Minimum Exact SER Precoding of Orthogonal Space-Time Block Codes for Correlated MIMO Channels

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***Abstract*— A memoryless precoder is designed for orthogonal space-time block codes for multiple-input multiple-output channels exhibiting joint transmit-receive correlation. Unlike most previous similar work which concentrate on transmit correlation only and pair-wise error probability metrics, the precoder is designed to minimize the *exact* symbol error rate as function of the channel correlation coefficients, which are fed back to the transmitter, and the correlation may or may not follow the so-called *Kronecker structure*. The proposed method can handle general propagation settings including those arising form a cooperative macro-diversity (multi-base) scenario. We present two algorithms. This first is suboptimal, but provide a simple closed-form precoder that handles the case of uncorrelated transmitters, correlated receivers. The second is a fast-converging numerical optimization which covers the general case. The results show the superiority of the exact SER metric over the PEP as a precoding metric and the impact of "non-Kronecker" channels in the overall performance.**

I. Introduction

In the area of efficient communications over non-reciprocal MIMO channels, recent research has demonstrated the value of feeding back to the transmitter information about channel state observed at the receiver. Among those, there has been a growing interest in transmitter schemes that can exploit low-rate long-term statistical channel state information in the form of antenna correlation coefficients. So far, emphasis has been on designing precoders for space-time block coded (STBC) [1] signals or spatially multiplexed streams that are adjusted based on the knowledge of the transmit correlation only while the receiving antennas are uncorrelated [2], [3], [4], [5]. These techniques are well suited to downlink situation where an elevated access point (situated above the surrounding clutter) transmits to a subscriber placed in a rich scattering environment. Although simple models exist for the joint transmit receiver correlation based on the well known Kronecker structure [1], the accuracy of these models has recently been questioned in the literature based on measurement campaigns [6]. Therefore, there is interest in investigating the precoding of orthogonal space-time coded (OSTBC) signals for MIMO channels that *do not* necessarily follow the Kronecker structure.

Methods have been proposed previously in the field of precoder design based on correlation knowledge on the transmitter side. For instance, an upper bound of the PEP is minimized in [2] for transmit-only correlation, and for full channel correlation in [7], [8]. In [9], the exact SER expressions were derived for *one* receiver antenna employing maximum ratio combiner at the receiver and a bound of the exact error probability was used as the optimization criterion. With only one receiver antenna, no receiver correlation can be included in the model unfortunately. In [10], exact SER expressions were found for uncorrelated MIMO channels that are precoded with the identity matrix as the precoder.

In this paper, we address the problem of linear precoding of OSTBC signals launched over a jointly transmit-receive correlated MIMO channel. Our main contributions are: We derive *exact* expressions for the average SER for a system where the transmitter has an OSTBC followed by a full precoder matrix and where the receiver also has multiple antennas and is using MLD. The transmitter knows the correlation matrix of the channel transfer matrix and the receiver knows the channel realization exactly. We propose an iterative numerical technique for minimizing the exact SER with respect to the precoder matrix. This is contrast with previous PEP-based precoders. Several properties of the optimal precoder are presented. We identify cases in which the precoder is dependent *or not* of the *receive* correlation matrix. An analytical closed-form precoder is proposed as an approximation based on the hereby proposed *maximum diversity principle* in the particular case of cooperative diversity. This solution is also easily interpretable.

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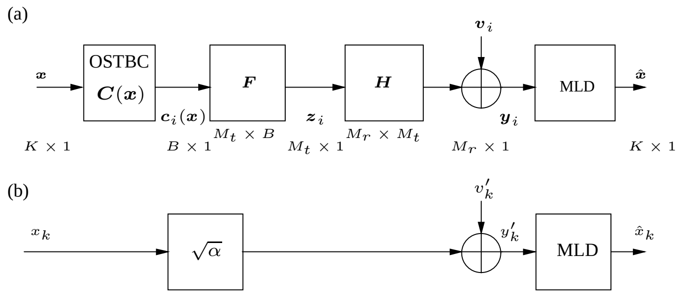

Fig. 1. Block model of the linear precoded OSTBC MIMO system.

II. System Description

A. OSTBC Signal Model

Figure 1 (a) shows the block MIMO system model with M_t and M_r transmitter and receiver antennas, respectively. One block of K signal samples $x_0, x_1, \ldots, x_{K-1}$ is transmitted by means of an OSTBC matrix $\boldsymbol{C}(\boldsymbol{x})$ of size $B \times N$, where B and N are the space and time dimension of the given OSTBC, respectively, and $\boldsymbol{x} = [x_0, x_1, \ldots, x_{K-1}]^T$. It is assumed that the OSTBC is given. Let $x_i \in \mathcal{A}$, where $\mathcal{A}$ is a signal constellation set such as M-PAM, M-QAM, or M-PSK. The OSTBC returns an $B \times N$ matrix $\boldsymbol{C}(\boldsymbol{x})$ that is dependent on $\boldsymbol{x}$. If bits are used as inputs to the system, $K \log_2 |\mathcal{A}|$ bits are used to produce the vector $\boldsymbol{x}$, where $|\cdot|$ denotes cardinality. Assume that $E\left[|x_i|^2\right] = \sigma_x^2$, and that the matrix that comes out of the OSTBC is denoted $\boldsymbol{C}(\boldsymbol{x})$ is of size $B \times N$. Since the OSTBC is orthogonal, the following holds

