

# Exact SER-Precoding of Orthogonal Space-Time Block Coded Correlated MIMO Channels: An Iterative Approach

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**Abstract**—Exact expressions are derived for the average symbol error rate (SER) for correlated non-frequency selective quasi-static multiple-input multiple-output (MIMO) Rayleigh fading channels where the transmitter employs orthogonal space-time block coding (OSTBC) and precoding with a full complex-valued precoder matrix. Expressions are given for  $M$ -PSK,  $M$ -PAM, and  $M$ -QAM signal constellations. An iterative optimization technique is proposed for finding the minimum exact SER precoder. The results show that the proposed precoder performs better than a system using the trivial precoder and a system using the precoder that minimizes an upper bound on the pair-wise error probability (PEP).

## I. INTRODUCTION

In the area of efficient communications over non-reciprocal MIMO channels, recent research has demonstrated the value of feeding back to the transmitter information about channel state observed at the receiver. Clearly, the type of feedback may vary largely, depending on its nature, e.g., required rate, instantaneous, or statistical channel state information (CSI), leading to various transmitter design schemes, see e.g., [1], [2], [3]. There has been a growing interest in transmitter schemes that can exploit low-rate long-term statistical CSI in the form of antenna correlation coefficients. So far, emphasis has been on designing precoders for space-time block coded (STBC) [2] signals or spatially multiplexed streams that are adjusted based on the knowledge of the transmit correlation only while the receiving antennas are uncorrelated [4], [5], [6], [7]. These techniques are well suited to downlink situation where an elevated access point (situated above the surrounding clutter) transmits to a subscriber placed in a rich scattering environment. Although simple models exist for the joint transmit receiver correlation based on the well known Kronecker structure [2], the accuracy of these models has recently been questioned in the literature based on measurement campaigns [8]. Therefore, there is interest in investigating the precoding of OSTBC signals for MIMO channels that *do not* necessarily follow the Kronecker structure.

Previously developed theory in the field: An upper bounds on the PEP is minimized in [4], [9] for transmitter correlation and for full channel correlation in [3], [10]. In [9], the exact SER expressions were derived for when there is *no receiver correlation* and maximum ratio combining is used at the receiver. A bound of the exact error probability was used as the optimization criterion in [9]. The no receiver correlation assumption might be an unrealistic channel model for example in uplink communications, where the access point (receiver) is equipped with several receiver antennas and where the direction of arrival has a small spread at the receiver antennas. In [11], exact SER expressions were found for uncorrelated MIMO channels that are precoded with the identity matrix.

In this paper, we address the problem of linear precoding of OSTBC signals launched over a jointly transmit-receive correlated

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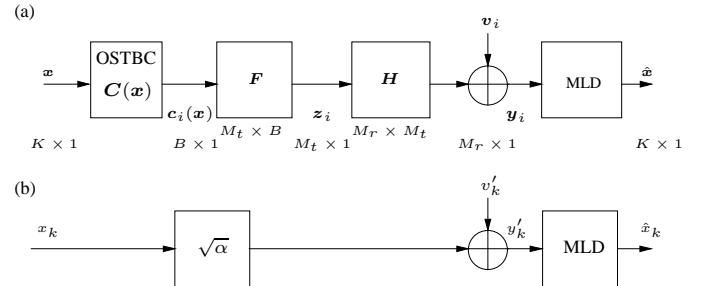


Fig. 1. Block model of the linear precoded OSTBC MIMO system.

MIMO channel. Our two main contributions are: Firstly, we derive *exact* expressions for the average SER for a system where the transmitter has an OSTBC and a full complex-valued precoder matrix equipped with multiple antennas and where the receiver also has multiple antennas and is using maximum likelihood decoding (MLD). The channel correlation matrix is general such that receiver correlation might be present. The transmitter knows the correlation matrix of the channel transfer matrix and the receiver knows the channel realization exactly. Secondly, we propose an iterative numerical technique for minimizing the *exact* SER with respect to the precoder matrix. In earlier works, an upper bound of the exact symbol error rate or the pairwise error probability are used. The precoder is obtained via an iterative algorithm which uses the knowledge of the full transmit-receive correlation, regardless of whether the Kronecker structure is valid or not.

