rank}(\mathbf{H}) \geq$

$m$ , where  $m$  represents the number of transmitted streams. The presence of severe correlations has important detrimental effects on the capacity and performance of MIMO systems. In fact, most Space-time code designs assume independent Rayleigh fading for each stream, which in practice is not true as shown in [2]. The problem has been addressed by transmitting on the eigen-modes of the transmit antenna correlation matrix [3], which provides performance and capacity gains. In [4], a prefiltering approach is proposed assuming partial Channel State Information at the Transmitter (CSIT), where knowledge of the transmit antenna correlations is successfully exploited to improve the Pairwise Error Probability (PEP) of a ST coded system.

MIMO channels allow to increase capacity by introducing spatial multiplexing. This consists of transmitting multiple data streams simultaneously. A scheme like V-BLAST puts these streams directly on the transmit antennas. If these streams form a spatiotemporally white Gaussian vector signal, then such spatial multiplexing allows to attain the MIMO ergodic capacity. Let us now focus on the vector channel seen by each transmitted stream, called transmit channel for short. The transmit channels in V-BLAST are the columns of the MIMO channel  $\mathbf{H}$ . Partial channel knowledge at the transmitter will typically lead to differences in power of the different transmit channels and possible dependencies between them. To exploit this information, a weighting matrix  $\mathbf{W}$  gets introduced to weigh and transform the transmit channels. The modified transmit channels are then the columns of  $\mathbf{H}\mathbf{W}$ .

Another important aspect of multi-antenna channels is an increase of diversity due to the multiplicity of links. The original transmit channels may have equal or different diversity orders. If the weighting matrix  $\mathbf{W}$  has all elements non-zero, then the transformed transmit channels are all combinations of all original transmit channels and hence their diversity order will tend to increase. The diversity orders may get modified more substantially if  $\mathbf{W}$  depends on the channel  $\mathbf{H}$  directly. Consider the SVD of the original channel matrix  $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$  where the diagonal matrix  $\mathbf{\Sigma}$  contains the singular values  $\sqrt{\lambda_i}$  in decreasing order. The weighting matrix  $\mathbf{W}$  that transforms the MIMO channel in orthogonal

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transmit channels (eigenmodes) is  $\mathbf{W} = \mathbf{V}$ . It is probably well-known that the strongest eigenmode (corresponding to  $\lambda_1$ ) exhibits full diversity in the sense that it combines the diversity orders of all transmit channels: the strongest eigenmode does better than the strongest transmit channel (selection diversity) which is known to exhibit full diversity. So a stream can be put directly into the strongest eigenmode and be received with full diversity. However, achieving capacity requires spatial multiplexing and hence streams need to be input also to the other eigenmodes. Perhaps it may be mistakenly believed that all eigenmodes exhibit this full diversity. It is true that the question actually does not arise in capacity achieving transmission for the case of full Channel State Information at the Transmitter (CSIT) because in that case the various eigenstreams get an adjusted power assigned, according to the well-known water-pouring principle, which in fact gives all streams infinite diversity (fast power control). Now consider a case of partial CSIT in which  $\mathbf{V}$  would be perfectly known (eigenmodes perfectly defined) but only the mean of the  $\lambda_i$  (mean power of each eigenmode) would be known (this is related to the scenario considered in [12] where quantization of (only) the eigenvalues is considered). So continue considering  $\mathbf{W} = \mathbf{V}$ . Then the diversity orders of the different eigenstreams are those of the channel eigenvalues  $\lambda_i$ . If the various channel ( $\mathbf{H}$ ) entries have an i.i.d. distribution, then the diversity order of eigenstream  $i$  is  $(N_T - i + 1)(N_R - i + 1)$ , which shows that consecutive eigenmodes have decreasing diversity. So, in order to maximize the diversity of all streams, spatiotemporal precoding is required before entering the weighted channel  $\mathbf{H}\mathbf{W}$ . In this paper, we propose to perform this precoding linearly via a space-time spreading (STS) scheme we have introduced earlier. The overall scheme is a linear precoding scheme combining STS and weighting, to optimize probability of error with full diversity.