$$\boldsymbol{C}(\boldsymbol{x})\boldsymbol{C}^H(\boldsymbol{x}) = a \sum_{i=0}^{K-1} |x_i|^2 \boldsymbol{I}_B, \tag{1}$$

where $a = 1$ if $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{G}_2^T$, $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{H}_3^T$, or $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{H}_4^T$ in [11] and $a = 2$ if $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{G}_3^T$ or $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{G}_4^T$ in [11]. The rate of the code is K/N. Other OSTBC can be used as well. The codeword matrix $\boldsymbol{C}(\boldsymbol{x})$ has size $B \times N$ and can be expressed as:

$$\boldsymbol{C}(\boldsymbol{x}) = \left[\boldsymbol{c}_0(\boldsymbol{x}) \;\; \boldsymbol{c}_1(\boldsymbol{x}) \;\; \cdots \;\; \boldsymbol{c}_{N-1}(\boldsymbol{x})\right], \tag{2}$$

where $\boldsymbol{c}_i(\boldsymbol{x})$ is the ith column vector of $\boldsymbol{C}(\boldsymbol{x})$ and it has size $B\times 1$.

Before each code vector $\boldsymbol{c}_i(\boldsymbol{x})$ is launched into the channel, it is precoded with a memoryless complex-valued matrix $\boldsymbol{F}$ of size $M_t\times B$, so the $M_r\times 1$ receive signal vector $\boldsymbol{y}_i$ becomes

$$\boldsymbol{y}_i = \boldsymbol{H}\boldsymbol{F}\boldsymbol{c}_i(\boldsymbol{x}) + \boldsymbol{v}_i, \tag{3}$$

where the additive noise on the channel $\boldsymbol{v}_i$ is complex Gaussian circularly distributed with independent components having variance N_0 and $\boldsymbol{H}$ is the channel transfer MIMO matrix. Let the vectors $\boldsymbol{y}_i$ and $\boldsymbol{v}_i$ be collected into the matrices $\boldsymbol{Y}$ and $\boldsymbol{V}$, respectively, of size $M_r\times N$ in the following way:

$$\boldsymbol{Y} = \begin{bmatrix}\boldsymbol{y}_0 & \boldsymbol{y}_1 & \cdots & \boldsymbol{y}_{N-1}\end{bmatrix}, \tag{4}$$

$$\boldsymbol{V} = \begin{bmatrix}\boldsymbol{v}_0 & \boldsymbol{v}_1 & \cdots & \boldsymbol{v}_{N-1}\end{bmatrix}. \tag{5}$$

Then, the input-output relationship for the MIMO system can be expressed:

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{F}\boldsymbol{C}(\boldsymbol{x}) + \boldsymbol{V}. \tag{6}$$

The receiver is assumed to know the channel matrix $\boldsymbol{H}$ and the precoding matrix $\boldsymbol{F}$ exactly, and it performs maximum likelihood decoding (MLD) of blocks of length N.

B. Correlated Channel Models

A quasi-static non-frequency selective correlated Rayleigh fading channel model [1] is assumed. Let $\boldsymbol{R}$ be the general $M_tM_r\times M_tM_r$ positive definite autocorrelation matrix for the channel coefficients. A channel realization of the correlated channel can then be found by

$$\operatorname{vec}(\boldsymbol{H}) = \boldsymbol{R}^{1/2}\operatorname{vec}(\boldsymbol{H}_w), \tag{7}$$

where $\boldsymbol{R}^{1/2}$ is the unique positive definite matrix square root [12] of $\boldsymbol{R}$, $\boldsymbol{H}_w$ has size $M_r\times M_t$ and is complex Gaussian circularly distributed with independent components all having unit variance, and the operator $\operatorname{vec}(\cdot)$ stacks the columns of the matrix it is applied to into a long column vector [12].

Kronecker model: A special case of the model above is as follows [1]

$$\boldsymbol{H} = \boldsymbol{R}_r^{1/2}\boldsymbol{H}_{\boldsymbol{w}}\boldsymbol{R}_t^{1/2}, \tag{8}$$

where the matrices $\boldsymbol{R}_r$ and $\boldsymbol{R}_t$ are the correlations matrices of the receiver and transmitter, respectively, and their sizes are $M_r\times M_r$ and $M_t\times M_t$. The full autocorrelation matrix $\boldsymbol{R}$ of the model in Equation (8) is then given by

$$\boldsymbol{R} = E\left[\operatorname{vec}(\boldsymbol{H})\operatorname{vec}^H(\boldsymbol{H})\right] = \boldsymbol{R}_t^T\otimes\boldsymbol{R}_r, \tag{9}$$

where the operator $(\cdot)^T$ denotes transposition and $\otimes$ is the Kronecker product. Unlike Equation (9), the general model considers that the receive (or transmit) correlation depends on at which transmit (or receive) antenna the measurements are performed.

