MULTIUSER-MIMO DOWNLINK TX-RX DESIGN BASED ON SVD CHANNEL DIAGONALIZATION AND MULTIUSER DIVERSITY

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ABSTRACT

We address the problem of transmitter and receiver design in Multi-User Multi-Input Multi-Output (MU-MIMO) communications over fading wireless channels. We assume perfect channel state information at the transmitter (CSIT). Attractive algorithmic solutions have been proposed in the past for the problem of minimizing total transmit power through joint beamforming and power control for MU-MISO systems with SINR constraints. Extensions of this approach to the MIMO case are not immediate, due to dependency on the receiver used for the multiple streams of a given user. By introducing a limited number of zero-forcing constraints, only between streams of the same user, the SINRs for all streams become decoupled, and the MU-MISO solutions can be straightforwardly adapted to the MU-MIMO problem. A multi-user diversity aspect can be introduced also, since the MU-MISO solution allows for straightforward identification of the most power hungry streams. When optimizing the number of streams in this way for maximum sum rate, the solution thus obtained is not far away from the Sato bound. Simulations are presented to also show the increased performance of the proposed approach over some alternative proposals.

1. INTRODUCTION

Wireless Multiple-Input Multiple-Output (MIMO) systems with multiple transmit and receive antennas offer significant advantage in terms of rate and robustness. The presence of multiple users (MU) leads to a Spatial Division Multiple Access (SDMA) aspect. We consider here the case of a streaming application, in which case multiuser diversity in time cannot be exploited. On the contrary, users need to be constantly connected and be provided with an acceptable SINR. When channel knowledge is available at the transmitter, the design of a downlink spatial prefiltering matrix or beamforming vectors is important to improve the quality of the system. The problem has been adressed in [1, 2] for the MU Multiple-Input Single-Output (MISO) case where each receiver has a single antenna. For the multi-user case with several antennas at each receiver, some proposed solutions based on Zero-Forcing ([3]) are proposed. A successive encoding and zero-forcing technique is proposed in ([4]). These techniques introduce zero-forcing to the streams of all or a subset of the other users, and hence are quite suboptimal. We consider flat fading MIMO channels and suppose perfect channel state information at transmitter (CSIT). Once a MIMO channel is known, we use its singular value decomposition (SVD) to decompose the channel into user-wise parallel subchannels. The solution proposed in [1, 2] for the MU MISO case involves a convex cost function, allowing simple globally convergent iterative solution algorithms. The problem addressed is either to minimize the total transmit power used for providing each user with a desired SINR (or hence throughput), or to maximize the minimum of the SINRs among users subject to a given total power constraint. The solution of these problems is intimately related to a network congestion measure that has been introduced by Hanly in [5], and which correspond to the maximal eigenvalue of a matrix of SINR crosscouplings between users. The application of this approach to the MU MIMO case is not straightforward since now the received signals are vectors and the SINRs become receiver-dependent. We propose to introduce a minimal amount of suboptimality by requiring zero-forcing between streams of a same user, which corresponds to a limited number of zero-forcing constraints. As a result, the reception of the streams becomes decoupled and the MU MIMO problem gets reduced to the MU MISO problem. Existing algorithmic solutions can hence be adapted and applied. The MU MISO problem is convex and hence these iterative techniques exhibit globally convergence. We also introduce a multiuser diversity aspect in this approach since it becomes relatively straightforward to identify which streams are the most difficult to serve.

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2. SYSTEM MODEL

We consider a multi-user MIMO system with N transmit antennas at the base station and K users, each k^{th} user (mobile station, MS) has N_k antennas. We consider the downlink, hence the Broadcast Channel (BC) problem. Let $\mathbf{x}_k$ represent the $L_k x1$ transmit data vector for user k , where L_k is the number of parallel data streams transmitted simultaneously for user k $(k = 1, \cdots, K)$. Before transmission, the data gets preprocessed through NxL_k matrices $\mathbf{T}_k$, $k = 1, \cdots, K$. We assume that the MIMO flat channel for the k^{th} user denoted by the $N_k xN$ matrix $\mathbf{H}_k$, is perfectly known at the transmitter. The noise is $\mathbf{n}_k$ and $\mathbf{y}_k$, the received signal at the k^{th} mobile station, is given by:

$$\mathbf{y}_k = \mathbf{H}_k \sum_{i=1}^{K} \mathbf{T}_i \mathbf{x}_i + \mathbf{n}_k = \mathbf{H}_k \mathbf{T}_k \mathbf{x}_k + \mathbf{H}_k \sum_{i \neq k} \mathbf{T}_i \mathbf{x}_i + \mathbf{n}_k \quad (1)$$

