Precoding of Orthogonal Space-Time Block Codes over Correlated Ricean MIMO Channels

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***Abstract*— A precoder is designed for orthogonal space-time block codes (OSTBCs) for arbitrarily correlated Ricean multiple-input multiple-output (MIMO) channels. Unlike previous works, the precoder can be designed to minimize the *exact* symbol error rate (SER) as function of both a) the joint transmit-receive channel correlation coefficients, and b) the MIMO Rice component, which are fed back to the transmitter. Importantly, the covariance may or may not follow the so-called *Kronecker structure*. Exact SER expressions are given for multi-level PAM, PSK, and QAM signaling. Several properties of the minimum exact precoder are provided. An iterative numerical optimization algorithm is proposed for finding the exact minimum SER precoder under a power constraint.**

I. Introduction

In the area of efficient communications over non-reciprocal MIMO channels, recent research has demonstrated the value of feeding back information about the channel state observed at the receiver to the transmitter. There has been a growing interest in transmitter schemes that can exploit low-rate long-term statistical or structural channel state information in the form of antenna correlation coefficients and line-of-sight MIMO coefficients. So far, emphasis has been on designing precoders for space-time block coded (STBC) [1] signals or spatially multiplexed streams that are adjusted based on the knowledge of the transmit correlation only while the receiving antennas are uncorrelated [2], [3], [4]. However, in practice both transmitter and receiver may exhibit correlation and the precoder can take this into account [5]. Furthermore, although simple models exist for the joint transmit receiver correlation based on the well known Kronecker structure [1], the accuracy of these models has recently been questioned in the literature based on measurement campaigns [6]. Therefore, it is of interest to investigate the precoding of OSTBC signals for MIMO channels that *do not* necessarily follow the Kronecker structure over Ricean channels.

Previous related research: An upper bound of the pair-wise error probability (PEP) is minimized in [2] for a Rayleigh fading channel with transmit-only correlation. In [5], an upper bound for the PEP is found for a Ricean fading channel with arbitrary correlation and some asymptotic results are provided. In [7], the exact SER expressions were derived when the receiver antennas employing maximum ratio combiner and a bound of the exact error probability was used as the optimization criterion for a correlated Rayleigh channel. No receiver correlation was included in the Rayleigh channel model used in [7]. In [8], the precoder matrix was designed for minimizing the exact SER for correlated Rayleigh MIMO channels. In [9], the precoder was designed for uncorrelated Ricean channels. Unfortunately, in practice correlation is present. In [10], exact expressions where proposed for correlated Ricean channels *not* employing precoding for arbitrary input signal constellations.

Here, we find the minimum exact SER linear precoder for OSTBC signals for communication over MIMO channels which are simultaneously correlated and have a line-of-sight (LOS) component such

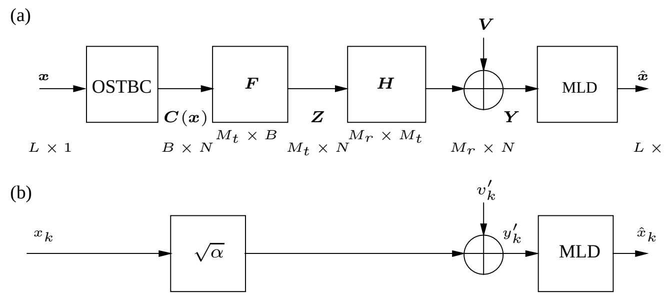

Fig. 1. Block model of the linearly precoded OSTBC MIMO system.

that they are Rice channels. More specifically, our main contributions and assumptions are: We derive *exact* expressions for the average SER for a system where the transmitter has an OSTBC followed by a full precoder matrix and where the receiver also has multiple antennas and is using maximum likelihood decoding (MLD). The SER expressions are found for regular multi-level PAM, PSK, and QAM, and they are easy to evaluate. The transmitter knows the LOS component and the invertible correlation matrix of the fading portion of the channel transfer matrix and the receiver knows the channel realization exactly. We propose an iterative numerical technique for minimizing the exact SER with respect to the precoder matrix. Several key properties of the optimal precoder are presented.

II. System Description

A. OSTBC Signal Model

Figure 1 (a) shows the block MIMO system model with M_t transmitter and M_r receiver antennas. One block of L signal samples $x_0, x_1, \dots, x_{L-1}$ is transmitted by means of an OSTBC matrix $\boldsymbol{C}(\boldsymbol{x})$ of size $B \times N$, where B and N are the space and time dimension of the given OSTBC, respectively, and $\boldsymbol{x} = [x_0, x_1, \dots, x_{L-1}]^T$. It is assumed that the OSTBC is given. Let $x_i \in \mathcal{A}$, where $\mathcal{A}$ is a signal constellation set such as M-PAM, M-QAM, or M-PSK. If bits are used as inputs to the system, $L \log_2 |\mathcal{A}|$ bits are used to produce the vector $\boldsymbol{x}$, where $|\cdot|$ denotes cardinality. Assume that $E\left[|x_i|^2\right] = \sigma_x^2$. Since the OSTBC $\boldsymbol{C}(\boldsymbol{x})$ is orthogonal, the following holds

