THE INSTRUMENTAL VARIABLE MULTICHANNEL FAP-RLS AND FAP ALGORITHMS

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ABSTRACT

The LMS and Affine Projection Algorithms (APA) and its Fast version (FAP) may suffer convergence slowdown in the presence of strong correlation between the input samples (colored input signals). In the case multichannel adaptive filtering, this problem may be compounded by strong correlation between the channel signals. Recently, improved multichannel APA's have been introduced (without a FAP counterpart), involving decorrelation operations between the channels. We show a connection between the improved APA and a certain choice of Instrumental Variable (IV) in an IV version of multichannel APA. There is also a connection to Fast Newton Transversal Filters (FNTF's). We suggest a particular choice for the IV that will make Fast multichannel IV APA possible and derive multichannel IV FAPs for the case of IVs with the usual shift-invariance structure. Whereas we have introduced a singlechannel IV FAP before, based on the Fast Transversal Filter (FTF) algorithm, we propose here a multichannel IV FAP based on the numerically more robust RLS algorithm.

1. THE MULTICHANNEL AFFINE PROJECTION ALGORITHM

The following criterion (introduced for $D = 1$ in [1]) encompasses quite a number of different adaptation techniques:

$$\min_{W_k} \left\{ \left\| d_{L,D,k} - X_{L,N,D,k} W_k^H \right\|^2_{S_k^{-1}} + \left\| W_k^H - W_{k-M}^H \right\|^2_{T_k^{-1}} \right\}$$

$$d_{L,D,k} = \begin{bmatrix} d_k^H \\ d_{k-D}^H \\ \vdots \\ d_{K-(L-1)D}^H \end{bmatrix}, \; X_{L,N,D,k} = \begin{bmatrix} X_k^H \\ X_{k-D}^H \\ \vdots \\ X_{K-(L-1)D}^H \end{bmatrix} \quad (1)$$

$$X_{L,N,D,k} = [x_{L,D,k} \; x_{L,D,k-1} \cdots x_{L,D,k-N+1}]$$

where $\|v\|_S^2 = v^H S v$, and S_k, T_k are Hermitian positive definite matrices, D is a subsampling factor for the error signal samples to be included in the sum of L squared errors, d_k is the desired-repsonse signal, x_k is the input signal, M is the update period and N is the adaptive filter length.

Consider taking $T_k = I$, $S_k = \nu_k X_{L,N,D,k} X_{L,N,D,k}^H = \nu_k R_{L,N,D,k}$. With $L < N$, we have an "underdetermined" problem (strictly speaking, only as $\nu_k \to 0$). The (row vector) filter solution can now be written as

$$\begin{aligned} W_k &= W_{k-M} + \\ \mu_k & \left(d_{L,D,k}^H - W_{k-M} X_{L,N,D,k}^H \right) R_{L,N,D,k}^{-1} X_{L,N,D,k} \end{aligned} \quad (2)$$

where the stepsize (relaxation parameter) $\mu_k = \frac{1}{1+\nu_k}$, and $R_{L,N,D,k}$ is the sample covariance matrix for a Sliding Window Covariance (SWC) LS problem with filter length L and window length N interchanged (and subsampling factor D). This, (2), (for $M = 1$) is the Affine Projection Algorithm (APA) which has been introduced in [2] with refinements in [3],[4]. (2) is in fact obtained by setting the gradient of 1) w.r.t. W_k to zero which yields for the filter update

$$\begin{aligned} \Delta W_k &= W_k - W_{k-M} = \\ \tfrac{1}{\nu_k} & \left(d_{L,D,k}^H - W_k X_{L,N,D,k}^H \right) R_{L,N,D,k}^{-1} X_{L,N,D,k} \,. \end{aligned} \quad (3)$$