C. Equivalent Single-Input Single-Output Model

Define the matrix $\boldsymbol{\Phi}$ of size $M_tM_r\times M_tM_r$ as:

$$\boldsymbol{\Phi} = \boldsymbol{R}^{1/2}\left[\left(\boldsymbol{F}^*\boldsymbol{F}^T\right)\otimes\boldsymbol{I}_{M_r}\right]\boldsymbol{R}^{1/2}. \tag{10}$$

This matrix plays an important role in the developed theory. Define the real non-negative scalar α by

$$\alpha = \|\boldsymbol{H}\boldsymbol{F}\|_F^2 = \operatorname{vec}^H(\boldsymbol{H}_w)\,\boldsymbol{\Phi}\operatorname{vec}(\boldsymbol{H}_w), \tag{11}$$

where $\|\cdot\|_F$ is the Frobenius norm. Since the matrix $\boldsymbol{H}_w$ contains unit variance uncorrelated variables, $E\left[\operatorname{vec}(\boldsymbol{H}_w)\operatorname{vec}^H(\boldsymbol{H}_w)\right] = \boldsymbol{I}_{M_tM_r}$. The expected value of α can now be found:

$$\begin{aligned}E[\alpha] &= E\left[\operatorname{vec}^H(\boldsymbol{H}_w)\,\boldsymbol{\Phi}\operatorname{vec}(\boldsymbol{H}_w)\right]\\ &= \operatorname{Tr}\left\{\boldsymbol{\Phi}E\left[\operatorname{vec}(\boldsymbol{H}_w)\operatorname{vec}^H(\boldsymbol{H}_w)\right]\right\} = \operatorname{Tr}\{\boldsymbol{\Phi}\}.\end{aligned} \tag{12}$$

By generalizing the approach given in [10], [13] to include a *full* complex-valued precoder $\boldsymbol{F}$ of size $M_t\times B$ and having a *full* channel correlation matrix $\boldsymbol{R}$ the OSTBC system can be shown to be equivalent with a system having the following output input relationship

$$y'_k = \sqrt{\alpha}x_k + v'_k, \tag{13}$$

for $k\in\{0,1,\ldots,K-1\}$, and where $v'_k\sim\mathcal{CN}(0,N_0/a)$ is complex circularly distributed. This signal is fed into a memoryless MLD that is designed from the signal constellation of the source symbols $\mathcal{A}$. The equivalent single-input single-output (SISO) model is shown in Figure 1 (b).

III. SER Expressions for Given Received SNR

By considering the SISO system in Figure 1 (b), it is seen that the instantaneous received SNR γ per source symbol is given by

$$\gamma = \frac{a\sigma_x^2\alpha}{N_0} = \delta\alpha, \tag{14}$$

where $\delta = \frac{a\sigma_x^2}{N_0}$. The expected received signal to noise ratio is given by: $E[\gamma] = \frac{a\sigma_x^2\operatorname{Tr}\{\boldsymbol{\Phi}\}}{N_0}$.

In order to simplify the expressions, the following three signal constellation dependent constants are defined

$$g_{\text{PSK}} = \sin^2\frac{\pi}{M},\; g_{\text{PAM}} = \frac{3}{M^2-1},\; g_{\text{QAM}} = \frac{3}{2(M-1)}. \tag{15}$$

The symbol error probability $\text{SER}_\gamma \triangleq \Pr\{\text{Symbol error}|\gamma\}$ for a given γ for M-PSK, M-PAM, and M-QAM signalling given by [14]

$$\text{SER}_\gamma = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} e^{-\frac{g_{\text{PSK}}\gamma}{\sin^2(\theta)}}d\theta, \tag{16}$$

$$\text{SER}_\gamma = \frac{2}{\pi}\frac{M-1}{M}\int_0^{\frac{\pi}{2}} e^{-\frac{g_{\text{PAM}}\gamma}{\sin^2(\theta)}}d\theta, \tag{17}$$

$$\begin{aligned}\text{SER}_\gamma = \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)&\left[\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{-\frac{g_{\text{QAM}}\gamma}{\sin^2(\theta)}}d\theta\right.\\ &\left.+\frac{1}{\sqrt{M}}\int_0^{\frac{\pi}{4}} e^{-\frac{g_{\text{QAM}}\gamma}{\sin^2(\theta)}}d\theta\right],\end{aligned} \tag{18}$$

respectively.

IV. Exact SER Expressions

The moment generating function of the probability density function $p_\gamma(\gamma)$ is defined as $\phi_\gamma(\gamma) = \int_0^\infty p_\gamma(\gamma)e^{s\gamma}d\gamma$. Since all the K source symbols go through the same SISO system in Figure 1 (b), the average SER of the MIMO system can be found as

$$\begin{aligned}\text{SER} \triangleq \Pr\{\text{Error}\} &= \int_0^\infty \Pr\{\text{Error}|\gamma\}\,p_\gamma(\gamma)d\gamma\\ &= \int_0^\infty \text{SER}_\gamma\, p_\gamma(\gamma)d\gamma.\end{aligned} \tag{19}$$

This integral can be rewritten by means of the moment generating function of γ.