Notation: Bold letters are for vector or matrix, $(.)^T$ denotes transpose, $(.)^H$ is for complex conjugate transpose. Let $\mathbf{R}_k$ denote the linear receiver matrix at the k^{th} user, then we have:

$$\hat{\mathbf{x}}_k = \mathbf{R}_k\mathbf{H}_k\mathbf{T}_k\mathbf{x}_k + \mathbf{R}_k\mathbf{H}_k\sum_{i\neq k}^{K}\mathbf{T}_i\mathbf{x}_i + \mathbf{R}_k\mathbf{n}_k \tag{2}$$

3. TX-RX DESIGN USING SVD

The SVD of the channel matrix $\mathbf{H}_i$ consists of three matrices $(\mathbf{U}_i, \mathbf{\Sigma}_i, \mathbf{V}_i) = SVD(\mathbf{H}_i)$, where $\mathbf{U}_i, \mathbf{V}_i$ are unitary matrices, and $\mathbf{\Sigma}_i$ is a diagonal matrix of the singular values of $\mathbf{H}_i$ sorted in descending order, which verifies ([6]) $\mathbf{H}_i = \mathbf{U}_i\mathbf{\Sigma}_i\mathbf{V}_i^H$. With the SVD, the channel matrix can be decomposed into independent or orthogonal modes referred to as the eigenmodes of the channel ([7]). With this decomposition we need to recall that: $\mathbf{U}_i$ is $L_i \times L_i$ matrix, $\mathbf{\Sigma}_i$ is $L_i \times N$ matrix, and $\mathbf{V}_i$ a $N \times N$ matrix. We take initially $L_i = N_i$.

The SVD method is a popular technique for channel equalization in multi-antenna systems with perfect CSI ([8, 9]), and can be used for the multiuser case also. In this section we propose to use this decomposition to design transmit and receive matrices. The i^{th} received vector signal is given by (1). This signal is subject to inter-user interference, noise but also Interstream Interference (ISI) of the same user. Unlike in the single receive antenna case, since the received signal is a vector, a transformation through a receiver is required to obtain stream estimates and express streamwise SINR. In the case of perfect CSI we can introduce the (user-wise) channel SVD, absorb the unitary factors in transmitter and receiver and get decoupled SISO channels. This eliminates the ISI problem and reduces the MU MIMO problem to the MU MISO problem. However, instead of transmitting in the transformed domain, we shall continue to transmit in the original domain and force the beamformer for a particular stream to remain orthogonal to the right channel singular vectors of the other eigenmodes. So consider for user $i \in \{1, \dots, K\}$ the spatial prefiltering transmit matrix

$$\mathbf{T}_i = \mathbf{V}_i\,\mathbf{W}_i\,\mathbf{\Lambda}_i^{\frac{1}{2}}\,,\ \mathbf{V}_i = \begin{bmatrix}\mathbf{V}_i^1 & \mathbf{V}_i^0\end{bmatrix},\ \mathbf{W}_i = \begin{bmatrix}\mathbf{D}_{w_i} \\ \overline{\mathbf{W}}_i\end{bmatrix} \tag{3}$$

and the receiver

$$\mathbf{R}_i = \mathbf{U}_i^H \tag{4}$$

where $\mathbf{V}_i^1$ is a $N \times L_i$ matrix, containing the right channel singular vectors corresponding to the L_i intended streams, $\mathbf{V}_i^0$ is a $N \times (N - L_i)$ matrix containing the singular vectors that are orthogonal to the ones used for the streams of user i, $\mathbf{D}_{w_i}$ is a $L_i \times L_i$ diagonal matrix, $\overline{\mathbf{W}}_i$ is $(N - L_i) \times L_i$ matrix, and $\mathbf{\Lambda}_i = \text{diag}\{\lambda_{i,1}, \dots, \lambda_{i,L_i}\}$ is a diagonal power control matrix, the introduction of which allows the columns of $\mathbf{W}_i$ to be normalized. Column m of $\mathbf{W}_i$ contains the prefilter coefficients of stream m of user i, and due to its structure, the beamforming weights for this stream are orthogonal by construction to the singular vectors of the other streams of the same user. So at the receiver output for user i, we get $\hat{\mathbf{x}}_i =$