If we focus for a moment on the noiseless case and consider the desired response to be the output of the optimal filter W_k^o, then the APA filter update (3) is of the form

$$\Delta W_k = \frac{1}{\nu_k} \widetilde{W}_k \, P_{X_{L,N,D,k}^H} \quad (4)$$

where $P_X = X(X^H X)^{-1} X^H$ is an orthogonal projection matrix onto the column space of X and $\widetilde{W}_k = W_k^o - W_k$ is the filter error. In the Instrumental Variable (IV) APA [5], the orthogonal projection is replaced by an oblique projection:

$$\Delta W_k = \frac{1}{\nu_k} \widetilde{W}_k \; X_{L,N,D,k}^H (Z_{L,N,D,k} X_{L,N,D,k}^H)^{-1} Z_{L,N,D,k}$$

where $Z_{L,N,D,k}$ is like $X_{L,N,D,k}$ but with the input signal x_k replaced by an instrumental variable signal z_k. In the IV

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APA, the (mean) update stops when the filter output error becomes uncorrelated, not with the filter input, but with the instrumental variable.

In this paper we shall restrict attention to the case $M = D = 1$. We consider the multichannel input case, in which case x_k is a $p \times 1$ vector, containing one input sample per channel. To simplify things, we shall consider that the filter length is the same for the p channels. The extension to channels with different filter lengths can be done along the lines of [6]. Furthermore, we shall assume that the multichannel structure for the IV is identical to that of the filter inputs: same number of channels and same filter length in each channel (these restrictions can be removed). With these assumptions, the filter update for the IV APA becomes, similar to (2), with $\mu_k \equiv \mu$,

$$W_k = W_{k-1} + \mu \left(d^H_{L,k} - W_{k-1} X^H_{L,N,k} \right) R^{-1}_{L,N,k} Z_{L,N,k} \quad (5)$$

where we have redefined $R_{L,N,k} = Z_{L,N,k} X^H_{L,N,k}$. We get for the $L \times 1$ a priori and a posteriori error vectors

$$\begin{aligned} e^p_{L,k} &= d_{L,k} - X_{L,N,k} W^H_{k-1} && (6) \\ e_{L,k} &= d_{L,k} - X_{L,N,k} W^H_k = (1-\mu)\, e^p_{L,k} && (7) \end{aligned}$$

2. THE FAST MULTI-CHANNEL IV APA

Fast APA (FAP) algorithms have been derived in [7],[8] based on the FTF algorithm. A multichannel version based on RLS appears in [9]. An IV single-channel version based on FTF appears in [5] and its extension to multichannel is just a matter of reinterpreting the notation. Here we present a multichannel IV FAP based on RLS. We note that in each filter update L consecutive input vectors get combined. To make this more explicit, introduce

$$e^{p\,H}_{L,k} R^{-1}_{L,N,k} = g_{L,k} = [g_{L,k,0} \;\; g_{L,k,1} \cdots g_{L,k,L-1}] \quad (8)$$

where the entries $g_{L,k,n}$ are not necessarily scalars but can be blocks. Then we can rewrite the APA filter update as

$$W_k - W_{k-1} = \mu\, g_{L,k}\, X_{L,N,k} = \mu \sum_{i=0}^{L-1} g_{L,k,i}\, Z^H_{k-i} \quad (9)$$

which represents a complexity of LN (block operations), once $g_{L,k}$ is known. This means that the IV vector Z_{k-L+1} will appear in L consecutive filter updates, for W_{k-L+1} up to W_k. Hence, we can reduce the complexity by regrouping the L updates involving Z_{k-L+1}, namely $g_{L,k-L+1,0} Z_{k-L+1}$, $g_{L,k-L+2,1} Z_{k-L+1}$ up to $g_{L,k,L-1} Z_{k-L+1}$, into one single update involving Z_{k-L+1} with a combined scalar factor, namely $\left(\sum_{i=0}^{L-1} g_{L,k-L+1+i,i} \right) Z_{k-L+1}$. Let $\overline{W}_k$ be the auxiliary adaptive filter that gets updated with this combined term, then

$$\overline{W}_k = \overline{W}_{k-1} + \mu\, f_{L,k,L-1}\, Z^H_{k-L+1} \quad (10)$$