## II. SYSTEM DESCRIPTION

### A. OSTBC Signal Model

Figure 1 (a) shows the block MIMO system model with  $M_t$  and  $M_r$  transmitter and receiver antennas, respectively. One block of  $K$  source samples  $x_0, x_1, \dots, x_{K-1}$  is transmitted by means of an OSTBC matrix  $C(\mathbf{x})$  of size  $B \times N$ , where  $B$  and  $N$  are the space and time dimension of the given OSTBC, respectively, and where  $\mathbf{x} = [x_0, x_1, \dots, x_{K-1}]^T$  contains the source samples. It is assumed that the OSTBC is given. Let  $x_i \in \mathcal{A}$ , where  $\mathcal{A}$  is a signal constellation set such as  $M$ -PAM,  $M$ -QAM, or  $M$ -PSK. The OSTBC returns an  $B \times N$  matrix  $C(\mathbf{x})$  that is dependent on  $\mathbf{x}$ . If bits are used as inputs to the system,  $K \log_2 |\mathcal{A}|$  bits are used to produce the vector  $\mathbf{x}$ , where  $|\cdot|$  denotes cardinality. Assume that  $E[|x_i|^2] = \sigma_x^2$ , and that the matrix that comes out of the OSTBC is denoted  $C(\mathbf{x})$  is of size  $B \times N$ . Since the OSTBC is orthogonal, the following holds

$$C(\mathbf{x})C^H(\mathbf{x}) = a \sum_{i=0}^{K-1} |x_i|^2 \mathbf{I}_B, \quad (1)$$

where  $a = 1$  if  $C(\mathbf{x}) = \mathcal{G}_2^T$ ,  $C(\mathbf{x}) = \mathcal{H}_3^T$ , or  $C(\mathbf{x}) = \mathcal{H}_4^T$  in [13] and  $a = 2$  if  $C(\mathbf{x}) = \mathcal{G}_3^T$  or  $C(\mathbf{x}) = \mathcal{G}_4^T$  in [13]. The rate of the

<sup>1</sup>The OSTBC  $\mathcal{G}_2^T$  is the well-known Alamouti code [12].

code is  $K/N$ . Other OSTBC can be used as well. The codeword matrix  $\mathbf{C}(\mathbf{x})$  has size  $B \times N$  and can be expressed as:

$$\mathbf{C}(\mathbf{x}) = [\mathbf{c}_0(\mathbf{x}) \ \mathbf{c}_1(\mathbf{x}) \ \cdots \ \mathbf{c}_{N-1}(\mathbf{x})], \quad (2)$$

where  $\mathbf{c}_i(\mathbf{x})$  is the  $i$ th column vector of  $\mathbf{C}(\mathbf{x})$  and it has size  $B \times 1$ .

Before each code vector is launched into the channel, it is precoded with a memoryless complex-valued matrix  $\mathbf{F}$  of size  $M_t \times B$ , so the  $M_r \times 1$  receive signal vector  $\mathbf{y}_i$  becomes

$$\mathbf{y}_i = \mathbf{H}\mathbf{F}\mathbf{c}_i(\mathbf{x}) + \mathbf{v}_i, \quad (3)$$

where the additive noise on the channel  $\mathbf{v}_i$  is complex Gaussian circularly distributed with independent components having variance  $N_0$  and  $\mathbf{H}$  is the channel transfer MIMO matrix. Let the vectors  $\mathbf{y}_i$  be collected into the matrix  $\mathbf{Y}$  of size  $M_r \times N$  in the following way:

$$\mathbf{Y} = [\mathbf{y}_0 \ \mathbf{y}_1 \ \cdots \ \mathbf{y}_{N-1}], \quad (4)$$

and the vectors  $\mathbf{v}_i$  into the matrix  $\mathbf{V}$  of size  $M_r \times N$ , in the following way:

$$\mathbf{V} = [\mathbf{v}_0 \ \mathbf{v}_1 \ \cdots \ \mathbf{v}_{N-1}]. \quad (5)$$