From Equation (11) and the fact that all the elements of $\boldsymbol{H}_w$ is independent and complex Gaussian distributed with zero mean and unit variance, it follows that the moment generating function of α is given by:

$$\phi_\alpha(s) = \frac{1}{\prod_{i=0}^{M_tM_r-1}(1-\lambda_i s)}, \tag{20}$$

where λ_i is eigenvalue number i of the positive semi-definite matrix $\boldsymbol{\Phi}$. Since $\gamma = \delta\alpha$, the moment generating function of γ is given by:

$$\phi_\gamma(s) = \phi_\alpha(\delta s) = \frac{1}{\prod_{i=0}^{M_t M_r - 1} (1 - \delta\lambda_i s)}. \tag{21}$$

By using Equation (19) and the definition of the moment generating function together with the result in Equation (21) it is possible to express the exact SER for all the signal constellations in terms of the eigenvalues λ_i of the matrix $\boldsymbol{\Phi}$. When finding the necessary conditions for the optimal precoder, eigenvalues that are not simple might case difficulties in connection with calculations of derivatives. Therefore, it is useful to rewrite the expressions for the SER in terms of the full matrix $\boldsymbol{\Phi}$. This can be done by utilizing the eigen-decomposition of this matrix. The result of all these operations led to the following expressions for the SER for M-PSK, M-PAM, and M-QAM

$$\mathrm{SER} = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{d\theta}{\det\left(\boldsymbol{I}_{M_t M_r} + \delta\frac{g_{\mathrm{PSK}}}{\sin^2\theta}\boldsymbol{\Phi}\right)}, \tag{22}$$

$$\mathrm{SER} = \frac{2}{\pi}\frac{M-1}{M}\int_0^{\frac{\pi}{2}} \frac{d\theta}{\det\left(\boldsymbol{I}_{M_t M_r} + \delta\frac{g_{\mathrm{PAM}}}{\sin^2\theta}\boldsymbol{\Phi}\right)}, \tag{23}$$

$$\mathrm{SER} = \frac{4}{\pi}\frac{\sqrt{M}-1}{\sqrt{M}}\left[\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{d\theta}{\det\left(\boldsymbol{I}_{M_t M_r} + \delta\frac{g_{\mathrm{QAM}}}{\sin^2\theta}\boldsymbol{\Phi}\right)} + \frac{1}{\sqrt{M}}\int_0^{\frac{\pi}{4}} \frac{d\theta}{\det\left(\boldsymbol{I}_{M_t M_r} + \delta\frac{g_{\mathrm{QAM}}}{\sin^2\theta}\boldsymbol{\Phi}\right)}\right], \tag{24}$$

respectively. It is seen that Equations (22) and (23) gives the same result when $M = 2$. This is not surprising, since, the constellations of 2-PSK and 2-PAM are identical. When $M = 4$, it can be shown that Equations (22) and (24) return the same result. If $\boldsymbol{R} = \boldsymbol{I}_{M_t M_r}$ and $\boldsymbol{F} = \boldsymbol{I}_{M_t}$, then the performance expressions are reduced to the results found in [10] and simulations result in the exact same results as reported in [10].

V. Precoding of OSTBC signals

A. Power Constraint

Then OSTBC is used, Equation (1) holds and the average power constraint on the transmitted block $\boldsymbol{Z} \triangleq \boldsymbol{F}\boldsymbol{C}(\boldsymbol{x})$ can be expressed as

$$aK\sigma_x^2 \operatorname{Tr}\left\{\boldsymbol{F}\boldsymbol{F}^H\right\} = P, \tag{25}$$

where P is the average power used by the transmitted block $\boldsymbol{Z}$.

B. Optimal Precoder Problem Formulation

The goal is to find the matrix $\boldsymbol{F}$ such that the exact SER is minimized under the power constraint. We propose that the optimal precoder is given by the following optimization problem:

Problem 1:

$$\min_{\left\{\boldsymbol{F}\in\mathbb{C}^{M_t\times B}\right\}} \quad \mathrm{SER}$$
$$\text{subject to} \quad Ka\sigma_x^2 \operatorname{Tr}\left\{\boldsymbol{F}\boldsymbol{F}^H\right\} = P.$$

Remark 1: The optimal precoder is dependent on the value of N_0 and therefore also of the signal to noise ratio (SNR).

C. Properties of Optimal Precoder

Lemma 1: If $\boldsymbol{F}$ is an optimal solution of Problem 1, then the precoder $\boldsymbol{F}\boldsymbol{U}$, where $\boldsymbol{U} \in \mathbb{C}^{B\times B}$ is unitary, is also optimal.

Proof: Let $\boldsymbol{F}$ be an optimal solution of Problem 1 and $\boldsymbol{U} \in \mathbb{C}^{B\times B}$, be an arbitrary unitary matrix. It is then seen by insertion that the objective function and the power constraint are unaltered by the unitary matrix. ■

Lemma 2: If $N_0 \to 0^+$ and $B = M_t$, then the optimal precoder is given by the trivial precoder $\boldsymbol{F} = \sqrt{\frac{P}{Ka\sigma_x^2 M_t}}\boldsymbol{I}_{M_t}$ for the M-PSK, M-PAM, and M-QAM constellations.

Proof: See [15]. ■

Lemma 3: If $M_t = B$ and $\boldsymbol{R} = \boldsymbol{I}_{M_t M_r}$, then the optimal precoder is given by the trivial precoder $\boldsymbol{F} = \sqrt{\frac{P}{Ka\sigma_x^2 M_t}}\boldsymbol{I}_{M_t}$ for the M-PSK, M-PAM, and M-QAM constellations.