The paper is organized as follows. Section 2 introduces the system model with transmitter and channel. The linear precoder is described in Section 3. Section 4 shows the weighting matrix evaluating the performance in terms of PEP. A capacity analysis is shown in section 5. Finally, conclusions are drawn in Section 6.

## 2. SYSTEM DESCRIPTION

The MIMO system model is shown in Fig. 1. The binary information data sequence  $\mathbf{x}_k$  is fed to an ST channel encoder, which generates  $N_s$  symbol sequences (streams) at symbol rate. The ST channel encoder spatially demultiplexes  $\mathbf{b}_k$  (S/P conversion), either before or after the coding-mapping operation. The generated coded sequence  $\mathbf{b}_k$  contains  $N_s$  symbols per symbol period  $k$ . The  $N_s$  streams are linearly mapped into the  $N_T$  output streams by applying linear Space-Time precoding, which will be studied in detail later

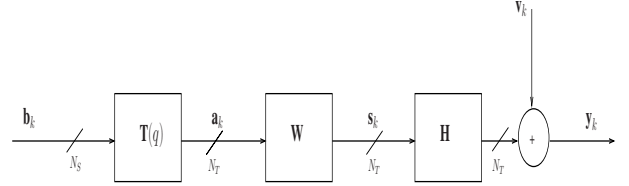


Figure 1: MIMO Transmission with Convolutional Linear Precoding and Adaptive Antenna Weighting.

on. After ST precoding, a constellation symbol  $\mathbf{b}_{m,k}$  of a given stream  $m$  appears in different time periods and output streams (spatiotemporal spreading). Thus, ST spreading let us take advantage of spatial diversity and frequency diversity (for channels with delay spread), while some coding gain can also be provided. Once spatiotemporal spreading has been applied, the transmitter exploits the CSIT, namely the transmit antenna correlation matrix, to improve the system performance. The precoded sequence  $\mathbf{a}_k$  is transformed by a linear  $N_T \times N_T$  stationary weighting matrix  $\mathbf{W}$ , to adapt the signal to the channel state. The linearly prefiltered and weighted sequence  $\mathbf{s}_k$  is transmitted over a MIMO wireless channel  $\mathbf{H}$  with  $N_R$  receive antennas, represented by the channel matrix  $\mathbf{H}$ . Each entry  $\mathbf{H}_{ij}$  of the channel matrix  $\mathbf{H}$  represents the channel response between the  $j$ -th transmit antenna and  $i$ -th receive antenna. The MIMO channel presents antenna correlation at the BS, due to limited local scattering. In the case of covariance (partial) CSIT, assuming uncorrelated receive antennas, the MIMO channel can be modeled as

$$\mathbf{H} = \mathbf{H}_w \mathbf{R}_T^{H/2} \quad (1)$$

where  $\mathbf{R}_T = \mathbf{E} \mathbf{H}^H \mathbf{H}$  is the  $N_T \times N_T$  transmit antenna correlation matrix (assumed stationary over time) and  $\mathbf{H}_w$  is a  $N_R \times N_T$  i.i.d. complex matrix. The channel entries  $\mathbf{H}_{w,ij}$  are considered zero-mean complex Gaussian variables with unit variance (Rayleigh flat-fading MIMO channel model). In the presence of additive white Gaussian noise, the received signal is

$$\mathbf{y}_k = \mathbf{H} \mathbf{s}_k + \mathbf{v}_k = \mathbf{H} \mathbf{W} \mathbf{T}(q) \mathbf{b}_k + \mathbf{v}_k \quad (2)$$

where the noise power spectral density matrix is  $S_{\mathbf{v}\mathbf{v}}(z) = \sigma_v^2 I$ ,  $q^{-1} \mathbf{b}_k = \mathbf{b}_{k-1}$ . After sampling the received signal, the channel output  $\mathbf{y}_k$  contains  $N_R$  symbol streams at symbol rate, which are passed on to a MIMO receiver. The transmitted data can be recovered by a ML receiver. In our analysis, we focus on the transmitter side, and specifically in the design of the linear ST precoder and weighting matrix. At the receiver side we assume perfect CSI.