Then the block input-output relationship for the MIMO system can be expressed:

$$\mathbf{Y} = \mathbf{H}\mathbf{F}\mathbf{C}(\mathbf{x}) + \mathbf{V}. \quad (6)$$

The receiver is assumed to know the channel matrix  $\mathbf{H}$  and the precoding matrix  $\mathbf{F}$  exactly, and it performs MLD of blocks of length  $N$ .

### B. Correlated Channel Models

A quasi-static non-frequency selective correlated Rayleigh fading channel model [2] is assumed. Let  $\mathbf{R} \triangleq E[\text{vec}(\mathbf{H})\text{vec}^H(\mathbf{H})]$  be the general  $M_t M_r \times M_t M_r$  positive definite autocorrelation matrix for the channel coefficients, where the operator  $\text{vec}(\cdot)$  stacks the columns of the matrix it is applied to into a long column vector [14]. A channel realization of the correlated channel can then be found by

$$\text{vec}(\mathbf{H}) = \mathbf{R}^{1/2} \text{vec}(\mathbf{H}_w), \quad (7)$$

where  $\mathbf{R}^{1/2}$  is the unique positive definite matrix square root [14] of  $\mathbf{R}$  and  $\mathbf{H}_w$  has size  $M_r \times M_t$  and is complex Gaussian circularly distributed with independent components all having unit variance.

**Kronecker model:** A special case of the model above is as follows [2]

$$\mathbf{H} = \mathbf{R}_r^{1/2} \mathbf{H}_w \mathbf{R}_t^{1/2}, \quad (8)$$

where the matrices  $\mathbf{R}_r$  and  $\mathbf{R}_t$  are the correlations matrices of the receiver and transmitter, respectively, and their sizes are  $M_r \times M_r$  and  $M_t \times M_t$ . The full autocorrelation matrix  $\mathbf{R}$  of the model in Equation (8) is then given by

$$\mathbf{R} = \mathbf{R}_t^T \otimes \mathbf{R}_r, \quad (9)$$

where the operator  $(\cdot)^T$  denotes transposition and  $\otimes$  is the Kronecker product. Unlike Equation (9), the general model considers that the receive (or transmit) correlation depends on at which transmit (or receive) antenna the measurements are performed.

### C. Equivalent Single-Input Single-Output Model

Define the matrix  $\Phi$  of size  $M_t M_r \times M_t M_r$  as:

$$\Phi = \mathbf{R}^{1/2} \left[ (\mathbf{F}^* \mathbf{F}^T) \otimes \mathbf{I}_{M_r} \right] \mathbf{R}^{1/2}. \quad (10)$$

This matrix plays an important role in the developed theory. Define the real non-negative scalar  $\alpha$  by

$$\alpha = \|\mathbf{H}\mathbf{F}\|_F^2 = \text{vec}^H(\mathbf{H}_w) \Phi \text{vec}(\mathbf{H}_w), \quad (11)$$

where  $\|\cdot\|_F$  is the Frobenius norm. Since the matrix  $\mathbf{H}_w$  contains unit variance uncorrelated variables,  $E[\text{vec}(\mathbf{H}_w)\text{vec}^H(\mathbf{H}_w)] = \mathbf{I}_{M_t M_r}$ . The expected value of  $\alpha$  can now be found:

$$\begin{aligned} E[\alpha] &= E[\text{vec}^H(\mathbf{H}_w) \Phi \text{vec}(\mathbf{H}_w)] \\ &= \text{Tr} \left\{ \Phi E[\text{vec}(\mathbf{H}_w)\text{vec}^H(\mathbf{H}_w)] \right\} = \text{Tr} \{ \Phi \}. \end{aligned} \quad (12)$$

