A Threshold-Based Channel State Feedback Algorithm for Modern Cellular Systems

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***Abstract*— In this paper we propose a channel state feedback algorithm that uses multiple feedback thresholds to reduce the number of users transmitting feedback to a minimum. The users are polled with lower and lower threshold values and only the users that are above a threshold value transmit feedback to the base station. We show how this feedback algorithm can be used for any scheduling algorithm and show how closed-form expressions for the optimal threshold values can be obtained for two well-known scheduling algorithms. Finally, we propose a two-step optimization procedure for optimizing the feedback algorithm for real-life cellular standards.**

***Index Terms*— Land mobile radio cellular systems, scheduling, state feedback.**

I. Introduction

In modern wireless networks, *adaptive coding and modulation* are implemented so that the mobile users and base stations can adapt their transmission rate to the quality of the wireless channel [1]. This adaptation does not only increase the spectral efficiency of the wireless links between the base station and the mobile users, but can also be exploited further by the base station through *opportunistic scheduling* [2]. Opportunistic scheduling increases the system spectral efficiency by giving priority to mobile users when they have good channel quality. Opportunistic scheduling algorithms in cellular networks are often executed by the base station. This means that the base station needs to know the instantaneous channel quality of the users in the system and schedule the users based on this knowledge. In modern wireless standards like Mobile WiMAX, HSPA, and 1xEVDO, opportunistic scheduling algorithms can be implemented to schedule users for every time-slot in the down-link [3]–[5]. Therefore, when the channels are rapidly varying, most of the opportunistic scheduling algorithms are based on having available channel quality estimates for *all* the mobile users in every time-slot. If all the mobile users are going to feed back their channel quality estimates to the base station for each time-slot, a significant share of the battery energy will be used on transmission of overhead information instead of useful data traffic. In addition, for many wireless systems, collecting carrier-to-noise ratio (CNR) estimates from all the users will lead to a significant delay before the transmission of useful data can start.

Three main directions have previously been pursued to reduce the degradation due to feedback, namely, (i) feedback quantization, (ii) feedback compression, and (iii) feedback load reduction. Publications investigating the first approach have shown that heavy quantization of the channel state information (CSI) being fed back, will not lead to a significant reduction of the system gain [6], [7]. Correspondingly, the quantization of the beamforming vector being fed back, has been investigated for multi-antenna systems [8]. The second approach exploits the channel correlation in time and frequency to design compression algorithms that reduce the feedback overhead significantly [9], [10]. Most algorithms trying to reduce the feedback load, i.e., the number of users feeding back channel state information, are based on CNR thresholds [11], [12]. One threshold-based algorithm that uses a single CNR threshold value was proposed by Gesbert and Alouini [12]. The mobile users that have a CNR above this threshold value transmit feedback to the scheduler. The algorithm in [12] does not always obtain feedback from the user with the highest CNR since it will always be a possibility that all users are below the threshold value and a random user has to be chosen.

The goal of this paper is to conduct a theoretical investigation of a novel feedback algorithm that is a generalization of the algorithm in [12]. This generalization is based on two main ideas, namely, (i) adapting the feedback threshold value to the *scheduling metric* of the scheduling algorithm, i.e. the metric that is used to decide which user is going to be scheduled in a time-slot, and (ii) using multiple feedback thresholds to collect feedback from the *preferred user*, i.e. the user that the scheduling algorithm prefers to schedule. For any scheduling algorithm, our proposed algorithm leads to a significant reduction of the number of users transmitting feedback. This will reduce the power consumption of the mobile users, and also reduce the time to collect feedback for many cellular systems.

Previous publications related to feedback load reduction are all based on the assumption that the base station always tries to collect feedback from the user with the highest CNR. This is because it is assumed that the scheduling algorithm used by the system is Max CNR Scheduling (MCS), where the user with the highest CNR is scheduled in every time-slot. Since the MCS algorithm can be unfair in many cases, other scheduling algorithms are often preferred, and we thus propose to adapt the feedback threshold values to account for any scheduling metric.

By employing multiple feedback thresholds, the base station

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can conduct the feedback collection process by polling the users sequentially from the highest threshold value down to the lowest threshold value until feedback from one or more users is received. Our numerical results show that by employing just a few number of thresholds, our algorithm will lead to a high probability for receiving feedback from only one user.