where we introduced

$$\begin{aligned} f_{L,k} &= [f_{L,k,0} \cdots f_{L,k,L-1}] \\ &= \left[g_{L,k,0} \quad g_{L,k-1,0} + g_{L,k,1} \cdots \sum_{i=0}^{L-1} g_{L,k-L+1+i,i} \right] \end{aligned} \quad (11)$$

which can be computed recursively as

$$f_{L,k} = [0 \;\; f_{L-1,k-1}] + g_{L,k} \quad (12)$$

where we shall simplify the notation with an abuse of notation as $f_{L,k,0:L-2} = f_{L-1,k}$. The difference between $\overline{W}_k$ and W_k is that W_k w.r.t. $\overline{W}_k$ has been updated with the IV vectors Z_{k-L+2} up to Z_k, namely

$$W_k = \overline{W}_k + \mu\, f_{L-1,k}\, Z_{L-1,N,k} \quad (13)$$

where indeed $L-1$ appears and not L.

So in the FAP algorithm, to reduce complexity, we shall update the auxiliary filter $\overline{W}_k$ instead of W_k. To update $\overline{W}_k$, we need $f_{L,k,L-1}$ (see (10)), hence we need $f_{L,k}$ (see (11)), hence we need $g_{L,k}$ (see (12)), hence we need $e^p_{L,k}$ and $R^{-1}_{L,N,k}$ (see (8)). We can compute $e^p_{L,k}$ recursively since we have

$$e^p_{L,k} = \begin{bmatrix} e^{p\,H}_k \\ e_{L-1,k-1} \end{bmatrix} = \begin{bmatrix} e^{p\,H}_k \\ (1-\mu)\, e^p_{L-1,k-1} \end{bmatrix} \quad (14)$$

where the first equality holds due to the definition of a priori and a posteriori errors, and the second equality results from (7). Note that the subscripts L or $L-1$ in the notation here (for e, d, x, X or R) refer to the size of the quantities and not to the use of APA with window length of variable size L or $L-1$: the window size in APA (determining the solutions W_k and $\overline{W}_k$) is assumed fixed at L. From (14) we see that it suffices to obtain e^p_k (when $\mu = 1$, a simplification occurs and e^p_k constitutes the only non-zero part of $e^p_{L,k}$). Using (13), we get

$$\begin{aligned} e^p_k &= d_k - W_{k-1} X_k \\ &= \underbrace{d_k - \overline{W}_{k-1} X_k}_{\overline{e}^p_k} - \mu\, f_{L-1,k-1} \underbrace{Z_{L-1,N,k-1} X_k}_{r_{L-1,k}} \\ &= \overline{e}^p_k - \mu\, f_{L-1,k-1}\, r_{L-1,k} \end{aligned} \quad (15)$$

where we introduced $r_{L-1,k}$ which can be computed recursively as

$$r_{L-1,k} = r_{L-1,k-1} + z_{L-1,k-1}\, x_k - z_{L-1,k-N-1}\, x_{k-N} \,.$$

So we can compute the error signal corresponding to the actual filter W_{k-1} by using only the auxiliary filter $\overline{W}_{k-1}$.

Remains to handle the factor $R^{-1}_{L,N,k}$, which can be updated with sliding rectangular window RLS. The updates for the p channels can be best performed sequentially.