$$\boldsymbol{C}(\boldsymbol{x})\boldsymbol{C}^H(\boldsymbol{x}) = a \sum_{i=0}^{L-1} |x_i|^2 \boldsymbol{I}_B, \tag{1}$$

where $a = 1$ if $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{G}_2^T$, $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{H}_3^T$, or $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{H}_4^T$ in [11] and $a = 2$ if $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{G}_3^T$ or $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{G}_4^T$ in [11], so the constant a is OSTBC dependent. The rate of the code is L/N. The proposed theory holds for any OSTBC.

Before each code word $\boldsymbol{C}(\boldsymbol{x})$ is launched into the channel, it is precoded with a memoryless complex-valued matrix $\boldsymbol{F}$ of size $M_t \times B$, so the $M_r \times N$ receive signal matrix $\boldsymbol{Y}$ becomes

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{F}\boldsymbol{C}(\boldsymbol{x}) + \boldsymbol{V}, \tag{2}$$

where the additive noise is contained in the block matrix $\boldsymbol{V}$ of size $M_r \times N$, with all the components are complex Gaussian circularly distributed with independent components having variance N_0, and $\boldsymbol{H}$ is the channel transfer MIMO matrix. The receiver is assumed to know the channel matrix $\boldsymbol{H}$ and the precoding matrix $\boldsymbol{F}$ exactly, and it performs MLD of blocks $\boldsymbol{Y}$ of size $M_r \times N$.

B. Correlated Channel Models

A quasi-static non-frequency selective correlated Rice fading channel model [1] is assumed. Let $\boldsymbol{R}$ be the general $M_tM_r \times M_tM_r$ positive definite autocorrelation matrix for the fading part of the channel coefficients and $\sqrt{\frac{K}{1+K}}\bar{\boldsymbol{H}}$ be the mean value of the channel coefficients. The mean value represents the LOS component of the MIMO channel. The factor $K \geq 0$ is called the Ricean factor [1]. A channel realization of the correlated channel can then be found by

$$\begin{aligned}\mathrm{vec}\left(\boldsymbol{H}\right) &= \sqrt{\frac{K}{1+K}}\,\mathrm{vec}\left(\bar{\boldsymbol{H}}\right) + \sqrt{\frac{1}{1+K}}\,\mathrm{vec}\left(\boldsymbol{H}_{\text{Fading}}\right)\\ &= \sqrt{\frac{K}{1+K}}\,\mathrm{vec}\left(\bar{\boldsymbol{H}}\right) + \sqrt{\frac{1}{1+K}}\boldsymbol{R}^{1/2}\,\mathrm{vec}\left(\boldsymbol{H}_w\right), \end{aligned} \tag{3}$$

where $\boldsymbol{R}^{1/2}$ is the unique positive definite matrix square root [12] of the assumed invertible matrix $\boldsymbol{R}$, where $\boldsymbol{R} = E\left[\mathrm{vec}\left(\boldsymbol{H}_{\text{Fading}}\right)\mathrm{vec}^H\left(\boldsymbol{H}_{\text{Fading}}\right)\right]$ is the correlation matrix of the $M_r \times M_t$ fading component $\boldsymbol{H}_{\text{Fading}}$ of the channel, $\boldsymbol{H}_w$ has size $M_r \times M_t$ and is complex Gaussian circularly distributed with independent components all having unit variance and zero mean, and the operator $\mathrm{vec}(\cdot)$ stacks the columns of the matrix it is applied to into a long column vector [12]. The notation $\mathrm{vec}\left(\boldsymbol{H}_w\right) \sim \mathcal{CN}\left(\boldsymbol{0}_{M_tM_r\times 1}, \boldsymbol{I}_{M_tM_r}\right)$ is used to indicate the distribution of the vector $\mathrm{vec}\left(\boldsymbol{H}_w\right)$. Using the same notation $\mathrm{vec}\left(\boldsymbol{H}\right) \sim \mathcal{CN}\left(\sqrt{\frac{K}{1+K}}\,\mathrm{vec}\left(\bar{\boldsymbol{H}}\right), \frac{1}{1+K}\boldsymbol{R}\right)$.