Contributions. We propose the novel channel state feedback algorithm as described above. In Section III, we assume that the time to collect feedback is negligible compared to the time used to transmit user data. We find an analytical expression for the feedback load when the number of threshold values L is predetermined and when a general scheduling metric is assumed. Based on this general expression, we obtain specific expressions for the feedback load for the MCS and the NCS algorithms, and use these expressions to find the feedback threshold values minimizing the feedback load for each of these two algorithms. In Section IV, we argue that the time used to collect feedback is in fact non-negligible in many real-life systems. For such systems, we propose a two-step method for optimizing both the threshold values and the number of thresholds L.

II. System Model

We consider a time-division multiplexed (TDM) based wireless system with a single base station scheduling N users. The transmission rate and the scheduling decision is based on CNR estimates being fed back from one or more users for each time-slot. If we assume that we have reciprocity between up-link and down-link, which can be assumed in e.g. Mobile WiMAX, these CNR estimates can be used to schedule users in both the down-link and up-link. If this is not the case, the system model is only valid for down-link scheduling. We assume that the channels of all users are independent, flat-fading channels with average CNR $\overline{\gamma}_i$, where the index i denotes the ith user. In order to have a roughly constant CNR level, $\gamma_i(t)$, within each time-slot, it is assumed that the duration of a time-slot is shorter than the coherence time of the channels.

The feedback threshold values vary with the scheduling metric $x_i(t)$ used by the scheduling algorithm. Examples of scheduling metrics of different scheduling algorithms will be given in the beginning of Section III. The base station searches for users in the whole range of their scheduling metric and we denote the feedback thresholds for user i by $x_{\text{th},i,L} > x_{\text{th},i,L-1} > \cdots > x_{\text{th},i,0}$. Assuming that the x_i-ranges are from zero to infinity, we set $x_{\text{th},i,L} = \infty$ and $x_{\text{th},i,0} = 0$, and let the base station start polling each of the users with $x_{\text{th},i,L-1}$ for user i. Since $x_{\text{th},i,L}$ is never used to search for the users, we say that we have L threshold values. Note that since the feedback thresholds are covering the whole x_i-range of the users, it is ensured that feedback will be received from at least one user. In practice, it is often a need for calculating the threshold values on-line. Since a threshold value can be expressed as a function of the CNR of a user, the optimization of the threshold values does not have to be performed by the users since the base station can always poll each of the users with CNR values that correspond to the threshold values $x_{\text{th},i,l}$, $l = 0, ..., L$.

The goal of this paper is to conduct a general theoretical analysis of the proposed feedback algorithm. However, to conduct an analysis of the true performance of the algorithm in different real-life networks, the system model needs further specification. For wireless systems based on WLAN standards, HSPA, Mobile WiMAX or 1xEVDO, a performance analysis of our algorithm has to be conducted by considering system specific parameters and protocols (See e.g. [13]). For HSPA, which uses code-division multiplexing (CDM) in both the up-link and the down-link, more users can transmit or receive data simultaneously. This means that CNR estimates can be fed back by several users simultaneously. Also for such systems our proposed feedback algorithm can be implemented to reduce the number of users transmitting feedback. The main gain for the system will be a lower power consumption for the users; in addition we also get less interference on the feedback channel, and hence also a lower bit-error-rate for the CNR estimates being fed back. For CDM-based systems, opportunistic scheduling algorithms often have to pick a subset of the users to transmit or receive within a time-slot. Our proposed feedback algorithm can also be used to obtain feedback from such a subset of preferred users.