### 3. LINEAR PRECODER

The linear precoding considered here consists of a modification of VBLAST, obtained by inserting a square matrix prefilter  $\mathbf{T}(z)$  before inputting the vector signal  $\mathbf{b}_k$  into the weighting matrix  $\mathbf{W}$ . The  $N_s = N_T$  ("full rate") component signals of  $\mathbf{b}_k$  are called streams or layers. The suggested prefilter is

$$\begin{aligned} \mathbf{T}(z) &= \mathbf{D}(z) \mathbf{Q}, & |\mathbf{Q}_{ij}| &= \frac{1}{\sqrt{N_T}} \\ \mathbf{D}(z) &= \text{diag}\{1, z^{-1}, \dots, z^{-(N_T-1)}\}, & \mathbf{Q}^H \mathbf{Q} &= \mathbf{I} \end{aligned} \quad (3)$$

$\mathbf{Q}$  is a Vandermonde matrix with dimension  $N_s \times N_T$ . As shown in [5], it minimizes an upper bound to the pairwise error probability at high SNR.

$$\mathbf{Q} = \frac{1}{\sqrt{N_T}} \begin{bmatrix} 1 & \theta_1 & \dots & \theta_1^{N_T-1} \\ 1 & \theta_2 & \dots & \theta_2^{N_T-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \theta_{N_T} & \dots & \theta_{N_T}^{N_T-1} \end{bmatrix} \quad (4)$$

which is unitary and has equal magnitude components and where  $\theta_i$  are the roots of  $\theta^{N_T} - j = 0$ ,  $j = \sqrt{-1}$ . Note that for a channel with a delay spread of  $L$  symbol periods, the prefilter can be immediately adapted by replacing the elementary delay  $z^{-1}$  by  $z^{-L}$  in  $\mathbf{D}(z)$ . It is important that the different columns of the channel matrix  $\mathbf{H}$  get spread out in time to get full diversity (otherwise the streams just pass through a linear combination of the columns, as in VBLAST, which offers limited diversity). The delay diversity only becomes effective by the introduction of the spatial spreading matrix  $\mathbf{Q}$ , which has equal magnitude elements for uniform diversity spreading. The specific Vandermonde choice for  $\mathbf{Q}$  shown in (4) corresponds to the DFT matrix multiplied by a diagonal matrix containing the elements of the first row of  $\mathbf{Q}$ . This choice for  $\mathbf{Q}$  can be shown to lead to maximum coding gain in case of QAM symbols [7], among all matrices with normalized columns.

The STS scheme discussed here has been introduced in [5] and further analyzed in [6] as a full (symbol) rate full diversity scheme without CSIT. It achieves the optimal diversity-vs-multiplexing tradeoff.

### 4. WEIGHTING MATRIX

The weighting matrix  $\mathbf{W}$  is designed so that an upper bound on the average Pairwise Error Probability (PEP) is minimized, as described in [4]. In the following, we drop the time index  $k$  for clarity. The PEP is defined as the error probability of choosing the nearest distinct codeword  $\mathbf{a}^l$  instead of  $\mathbf{a}^n$ . For the case here described, a convolutional linear precoder is applied to obtain  $\mathbf{a}$ , which can be interpreted as a block code of infinite length. The code error matrix can

be written as  $\tilde{\mathbf{E}}(l, n) := [\mathbf{a}^n - \mathbf{a}^l]$ . If  $\tilde{\mathbf{E}}(l, n)$  is full-rank for all distinct  $l, n$ , maximum diversity is obtained [8]. The coding gain is dictated by the minimum distance code error matrix given by

$$\mathbf{E} = \arg \min_{\tilde{\mathbf{E}}(l, n)} \det \left[ \tilde{\mathbf{E}}(l, n) \tilde{\mathbf{E}}^H(l, n) \right] \quad (5)$$