Kronecker model: A special case of the model above is as follows [1]

$$\boldsymbol{H} = \sqrt{\frac{K}{1+K}}\bar{\boldsymbol{H}} + \sqrt{\frac{1}{1+K}}\boldsymbol{R}_r^{1/2}\boldsymbol{H}_{\boldsymbol{w}}\boldsymbol{R}_t^{1/2}, \tag{4}$$

where the matrices $\boldsymbol{R}_r$ and $\boldsymbol{R}_t$ are the covariance matrices of the receiver and transmitter, respectively, and their sizes are $M_r \times M_r$ and $M_t \times M_t$. The autocorrelation matrix of the fading component $\boldsymbol{R}$ of the model in (4) is then given by

$$\boldsymbol{R} = \boldsymbol{R}_t^T \otimes \boldsymbol{R}_r, \tag{5}$$

where the operator $(\cdot)^T$ denotes transposition and $\otimes$ is the Kronecker product. Unlike (5), the general model considers that the receive (or transmit) covariance depends on which transmit (or receive) antenna the measurements are performed at.

C. Equivalent Single-Input Single-Output Model

Define the matrix $\boldsymbol{\Phi}$ of size $M_tM_r \times M_tM_r$ as:

$$\boldsymbol{\Phi} = \boldsymbol{R}^{1/2}\left[\left(\boldsymbol{F}^*\boldsymbol{F}^T\right)\otimes \boldsymbol{I}_{M_r}\right]\boldsymbol{R}^{1/2}. \tag{6}$$

This matrix plays an important role in the developed theory. Let the eigenvalue decomposition of this Hermitian non-negative definite matrix be given by:

$$\boldsymbol{\Phi} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H, \tag{7}$$

where $\boldsymbol{U} \in \mathbb{C}^{M_tM_r\times M_tM_r}$ is unitary and $\boldsymbol{\Lambda} \in \mathbb{R}^{M_tM_r\times M_tM_r}$ is a diagonal matrix containing the non-negative eigenvalues λ_i of $\boldsymbol{\Phi}$ on its main diagonal.

Define the real non-negative scalar α by

$$\begin{aligned}\alpha = \|\boldsymbol{H}\boldsymbol{F}\|_F^2 = &\left[\sqrt{\frac{1}{1+K}}\,\mathrm{vec}^H\left(\boldsymbol{H}_w\right)\boldsymbol{R}^{1/2} + \sqrt{\frac{K}{1+K}}\,\mathrm{vec}^H\left(\bar{\boldsymbol{H}}\right)\right]\\ &\left[\left(\boldsymbol{F}^*\boldsymbol{F}^T\right)\otimes\boldsymbol{I}_{M_r}\right]\left[\sqrt{\frac{1}{1+K}}\boldsymbol{R}^{1/2}\mathrm{vec}\left(\boldsymbol{H}_w\right) + \sqrt{\frac{K}{1+K}}\mathrm{vec}\left(\bar{\boldsymbol{H}}\right)\right], \end{aligned} \tag{8}$$

where $\|\cdot\|_F$ is the Frobenius norm. Since the matrix $\boldsymbol{H}_w$ contains zero mean, unit variance uncorrelated variables, $E\left[\mathrm{vec}\left(\boldsymbol{H}_w\right)\mathrm{vec}^H\left(\boldsymbol{H}_w\right)\right] = \boldsymbol{I}_{M_tM_r}$. Since it is assumed that $\boldsymbol{R}$ is invertible, α can be rewritten by means of the eigen-decomposition, in (7), as:

$$\alpha = \sum_{i=0}^{M_tM_r-1}\frac{\lambda_i}{1+K}\left|\left(\mathrm{vec}\left(\boldsymbol{H}'_w\right) + \sqrt{K}\boldsymbol{U}^H\boldsymbol{R}^{-1/2}\,\mathrm{vec}\left(\bar{\boldsymbol{H}}\right)\right)_i\right|^2, \tag{9}$$

where $\mathrm{vec}\left(\boldsymbol{H}'_w\right) \sim \mathcal{CN}\left(\boldsymbol{0}_{M_tM_r\times 1}, \boldsymbol{I}_{M_tM_r}\right)$ has the same distribution as $\mathrm{vec}\left(\boldsymbol{H}_w\right)$.

By generalizing the approach given in [13], [14] to include a *full* complex-valued precoder $\boldsymbol{F}$ of size $M_t \times B$ and having a *full* channel correlation matrix $1/(1+K)\boldsymbol{R}$ and mean $\sqrt{K/(1+K)}\bar{\boldsymbol{H}}$, the OSTBC system can be shown to be equivalent with a system having the following input-output relationship

$$y'_k = \sqrt{\alpha}x_k + v'_k, \tag{10}$$

for $k \in \{0, 1, \ldots, L-1\}$, and where $v'_k \sim \mathcal{CN}(0, N_0/a)$ is complex circularly distributed. This signal is fed into a memoryless MLD that is designed from the signal constellation of the source symbols $\mathcal{A}$. The equivalent single-input single-output (SISO) model is shown in Figure 1 (b). The equivalent SISO model is valid for any realization of $\boldsymbol{H}$.