Proof: See [15]. ■

Lemma 4: Let $B = M_t$. If only receiver correlation is present, the the total correlation matrix can be expressed as

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_{r_0} & \boldsymbol{0}_{M_r\times M_r} & \cdots & \boldsymbol{0}_{M_r\times M_r} \\ \boldsymbol{0}_{M_r\times M_r} & \boldsymbol{R}_{r_1} & \cdots & \boldsymbol{0}_{M_r\times M_r} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0}_{M_r\times M_r} & \boldsymbol{0}_{M_r\times M_r} & \cdots & \boldsymbol{R}_{r_{M_t-1}} \end{bmatrix}, \tag{26}$$

where $\boldsymbol{R}_{r_i}$ is the receive correlation matrix seen by transmitter number i and the matrix $\boldsymbol{0}_{k\times l}$ has size $k\times l$ containing only zeroes. Then, the optimal $\boldsymbol{F}$ can be chosen diagonal up to a unitary matrix. Without loss of optimality, the precoding matrix can also be chosen real with non-negative diagonal elements.

Proof: See [15]. ■

Lemma 5: Let the correlation model of the channel follow the Kronecker model in Equation (9) and assume that $B = M_t$. If $\boldsymbol{R}_t = \boldsymbol{I}_{M_t}$, then the optimal precoder is independent of the receiver correlation matrix $\boldsymbol{R}_r$ and given by $\boldsymbol{F} = \sqrt{\frac{P}{Ka\sigma_x^2 M_t}}\boldsymbol{I}_{M_t}$.

Proof: See [15]. ■

D. A Closed-Form Solution for Independent-Transmit Correlated-Receive Antennas

In this subsection, we derive a method to obtain a closed-form expression for the precoder in the particular case when the transmit antennas are uncorrelated but the receive antennas are not. The examples of this situation include the possibly non-Kronecker scenario described in [6] where the the transmit antennas are sufficiently spaced to be (close to) uncorrelated. For the sake of exposition we limit ourselves to the case of $B = M_t = 2$, however, the approach can be extended to $B = M_t > 2$ as well. The number of receive antenna remains arbitrary.

From Lemma 4, we know that when the transmit antennas are uncorrelated, the optimal precoder boils down to a diagonal precoder, i.e., the precoder amounts to a power allocation strategy. Here, we attempt to find the optimal power weights analytically. For the sake of exposition, we assume Alamouti [16] OSTBC, i.e., $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{G}_2^T$ from [11]. We also take the following normalization: $\frac{P}{aK\sigma_x^2} = 1$.

1) Equivalent I.I.D. Channel Formulation: Our strategy below consists in rewriting the independent-transmit correlated-receive channel model into an equivalent i.i.d. MIMO channel with power weights depending on both i) the correlation matrix, ii) the precoder coefficients.

Let $\boldsymbol{H} = [\boldsymbol{h}_0 \;\; \boldsymbol{h}_0]$. From Equation (7), it follows that

$$\boldsymbol{h}_i = \boldsymbol{R}_{r_i}^{1/2}\boldsymbol{h}_{w_i}, \tag{27}$$

where $\boldsymbol{h}_{w_i} = \left[h_{w_{i_0}}, h_{w_{i_1}}, \ldots, h_{w_{i_{M_r-1}}}\right]^T$ contains unit variance complex Gaussian circularly distributed independent variables. Let $\boldsymbol{R}_{r_i} = E\left[\boldsymbol{h}_i\boldsymbol{h}_i^H\right]$ have the following eigenvalue decomposition:

$$\boldsymbol{R}_{r_i} = \boldsymbol{V}_{r_i}\boldsymbol{\Lambda}_{r_i}\boldsymbol{V}_{r_i}^H. \tag{28}$$

From the equivalent SISO model in Equation (13), it is seen that all the K original samples are going through the same SISO system. Therefore, it is sufficient to consider any of the K samples x_k. Since the precoder matrix $\boldsymbol{F}$, of size 2×2, is a diagonal matrix satisfying the power constraint in Equation (25) with $\frac{P}{aK\sigma_x^2} = 1$, it follows that $f_0^2 + f_1^2 = 1$, where f_i is diagonal element number i of the matrix $\boldsymbol{F}$. From Equations (10) and (11), it is seen that the signal amplification (α) of x_k in the equivalent SISO model can be expressed as:

$$\begin{aligned}\alpha =& f_0^2\|\boldsymbol{h}_0\|^2 + f_1^2\|\boldsymbol{h}_1\|^2 \\ =& f_0^2 \sum_{j=0}^{M_r-1} \lambda_{r_{0_j}} |h'_{w_{0_j}}|^2 + f_1^2 \sum_{j=0}^{M_r-1} \lambda_{r_{1_j}} |h'_{w_{1_j}}|^2. \end{aligned} \quad (29)$$

where the variable $h'_{w_{i_j}}$ is the jth component of the vector $\boldsymbol{V}_{r_i}^H \boldsymbol{h}_{w_i}$. Since $\boldsymbol{V}_{r_i}$ is unitary, each of the variables $h'_{w_{i_j}}$ has the same distribution as the variables $h_{w_{i_j}}$, i.e., they are independent complex Gaussian distributed with zero mean and unit variance.