$$\mathbf{U}_i^H\mathbf{H}_i\begin{bmatrix}\mathbf{V}_i^1 & \mathbf{V}_i^0\end{bmatrix}\begin{bmatrix}\mathbf{D}_{w_i} \\ \overline{\mathbf{W}}_i\end{bmatrix}\mathbf{\Lambda}_i^{\frac{1}{2}}\mathbf{x}_i + \mathbf{U}_i^H\mathbf{H}_i\sum_{k\neq i}^{K}\mathbf{T}_k\mathbf{x}_k + \mathbf{U}_i^H\mathbf{n}_i. \tag{5}$$

By replacing $\mathbf{H}_i$ by it's SVD decomposition we get $\hat{\mathbf{x}}_i =$

$$\overline{\mathbf{\Sigma}}_i\mathbf{D}_{w_i}\mathbf{\Lambda}_i^{\frac{1}{2}}\mathbf{x}_i + \sum_{k\neq i}^{K}\overline{\mathbf{\Sigma}}_i\mathbf{V}_i^{1H}\begin{bmatrix}\mathbf{V}_k^1 & \mathbf{V}_k^0\end{bmatrix}\begin{bmatrix}\mathbf{D}_{w_k} \\ \overline{\mathbf{W}}_k\end{bmatrix}\mathbf{\Lambda}_k^{\frac{1}{2}}\mathbf{x}_k + \mathbf{U}_i^H\mathbf{n}_i \tag{6}$$

where $\overline{\mathbf{\Sigma}}_i = \text{diag}\{\sigma_{i,1}, \sigma_{i,2}, \dots, \sigma_{i,L_i}\}$ contains the square non-singular part of $\mathbf{\Sigma}_i$. We recall that $\hat{\mathbf{x}}_i = \begin{bmatrix}\hat{x}_{i,1} \\ \vdots \\ \hat{x}_{i,L_i}\end{bmatrix}$, so we get for stream m of user i

$$\hat{x}_{i,m} = \sigma_{i,m}\mathbf{e}_1^H\mathbf{w}_{i,m}\sqrt{\lambda_{i,m}}x_{i,m} + \sum_{k\neq i}\sum_{n=1}^{L_k}\sigma_{i,m}\mathbf{V}_{i,m}^H\begin{bmatrix}\mathbf{V}_{k,n}^1 & \mathbf{V}_k^0\end{bmatrix}\mathbf{w}_{k,n}\sqrt{\lambda_{k,n}}x_{k,n} + \mathbf{U}_{i,m}^H\mathbf{n}_i \tag{7}$$

where $\mathbf{e}_1$ is the first column of the identity matrix, $\mathbf{w}_{k,l}$ is a vector of size $N - L_k + 1$ matrix, its first element is $(\mathbf{D}_{w_k})_{l,l}$ and the last $N - L_k$ elements constitute column l of $\overline{\mathbf{W}}_k$. With this diagonalization of the channel we cancel all ISI at each receiver, so the interference is only due to inter-user interference. This means that the solution for the multi-user multi-input single-output (MU-MISO) problem can now straightforwadly be applied to the constrained MU-MIMO problem considered. In the next section we compute the SINR for each substream of each user.

4. SUBSTREAM SINR OPTIMIZATION

The receiver output for subtream m of the i^{th} user is given by (7), and can be rewritten as

$$\begin{aligned}\hat{x}_{i,m} =& \mathbf{g}_{i,m,i,m}\mathbf{w}_{i,m}\sqrt{\lambda_{i,m}}\,x_{i,m} + \\ & \sum_{k\neq i}\sum_{n=1}^{L_k}\mathbf{g}_{i,m,k,n}\mathbf{w}_{k,n}\sqrt{\lambda_{k,n}}\,x_{k,n} + \mathbf{U}_{i,m}^H\mathbf{n}_i\end{aligned} \tag{8}$$