III. Optimizing the Algorithm for a Fixed Number of Feedback Thresholds

In this section we assume that the time to collect feedback is negligible compared to the time used to transmit user data. This means that we focus on how the number of users transmitting feedback can be minimized for a given value of L. By denoting the scheduling metric as $x_i(t)$ for user i in time-slot t, we have $x_i(t) = \gamma_i(t)$ for the MCS algorithm, where $\gamma_i(t)$ denotes the instantaneous CNR of user i in time-slot t. For the Normalized CNR Scheduling algorithm (NCS) the corresponding scheduling metric is $x_i(t) = \gamma_i(t)/\overline{\gamma}_i$, where $\overline{\gamma}_i$ is the average CNR of user i [14]. For this scheduling algorithm the threshold values have to be optimized for searching for the preferred users in the $\gamma_i(t)/\overline{\gamma}_i$-range of the users. Likewise, the feedback algorithm can be designed for the Proportional Fair Scheduling (PFS) algorithm by optimizing the feedback thresholds to search for the preferred user in the $r_i(t)/T_i$-range of the users, where $r_i(t)$ denotes the instantaneous rate of user i and T_i denotes a weighted sum of the rate allocated to this user [15].

A. Feedback Thresholds for a General Scheduling Metric

To evaluate the performance of our feedback algorithm for a general scheduling metric $x_i(t)$, we have to find an expression for the *normalized feedback load* (NFL), which expresses the average share of users that give feedback for each time-slot. It can be shown that the NFL can be obtained as:

$$\bar{\mathrm{F}}_{\text{gen}} = \frac{1}{N}\sum_{l=0}^{L-1}\sum_{\Psi}|\Psi|\prod_{i\in\Psi}(P_{x_i}(x_{\text{th},i,l+1})-P_{x_i}(x_{\text{th},i,l})) \times \prod_{j\notin\Psi}P_{x_j}(x_{\text{th},j,l}), \quad (1)$$

where $\Psi \neq \emptyset$ denotes any subset of users, including the set of all users, while $P_{x_i}(\cdot)$ denotes the cumulative distribution

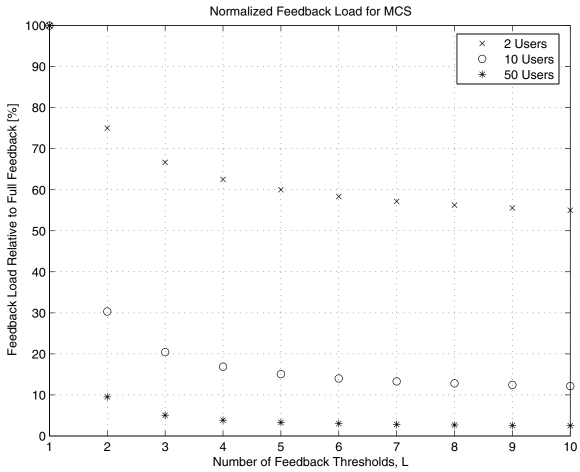

Fig. 1. Minimum normalized feedback load for the MCS algorithm as a function of L for different number of users with Rayleigh fading channels with $\overline{\gamma} = 15$ dB.

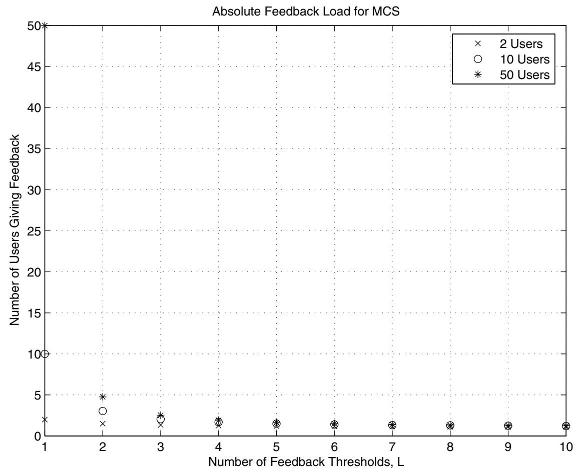

Fig. 2. Minimum absolute feedback load for the MCS algorithm as a function of L for different number of users with Rayleigh channels with $\overline{\gamma} = 15$ dB.

function (CDF) of the scheduling metric of user i. For many scheduling algorithms it can be hard to find closed-form expressions for the distributions $P_{x_i}(\cdot)$ for $i = 1, ..., N$. For such scheduling algorithms these distributions have to be estimated on-line. Such estimations can be performed at the base station, based on the CNR estimates being fed back, by using for example *order statistic filter banks* [16]. To obtain the optimal feedback thresholds, the feedback thresholds that minimize (1) have to be obtained. For the PFS algorithm and many other algorithms, the distributions $P_{x_i}(\cdot)$ are not known and a numerical optimization procedure has to be employed. However, for the MCS and the NCS algorithms, the distributions $P_{x_i}(\cdot)$ are known and we can obtain closed-form expressions for the optimal feedback threshold values.