As shown in Appendix A, the minimum distance code error matrix is paraunitary for the proposed STS scheme:

$$\mathbf{E} \mathbf{E}^H = \alpha \mathbf{I} \quad (6)$$

where  $\alpha$  is a scalar.  $\mathbf{E} \mathbf{E}^H$  is called a *Gram* matrix. As proven in [8], the PEP over a Rayleigh flat-fading MIMO channel can be upper-bounded by

$$PEP \leq e^{-d_{min}^2/2} \quad (7)$$

where  $d_{min}^2$  is defined as

$$d_{min}^2 = \frac{1}{\sigma_v^2} \|\mathbf{H}_w \mathbf{R}_T^{H/2} \bar{\mathbf{E}}\|_F^2 \quad (8)$$

The effective minimum distance error matrix is  $\bar{\mathbf{E}} = \mathbf{W} \mathbf{E}$ , and  $\|\cdot\|_F$  is the Frobenius norm. Let  $\mathbf{A}$  be

$$\mathbf{A} = \frac{E_s}{\sigma_v^2} \mathbf{R}_T^{H/2} \mathbf{W} \mathbf{E} \mathbf{E}^H \mathbf{W}^H \mathbf{R}_T^{1/2}, \quad (9)$$

where  $E_s$  is the energy per symbol. An upper-bound on the average PEP can be obtained by taking expectation of the PEP w.r.t.  $\mathbf{H}_w$  [8].

$$\overline{PEP} \leq [\det(\mathbf{I} + \mathbf{A})]^{-N} \quad (10)$$

Therefore, the weighting matrix  $\mathbf{W}$  that minimizes the average PEP can be found by solving the optimization problem given by

$$\begin{cases} \max_{\mathbf{W}} \det \left( \mathbf{I} + \frac{E_s}{\sigma_v^2} \mathbf{R}_T^{H/2} \mathbf{W} \mathbf{E} \mathbf{E}^H \mathbf{W}^H \mathbf{R}_T^{1/2} \right) \\ \text{subject to: } Tr(\mathbf{W} \mathbf{W}^H) = P_0 \end{cases} \quad (11)$$

By introducing the following singular value decomposition (SVD)

$$\mathbf{R}_T^{H/2} = \mathbf{U}_r \Sigma \mathbf{V}^H \quad (12)$$

and taking into account (6), the optimal weighting matrix  $\mathbf{W}$  that maximizes (11) in the proposed system is [4]

$$\mathbf{W} = \mathbf{V} \Lambda_w, \quad \Lambda_w^2 = \left[ \gamma \mathbf{I} - N_T \left( \frac{E_s}{\sigma_v^2} \right)^{-1} \Sigma^{-2} \right]_+ \quad (13)$$

where  $[\cdot]_+$  means  $\max(\cdot, 0)$  and  $\gamma$  is a constant that is computed from the trace constraint. The weighting matrix described in (13) corresponds to a statistical eigenbeamformer.

The rotation matrix  $\mathbf{V}$  ensures that  $\mathbf{W}$  transmits on the eigenmodes of  $\mathbf{R}_T$ .

One may wonder about the optimal structure of the error Grammian  $\mathbf{E}\mathbf{E}^H$  from the point of view of average PEP. One may remark that the  $\mathbf{E}$  considered corresponds to an error on one symbol when interpreting the linear precoding scheme as a linear dispersion code. So  $\mathbf{E}\mathbf{E}^H$  is not only the Grammian of a minimum distance code error matrix, it is also proportional to the contribution of one symbol to the transmit covariance matrix. Since the power should be distributed evenly over all symbols (in a properly designed transmission scheme), an overall power constraint leads to a power constraint per symbol and hence to a constraint on  $\text{tr}(\mathbf{E}\mathbf{E}^H)$ . Now, at high SNR, we have  $\det(\mathbf{I} + \mathbf{A}) \approx \det(\mathbf{A})$  and hence the average PEP is minimized when  $\det(\mathbf{E}\mathbf{E}^H)$  is maximized. For given diagonal elements, the determinant of a Grammian is maximized when all its non-diagonal elements are zero (Hadamard inequality). So, given the trace constraint  $\text{tr}(\mathbf{E}\mathbf{E}^H) = P$ ,  $\det(\mathbf{E}\mathbf{E}^H)$  is maximized when  $\mathbf{E}\mathbf{E}^H = \frac{P}{N_T}\mathbf{I}$ . Hence, the proposed full-stream precoding scheme has the optimal  $\mathbf{E}\mathbf{E}^H$  structure as shown in Appendix A. Previous works assuming ST encoding with  $\mathbf{E}\mathbf{E}^H = \alpha\mathbf{I}$ , assume in fact an O-STBC scheme, thus limiting the system to the single stream case.