By generalizing the approach given in [11], [15] to include a full complex-valued precoder  $\mathbf{F}$  of size  $M_t \times B$  and having an arbitrary channel correlation matrix  $\mathbf{R}$  the OSTBC system can be shown to be equivalent with a system having the following output input relationship

$$y'_k = \sqrt{\alpha} x_k + v'_k, \quad (13)$$

for  $k \in \{0, 1, \dots, K-1\}$ . This signal is fed into a memoryless MLD that is designed from the signal constellation of the source symbols  $\mathcal{A}$ .  $v'_k \sim \mathcal{CN}(0, N_0/\alpha)$  is complex circularly distributed. The equivalent single-input single-output (SISO) model is shown in Figure 1 (b).

### III. SER EXPRESSIONS FOR GIVEN RECEIVED SNR

By considering the SISO system in Figure 1 (b), it is seen that the instantaneous received SNR  $\gamma$  per source symbol is given by

$$\gamma = \frac{a\sigma_x^2 \alpha}{N_0} = \delta \alpha, \quad (14)$$

where  $\delta \triangleq \frac{a\sigma_x^2}{N_0}$ . The expected received signal to noise ratio is given by:  $E[\gamma] = \frac{a\sigma_x^2 \text{Tr}\{\Phi\}}{N_0} = \delta \text{Tr}\{\Phi\}$ .

In order to simplify the expressions, the following three signal constellation dependent constants are defined

$$g_{\text{PSK}} = \sin^2 \frac{\pi}{M}, \quad g_{\text{PAM}} = \frac{3}{M^2 - 1}, \quad g_{\text{QAM}} = \frac{3}{2(M-1)}. \quad (15)$$

The symbol error probability  $\text{SER}_\gamma \triangleq \Pr\{\text{Symbol error}|\gamma\}$  for a given  $\gamma$  for  $M$ -PSK,  $M$ -PAM, and  $M$ -QAM signalling given by [16]

$$\text{SER}_\gamma = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} e^{-\frac{g_{\text{PSK}} \gamma}{\sin^2(\theta)}} d\theta, \quad (16)$$

$$\text{SER}_\gamma = \frac{2}{\pi} \frac{M-1}{M} \int_0^{\frac{\pi}{2}} e^{-\frac{g_{\text{PAM}} \gamma}{\sin^2(\theta)}} d\theta, \quad (17)$$

$$\text{SER}_\gamma = \frac{4}{\pi} \left( 1 - \frac{1}{\sqrt{M}} \right) \left[ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{-\frac{g_{\text{QAM}} \gamma}{\sin^2(\theta)}} d\theta + \frac{1}{\sqrt{M}} \int_0^{\frac{\pi}{4}} e^{-\frac{g_{\text{QAM}} \gamma}{\sin^2(\theta)}} d\theta \right], \quad (18)$$

respectively.

### IV. EXACT SER EXPRESSIONS

The moment generating function  $\phi_\gamma(s)$  of the probability density function  $p_\gamma(\gamma)$  is defined as  $\phi_\gamma(s) = \int_0^\infty p_\gamma(\gamma) e^{s\gamma} d\gamma$ . Since all the  $K$  source symbols go through the same SISO system in Figure 1 (b), the average SER of the MIMO system can be found as

$$\begin{aligned} \text{SER} &\triangleq \Pr\{\text{Error}\} = \int_0^\infty \Pr\{\text{Error}|\gamma\} p_\gamma(\gamma) d\gamma \\ &= \int_0^\infty \text{SER}_\gamma p_\gamma(\gamma) d\gamma. \end{aligned} \quad (19)$$

This integral can be rewritten by means of the moment generating function of  $\gamma$ .