III. SER Expressions for Given Received SNR

By considering the SISO system in Figure 1 (b), it is seen that the instantaneous received SNR γ per source symbol is given by

$$\gamma = \frac{a\sigma_x^2\alpha}{N_0} = \delta\alpha, \tag{11}$$

where $\delta \triangleq \frac{a\sigma_x^2}{N_0}$.

In order to simplify the expressions, the following three signal constellation dependent constants are defined

$$g_{\text{PSK}} = \sin^2\frac{\pi}{M},\ g_{\text{PAM}} = \frac{3}{M^2-1},\ g_{\text{QAM}} = \frac{3}{2(M-1)}. \tag{12}$$

Define the positive definite matrix $\boldsymbol{A}$ of size $M_tM_r \times M_tM_r$ as

$$\boldsymbol{A} = \boldsymbol{I}_{M_tM_r} + \frac{\delta g}{(1+K)\sin^2(\theta)}\boldsymbol{\Phi}, \tag{13}$$

where g takes on the form in (12). The symbols $\boldsymbol{A}^{(\text{PSK})}$, $\boldsymbol{A}^{(\text{PAM})}$, and $\boldsymbol{A}^{(\text{QAM})}$ are used for the PSK, PAM, and QAM constellations, respectively.

The symbol error probability $\mathrm{SER}_\gamma \triangleq \Pr\{\mathrm{Error}|\gamma\}$ for a given γ for M-PSK, M-PAM, and M-QAM signaling is given by [15]

$$\mathrm{SER}_\gamma = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} e^{-\frac{g_{\mathrm{PSK}}\gamma}{\sin^2\theta}}d\theta, \tag{14}$$

$$\mathrm{SER}_\gamma = \frac{2}{\pi}\frac{M-1}{M}\int_0^{\frac{\pi}{2}} e^{-\frac{g_{\mathrm{PAM}}\gamma}{\sin^2\theta}}d\theta, \tag{15}$$

$$\mathrm{SER}_\gamma = \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\left[\frac{1}{\sqrt{M}}\int_0^{\frac{\pi}{4}} e^{-\frac{g_{\mathrm{QAM}}\gamma}{\sin^2\theta}}d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{-\frac{g_{\mathrm{QAM}}\gamma}{\sin^2\theta}}d\theta\right], \tag{16}$$

respectively.

IV. Exact SER Expressions

The moment generating function of the probability density function $p_\gamma(\gamma)$ is defined as $\phi_\gamma(s) = \int_0^\infty p_\gamma(\gamma)e^{s\gamma}d\gamma$. Since all the L source symbols go through the same SISO system in Figure 1 (b), the average SER of the MIMO system can be found as

$$\mathrm{SER} \triangleq \Pr\{\mathrm{Error}\} = \int_0^\infty \mathrm{SER}_\gamma\, p_\gamma(\gamma)d\gamma. \tag{17}$$

This integral can be rewritten by means of the moment generating function of γ.

In order to find the moment generating function of γ, the following results will be useful.

Lemma 1: Let $X \sim \mathcal{N}(m_X, \sigma_X^2)$ and $Y = X^2$. The moment generating function of Y is given by:

$$\phi_Y(s) = \frac{e^{\frac{m_X^2 s}{1-2\sigma_X^2 s}}}{\sqrt{1-2\sigma_X^2 s}}. \tag{18}$$

Proof: This result can be shown by using the Laplace transform formulas in Chapter 29 of [16] of Equation (2.1-115) in [17]. ■

Lemma 2: Let $Z = X + jY$ be a Gaussian complex circularly distributed process $\mathcal{CN}(m_Z, \sigma_Z^2)$. The moment generating function of $|Z|^2 = X^2 + Y^2$ is given by:

$$\phi_{|Z|^2}(s) = \frac{e^{\frac{|m_Z|^2 s}{1-\sigma_Z^2 s}}}{1-\sigma_Z^2 s}. \tag{19}$$

Proof: Since Z is circularly distributed, the real and imaginary part of Z are statistically independent, real and Gaussian distributed with equal variance $\frac{\sigma_Z^2}{2}$. Since the moment generating function of a sum of two random variables is equal to the product of the moment generating functions of the two variables, the result follows by using Lemma 1. ■

The moment generating function in Lemma 2 has the same shape as the moment generating function for the Rice distribution given in Equation (2.17) in [15].