2) Maximum Diversity Principle: In this subsection, we examine the factor multiplying x_k in Equation (29) and invoke the *maximum diversity principle* in order to determine optimal power weights f_0 and f_1 in closed form. Note that we do not claim optimality of the approach below in terms of symbol error rate, although we do conjecture the obtained coefficients are (close to) optimal in that sense as well. We observe that for diagonal precoders α is equal to a sum of $2M_r$ uncorrelated diversity branches weighted by power terms. According to our proposed maximum diversity principle, we make those weights as similar to each other as possible in order to spread the symbol energy evenly across all diversity branches. Mathematically, this is realized through the following minimum variance problem:

$$\min_{f_0, f_1 \geq 0} \sum_{i=0}^{1} \sum_{j=0}^{M_r-1} \left(f_i^2 \lambda_{r_{i_j}} - \frac{1}{2M_r} \sum_{i=0}^{1} \sum_{j=0}^{M_r-1} f_i^2 \lambda_{r_{i_j}} \right)^2 \quad (30)$$
$$\text{subject to} \quad f_0^2 + f_1^2 = 1,$$

Interestingly, we notice that the empirical mean, defined as m, used in the expression above is independent of the precoder: Since $\operatorname{Tr}\{\boldsymbol{R}_{r_i}\} = M_r \ \forall \ i$ we have

$$m = \frac{1}{2M_r} \sum_{i=0}^{1} \sum_{j=0}^{M_r-1} f_i^2 \lambda_{r_{i_j}} = \frac{1}{2M_r}(f_0^2 M_r + f_1^2 M_r) = \frac{1}{2}.$$

So our problem can be rewritten simply into

Problem 2:

$$\min_{f_0, f_1 \geq 0} \sum_{i=0}^{1} \sum_{j=0}^{M_r-1} \left(f_i^2 \lambda_{r_{i_j}} - \frac{1}{2} \right)^2 \quad (31)$$
$$\text{subject to} \quad f_0^2 + f_1^2 = 1.$$

Lemma 6: We parametrize the precoder according to $f_0 = \cos(\theta)$, $f_1 = \sin(\theta)$ where θ is arbitrary in $\left[0, \frac{\pi}{2}\right]$. The solution to Problem 2 is given in terms of θ by:

$$\tan\theta = \sqrt{\frac{\sum_{j=0}^{M_r-1} \lambda_{r_{0_j}}^2}{\sum_{j=0}^{M_r-1} \lambda_{r_{1_j}}^2}}. \quad (32)$$

Proof: See [15]. ■

This results can be interpreted as follows: The power allocation on a given transmit antenna is proportional to the receive-correlation eigenvalue spread "experienced" by this antenna.

3) Examples:

Example 1 (Precoding for Kronecker Correlation): Let $\boldsymbol{R}$ be modeled according to Equation (26) with $\boldsymbol{R}_{r_0} = \boldsymbol{R}_{r_1}$. In this case, the eigenvalues are characterized by

$$\lambda_{r_{0_j}} = \lambda_{r_{1_j}} \quad (33)$$

which according to Equation (32) yields $f_0^2 = f_1^2 = \frac{1}{2}$. In other words, if the transmit antennas are uncorrelated and the receive antenna are correlated but in a way that is independent of which transmit antenna is taken, then the best strategy is to pour power equally across the transmit antennas, which makes good intuitive sense. It means that the fact that the receive antennas are correlated, cannot be compensated for at the transmitter through precoding of the orthogonal STBC signals in the Kronecker case.

Example 2 (Precoding for Non-Kronecker Correlation): Here, we assume that the two transmit antennas are uncorrelated and "see" two widely different *receive* correlation matrices. This may happen for instance for widely spaced transmit antennas, or transmit antennas located on distinct access points, such as the cooperative diversity scenario in [17]. We assume an extreme case where transmit antenna number 0 sees an uncorrelated receiver $\boldsymbol{R}_{r_0} = \boldsymbol{I}$. This corresponds to a link with M_r orders of diversity with a wide angle spread in the direction of arrival. While antenna number 1 sees a fully correlated receiver $\boldsymbol{R}_{r_1} = \boldsymbol{1}_{M_r \times M_r}$, where the matrix $\boldsymbol{1}_{M_r \times M_r}$ contains only ones and has size $M_r \times M_r$.

In this case, $\lambda_{r_{0_i}} = 1 \ \forall \ i \in \{0, 1, \ldots, M_r - 1\}$ and $\lambda_{r_{1_1}} = M_r$ and $\lambda_{r_{1_i}} = 0 \ \forall \ i \in \{1, \ldots, M_r - 1\}$. This yields directly $\tan\theta = \sqrt{\frac{1}{M_r}}$, thus,

$$f_0^2 = \frac{M_r}{M_r + 1}, \quad f_1^2 = \frac{1}{M_r + 1}.$$

VI. Optimization Algorithm

Let the matrix $\boldsymbol{K}_{k,l}$ be the commutation matrix [18] of size $kl \times kl$. The constrained maximization Problem 1 can be converted into an unconstrained optimization problem by introducing a Lagrange multiplier μ'. This is done by defining the following Lagrange function:

$$\mathcal{L}(\boldsymbol{F}) = \text{SER} + \mu' \operatorname{Tr}\left\{\boldsymbol{F}\boldsymbol{F}^H\right\}. \quad (34)$$