Sequential Processing Multichannel IV FAP-RLS	
Computation	**Cost** $(\times)$
Initialization	
$r_{L-1,-N-1}=0\,,\;\; R^{-1}_{L,N,-N-1}=\frac{1}{\mu}I_L\,,$ $x_k=0=z_k,\, k<-N\,,\; \overline{W}_{-1}=$ initial value, $e^{p\,H}_{L-1,-1}=0\,,\;\; f_{L-1,-1}=0$	
Adaptation	
if $k\geq -N$ do	
$r_{L-1,k}=r_{L-1,k-1}\;+z_{L-1,k-1}\,x_k - z_{L-1,k-N-1}\,x_{k-N}$	$2(L{-}1)p$
for $i=1:p$ do $\quad R_{L,N,0,k}=R_{L,N,k-1}$	
$C_{i,k}=x^H_{L,i,k}\,R^{-1}_{L,N,i-1,k}$	L^2
$G_{i,k}=R^{-1}_{L,N,i-1,k}\,z_{L,i,k}$	L^2
$\gamma_{i,k}=1+x^H_{L,i,k}\,G_{i,k}$	L
$R^{-1}_{L,N+\frac{1}{p},i,k}=R^{-1}_{L,N,i-1,k}-G_{i,k}\,\gamma^{-1}_{i,k}\,C_{i,k}$	L^2+L
$D_{i,k}=x^H_{L,i,k-N}\,R^{-1}_{L,N+\frac{1}{p},i,k}$	L^2
$F_{i,k}=R^{-1}_{L,N+\frac{1}{p},i,k}\,z_{L,i,k-N}$	L^2
$\delta_{i,k}=1-x^H_{L,i,k-N}\,F_{i,k}$	L
$R^{-1}_{L,N,i,k}=R^{-1}_{L,N+\frac{1}{p},i,k}+F_{i,k}\,\delta^{-1}_{i,k}\,D_{i,k}$	L^2+L
end for $\quad R_{L,N,k}=R_{L,N,p,k}$	
end if	
if $k\geq 0$ do	
$\overline{e}^p_k=d_k-\overline{W}_{k-1}\,X_k$	Np
$e^p_k=\overline{e}^p_k-\mu\,f_{L-1,k-1}\,r_{L-1,k}$	$2(L{-}1)$
$e^{p\,H}_{L,k}=\left[e^p_k \quad (1{-}\mu)\,e^{p\,H}_{L-1,k-1}\right]$	$L{-}1$
$g_{L,k}=e^{p\,H}_{L,k}\,R^{-1}_{L,N,k}$	L^2
$f_{L,k}=[0\;\; f_{L-1,k-1}]+g_{L,k}$	$L{-}1$
$\overline{W}_k=\overline{W}_{k-1}+\mu\,f_{L,k,L-1}\,Z^H_{k-L+1}$	$Np+1$
end if	
cost/update ($2p\,\div$): $2pN+(6p{+}1)L^2+(6p{+}4)L-2p-3\;(\times)$	

3. THE IMPROVED MULTICHANNEL APA

The development of the multichannel IV FAP algorithm presented here was motivated by the improved multichannel APA algorithm presented in [10],[11], and for which no FAP algorithm exists. The application considered is that of multichannel acoustic echo cancellation but in fact the multichannel adaptive filtering algorithms to be discussed here apply to any situation in which there is strong correlation between the input signals of the various channels. Such correlation can be due to the fact that the channel input signals are different mixtures of some sources signals that are smaller in number than the number of channels (overdetermined mixture). Such a situation arises e.g. in stereo acoustic echo cancellation with a mono source generating the stereo echos. To put the improved multichannel APA into the IV multichannel APA context, it will be necessary to temporarily switch to different, permuted notation (that coincidentally would also be the right notation to handle the case of different channel filter lengths). Let $W_{i,k}$ be the filter coefficients in channel i, filtering $x_{i,k}$, input signal i, so that e.g.

$$d_{L,k}-e_{L,k}=\sum_{i=1}^{p}X_{i,L,N,k}\,W^H_{i,k}=\widehat{X}_{L,N,k}\,\widehat{W}_k \qquad (16)$$

where $X_{i,L,N,k}$ is like $X_{L,N,k}$ except that it is filled up with the scalar signal $x_{i,k}$ instead of the vector signal x_k and $\widehat{W}_k=[W_{1,k}\cdots W_{p,k}]$, $\widehat{X}_{L,N,k}=[X_{1,L,N,k}\cdots X_{p,L,N,k}]$. Let us introduce notation for signals in the other channels:

$$\begin{aligned}&\overline{X}_{i,L,N,k}=\\&\left[X^H_{1,L,N,k}\cdots X^H_{i-1,L,N,k}\;X^H_{i+1,L,N,k}\cdots X^H_{p,L,N,k}\right]^H\end{aligned} \qquad (17)$$