From $\mathrm{vec}(\boldsymbol{H}'_w) + \sqrt{K}\boldsymbol{U}^H\boldsymbol{R}^{-1/2}\mathrm{vec}(\bar{\boldsymbol{H}}) \sim \mathcal{CN}\left(\sqrt{K}\boldsymbol{U}^H\boldsymbol{R}^{-1/2}\mathrm{vec}(\bar{\boldsymbol{H}}), \boldsymbol{I}_{M_tM_r}\right)$ and (9), it follows from Lemma 2 that the moment generating function of α is given by:

$$\phi_\alpha(s) = \frac{e^{\sum_{i=0}^{M_tM_r-1}\frac{|\boldsymbol{u}_i^H\boldsymbol{R}^{-1/2}\mathrm{vec}(\bar{\boldsymbol{H}})|^2\frac{K}{1+K}\lambda_i s}{1-\frac{\lambda_i}{1+K}s}}}{\prod_{i=0}^{M_tM_r-1}\left(1-\frac{\lambda_i}{1+K}s\right)}, \tag{20}$$

where λ_i is eigenvalue number i of the matrix $\boldsymbol{\Phi}$, and $\boldsymbol{u}_i$ is the ith column vector of the matrix $\boldsymbol{U}$. Since $\gamma = \delta\alpha$, the moment generating function of γ is given by:

$$\phi_\gamma(s) = \phi_\alpha(\delta s) = \frac{e^{\sum_{i=0}^{M_tM_r-1}\frac{|\boldsymbol{u}_i^H\boldsymbol{R}^{-1/2}\mathrm{vec}(\bar{\boldsymbol{H}})|^2\frac{K\delta\lambda_i}{1+K}s}{1-\frac{\delta\lambda_i}{1+K}s}}}{\prod_{i=0}^{M_tM_r-1}\left(1-\frac{\delta\lambda_i}{1+K}s\right)}. \tag{21}$$

By using (17) and the definition of the moment generating function together with the result in (21) it is possible to express the exact SER for all the signal constellations in terms of the eigenvalues λ_i and eigenvectors $\boldsymbol{u}_i$ of the matrix $\boldsymbol{\Phi}$:

$$\mathrm{SER} = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\phi_\gamma\left(-\frac{g_{\mathrm{PSK}}}{\sin^2\theta}\right)d\theta, \tag{22}$$

$$\mathrm{SER} = \frac{2}{\pi}\frac{M-1}{M}\int_0^{\frac{\pi}{2}}\phi_\gamma\left(-\frac{g_{\mathrm{PAM}}}{\sin^2\theta}\right)d\theta, \tag{23}$$

$$\mathrm{SER} = \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\left[\frac{1}{\sqrt{M}}\int_0^{\frac{\pi}{4}}\phi_\gamma\left(-\frac{g_{\mathrm{QAM}}}{\sin^2\theta}\right)d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\phi_\gamma\left(-\frac{g_{\mathrm{QAM}}}{\sin^2\theta}\right)d\theta\right]. \tag{24}$$

In order to present the SER expressions compactly, define the following real non-negative scalar function, which is dependent on the LOS component, as:

$$f(K, \boldsymbol{R}, \boldsymbol{A}) = \frac{e^{K\,\mathrm{vec}^H(\bar{\boldsymbol{H}})\boldsymbol{R}^{-1/2}\boldsymbol{A}^{-1}\boldsymbol{R}^{-1/2}\mathrm{vec}(\bar{\boldsymbol{H}})}}{\det\boldsymbol{A}}. \tag{25}$$

Optimization of the system might be done through differentiation. The eigenvalues that are not simple can cause problems since the differentiation with respect to these are difficult to find. Therefore, it is useful to rewrite the SER expressions such that the expressions become independent of $\boldsymbol{U}$ and λ_i. If the eigen-decomposition in (7) is utilized, it is possible to express the SER as a function of $\boldsymbol{\Phi}$:

$$\mathrm{SER} = \frac{f(-K, \boldsymbol{R}, \boldsymbol{I}_{M_tM_r})}{\pi}\int_0^{\frac{M-1}{M}\pi} f(K, \boldsymbol{R}, \boldsymbol{A}^{(\mathrm{PSK})})d\theta, \tag{26}$$

$$\mathrm{SER} = \frac{2f(-K, \boldsymbol{R}, \boldsymbol{I}_{M_tM_r})}{\pi}\frac{M-1}{M}\int_0^{\frac{\pi}{2}} f(K, \boldsymbol{R}, \boldsymbol{A}^{(\mathrm{PAM})})d\theta, \tag{27}$$

$$\mathrm{SER} = \frac{4f(-K, \boldsymbol{R}, \boldsymbol{I}_{M_tM_r})}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\left[\frac{1}{\sqrt{M}}\int_0^{\frac{\pi}{4}} f(K, \boldsymbol{R}, \boldsymbol{A}^{(\mathrm{QAM})})d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f(K, \boldsymbol{R}, \boldsymbol{A}^{(\mathrm{QAM})})d\theta\right], \tag{28}$$

for PSK, PAM, and QAM signaling, respectively.