where

$$\begin{aligned}\mathbf{g}_{i,m,i,m} =& \sigma_{i,m}\mathbf{e}_1^H \\ \mathbf{g}_{i,m,k,n} =& \sigma_{i,m}\mathbf{V}_{i,m}^H\begin{bmatrix}\mathbf{V}_{k,n}^1 & \mathbf{V}_k^0\end{bmatrix}\end{aligned} \tag{9}$$

can be interpreted as equivalent channel impulse responses and $\mathbf{w}_{i,m}$ as the beamforming vector for the substream m of the i^{th} user. With the considered scenario, $\gamma_{i,m}$, the SINR of the substream m of the i^{th} user, is given by:

$$\begin{aligned}\gamma_{i,m} =& \frac{\lambda_{i,m}\,\mathbf{g}_{i,m,i,m}\mathbf{w}_{i,m}\mathbf{w}_{i,m}^H\mathbf{g}_{i,m,i,m}^H}{\sum_{k\neq i}\sum_{n=1}^{L_k}\lambda_{k,n}\,\mathbf{g}_{i,m,k,n}\mathbf{w}_{k,n}\mathbf{w}_{k,n}^H\mathbf{g}_{i,m,k,n}^H + \sigma_{n_i}^2} \\ =& \frac{\lambda_{i,m}\,\mathbf{w}_{i,m}^H\mathbf{R}_{i,m,i,m}\mathbf{w}_{i,m}}{\sum_{k\neq i}\sum_{n=1}^{L_k}\lambda_{k,n}\,\mathbf{w}_{k,n}^H\mathbf{R}_{i,m,k,n}\mathbf{w}_{k,n} + \sigma_{n_i}^2}\end{aligned} \tag{10}$$

where $E|x_{i,m}|^2 = 1$, $E|\mathbf{U}_{i,m}^H\mathbf{n}_i|^2 = \sigma_{n_i}^2$, and $\mathbf{R}_{i,m,k,n} = \mathbf{g}_{i,m,k,n}^H\mathbf{g}_{i,m,k,n}$, $1 \le i,k \le K$, $1 \le m,n \le L_i$.

The maxmin SINR optimization problem is given by

$$\begin{aligned}&\max_{\boldsymbol{W},\boldsymbol{\lambda}}\min_{(i,m)}\gamma_{i,m} \\ &s.t. \sum_{k=1}^{K}\sum_{n=1}^{L_k}|\lambda_{k,n}|^2 \le P_{max} \\ &\|\mathbf{w}_{i,m}\| = 1,\ \ 1 \le i \le K,\ \ 1 \le m \le L_i\end{aligned} \tag{11}$$

where P_{max} is the total available transmit power, $\boldsymbol{W} = \{\mathbf{w}_{1,1}, \mathbf{w}_{1,2}, \dots, \mathbf{w}_{1,L_1}, \mathbf{w}_{2,1}, \dots, \mathbf{w}_{K,L_K}\}$, $\boldsymbol{\lambda} = [\lambda_{1,1}, \lambda_{1,2}, \dots, \lambda_{1,L_1}, \lambda_{2,1}, \dots, \lambda_{K,L_K}]^T$.

Consider that for each stream (i,m) individual SINR targets $\alpha_{i,m}$ should be achieved. The problem is feasible if it is possible to have

$$\min_{\substack{1 \le i \le K \\ 1 \le m \le L_i}} \frac{\gamma_{i,m}}{\alpha_{i,m}} \ge 1 \qquad s.t. \sum_{k=1}^{K} \sum_{n=1}^{L_k} \lambda_{k,n} \le P_{max} \tag{12}$$

The feasibility of this problem is studied by Boche & Schubert in ([2]). The SINR $\gamma_{i,m}$ depend on the power P_{max} and its distribution $\boldsymbol{\lambda}$, and on the choice of transmit beamformes $\mathbf{W}$. The above problems are solved in the absence of noise case in ([1]) and in the general case in ([2]). The solution can be found through an uplink downlink duality. For a given $\mathbf{W}$ we consider the following power optimization problem:

$$\max_{\boldsymbol{\lambda}} \min_{(i,m)} \frac{\gamma_{i,m}(\mathbf{W})}{\alpha_{i,m}} \qquad s.t. \sum_{k=1}^{K} \sum_{n=1}^{L_k} \lambda_{k,n} \le P_{max} \tag{13}$$