From Equation (11) and the fact that all the elements of  $\mathbf{H}_w$  is independent and complex Gaussian distributed with zero mean and unit variance, it follows that the moment generating function of  $\alpha$  is given by:

$$\phi_\alpha(s) = \frac{1}{M_t M_r - 1} \prod_{i=0}^{M_t M_r - 2} (1 - \lambda_i s), \quad (20)$$

where  $\lambda_i$  is eigenvalue number  $i$  of the positive semi-definite matrix  $\Phi$ . Since  $\gamma = \delta\alpha$ , the moment generating function of  $\gamma$  is given by:

$$\phi_\gamma(s) = \phi_\alpha(\delta s) = \frac{1}{M_t M_r - 1} \prod_{i=0}^{M_t M_r - 2} (1 - \delta \lambda_i s). \quad (21)$$

By using Equation (19) and the definition of the moment generating function together with the result in Equation (21) it is possible to express the exact SER for all the signal constellations in terms of the eigenvalues  $\lambda_i$  of the matrix  $\Phi$ . When finding the necessary conditions for the optimal precoder, eigenvalues that are not simple might case difficulties in connection with calculations of derivatives. Therefore, it is useful to rewrite the expressions for the SER in terms of the full matrix  $\Phi$ . This can be done by utilizing the eigen-decomposition of this matrix. The results of all these operations led to the following expressions for the SER for  $M$ -PSK,  $M$ -PAM, and  $M$ -QAM

$$\text{SER} = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \frac{d\theta}{\det(\mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{PSK}}}{\sin^2 \theta} \Phi)}, \quad (22)$$

$$\text{SER} = \frac{2}{\pi} \frac{M-1}{M} \int_0^{\frac{\pi}{2}} \frac{d\theta}{\det(\mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{PAM}}}{\sin^2 \theta} \Phi)}, \quad (23)$$

$$\text{SER} = \frac{4}{\pi} \frac{\sqrt{M}-1}{\sqrt{M}} \left[ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{d\theta}{\det(\mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{QAM}}}{\sin^2 \theta} \Phi)} + \frac{1}{\sqrt{M}} \int_0^{\frac{\pi}{4}} \frac{d\theta}{\det(\mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{QAM}}}{\sin^2 \theta} \Phi)} \right], \quad (24)$$

respectively. It is seen that Equations (22) and (23) gives the same result when  $M = 2$ . This is not surprising, since, the constellations of 2-PSK and 2-PAM are identical. When  $M = 4$ , it can be shown that Equations (22) and (24) return the same result. If  $\mathbf{R} = \mathbf{I}_{M_t M_r}$  and  $\mathbf{F} = \mathbf{I}_{M_t}$ , then the performance expressions in Equations (22) and (24) are reduced to the results found in [11]. If  $M_r = 1$  and no receiver correlation is present, Equations (22), (23), and (24) are in accordance with expressions derived in [9].

## V. PRECODING OF OSTBC SIGNALS

### A. Power Constraint

Then OSTBC is used, Equation (1) holds and the average power constraint on the transmitted block  $\mathbf{Z} \triangleq \mathbf{F}\mathbf{C}(\mathbf{x})$  can be expressed as

$$aK\sigma_x^2 \text{Tr} \{ \mathbf{F}\mathbf{F}^H \} = P, \quad (25)$$

where  $P$  is the average power used by the transmitted block  $\mathbf{Z}$ .

### B. Optimal Precoder Problem Formulation

The goal is to find the matrix  $\mathbf{F}$  such that the exact SER is minimized under the power constraint. We propose that the optimal precoder is given by the following optimization problem:

*Problem 1:*

$$\begin{aligned} & \min_{\{\mathbf{F} \in \mathbb{C}^{M_t \times B}\}} \text{SER} \\ & \text{subject to } K a \sigma_x^2 \text{Tr} \{ \mathbf{F}\mathbf{F}^H \} = P. \end{aligned}$$

*Remark 1:* The optimal precoder is dependent on the value of  $N_0$  and therefore also on SNR.