Remark 1: If $K = 0$, then the above SER expressions reduce to the SER expressions derived for Rayleigh fading channels in [8]. The above SER expressions were verified by Monte Carlo simulations. The proposed SER expressions are only valid for invertible $\boldsymbol{R}$ when there is a non-zero LOS component present, but if the LOS component is zero, R can be singular. It is seen from the above expressions, that if $\boldsymbol{F} = \boldsymbol{0}_{M_t\times B}$, then all three expressions give $\mathrm{SER} = \frac{M-1}{M}$. Intuitively, this makes sense, since in this case, the receiver will only receive noise and then on average one of M symbol decisions will be correct. The above expressions have a similar form as the PEP expressions proposed in [5]. The expressions in [10] are not as easy to evaluate as the proposed expressions since, in [10], the input signal constellation was arbitrary and then the SER expressions must be found by performing two-dimensional integrals over possibly complicated regions in the complex plane. The proposed expressions are very easy to evaluate.

V. Precoding of OSTBC signals

A. Power Constraint

When an OSTBC is used, (1) holds and the average power constraint on the transmitted block $\boldsymbol{Z} \triangleq \boldsymbol{F}\boldsymbol{C}(\boldsymbol{x})$ can be expressed as

$$aL\sigma_x^2 \operatorname{Tr}\left\{\boldsymbol{F}\boldsymbol{F}^H\right\} = P, \tag{29}$$

where P is the average power used by the transmitted block $\boldsymbol{Z}$.

B. Optimal Precoder Problem Formulation

The goal is to find the matrix $\boldsymbol{F}$ such that the exact SER is minimized under the power constraint. We propose that the optimal precoder is given by the following optimization problem:

Problem 1:

$$\min_{\left\{\boldsymbol{F}\in\mathbb{C}^{M_t\times B}\right\}} \quad \text{SER}$$
$$\text{subject to} \quad La\sigma_x^2 \operatorname{Tr}\left\{\boldsymbol{F}\boldsymbol{F}^H\right\} = P.$$

In general, the optimal precoder is dependent on the value of N_0 and, therefore, also of the signal to noise ratio (SNR).

C. Properties of Optimal Precoder

Remark 2: When $K = 0$, the channel has no LOS component and then all the properties given in [8] are applicable.

Lemma 3: If $\boldsymbol{F}$ is an optimal solution of Problem 1, then the precoder $\boldsymbol{F}\boldsymbol{W}$, where $\boldsymbol{W} \in \mathbb{C}^{B\times B}$ is unitary, is also optimal.

Proof: Let $\boldsymbol{F}$ be an optimal solution of Problem 1 and $\boldsymbol{W} \in \mathbb{C}^{B\times B}$, be an arbitrary unitary matrix. It is then seen by insertion that the objective function and the power constraint are unaltered by the unitary matrix. ■

Lemma 4: If $\text{SNR} \to \infty$ and $B = M_t$, then the optimal precoder is given by the trivial precoder $\boldsymbol{F} = \sqrt{\frac{P}{La\sigma_x^2 M_t}}\boldsymbol{I}_{M_t}$ for the M-PSK, M-PAM, and M-QAM constellations.

Proof: When $\text{SNR} \to \infty$, then $\delta \to \infty$, and in this case, the integrand of SER can be simplified as $f(K, \boldsymbol{R}, \boldsymbol{A}) \to 1/\det\left(\delta g/((1+K)\sin^2\theta)\boldsymbol{\Phi}\right)$. Then, the problem can be rewritten as finding the maximum of $\det(\boldsymbol{\Phi})$ under the power constraint. This problem is again equivalent to maximize $\det\left(\boldsymbol{F}\boldsymbol{F}^H\right)$ subject to $\operatorname{Tr}\left\{\boldsymbol{F}\boldsymbol{F}^H\right\} = \frac{P}{aL\sigma_x^2}$. It can be shown that the solution of this symmetrical equivalent problem is the trivial precoder. ■

Let the matrix $\boldsymbol{K}_{k,l}$ be the commutation matrix[1] of size $kl \times kl$. Let $\boldsymbol{G} \triangleq \boldsymbol{K}_{M_r,M_t}\left[K\operatorname{vec}\left(\bar{\boldsymbol{H}}\right)\operatorname{vec}^H\left(\bar{\boldsymbol{H}}\right) + \boldsymbol{R}\right]\boldsymbol{K}_{M_t,M_r}$, and let the ith block diagonal of this matrix of size $M_t \times M_t$ be denoted $\boldsymbol{G}_i$, i.e., $\boldsymbol{G}_i = (\boldsymbol{G})_{iM_t:(i+1)M_t-1,iM_t:(i+1)M_t-1}$. Define the $M_t \times M_t$ matrix $\boldsymbol{\beta}$ as:

$$\boldsymbol{\beta} = \sum_{i=0}^{M_r-1}\boldsymbol{G}_i^T. \tag{30}$$

[1]The *commutation matrix* $\boldsymbol{K}_{k,l}$ is the unique $kl \times kl$ permutation matrix satisfying $\boldsymbol{K}_{k,l}\operatorname{vec}(\boldsymbol{S}) = \operatorname{vec}\left(\boldsymbol{S}^T\right)$ for all matrices $\boldsymbol{S} \in \mathbb{C}^{k\times l}$.