B. Feedback Thresholds for the MCS Algorithm

If we assume that the users have the same distribution of their CNRs with an average of $\overline{\gamma}$, the NFL for the MCS algorithm can be expressed as:

$$\bar{\text{F}}_{\text{MCS}} = \frac{1}{N}\sum_{l=0}^{L-1}\sum_{n=1}^{N} n\binom{N}{n}\left(P_\gamma(\gamma_{\text{th},l+1}) - P_\gamma(\gamma_{\text{th},l})\right)^n \times P_\gamma^{N-n}(\gamma_{\text{th},l}), \quad (2)$$

where $P_\gamma(\gamma)$ is the CDF of the CNR for a single user and $\gamma_{\text{th},l}$ denotes the lth threshold value. This expression was found by calculating the expected number of users that give feedback for each threshold value, and summing all these feedback loads. The expression is normalized by dividing by the number of users. Using the binomial expansion formula [17], (2) can be written as:

$$\bar{\text{F}}_{\text{MCS}} = \sum_{l=0}^{L-1}\left(P_\gamma(\gamma_{\text{th},l+1}) - P_\gamma(\gamma_{\text{th},l})\right)\cdot P_\gamma^{N-1}(\gamma_{\text{th},l+1}). \quad (3)$$

A plot of the NFL for the optimal threshold values is shown in Fig. 1 as a function of L for different number of users with Rayleigh channels with $\overline{\gamma} = 15$ dB. We see that the NFL converges to $1/N$ as L grows large. This is logical since the more thresholds there are in the system, the more likely is it that only one user will have a CNR value between two adjacent thresholds. To investigate how the feedback load scales with the number of users, we have also plotted the *absolute feedback load* (AFL) in Fig. 2. The AFL expresses the average number of users transmitting feedback and we can observe that the decrease in AFL as a function of the number of thresholds is higher for a high number of users.

To optimize the thresholds, we take the gradient of (3) with respect to the threshold values and set it equal to zero, which gives the following expression for the optimal threshold values:

$$\gamma_{\text{th},l}^{*} = P_\gamma^{-1}\left(S_l \cdot P_\gamma(\gamma_{\text{th},l+1}^{*})\right), \quad l = 1, 2, 3, \cdots, L-1, \quad (4)$$

where $P_\gamma^{-1}(\cdot)$ is the inverse CDF of the CNR for a single user, and the constants S_l are given by:

$$S_l = \begin{cases} N^{\frac{1}{1-N}}, & l = 1 \\ \left[N - (N-1)S_{l-1}\right]^{\frac{1}{1-N}}, & l = 2, 3, \cdots, L-1, \end{cases} \quad (5)$$

with $N \geq 2$. The set of equations in (4) has a recursive nature. One way to calculate these threshold values is to start by calculating $\gamma_{\text{th},L-1}$. This value can easily be found since $\gamma_{\text{th},L}$ is defined to be infinity. Knowing $\gamma_{\text{th},L-1}$, (4) can be used to calculate all threshold values down to $\gamma_{\text{th},1}$. It is also possible to express the threshold values as the sum of the average CNR and a constant (in dB). By writing (4) in the form:

$$P_\gamma(\gamma_{\text{th},l}^{*}) = S_l \cdot P_\gamma(\gamma_{\text{th},l+1}^{*}), \quad l = 1, 2, 3, \cdots, L-1, \quad (6)$$

and exploiting the fact that $P_\gamma(\gamma_{\text{th},L}) = 1$, we can write (4) as:

$$\gamma_{\text{th},l}^{*} = P_\gamma^{-1}\left(\prod_{i=l}^{L-1} S_i\right), \quad l = 1, 2, 3, \cdots, L-1. \quad (7)$$

For the Rayleigh, Nakagami, and Rice distributions, the inverse CDF $P_\gamma^{-1}(\cdot)$ equals $\overline{\gamma}$ multiplied by a constant which