The optimum is characterized by $c = \frac{\gamma_{i,m}(\mathbf{W})}{\alpha_{i,m}},\ \forall i,m$. Let $\boldsymbol{\lambda}$ be the power distribution that allows to achieved the optimum, we can optimize w.r.t. $\mathbf{W}$ to find the beamforming vectors. We have the following matrix representation of the problem ([1, 2]).

$$\begin{aligned} \boldsymbol{\lambda}\frac{1}{c} &= \mathbf{D}\boldsymbol{\Psi}\boldsymbol{\lambda} + \mathbf{D}\boldsymbol{\nu} \\ \frac{1}{c} &= \frac{1}{P_{max}}\mathbf{1}^T\mathbf{D}\boldsymbol{\Psi}\boldsymbol{\lambda} + \frac{1}{P_{max}}\mathbf{1}^T\mathbf{D}\boldsymbol{\nu} \end{aligned} \tag{14}$$

where $\boldsymbol{\nu} = [\nu_{1,1}, \ldots, \nu_{1,L_1}, \nu_{2,1}, \ldots, \nu_{K,L_K}]^T$, $\nu_{i,m} = \sigma_{n_i}^2$, $\mathbf{1}^T = [1, \ldots, 1]$,

$$\mathbf{D} = diag\left\{ \frac{\alpha_{1,1}}{\mathbf{w}_{1,1}^H \mathbf{R}_{1,1,1,1}\mathbf{w}_{1,1}}, \ldots, \frac{\alpha_{1,L_1}}{\mathbf{w}_{1,L_1}^H \mathbf{R}_{1,L_1,1,L_1}\mathbf{w}_{1,L_1}}, \ldots, \frac{\alpha_{2,1}}{\mathbf{w}_{2,1}^H \mathbf{R}_{2,1,2,1}\mathbf{w}_{2,1}}, \ldots, \frac{\alpha_{K,L_K}}{\mathbf{w}_{K,L_K}^H \mathbf{R}_{K,L_K,K,L_K}\mathbf{w}_{K,L_K}} \right\} \tag{15}$$

$\boldsymbol{\Psi} = [\boldsymbol{\Psi}_{i,k}]_{1 \le i,k \le K}$ (block (i,k) of a block matrix $\boldsymbol{\Psi}$) and

$$(\boldsymbol{\Psi}_{i,k})_{m,n} = \begin{cases} \mathbf{w}_{k,n}^H \mathbf{R}_{i,m,k,n}\mathbf{w}_{k,n} & \text{for } m \ne n \\ 0 & \text{if } m = n \end{cases} \tag{16}$$

Let us denote

$$\boldsymbol{\Phi} = \begin{bmatrix} \mathbf{D}\boldsymbol{\Psi} & \mathbf{D}\boldsymbol{\nu} \\ \frac{1}{P_{max}}\mathbf{1}^T\mathbf{D}\boldsymbol{\Psi} & \frac{1}{P_{max}}\mathbf{1}^T\mathbf{D}\boldsymbol{\nu} \end{bmatrix} \tag{17}$$

then

$$\boldsymbol{\Phi} \begin{bmatrix} \boldsymbol{\lambda} \\ 1 \end{bmatrix} = \frac{1}{c} \begin{bmatrix} \boldsymbol{\lambda} \\ 1 \end{bmatrix} \tag{18}$$

where $\frac{1}{c}$ is the maximum eigenvalue of Φ with corresponding eigenvector $[\boldsymbol{\lambda}^T\ 1]^T$. The problem is feasible is the maximum eigenvalue is less than one. This maximum eigenvalue has been given an interpretation as a congestion measure in [5].

5. SVD-SINR BALANCING ALGORITHM

The algorithm for the proposed scheme is given in Table 1. The design of this algorithm is based on non-negative matrices and properties of their maximum eigenvalue and corresponding eigenvectors. Similar algorithms have been proposed in ([1, 2]) for the MISO case. The global convergence of these algorithms has been shown in [2].

