# On the estimation of the Degrees of Freedom of In-door UWB channel

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# I. INTRODUCTION

Considering the growing interest of Personal Communication beyond 3G, UWB communication is presented as a serious candidates and a support technology for this kind of application. UWB systems are often defined as systems that have a relative bandwidth that is larger than 25% and/or an absolute bandwidth of more than 500MHz (FCC). The UWB using large absolute bandwidth, are robust to frequency-selective fading, which has significant implications on both, design and implementation. Additionnally, the spreading of the information over a very large frequency range decreases the spectral density and makes it compatible with existing systems. For designing and implementing any wireless system, channel sounding and modelling are a basic necessity. Several studies, theoretical and practical, have shown an extreme difference with to respect narrowband channels [1]. In previous work at Eurecom Institute, we characterized the second order statistics of indoor (UWB) channels using channel sounding techniques. The goal of this contribution is to analyse the impact of these extremely large systems bandwidth on the covariance matrix channel. We are primarily interested in assessing the growth in the number of degrees of freedom needed to characterize the channel as a function of the system bandwidth using the Akaike information criterion (AIC) and the Minimum Description Length (MDL). We are also interested in the root mean square (rms)delay spread behavior for a given threshold of received power (98% in the total received energy) as a function of the system bandwidth for both LOS and NLOS cases. The remaining of the paper is organized as follows. Section II describes the channel covariance matrix. In section III we outline the covariance matrix estimation and the two information theoretic criteria: the (AIC) and the (MDL) methods used for estimating the number of degrees of freedom (DoF) of the UWB propagation channel. Section IV describes the numerical results about the number of (DoF) and the (rms) delay spread. Finally section V presents the conclusions of this study.

# II. THE CHANNEL COVARIANCE MATRIX FORMULATION

The radio-propagation channel is randomly time-varying due to variations in the environment and mobility of transmitters and receivers. It is classically represented by its input delay-spread function  $h(t, \tau)$  called also, by abuse of language, the time-varying Channel Impulse Response (CIR). The variable t in the CIR notation represents the time-varying behavior of the channel caused by the mobility of either the transmitter, the receiver or the scatterers. The second variable  $\tau$  represents the delay domain in which we characterize the channel regarding the most important arriving paths. We consider for each measurement a fixed position at the transmitter and the receiver sides, and a static environment. We are thus considering a static channel and we can then simplify the notation of the CIR by dropping its dependence on t. Let  $\mathbf{h} = [h_{W,1}, h_{W,2}, ..., h_{W,N}]^T$ 

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be the matrix containing the N different impulse response for the N different antenna configurations, where  $h_{W,i}$  is expressed as

$$h_{W,i} = g_{W,i} + n_{W,i}(t), \ i = 1..N, \tag{1}$$

where  $n_{W,i}$  is zero-mean additive white Gaussian noise with power spectral density equal to  $\sigma_n^2$  at all frequencies in the bandwidth of interest. We neglect any non-linear perturbation caused by measurement elements, which were treated in a more general setting in [2]. In order to estimate the true covariance matrix  $\mathbf{K}_h$ , we use statistical averages based on observations from  $(20 \times 50)$  positions.

# III. ESTIMATION OF THE NUMBER OF DEGREES OF FREEDOM

# A. Covariance matrix estimation

The sample covariance matrix is a maximum-likelihood estimate, under the assumption of a large number of independent channel observations which arise from the different transmitting and receiving antenna positions. The covariance matrix of measured channel samples,  $\mathbf{h}$ , is written as

$$\mathbf{K}_{\mathbf{h}} = E[\mathbf{h}\mathbf{h}^{H}] = E[\mathbf{g}\mathbf{g}^{H}] + \sigma_{n}^{2}\mathbf{I}$$
(2)

where g is a vector of samples of the noise-free channel process, and I is the identity matrix. The maximum-likelihood covariance matrix estimate computed from N statistically independent channel observation with length p and p < N is given by

$$\mathbf{R} = \mathbf{K}_{\mathbf{h}}^{N} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{h}_{W,i} \mathbf{h}_{W,i}^{H}, \qquad (3)$$

In the context of our measurements, the multiple transmitter/receiver grid can equivalently be seen as a large( $50 \times 20$ ) MIMO system.