The idea now is to decorrelate the channel signals among themselves before use in an APA. Consider

$$\begin{aligned}\widehat{Z}_{L,N,k} &= \left[\widehat{Z}_{1,L,N,k}\cdots\widehat{Z}_{p,L,N,k}\right]\\ &= \left[X_{1,L,N,k}P^{\perp}_{\overline{X}^H_{1,L,N,k}}\cdots X_{p,L,N,k}P^{\perp}_{\overline{X}^H_{p,L,N,k}}\right]\end{aligned}$$

where $P^{\perp}_X=I-P_X$. Then the improved multichannel APA of [10] corresponds in fact to the multichannel IV APA of (5) with $\widehat{W}$, $\widehat{X}$, $\widehat{Z}$ replacing W, X and Z. To facilitate this identification, it is worth noting that $\widehat{Z}_{L,N,k}\widehat{X}^H_{L,N,k}=\widehat{Z}_{L,N,k}\widehat{Z}^H_{L,N,k}$.

Let us interpret the filtering operations in going from the input signals $\widehat{X}$ to the IV signals $\widehat{Z}$. Note that $\widehat{Z}_{i,L,N,k}=X_{i,L,N,k}P^{\perp}_{\overline{X}^H_{i,L,N,k}}=X_{i,L,N,k}-B_{i,k}\,\overline{X}_{i,L,N,k}$ where $B_{i,k}=X_{i,L,N,k}\overline{X}^H_{i,L,N,k}(\overline{X}_{i,L,N,k}\overline{X}^H_{i,L,N,k})^{-1}$. Let us focus on row $l{+}1$ of these matrix expressions, then we obtain for $l=0,\ldots,L{-}1$

$$\begin{aligned}\widehat{Z}_{l,i,L,N,k} &= X^H_{i,k-l}-B^H_{l,i,k}\overline{X}_{i,L,N,k}\\ &= X^H_{i,k-l}-\sum_{j=1,\neq i}^{p}B_{l,i,j,k}\,X_{j,L,N,k}\end{aligned} \qquad (18)$$

which after taking Hermitian transpose becomes

$$\widehat{Z}^H_{l,i,L,N,k}=X_{i,k-l}-\sum_{j=1,\neq i}^{p}\mathcal{T}(B^*_{l,i,j,k})\,\overline{X}_{j,k,N+L-1} \qquad (19)$$

where $\mathcal{T}(B)$ denotes a rectangular banded Toeplitz matrix with B as the first, non-zero part of the first row, B^* denotes complex conjugate, and $\overline{X}_{j,k,N+L-1}$ is a column vector containing the $N+L-1$ last samples at time k of the input signal $x_{j,k}$, so that $\overline{X}_{j,k,N+L-1} = X_{j,k}$. Hence, to obtain the IV vectors from the input signal vectors in the improved multichannel APA algorithm, the IV signal for channel i is obtained by replacing the input signal for channel i with its FIR Wiener filtering (LMMSE estimation) error in terms of the signals of the other channels. This filtering is time-invariant for the N samples of an IV vector. However, the filter coefficients are recalculated for every updating instant k according to a least-squares criterion over the last N samples. Although the FIR Wiener filter length is fixed to L, the delay used in the Wiener filtering varies for each of the L IV vectors involved in the construction of the L dimensional subspace for the APA projection. The delay equals l, hence it is zero for the first vector ($l = 0$, causal filtering) and it is maximal ($L-1$) for the last vector ($l = L-1$, anticausal filtering). For the intermediate columns the Wiener filtering corresponds to fixed-lag smoothing.

Some comments are in order. The fact that the Wiener filter is computed with a LS criterion with rectangular window of length N, and that the same filter coefficients are applied to the N samples in the IV vectors is probably not of crucial importance. The fact that the smoothing delay is varied for the L vectors is probably also not crucial. Finally, we may remark that there is no intrachannel decorrelation, only interchannel decorrelation.