1:	Initialize $\boldsymbol{\lambda} = \frac{1}{L\,P_{max}}\mathbf{1}$, $L = \sum_{k=1}^{K} L_k$, $\boldsymbol{\nu} = [\nu_{1,1}, \ldots, \nu_{K,L_K}]^T$
2:	Compute the SVDs $[\mathbf{U}_i, \boldsymbol{\Sigma}_i, \mathbf{V}_i] = SVD(\mathbf{H}_i)$
3:	Compute $\mathbf{R}_{i,m,k,n} = \mathbf{g}_{i,m,k,n}^H \mathbf{g}_{i,m,k,n}$, $1 \le i,k \le K$, $1 \le m,n \le L_i$
4:	Repeat
5:	$\overline{\mathbf{R}}_{i,m} = \sum_{k \ne i}^{K} \sum_{n=1}^{L_k} \lambda_{k,n}\mathbf{R}_{k,n,i,m} + \mathbf{I}$
6:	$\mathbf{w}_{i,m}(n) = V_{max}\left(\mathbf{R}_{i,m,i,m}, \overline{\mathbf{R}}_{i,m}\right)$, $1 \le i \le K$, $1 \le m \le L_i$
7:	Solve $\boldsymbol{\Phi}\,\left[\boldsymbol{\lambda}^T, 1\right]^T = \lambda_{max}\,\left[\boldsymbol{\lambda}^T, 1\right]^T$
8:	Untill convergence
9:	Compute $\mathbf{T}_i$, $1 \le i \le K$ using (3)

Table 1. MU-MIMO SVD-SINR balancing algorithm

In line 6, $V_{max}(A, B)$ is the normalized generalized eigenvector corresponding to the maximum generalized eigenvalue of the matrices A and B.

6. TX DESIGN BASED ON SVD AND MULTIUSER DIVERITY

In this section we will introduce a scheduling aspect that will allow to improve the performance of the proposed approach. For this we assume that the base station has multiple frequency bands available and can select within a certain frequency band to serve only the users that go well together in the sense of requiring minimal power to satisfy their SINR requirements. Although the global optimization problem requires to consider all frequency bands jointly, we shall here just concentrate on optimizing performance within one frequency band. The solution of the problem (13) will provide each substream with the same ratio of achieved SINR over desired SINR for the substream, but with a different required power contribution $\lambda_{i,m}$ from the base station. Clearly, the stream requiring the largest power $\lambda_{i,m}$ to achieve the same SINR as the other streams has the channel that is the least compatible with the others (clearly, in a more sophisticated version, the product of power $\lambda_{i,m}$ times (average) attenuation of the user i should be compared instead of just the instantaneous required power for the stream). Although for the purpose of the streaming application considered, this is of limited relevance, we shall consider the effect of eliminating incompatible streams on the sum capacity of the system. By eliminating the least compatible stream and resolving the maxmin SINR problem again using the algorithm in Table 1, it is possible that the sum capacity improves in spite of feeding one less stream because of the eliminated interference from the most interfering stream. With the system model described above, the i^{th} user with N_i antennas is looking for L_i data streams with a required SINR target. So for the overall system we have $L = \sum_{i=1}^{K} L_i$ substreams. We recall that we take initially $L_i = N_i$. After the successive scheduling optimization steps, the system will send less than L streams with the best QoS. Thus the proposed scheme will enhance the throughput of the system. The algorithm in Table 2 gives the description of the procedure of this technique.

1:	$C^0 = 0,\ L = \sum_{i=1}^{K} L_i$
2:	Run the algorithm in Table 1
3:	Compute $C = \sum_{i,l} \log\left(1 + \gamma_{i,l}\right)$
4:	If $C^0 \geq C$ keep solution at $L+1$ and stop
5:	Find $(k, l) = \arg\max_{(i,l)} \lambda_{i,l}^1$
6:	$\mathbf{H}_k \leftarrow \left[\mathbf{h}_{k,1}^T, \ldots, \mathbf{h}_{k,l-1}^T, \mathbf{h}_{k,l+1}^T, \ldots, \mathbf{h}_{k,L_k}^T\right]^T$,
	$\mathbf{H}_i \leftarrow \mathbf{H}_i$ for $i \neq k$
	$L_k = L_k - 1,\ L = L - 1,\ C^0 = C$, go to step 2