### C. Properties of Optimal Precoder

*Lemma 1:* If  $\mathbf{F}$  is an optimal solution of Problem 1, then the precoder  $\mathbf{F}\mathbf{U}$ , where  $\mathbf{U} \in \mathbb{C}^{B \times B}$  is unitary, is also optimal.

*Proof:* Let  $\mathbf{F}$  be an optimal solution of Problem 1 and  $\mathbf{U} \in \mathbb{C}^{B \times B}$ , be an arbitrary unitary matrix. It is then seen by insertion that the objective function and the power constraint are unaltered by the unitary matrix. ■

*Lemma 2:* If  $N_0 \rightarrow 0^+$ ,  $B = M_t$ , and  $\mathbf{R}$  is non-singular, then the optimal precoder is given by the trivial precoder  $\mathbf{F} = \sqrt{\frac{P}{K a \sigma_x^2 M_t}} \mathbf{I}_{M_t}$  for the  $M$ -PSK,  $M$ -PAM, and  $M$ -QAM constellations.

*Proof:* See [17]. ■

*Lemma 3:* If  $M_t = B$  and  $\mathbf{R} = \mathbf{I}_{M_t M_r}$ , then the optimal precoder is given by the trivial precoder  $\mathbf{F} = \sqrt{\frac{P}{K a \sigma_x^2 M_t}} \mathbf{I}_{M_t}$  for the  $M$ -PSK,  $M$ -PAM, and  $M$ -QAM constellations.

*Proof:* See [17]. ■

## VI. OPTIMIZATION ALGORITHM

Let the matrix  $\mathbf{K}_{k,l}$  be the commutation matrix [18] of size  $kl \times kl$ . The constrained maximization Problem 1 can be converted into an unconstrained optimization problem by introducing a Lagrange multiplier  $\mu'$ . This is done by defining the following Lagrange function:

$$\mathcal{L}(\mathbf{F}) = \text{SER} + \mu' \text{Tr} \{ \mathbf{F}\mathbf{F}^H \}. \quad (26)$$

Since the objective function should be minimized,  $\mu' > 0$ . Define the  $M_t^2 \times M_t^2 M_r$  matrix  $\mathbf{L}$  as

$$\mathbf{L} = \left[ \mathbf{I}_{M_t^2} \otimes \text{vec}^T(\mathbf{I}_{M_r}) \right] \left[ \mathbf{I}_{M_t} \otimes \mathbf{K}_{M_t, M_r} \otimes \mathbf{I}_{M_r} \right]. \quad (27)$$

*Lemma 4:* The precoder that is optimal for Problem 1 must satisfy Equations (28), (29), and (30) in the bottom of the next page for the  $M$ -PSK,  $M$ -PAM, and  $M$ -QAM constellations, respectively.  $\mu$  is a positive scalar chosen such that the power constraint in Equation (25) is satisfied.

*Proof:* See [17]. ■

Equations (28), (29), and (30) can be used in a fixed point iteration for finding the precoder that solves Problem 1. Notice that the positive constants  $\mu'$  and  $\mu$  are different.

## VII. RESULTS AND COMPARISONS

SNR is here defined as:  $\text{SNR} = 10 \log_{10} \frac{P}{N_0}$ . Comparisons are made against a system not employing any precoding, i.e.,  $\mathbf{F} = \sqrt{\frac{P}{K a \sigma_x^2 M_t}} \mathbf{I}_{M_t}$  and the system minimizing an upper bound of the PEP [10].

The following parameters are used in the examples:  $P = 1$ , and  $M_r = 6$ . The signal constellation is 8-PAM with  $\sigma_x^2 = 1/2$ . As OSTBC the code  $\mathbf{C}(\mathbf{x}) = \mathcal{G}_4^T$  in [13] was used such that  $a = 2$ ,  $K = 4$ ,  $M_t = B = 4$ , and  $N = 8$ .

Let the correlation matrix  $\mathbf{R}$  be given by

$$(\mathbf{R})_{k,l} = 0.9^{|k-l|}, \quad (31)$$

where the notation  $(\cdot)_{k,l}$  picks out element with row number  $k$  and column number  $l$ .