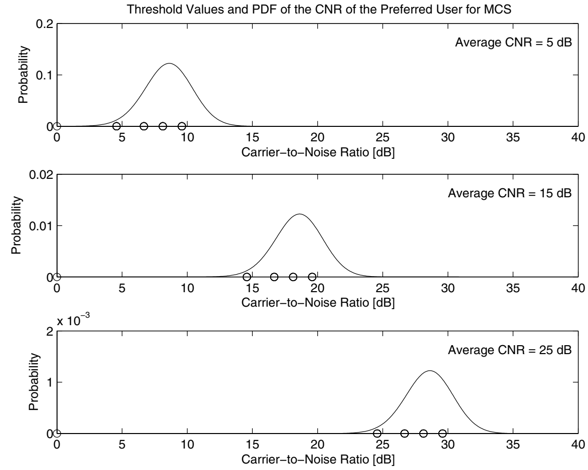

Fig. 3. Three sets of feedback threshold values for the MCS algorithm for $L = 5$ and 10 users with Rayleigh fading channels with $\overline{\gamma} = 5$ dB, $\overline{\gamma} = 15$ dB, and $\overline{\gamma} = 25$ dB, respectively. The PDF of the CNR of the user with the highest CNR is also shown for $\overline{\gamma} = 5$ dB, $\overline{\gamma} = 15$ dB, and $\overline{\gamma} = 25$ dB, respectively.

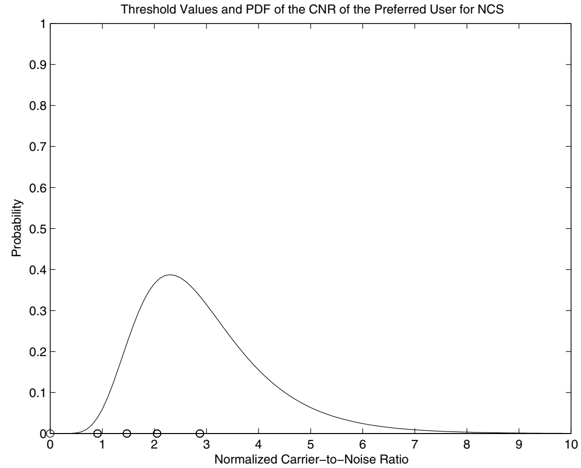

Fig. 4. One set of feedback threshold values for the NCS algorithm for $L = 5$ and 10 users with Rayleigh fading channels with different average CNRs. The PDF of the normalized CNR of the user with the highest normalized CNR is also shown.

is only dependent on the number of users [18]. Consequently, the threshold values in dB will be a sum of $\overline{\gamma}$ and a constant.

Fig. 3 shows three sets of threshold values for three cases where we have ten users having Rayleigh distributed channels with $\overline{\gamma} = 5$ dB, $\overline{\gamma} = 15$ dB, and $\overline{\gamma} = 25$ dB, respectively. The threshold values are identical for all users and are shown as small rings. Each set of threshold values contains five CNR values ($L = 5$). By comparing the threshold values of the three different sets, we see that the threshold values in dB are a sum of $\overline{\gamma}$ and a constant. The probability density functions (PDF) of the best user among ten users is also shown for each of the three $\overline{\gamma}$-values. These plots show that the probability of finding the best user below $\gamma_{\text{th},1}$ is quite small. Consequently, the probability of full feedback is low.

C. Feedback Thresholds for the NCS Algorithm

The PDFs of the scheduling metric $x_i(t) = \chi_i(t) = \gamma_i(t)/\overline{\gamma}_i$ can be obtained by doing a transformation of the PDF of the CNR of user i, γ_i [17, (2.1.8)]. If the Rayleigh, Nakagami or Rice distributions listed in [18] are used in this transformation, it can be shown that the resulting PDFs are independent of i. Therefore, if we assume that the users' CNRs have the same distributions with different averages, it can be shown that the PDF of $\chi_i(t)$ is the same for all users. Since this PDF and the corresponding CDF are independent of i, they can be denoted $p_\chi(\chi)$ and $P_\chi(\chi)$, respectively. Using these distributions for χ, we can obtain an expression for the NFL in similar way as in the previous section:

$$\bar{\mathrm{F}}_{\text{NCS}} = \sum_{l=0}^{L-1} \left(P_\chi(\chi_{\text{th},l+1}) - P_\chi(\chi_{\text{th},l})\right) \cdot P_\chi^{N-1}(\chi_{\text{th},l+1}), \quad (8)$$

where $\chi_{\text{th},l}$ denotes the lth threshold value. Taking the gradient of (8) with respect to the threshold values, we obtain a similar expression for the threshold values as we did for the MCS feedback threshold values:

$$\chi^*_{\text{th},l} = P_\chi^{-1}\left(S_l \cdot P_\chi(\chi^*_{\text{th},l+1})\right), \quad l = 1, 2, 3, \cdots, L-1, \quad (9)$$

where $P_\chi^{-1}(\cdot)$ is the inverse CDF of $\chi(t)$, and the constants S_l are given by (5).

The plots of the NFL and AFL for the optimal threshold values in (8) as a function of L for different number of users with Rayleigh distributed channels, is identical to Figs. 1 and 2, respectively. It can be shown that the identical feedback load for the MCS and NCS algorithms arises as a consequence of the similarities between the CDFs of the scheduling metrics when the users have the same average CNR for the MCS algorithm. Since the feedback thresholds are adapted to the scheduling metric of the NCS algorithm, the feedback load is independent of the average CNRs of the users.

Fig. 4 shows five feedback threshold values of the NCS algorithm for ten users with Rayleigh fading channels with different average CNRs. The corresponding PDF of χ for the preferred user is also shown. It should be noted that the threshold values are identical for all users. However, if the threshold values are converted to the corresponding CNR values, $\gamma_{\text{th},i,l} = \overline{\gamma}_i \chi_{\text{th},l}$, they will differ from user to user.

IV. A Two-Step Procedure for Optimizing the Threshold Values and the Number of Thresholds

In the previous section, it was assumed that the time it takes for the scheduler to conduct the polling process, take a scheduling decision, and distribute this decision is negligible. In practical systems this process will have to be conducted within a *guard time* at the beginning of the time-slots. To have the highest possible utilization of the system, we thus want that (a) feedback is received from the preferred user (or subset of users), (b) the power consumption of the users is minimized and (c) the guard time is reduced. As previously explained, both (a) and (b) are achieved by using our feedback algorithm. The guard time can be split into two components, namely, (i) the time used to poll the users with lower and lower feedback thresholds, and (ii) the time used to receive feedback from one or more users. Both the number of users, the value of L, and the threshold values will affect these two time contributions. Setting all the thresholds to zero will minimize (i). However, this will maximize the feedback load and hence also (a). Consequently, the threshold values have to be set to non-zero values and thus (i) is strongly dependent on the number of users and the value of L.

Based on the discussion above, we see that it is often favorable both for the power consumption and the guard time length that the threshold values are set to minimize the feedback load. However, the guard time will also be affected by the value of L. We therefore propose a two-step optimization procedure to obtain the threshold values and the value of L. In the first step, the threshold values are set to those who minimize the feedback load. In the second step, the value of L that minimizes the guard time is found numerically, based on the threshold values from step one. In [13], we performed the two-step optimization procedure described above, for an IEEE 802.11 system, and we refer to [13] for numerical results on this procedure.

V. Conclusions

We have proposed a new channel state feedback algorithm for modern cellular networks. Compared to previously published works, our algorithm is based on two novel concepts, namely, (i) adapting the feedback threshold value to the scheduling algorithm implemented in the system, and (ii) employing multiple feedback thresholds to reduce the number of users transmitting feedback to a minimum. Our feedback algorithm leads to a significant decrease in the power consumption of the mobile users and also in the time used to collect feedback for many systems. The proposed feedback algorithm can be implemented for any scheduling metric, but in most cases the optimal threshold values have to be found numerically. However, for the MCS and the NCS algorithms we obtained elegant closed-form expressions for the optimal threshold values. Finally, we proposed a two-step optimization procedure for obtaining the threshold values and the number of thresholds in real-life wireless networks.

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