Lemma 5: Assume that β has a simple maximum eigenvalue, with the unit-norm vector $\boldsymbol{v}$ as the corresponding eigenvector. If $\text{SNR} \to -\infty$ dB, then the optimal precoder is given by

$$\boldsymbol{F} = \sqrt{\frac{P}{La\sigma_x^2}}\left[\boldsymbol{v} \;\; \boldsymbol{0}_{M_t\times B-1}\right], \tag{31}$$

for the M-PSK, M-PAM, and M-QAM constellations.

Proof: This can be proven using a similar strategy as was used in [5], where the same result is proved when PEP is the optimization criterion. ■

With the precoder in Lemmas 5, the transmitted signal from the transmitter has the shape $\boldsymbol{v}\left(\boldsymbol{C}(\boldsymbol{x})\right)_{0,:}$. This shows that the first row of $\boldsymbol{C}(\boldsymbol{x})$ is beamformed in the direction of $\boldsymbol{v}$ which is the eigenvector corresponding to the largest eigenvalue of $\boldsymbol{G} = K\bar{\boldsymbol{H}}\bar{\boldsymbol{H}}^H + E\left[\boldsymbol{H}_{\text{Fading}}^H\boldsymbol{H}_{\text{Fading}}\right]$.

Lemma 6: Assume that the $M_t \times M_t$ matrix $\bar{\boldsymbol{H}}^H\bar{\boldsymbol{H}}$ has a simple maximum eigenvalue with corresponding normalized eigenvector $\boldsymbol{w}$. If $K \to \infty$, then the minimum SER precoder is given by:

$$\boldsymbol{F} = \sqrt{\frac{P}{La\sigma_x^2}}\left[\boldsymbol{w} \;\; \boldsymbol{0}_{M_t\times B-1}\right], \tag{32}$$

for the M-PSK, M-PAM, and M-QAM constellations.

Proof: This can be proven using a similar method as in the previous lemma. ■

With the precoder in Lemma 6, the transmitted signal from the transmitter has the shape $\boldsymbol{w}\left(\boldsymbol{C}(\boldsymbol{x})\right)_{0,:}$, where $\boldsymbol{w}$. This shows that the first row of $\boldsymbol{C}(\boldsymbol{x})$ is beamformed in the direction of $\boldsymbol{w}$, which corresponds to the leading right singular vector of the LOS matrix $\bar{\boldsymbol{H}}$. Notice that, the precoders in Lemmas 5 and 6 are not unique since any precoder can be postmultiplied with a unitary matrix, see Lemma 3.

VI. Optimization Algorithm

The constrained maximization Problem 1 can be converted into an unconstrained optimization problem by introducing a Lagrange multiplier μ'. This is done by defining the following Lagrange function:

$$\mathcal{L}(\boldsymbol{F}) = \text{SER} + \mu'\operatorname{Tr}\left\{\boldsymbol{F}\boldsymbol{F}^H\right\}. \tag{33}$$

Since the objective function should be minimized, $\mu' > 0$. Define the $M_t^2 \times M_t^2 M_r^2$ matrix $\boldsymbol{\Pi}$ as

$$\boldsymbol{\Pi} = \left[\boldsymbol{I}_{M_t^2} \otimes \operatorname{vec}^T\left(\boldsymbol{I}_{M_r}\right)\right]\left[\boldsymbol{I}_{M_t} \otimes \boldsymbol{K}_{M_t,M_r} \otimes \boldsymbol{I}_{M_r}\right]. \tag{34}$$

In order to present the results compactly, define the following $BM_t \times 1$ vector $\boldsymbol{s}(\boldsymbol{F}, \theta, g, \mu)$:

$$\boldsymbol{s}(\boldsymbol{F},\theta,g,\mu) = \mu\left[\boldsymbol{F}^T \otimes \boldsymbol{I}_{M_t}\right]\boldsymbol{\Pi}\left[\boldsymbol{R}^{1/2} \otimes \left(\boldsymbol{R}^{1/2}\right)^T\right]$$
$$\operatorname{vec}^*\left(\boldsymbol{A}^{-1} + K\boldsymbol{A}^{-1}\boldsymbol{R}^{-1/2}\operatorname{vec}\left(\bar{\boldsymbol{H}}\right)\operatorname{vec}^H\left(\bar{\boldsymbol{H}}\right)\boldsymbol{R}^{-1/2}\boldsymbol{A}^{-1}\right)$$
$$\frac{e^{K\operatorname{vec}^H\left(\bar{\boldsymbol{H}}\right)\boldsymbol{R}^{-1/2}\boldsymbol{A}^{-1}\boldsymbol{R}^{-1/2}\operatorname{vec}\left(\bar{\boldsymbol{H}}\right)}}{\sin^2(\theta)\det(\boldsymbol{A})}. \tag{35}$$