4. PREWHITENING AND IV-FAP-RLS

The problem with the IV signal matrix $\widehat{Z}$, if a FAP algorithm is desired, is that it does not exhibit the usual shift invariance (Hankel block structure). To do so with the least possible deviation from the original improved multichannel algorithm could be achieved by fixing the smoothing delay to some intermediate value (around $\frac{L-1}{2}$) and applying the resulting filter setting $B_{l,i,k}$ to obtain a sample in the middle of $\widehat{Z}_{i,L,N,k}$, and varying the filter $B_{l,i,k}$ time index in the same way as the sample time index within $\widehat{Z}_{i,L,N,k}$ to obtain a Hankel block. However, it is not necessary to stick that closely to the details of the improved APA algorithm to obtain improved convergence behavior. In the monochannel case, the doubly prewhitened input signal $z_k = A^\dagger(q)A(q)x_k$ leads to a good results [5], where $A(q)$ is the prediction error filter on the signal x_k and $A^\dagger(q)$ is its matched filter version (paraconjugate). In the multi-channel case, x_k is a $p\times 1$ vector signal, and hence $A(q)$ is $p\times p$ matrix filter. Then, an extension of this Instrumental Variable in the multi-channel case is:

$$z_k = A^\dagger(q)R^{-1}_{\widetilde{x}\widetilde{x}}A(q)\; x_k \tag{20}$$

where : $A(q) = I_p + A_1 q^{-1} + \cdots + A_M q^{-M}$ is a $p\times p$ matrix characterizing the multichannel prediction error filter (here of order M), $\widetilde{x}_k = A(q)x_k$ is the associated prediction error and $R_{\widetilde{x}\widetilde{x}}$ is the $p \times p$ prediction error covariance matrix. The infinite length (and infinite delay) smoothing filter for handling both inter- and intrachannel correlation is simply $z_k = S^{-1}_{xx}(q)\, x_k$ where $S_{xx}(z)$ is the matrix spectrum of x_k and $S^{-1}_{xx}(z) = A^\dagger(z)R^{-1}_{\widetilde{x}\widetilde{x}}A(z)$ when the prediction quantities are taken to be of infinite order. The reason why such an IV is desirable is that the resulting cross spectral density is $S_{zx}(z) = I_p$. This means that in principle the cascade of adaptive filter transition matrices should behave as in the white input case.

To perform smoothing of finite order (e.g. $M = L-1$), it would suffice to replace $S^{-1}_{xx}(q)$ by a matrix inverse R^{-1}_{XX}. However, taking into account that this matrix inverse needs to be updated with e.g. the Riccati equation (RLS), we may as well update a prediction filter of same order and perform smoothing as in (20) which should work somewhat better since the smoothing order gets doubled. The proposed multichannel IV FAP is quite flexible. By choosing a larger prediction order M, the IV in (20) should allow to attain better performance than the improved multichannel APA.

For readers familiar with the Fast Newton Transversal filter (FNTF), an approximation of the RLS and FTF algorithms in case the input signal can be modeled as an autoregressive process of reduced order, it will be clear that the FNTF algorithm can be approximately viewed as an IV LMS algorithm with the IV of (20). Hence, the proposed IV APA algorithm combines the advantages of APA and FNTF algorithms.

For a prediction order $M = 0$, the IV in (20) gives an IV that is very close to the one in the improved multichannel APA for $L = 1$ (which is hence the improved multichannel NLMS). Indeed, let

$$\begin{aligned} X_k &= [X_{1,k} \cdots X_{p,k}]\;, \\ \overline{X}_{i,k} &= [X_{1,k} \cdots X_{i-1,k}\, X_{i+1,k} \cdots X_{p,k}] \end{aligned} \tag{21}$$

then $\frac{1}{N}X_k^H X_k$ is an estimate for R^*_{xx}. The following identity can be shown