## 5. CAPACITY AND MFB ANALYSIS

The **ergodic capacity** with partial CSIT and perfect CSIR is given by:

$$\begin{aligned} C(\mathbf{W}) &= E_H \frac{1}{2\pi j} \oint \frac{dz}{z} \ln \det \left( I + \frac{1}{\sigma_v^2} \mathbf{H} S_{ss}(z) \mathbf{H}^H \right) \\ &= E_H \frac{1}{2\pi j} \oint \frac{dz}{z} \ln \det \left( I + \frac{1}{\sigma_v^2} \mathbf{H} \mathbf{W} \mathbf{T}(z) S_{bb}(z) \mathbf{T}^\dagger(z) \mathbf{W}^H \mathbf{H}^H \right) \\ &= E_H \frac{1}{2\pi j} \oint \frac{dz}{z} \ln \det \left( I + \rho \mathbf{H} \mathbf{W} \mathbf{W}^H \mathbf{H}^H \right) \end{aligned} \quad (14)$$

where we assume that the channel coding and interleaving per stream leads to spatially and temporally white symbols:  $S_{bb}(z) = \sigma_b^2 I$ ,  $\rho = \frac{\sigma_b^2}{\sigma_v^2}$ , and since the prefilter  $\mathbf{T}(z)$  is paraunitary,  $\mathbf{T}(z)\mathbf{T}^\dagger(z) = \mathbf{I}$ , it transforms the white vector stream  $\mathbf{b}_k$  into the white vector stream  $\mathbf{a}_k$  at the input of the weighting matrix  $\mathbf{W}$ . The expectation  $E_H$  is here w.r.t. the distribution of the channel. Now, it has been shown [11] that capacity is achieved by a weighting matrix  $\mathbf{W}$  that transmits on the eigenmodes of  $\mathbf{R}_T$  also. The singular values of  $\mathbf{W}$  though that allow to achieve capacity are found by solving a different water filling problem from the one that optimizes the average PEP in (13) (the two cost functions are related by exchanging the order of the  $\ln \det$  and  $E_H$  operations).

For the flat propagation channel  $\mathbf{H}$  combined with the prefilter  $\mathbf{T}(z)$  and the weighting matrix  $\mathbf{W}$ , symbol stream

$n(\mathbf{b}_{n,k})$  passes through the equivalent SIMO channel

$$\sum_{i=1}^{N_T} z^{-(i-1)} \bar{\mathbf{H}}_{:,i} \mathbf{Q}_{i,n} \quad (15)$$

where the definition  $\bar{\mathbf{H}} = \mathbf{H}\mathbf{W}$  has been used. The equivalent channel in (15) has memory due to the delay diversity introduced by  $\mathbf{D}(z)$ , which allows the Matched Filter Bound (MFB) to exhibit full diversity.