Since the objective function should be minimized, $\mu' > 0$. Define the $M_t^2 \times M_t^2 M_r^2$ matrix $\boldsymbol{L}$ as

$$\boldsymbol{L} = \left[\boldsymbol{I}_{M_t^2} \otimes \operatorname{vec}^T(\boldsymbol{I}_{M_r})\right] \left[\boldsymbol{I}_{M_t} \otimes \boldsymbol{K}_{M_t, M_r} \otimes \boldsymbol{I}_{M_r}\right]. \quad (35)$$

In order to present the results compactly, define the following $BM_t \times 1$ vector $\boldsymbol{s}(\boldsymbol{F}, \theta, g, \mu)$:

$$\begin{aligned}\boldsymbol{s}(\boldsymbol{F}, \theta, g, \mu) =& \mu \left[\boldsymbol{F}^T \otimes \boldsymbol{I}_{M_t}\right] \boldsymbol{L} \left[\boldsymbol{R}^{1/2} \otimes \left(\boldsymbol{R}^{1/2}\right)^*\right] \\ &\times \frac{\operatorname{vec}\left(\left[\boldsymbol{I}_{M_t M_r} + \delta \frac{g}{\sin^2(\theta)} \boldsymbol{\Phi}^*\right]^{-1}\right)}{\sin^2(\theta) \det\left(\boldsymbol{I}_{M_t M_r} + \delta \frac{g}{\sin^2(\theta)} \boldsymbol{\Phi}\right)}. \end{aligned} \quad (36)$$

Lemma 7: The precoder that is optimal for Problem 1 must satisfy:

$$\operatorname{vec}(\boldsymbol{F}) = \int_0^{\frac{M-1}{M}\pi} \boldsymbol{s}(\boldsymbol{F}, \theta, g_{\text{PSK}}, \mu) d\theta, \quad (37)$$

$$\operatorname{vec}(\boldsymbol{F}) = \int_0^{\frac{\pi}{2}} \boldsymbol{s}(\boldsymbol{F}, \theta, g_{\text{PAM}}, \mu) d\theta, \quad (38)$$

$$\operatorname{vec}(\boldsymbol{F}) = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \boldsymbol{s}(\boldsymbol{F}, \theta, g_{\text{QAM}}, \mu) d\theta + \frac{1}{\sqrt{M}} \int_0^{\frac{\pi}{4}} \boldsymbol{s}(\boldsymbol{F}, \theta, g_{\text{QAM}}, \mu) d\theta. \quad (39)$$

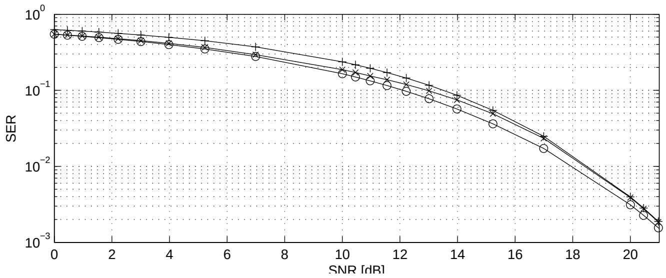

Fig. 2. Scenario 1: SER versus SNR performance of the proposed minimum SER precoder $-\circ-$, the trivial precoder $-+-$, and the minimum PEP precoder $-\times-$ proposed in [8].

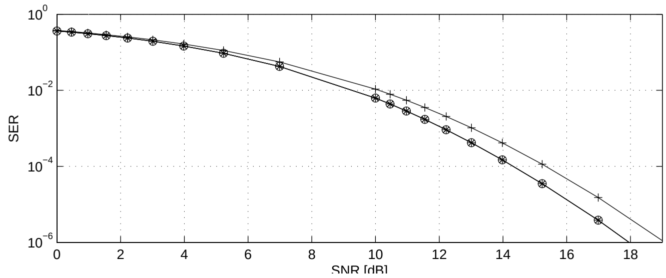

Fig. 3. Scenario 2: SER versus SNR performance of the proposed precoder $-\circ-$ in Lemma 7, the PEP precoder $-\times-$, the precoder in Lemma 6, and the trivial precoder $-+-$.

for the M-PSK, M-PAM, and M-QAM constellations, respectively. μ is a positive scalar chosen such that the power constraint in Equation (25) is satisfied.

Proof: See [15]. ■

Equations (37), (38), and (39) can be used in a fixed point iteration for finding the precoder that solves Problem 1. Notice that the positive constants μ' and μ are different.

VII. Results and Comparisons

Comparisons are made against a system not employing any precoding, i.e., $\boldsymbol{F} = \sqrt{\frac{P}{Ka\sigma_x^2 M_t}}\boldsymbol{I}_{M_t}$ and the system minimizing an upper bound of the PEP [8]. The SNR is defined as: $\mathrm{SNR} = 10\log_{10}\frac{P}{N_0}$. $\sigma_x^2 = 1/2$, $P = 1$, and $M_r = 6$ are used.

Scenario 1: The following parameters are used in Scenario 1: The signal constellation is 8-PAM. As OSTBC the code $\boldsymbol{C}(\boldsymbol{x}) = \mathcal{G}_4^T$ in [11] was used such that $a = 2$, $K = 4$, $M_t = B = 4$, and $N = 8$. Let the correlation matrix $\boldsymbol{R}$ be given by

$$(\boldsymbol{R})_{k,l} = 0.9^{|k-l|}, \tag{40}$$

where the notation $(\cdot)_{k,l}$ picks out element with row number k and column number l.