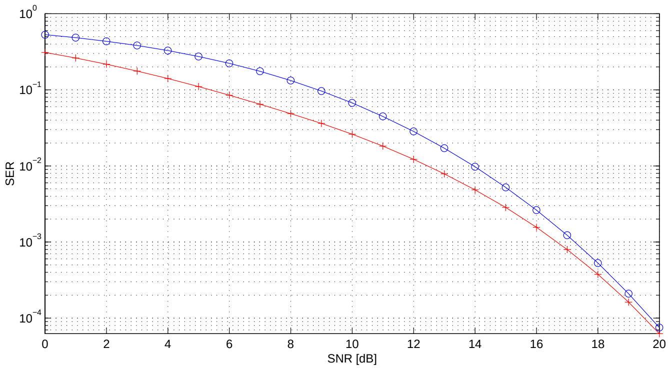

Fig. 2. SER versus SNR performance of the proposed minimum SER precoder $-+-$ and the trivial precoder $-\circ-$.

Theorem 1: The precoder that is optimal for Problem 1 must satisfy:

$$\mathrm{vec}\left(\boldsymbol{F}\right) = \int_0^{\frac{M-1}{M}\pi} \boldsymbol{s}(\boldsymbol{F}, \theta, g_{\mathrm{PSK}}, \mu) d\theta, \tag{36}$$

$$\mathrm{vec}\left(\boldsymbol{F}\right) = \int_0^{\frac{\pi}{2}} \boldsymbol{s}(\boldsymbol{F}, \theta, g_{\mathrm{PAM}}, \mu) d\theta, \tag{37}$$

$$\mathrm{vec}\left(\boldsymbol{F}\right) = \frac{1}{\sqrt{M}} \int_0^{\frac{\pi}{4}} \boldsymbol{s}(\boldsymbol{F}, \theta, g_{\mathrm{QAM}}, \mu) d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \boldsymbol{s}(\boldsymbol{F}, \theta, g_{\mathrm{QAM}}, \mu) d\theta. \tag{38}$$

for the M-PSK, M-PAM, and M-QAM constellations, respectively. μ is a positive scalar chosen such that the power constraint in (29) is satisfied.

Proof: The necessary condition for the optimality of Problem 1 is found by setting the derivative of the Lagrangian in (33) with respect to $\mathrm{vec}\left(\boldsymbol{F}^*\right)$ equal to the zero vector of the same size. Finding the derivative with respect to the complex valued vector $\mathrm{vec}\left(\boldsymbol{F}^*\right)$ can be done by generalizing the works in [18], [19] when the differentials of $\boldsymbol{F}$ and $\boldsymbol{F}^*$ are treated as independent. ■

Equations (36), (37), and (38) can be used in a fixed point iteration for finding the precoder that solves Problem 1. Notice that the positive constants μ' and μ are different.

VII. Results and Comparisons

Comparisons are made against a system not employing any precoding, i.e., $\boldsymbol{F} = \sqrt{\frac{P}{La\sigma_x^2 M_t}} \boldsymbol{I}_{M_t}$, since we have not found any explicit algorithm for minimizing PEP in the literature for precoded *correlated* Ricean channels. The SNR is defined as: $\mathrm{SNR} = 10 \log_{10} \frac{P}{N_0}$. $\sigma_x^2 = 1/2$, $P = 1$, $M_r = 6$, and 9-QAM were used. The OSTBC code $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{G}_4^T$ in [11] was used such that $a = 2$, $L = M_t = B = 4$, and $N = 8$. The channel statistics is given by $\left(\boldsymbol{R}\right)_{k,l} = 0.9^{|k-l|}$, where the notation $(\cdot)_{k,l}$ picks out element with row number k and column number l, $K = 1$ and $\bar{\boldsymbol{H}} = \boldsymbol{1}_{M_r \times M_t}$, where the matrix $\boldsymbol{1}_{k \times l}$ has size $k \times l$ containing only ones. Although $\boldsymbol{R}$ is not necessarily a practical case, this serves as a test-case for the proposed algorithm.

Figure 2 shows the SER versus SNR performance of the proposed system and a system not using precoding. It is seen from the figure that the proposed precoder outperforms the system not employing a precoder. It is seen from Figure 2 that when $\mathrm{SNR} \to \infty$, the performances of the systems approach each other. This is in accordance with Lemma 4.

VIII. Conclusions

For an arbitrary given OSTBC, exact SER expressions have been derived for a precoded MIMO system for communication over correlated Ricean channels. The receiver employs MLD and has knowledge of the exact channel coefficients, while the transmitter only knows the channel statistics, i.e., the Ricean factor, the LOS component, and the autocorrelation matrix of the fading component of the channel. An iterative method is proposed for finding the exact minimum SER precoder for M-PSK, M-PAM, and M-QAM signaling. The proposed precoders outperforms the trivially precoded OSTBC system. Several properties of the optimal precoder were identified.

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