Table 2. MU-MIMO SVD-SINR balancing and MU Diversity algorithm

7. SIMULATION AND RESULTS

The performance of the proposed scheme is investigated through computer simulations. The noise power is assumed the same for all users ($\sigma_n^2 = 1$). The flat channel matrices are chosen as $\mathbf{H}_i = \mathbf{H}_{0i}\mathbf{R}_{T,i}^{H/2}$. The elements of $\mathbf{H}_{0i}$ are i.i.d. complex Gaussian with $E\left(\mathbf{H}_{0i}^H\mathbf{H}_{0i}\right) = \mathbf{I}_N$ and $\mathbf{R}_{T,i}$ is chosen as follows: $\mathbf{R}_{T,i} = \text{diag}\left(\mathbf{V}\left(\theta_i\right)\right)\mathbf{\Pi}\text{diag}\left(\mathbf{V}\left(\theta_i\right)\right)^H$, where $\mathbf{\Pi}$ is Hermitian Toeplitz with first row $[1\,\rho\,\rho^2\cdots\rho^{N-1}]$, $\rho = \exp\left(-\sigma^2\right)$, $\varphi_i = \frac{2\pi\sin\theta_i d}{\lambda}$, $\mathbf{V}\left(\theta_i\right) = [1\ \ \exp j\varphi_i\ \ \exp 2j\varphi_i\ \ \ldots\ \ \exp\left(N-1\right)j\varphi_i]^T$. θ_i is the nominal angle of the i^{th} user, d is the distance between antennas. For the simulations we use $\rho = 0.5$ and $d = \lambda/2$. The figures show the performance of the proposed multi-user MIMO downlink Tx-Rx scheme (SVD-SINR balancing). Since we have parallel subchannels the average sum capacity of the system is $C = E\left\{\sum_{i=1}^{K}\sum_{m=1}^{L_i}\log\left(1+\gamma_{i,m}\right)\right\}$. We compare the result to some proposed schemes and to the Sato upper bound which can be achieved by using an iterative water-filling algorithm proposed in [10]. The proposed algorithm gets much closer to the Sato upper bound than the other algorithms considered, which are CZF-SESAM (cooperative zero-forcing with successive encoding and allocation method [4]), Joint MU-MIMO Decomposition design [11] and block diagonalization zero-forcing (BD-ZF) [3]. These techniques perform signicantly worse than the proposed technique.

In the figures 1 and 1 1000 channel realizations are averaged. For CZF-SESAM, the sum capacity in the MISO case is given by $C = E\left\{\sum_{i=1}^{K}\log\left(1+\gamma_i\right)\right\}$. For BD-ZF and Joint MU-MIMO Decomposition case, the expression used for the sum capacity is $\sum_{i=1}^{K} E\left(log(\frac{det(\mathbf{H}_i\mathbf{T}_i\mathbf{T}_i^H\mathbf{H}_i^H + \mathbf{H}_i\sum_{k\neq i}\mathbf{T}_k\mathbf{T}_k^H\mathbf{H}_i^H + \boldsymbol{\sigma}_{n_i}^2\mathbf{I})}{det(\mathbf{H}_i\sum_{k\neq i}\mathbf{T}_k\mathbf{T}_k^H\mathbf{H}_i^H + \boldsymbol{\sigma}_{n_i}^2\mathbf{I})})\right)$, the preprocess transmit and receive matrices are computed as described in [3, 11]. The Figure 3 presents the sum capacity when the number of total transmitted streams is varying at $SNR = 10dB$. This figure is plotted for 100 channel realizations. The system configuration used is $K = 5$ users, $N = 5$ antennas at the base station, $N_i = 3$ antennas for each user, and the initial number of streams for each user is $L_i = 3$. We can observe that the proposed scheme gets closer to the Sato bound when we eliminate the worst channels. Figure 4 gives the capacity for each step of the SVD-SINR balancing MU-Diversity algorithm, and the Sato bound. The system configuration is the same as for Figure 3. Thus the proposed algorithm outperforms the other approaches, which use zero-forcing of the interference between users, whereas the proposed approach only zero-forces between streams of a same user.

8. CONCLUSIONS

A novel method for multi-user MIMO channel decoupling is presented. The approach is based on full channel state information and the singular value decomposition (SVD). The signal to interference plus noise ratio of the resulting substreams is optimized w.r.t. quality of service requirements and the total available power at the transmitter contraints. Finally we introduce a multi-user (multi-stream) diversity aspect to improve the performance of the algorithm.