Scenario 2: Let the correlation matrix $\boldsymbol{R}$ be given by Equation (26) with $\boldsymbol{R}_{r_0} = \boldsymbol{I}_{M_r}$ and $\boldsymbol{R}_{r_1} = a\boldsymbol{1}_{M_r\times M_r} + (1-b)\boldsymbol{I}_{M_r}$, where $b = 0.9999$. 9-QAM is used with Alamouti coding. Since the PEP precoder is developed under the assumption that $\boldsymbol{R}$ is invertible, the parameter b is chosen close to one but different from one.

Figures 2 and 3 show the SER versus SNR performance for Scenario 1 and 2 for the trivially precoder, the minimum upper bound PEP precoder [8], and the proposed minimum SER precoder in Lemma 7. For Scenario 2, the precoder in Lemma 6 is also shown. From Figure 2, it is seen that proposed minimum SER precoder outperforms the reference systems for all values of SNR and that the performance of the proposed system is similar to the minimum PEP precoder for low and high values of SNR, but for moderate values of SNR, a gain up to 0.8 dB can be achieved. In Figure 3, the performance between the minimum SER, PEP and the precoder in Lemma 6 are indistinguishable in this example, and the performance of these three precoders are up to 2 dB better than the trivial precoder.

Monte Carlo simulations verify the exact theoretical SER expressions.

VIII. Conclusions

For an arbitrary given OSTBC, exact SER expressions have been derived for a precoded MIMO system with correlation both in the transmitter and the receiver. The receiver employs MLD and has knowledge of the exact channel coefficients, while the transmitter only knows the correlation matrix of the channel matrix. An iterative method is proposed for finding the minimum SER precoder for M-PSK, M-PAM, and M-QAM signalling. The proposed precoders outperforms the precoder that minimizes an upper bound for the PEP. In the particular case of cooperative diversity, we present a closed-form precoder which approximates well the optimal precoder.

References

[1] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge University Press, May 2003.

[2] H. Sampath and A. Paulraj, "Linear precoding for space-time cpded systems with known fading correlations," *IEEE Communications Letters*, vol. 6, no. 6, pp. 239–241, June 2002.

[3] J. Akhtar and D. Gesbert, "Spatial multiplexing over correlated MIMO channels with a closed form precoder," *IEEE Trans. Wireless Communications*, submitted 2003.

[4] R. U. Nabar, H. Bölcskei, and A. J. Paulraj, "Cut-off rate based transmit optimization for spatial multiplexing on general MIMO channels," in *Proc. Int. Conf. on Acoustics, Speech, and Signal Proc.*, vol. 5, 2003, pp. 61–64.

[5] R. U. Nabar, H. Bolcskei, and A. J. Paulraj, "Transmit optimization for spatial multiplexing in the presence of spatial fading correlation," in *Proc. IEEE GLOBECOM*, vol. 1, Nov. 2001, pp. 131–135.

[6] E. Bonek, H. Özcelik, M. Herdin, W. Weichselberger, and J. Wallace, "Deficiencies of a popular stochastic MIMO radio channel model," in *Proc. Int. Symp. on Wireless Personal Multimedia Communications*, Yokosuka, Japan, Oct. 2003.

[7] G. Jöngren, M. Skoglund, and B. Ottersten, "Combining beamforming and orthogonal space-time block coding," *IEEE Trans. Inform. Theory*, vol. 48, no. 3, pp. 611–627, Mar. 2002.

[8] A. Hjørungnes, D. Gesbert, and J. Akhtar, "Precoding of space-time block coded signals for joint transmit-receive correlated MIMO channels," *IEEE Trans. Wireless Communications*, submitted 18.02.2004.

[9] S. Zhou and G. B. Giannakis, "Optimal transmitter eigen-beamforming and space-time block coding based on channel correlations," *IEEE Trans. Inform. Theory*, vol. 49, no. 7, pp. 1673–1690, July 2003.

[10] H. Shin and J. H. Lee, "Exact symbol error probability of orthogonal space-time block codes," in *Proc. IEEE GLOBECOM*, vol. 2, Nov. 2002, pp. 1197–1201.

[11] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block coding for wireless communications: Performance results," *IEEE J. Select. Areas Commun.*, vol. 17, no. 3, pp. 451–460, Mar. 1999.

[12] R. A. Horn and C. R. Johnson, *Topics in Matrix Analysis*. Cambridge University Press Cambridge, UK, 1991, reprinted 1999.

[13] X. Li, T. Luo, G. Yue, and C. Yin, "A squaring method to simplify the decoding of orthogonal space-time block codes," *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 1700–1703, Oct. 2001.

[14] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*. Wiley Series in Telecommunications and Signal Processing, 2000.

[15] A. Hjørungnes and D. Gesbert, "Minimum exact SER precoder for orthogonal space-time block codes over correlated MIMO channels," *IEEE Trans. Wireless Communications*, submitted.

[16] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[17] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[18] J. R. Magnus and H. Neudecker, *Matrix Differential Calculus with Application in Statistics and Econometrics*. Essex, England: John Wiley & Sons, Inc., 1988.