## 6. MEAN-COVARIANCE PARTIAL CSIT

To combine mean channel information, typically modeled as a noisy estimate  $\hat{\mathbf{H}}$  with i.i.d. estimation errors  $\mathcal{N}(0, \sigma_h^2)$ , and covariance information  $\mathbf{R}_T$ , we can consider the posterior mean  $\hat{\mathbf{H}}(\mathbf{I} + \sigma_h^2 \mathbf{R}_T^{-1})^{-1}$  and posterior covariance  $(\sigma_h^{-2} \mathbf{I} + \mathbf{R}_T^{-1})^{-1}$  which can be combined into the posterior transmit correlation matrix

$$\hat{\mathbf{R}}_T = (\mathbf{I} + \sigma_h^2 \mathbf{R}_T^{-1})^{-1} \hat{\mathbf{H}}^H \hat{\mathbf{H}} (\mathbf{I} + \sigma_h^2 \mathbf{R}_T^{-1})^{-1} + (\sigma_h^{-2} \mathbf{I} + \mathbf{R}_T^{-1})^{-1}$$

One can verify that  $\hat{\mathbf{R}}_T$  becomes  $\hat{\mathbf{H}}^H \hat{\mathbf{H}}$  (mean information only) or  $\mathbf{R}_T$  (covariance information only) when  $\sigma_h^2 \rightarrow 0, \infty$  respectively. For the optimization of the precoder,  $\mathbf{R}_T$  can be replaced by  $\hat{\mathbf{R}}_T$ . See also a companion paper for a more detailed discussion on partial CSIT.

## 7. CONCLUSIONS

In this work, a new approach for linear precoding with Partial Channel State Information at the Transmitter (CSIT) has been introduced. It is based on a combination of linear precoding and weighting in the transmit correlation eigenspace. The proposed transmitter minimizes an upperbound on the average Pairwise Error Probability (PEP) by allocating power on the eigenmodes of the transmit antenna correlation. On the other hand, diversity of all streams is maximized by introducing delay diversity and spatial spreading in the linear precoder. The described linear precoder has optimal *Gram* matrix  $\mathbf{E}\mathbf{E}^H$  in the full-stream case. In the literature this property has been only shown in orthogonal ST block coded systems, which are single stream. The proposed precoder is also closely related to the optimization of mutual information.

### Appendix A.

The linearly precoded sequence is given by

$$\mathbf{a}_k = \mathbf{T}(q) \mathbf{b}_k = \mathbf{D}(q) \mathbf{Q} \mathbf{b}_k = \mathbf{D}(q) \mathbf{c}_k \quad (16)$$

where  $\mathbf{c}_k = [c_1(k) c_2(k) \dots c_{N_T}(k)]^T$ . We consider now the transmission of the coded symbols over a duration of  $T$

symbol periods. The accumulated precoded signal  $\mathbf{a}_{1:T} = \mathbf{C}\mathbf{c}$  has dimension  $N_T \times T$ . The distance between two code-words  $\mathbf{C}$  and  $\mathbf{C}'$  is defined as  $\mathbf{C} - \mathbf{C}' =$

$$\frac{1}{\sigma_b} \begin{bmatrix} c_1(0) - c'_1(0) & c_1(1) - c'_1(1) & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 & c_{N_T}(0) - c'_{N_T}(0) & c_{N_T}(1) - c'_{N_T}(1) & \dots \end{bmatrix}$$

By considering a single error event  $i$ , the upper bound on the pairwise error probability becomes maximized. Let  $i$  be the time index of the first error, and introduce the minimum distance code error matrix  $\mathbf{E}$

$$\mathbf{E} = \mathbf{C} - \mathbf{C}' = \begin{bmatrix} 0 & \dots & 0 & \dots & e_1(i) & \dots & \dots & \dots \\ \vdots & \ddots & \vdots & \ddots & \vdots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & 0 & e_{N_T}(i) & \dots & \dots \end{bmatrix} \quad (17)$$

where  $e_k = \frac{1}{\sigma_b}(\mathbf{c}_k - \mathbf{c}'_k)$  and it is assumed that  $\prod_{i=1}^{N_T} e_n(i) \neq 0$ , a condition that is well-known in the design of lattice constellations [9][10], a field based on the theory of numbers. Let  $\mathbf{Q}$  the Vandermonde matrix shown in (4). For  $N_T = 2^{m_t} (n_t \in \mathbb{Z})$ ,  $\mathbf{Q}$  guarantees [10] for any constellation such that  $b_n(i) - b'_n(i) = a + jb \in \mathbb{Z}[j]$  ( $\mathbb{Z}[j] = \{a + jb \mid a, b \in \mathbb{Z}\}$ ), with  $\mathbf{b}_i - \mathbf{b}'_i \in (\mathbb{Z}[j])^{N_T}/0$ , that  $(N_T^{N_T/2} \prod_{n=1}^{N_T} e_n(i)) \in \mathbb{Z}[j]/0$ , and hence:

$$\min_{\mathbf{e}_i \neq 0} \prod_{n=1}^{N_T} |e_n(i)|^2 \geq \left(\frac{1}{N_T}\right)^{N_T} \quad (18)$$