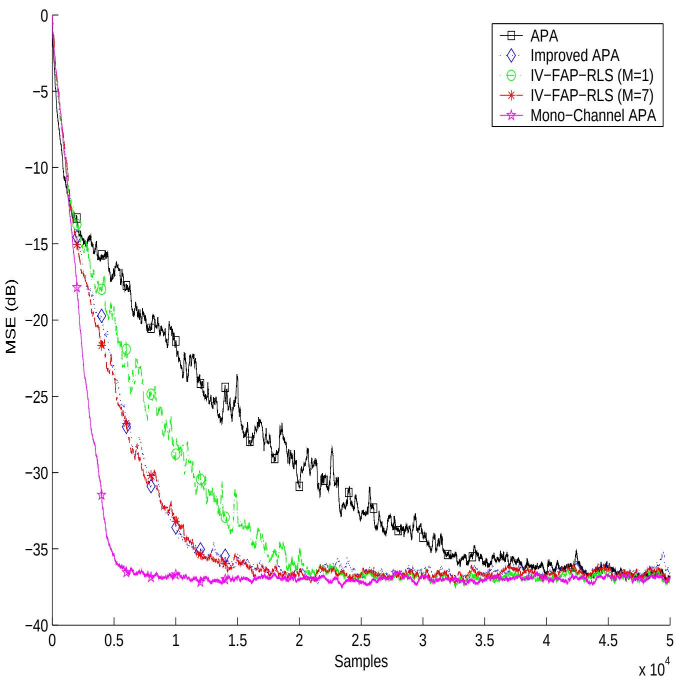
$$X_k(X_k^H X_k)^{-1} = \left[\frac{1}{X^H_{1,k}P^\perp_{\overline{X}_{1,k}}X_{1,k}} P^\perp_{\overline{X}_{1,k}} X_{1,k} \cdots \frac{1}{X^H_{p,k}P^\perp_{\overline{X}_{p,k}}X_{p,k}} P^\perp_{\overline{X}_{p,k}} X_{p,k}\right]$$

Hence, apart from the channel-wise scaling factors, 20) with $M = 0$ produces exactly the IV for the improved multichannel NLMS.

The prediction quantities can be adapted with RLS, using a sliding rectangular or exponential window of length of the order of N. Of course, any other adaptive filtering algorithm could be used also.

5. SIMULATION RESULTS

In order to compare the convergence of the proposed multichannel IV FAP algorithms with other variants of the Affine Projection Algorithm, we applied the algorithms to the stereo acoustic echo cancellation (AEC) problem. The AEC simulations were performed using Matlab with acoustic transfer functions obtained from measurements in a teleconference room (provided by André Gilloire from *France Télécom R&D*). The signals from the two loudspeakers get filtered by the (loudpspeaker to microphone) echo channels that got truncated to N=256 samples. White noise gets added to the thus obtained microphone signal at an SNR of 40dB. The loudpseaker signals themselves consist of white noise filtered by two closer distance louspeaker to stereo microphone transfer functions, and to each loudspeaker signal some independent white noise is added. The slowest curve corresponds to the original two-channel APA with $L = 8$. The next curve is the proposed twochannel IV FAP with an IV as in (20) and first-order prediction. When the prediction order is increased to 7 (=$L-1$), we obtain one of the next two curves which pretty much coincides with the other curve, corresponding to the improved twochannel APA. The fastest convergence curve is obtained with a singlechannel APA in which the two filter impulse responses have been put in cascade to yield a single filter of order $2N = 512$. This shows that not all correlation between the two channels has been removed in the twochannel algorithms.



6. CONCLUSION

In this paper, we have proposed a multichannel IV APA algorithm and its fast version multichannel IV FAP based on RLS. We also elaborated on a tight connection between the so-called improved multi-channel APA and certain IV versions of APA. It was shown through simulations that the proposed multichannel IV algorithms can provide a great increase in convergence speed over the standard multichannel APA algorithm and behave comparably to the improved multichannel APA, for which no fast version exists however.

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