# B. Information theoretic criteria

Wax and Kailath [3] presented a new approach for estimating the number of signals in multichannel time-series and frequencyseries, based on statistical classification criteria (AIC) and (MDL). The covariance matrix  $\mathbf{R}$  is Hermitian and positive definite. The (AIC) criterion is given by:

$$AIC(k) = -2log\left(\frac{\prod_{i=k+1}^{p} \lambda_i(\mathbf{h})^{\frac{1}{(p-k)}}}{\frac{1}{p-k} \sum_{i=k+1}^{p} \lambda_i(\mathbf{h})}\right)^{N(p-k)} + 2k(2p-k)$$
(4)

and in [4] the MDL criterion is given as follows:

$$MDL(k) = -log\left(\frac{\prod_{i=k+1}^{p}\lambda_{i}(\mathbf{h})^{\frac{1}{(p-k)}}}{\frac{1}{p-k}\sum_{i=k+1}^{p}\lambda_{i}(\mathbf{h})}\right)^{N(p-k)}$$
$$+log(N)\frac{k(2p-k+1)}{4}$$
(5)

where the  $\lambda_i(\mathbf{h})$  are the eigenvalues of the covariance matrix **R**. The number of (DoF), possibly the number of significant eigenvalues, is determined as the value of  $k \in \{0, 1, ..., p - 1\}$  which minimizes the value of (4) or (5). In this work, the number of DoF represents the number of unitary dimension independant channels that constitute an UWB channel.

# IV. RESULTS AND ANALYSIS ABOUT UWB CHANNEL

In this section, we present and analyze the results obtained from the UWB channel measurement conducted at Eurecom. Figure 1 considers LOS and NLOS measurement scenarios. We plot the (AIC) and (MDL) functions for two different bandwidths typically 200MHz and 6GHz. The minimum of (AIC) or (MDL) curves gives the number of significant eigenvalues. As a matter of fact, we see that the number of DoF increases with bandwidth but not linearly. Thus, for 200MHz bandwidth, we capture 98% of the energy with 25 significant eigenvalues wheras for 6GHz channel bandwidths the number of eigenvalues is 50. To illustrate the relationship between number of (DoF) and system bandwidth, we recall that for a signal with duration T and frequency band  $\Delta W$ , the number of (DoF) of the signal space  $N_{dof}$  is given by [5]

$$N_{dof} = T \cdot \Delta W + 1. \tag{6}$$

Generally [6], we find that if one transmits a band limited and time limited signal over a fading channel with rms delay spread  $T_d$ , the channel (DoF) N is approximately

$$N = T_d \cdot \Delta W. \tag{7}$$

To investigate deeply the validity of this relationship for UWB channels, we measure the evolution of the rms delay spread with the frequency bandwidth, for both LOS and NLOS cases, for one fixed threshold of received energy -20 dB attenuation regarding the first arriving path. Then we plot, on figure 2, the computed number of (DoF) following equation (7). We thene compare this result with the number of (DoF) obtained by (AIC) criterion from measurements. For 98% of the captured energy, we notice that the number of eigenvalues using the relationship in (7) increases linearly with the bandwidth for both LOS and NLOS case. In opposition, the number of eigenvalues calculated directly from measurements by (AIC) tends towards saturation beyond 2000MHz frequency bandwidth for LOS case and beyond 1500MHz frequency banwidth for NLOS case. We remark also, that for lower frequencies (below 800MHz for NLOS and 1500MHz for LOS settings), the number of DoF by (AIC) is higher than that one obtained following equation (7). In fact, the measured number of (DoF) based on (AIC) is computed from the total channel impulse response obtained by IFFT while in the other case we focus on the time limited channel impulse response troncated at  $T_d$ . Hence the difference between both computed DoF comes from energy outside the  $T_d$  interval.

### V. CONCLUSION

In this work, we showed the (AIC) and the (MDL) are two techniques to estimate the number of (DoF) of an UWB channel in an in-door environment. We also studied the evolution of the rmsdelay spread behavior,  $T_d$ , as a function of frequency bandwidth based on a measurements campaign carried out at Eurecom Mobile Communication laboratory. We compared the (AIC) result with the number of (DoF) obtained by the product of  $T_d$  by frequency bandwidth. This comparison, pointed out that the number of DoF for a given UWB channel saturates beyond a certain frequency and does not increase linearly.



Fig. 1. The min of AIC and MDL



Fig. 2. Evolution of the number of (DoF) for LOS and NLOS cases

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