For finite QAM constellations with  $(2M)^2$  points, any symbol can be written as:  $b_n(i) = d\{(2l-1) + j(2p-1)\}$  where  $d \in \mathbb{R}^{+*}$ ,  $l, p \in \{-M+1, -M+2, \dots, M\}$ . Then  $\frac{1}{\sigma_b}(b_n(i) - b'_n(i)) = \frac{2d}{\sigma_b}(l' + jp')$ ,  $l', p' \in \{-2M+1, -2M+2, \dots, 2M-1\}$  and  $\sigma_b^2 = \frac{2(4M^2-1)d^2}{3}$ . The lower bound of (18) becomes

$$\min_{\mathbf{e}_i \neq 0} \prod_{n=1}^{N_T} |e_n(i)|^2 \geq \left(\frac{4d^2}{\sigma_b^2}\right)^{N_T} \left(\frac{1}{N_T}\right)^{N_T} = \left(\frac{4d^2}{N_T \sigma_b^2}\right)^{N_T} \quad (19)$$

In what follows, we consider an upper bound for the coding gain for any matrix  $\mathbf{Q}$  with normalized columns. The minimal product of errors  $\prod_n |e_n(i)|^2$  is upper bounded by a particular error instance corresponding to a single error in the  $b$ 's, when  $\frac{1}{\sigma_b}(\mathbf{b}_i - \mathbf{b}'_i) = \frac{2d}{\sigma_b} \mathbf{w}_{n_0}$ , where  $\mathbf{w}_{n_0}$  is the vector with one in the  $n_0^{\text{th}}$  coefficient and zeros elsewhere, hence

$$\min_{\mathbf{e}_i \neq 0} \prod_{n=1}^{N_T} |e_n(i)|^2 \leq \left(\frac{4d^2}{\sigma_b^2}\right)^{N_T} \prod_{n=1}^{N_T} |Q_{n,n_0}|^2 \quad (20)$$

Now, given that  $\sum_{n=1}^{N_T} |Q_{n,n_0}|^2 = 1$ , then by applying Jensen's inequality, we get

$$\prod_{n=1}^{N_T} |Q_{n,n_0}|^2 \leq \left(\frac{1}{N_T}\right)^{N_T} \quad (21)$$

Hence,

$$\min_{\mathbf{e}_i \neq 0} \prod_{n=1}^{N_T} |e_n(i)|^2 \leq \left(\frac{4d^2}{\sigma_b^2}\right)^{N_T} \left(\frac{1}{N_T}\right)^{N_T} = \left(\frac{4d^2}{N_T \sigma_b^2}\right)^{N_T} \quad (22)$$

is an upper bound for the coding gain for any matrix  $\mathbf{Q}$  with normalized columns. Now, the intersection of upper and

lower bounds leads to  $\min_{\mathbf{e}_i \neq 0} \prod_{n=1}^{N_T} |e_n(i)|^2 = \left(\frac{4d^2}{N_T \sigma_b^2}\right)^{N_T}$

and  $\frac{1}{\sigma_b}(\mathbf{b}_i - \mathbf{b}'_i) = \frac{2d}{\sigma_b} \mathbf{w}_{n_0}$  for some  $n_0$ . Hence  $|e_n(i)|^2 = \frac{4d^2}{N_T \sigma_b^2}$  and therefore  $\mathbf{E}\mathbf{E}^H = \alpha \mathbf{I}$  with  $\alpha = \frac{4d^2}{N_T}$ .

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