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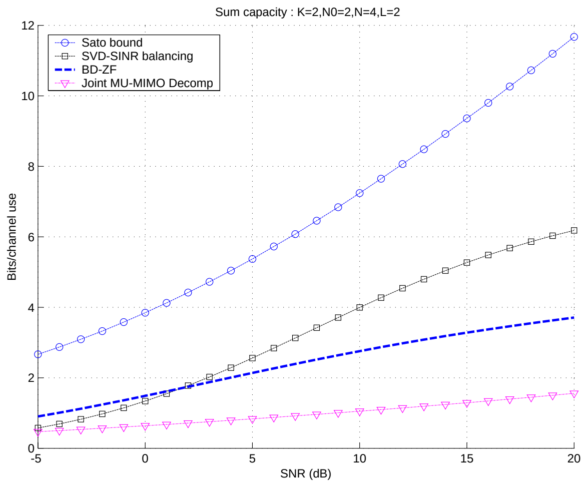

Fig. 1. MU-MIMO sum capacity K=2, N=4, $L_k = 2$, $k = 1 \ldots 5$.

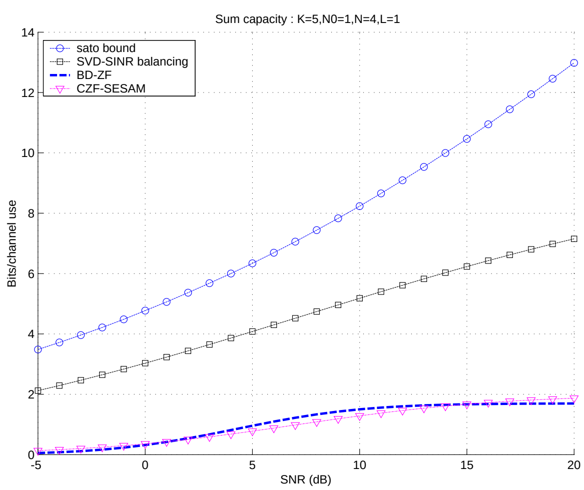

Fig. 2. MU-MISO sum capacity K=5, N=4, $L_k = 1$, $k = 1 \ldots 5$.

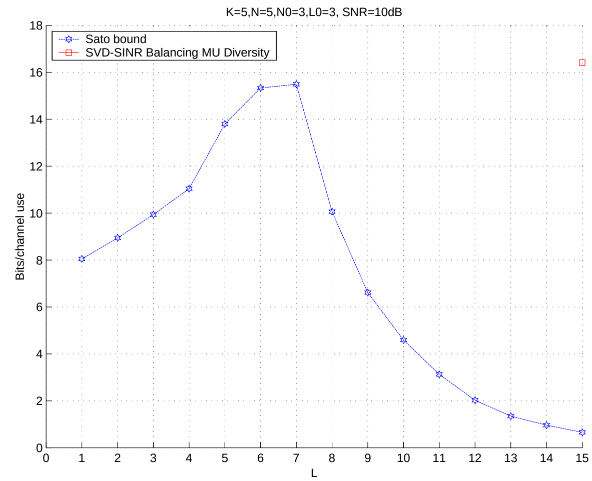

Fig. 3. MU-MIMO sum capacity K=5, N=5, $N_k = 3$, and initially $L_k = 3$, $k = 1 \ldots 5$, $L = \sum_{k=1}^{K} L_k^0$, L_k^0 is the number of streams sent to user k, SNR=10dB.

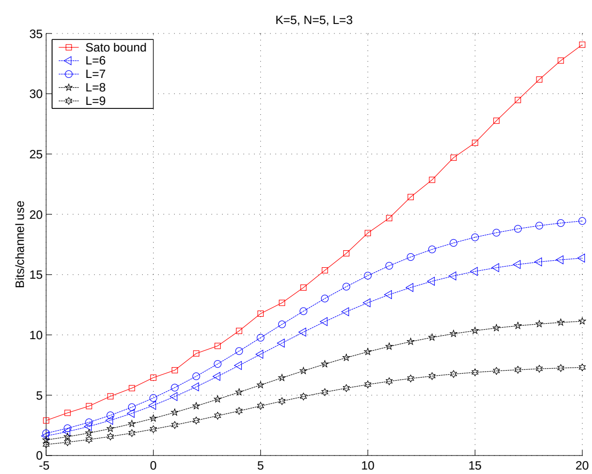

Fig. 4. MU-MIMO sum capacity K=5, N=5, $N_k = 3$, and initially $L_k = 3$, $k = 1 \ldots 5$.