Figure 2 show the SER versus SNR performance for the trivially precoder, the minimum upper bound PEP precoder [10], and the proposed minimum SER precoder. From the figures, it is seen that proposed minimum SER precoder outperforms the reference systems for all values of SNR. The performance of the proposed system is similar to the minimum PEP precoder for low and high values of SNR, but for moderate values of SNR, a gain up to 0.8 dB can be

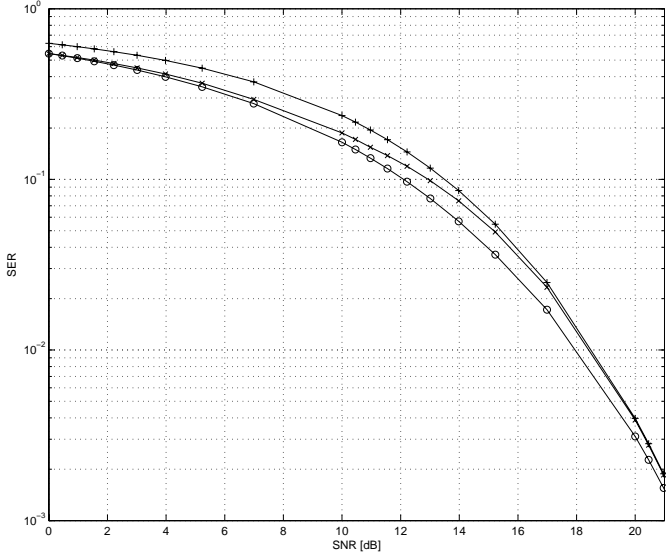


Fig. 2. SER versus SNR performance of the proposed minimum SER precoder — o —, the trivial precoder — + —, and the minimum PEP precoder — × — proposed in [10].

achieved over the minimum PEP precoder [10] and up to 3.5 dB is achieved over the trivial precoder.

If the same parameters are used as in [11] with  $\mathbf{F} = \mathbf{I}_{M_t}$  and  $\mathbf{R} = \mathbf{I}_{M_t M_r}$ , then the same results are found as reported in [11].

By Monte Carlo simulations, the exact theoretical SER expressions were verified.

### VIII. CONCLUSIONS

For an arbitrary given OSTBC, exact SER expressions has been derived for a precoded MIMO system equipped with multiple antennas both in the transmitter and the receiver. The receiver employs MLD and has knowledge of the exact channel coefficients, while the transmitter only knows the channel correlation matrix. A fixed point method is proposed for finding the minimum SER precoder for  $M$ -PSK,  $M$ -PAM, and  $M$ -QAM signalling. The proposed precoders outperforms the trivial precoder and the precoder that minimizes an upper bound for the PEP.

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$$\text{vec}(\mathbf{F}) = \mu \left[ \mathbf{F}^T \otimes \mathbf{I}_{M_t} \right] \mathbf{L} \left[ \mathbf{R}^{1/2} \otimes \left( \mathbf{R}^{1/2} \right)^* \right] \int_0^{\frac{M-1}{M}\pi} \frac{\text{vec} \left( \left[ \mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{PSK}}}{\sin^2(\theta)} \Phi^* \right]^{-1} \right) d\theta}{\sin^2(\theta) \det \left( \mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{PSK}}}{\sin^2(\theta)} \Phi \right)}, \quad (28)$$

$$\text{vec}(\mathbf{F}) = \mu \left[ \mathbf{F}^T \otimes \mathbf{I}_{M_t} \right] \mathbf{L} \left[ \mathbf{R}^{1/2} \otimes \left( \mathbf{R}^{1/2} \right)^* \right] \int_0^{\frac{\pi}{2}} \frac{\text{vec} \left( \left[ \mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{PAM}}}{\sin^2(\theta)} \Phi^* \right]^{-1} \right) d\theta}{\sin^2(\theta) \det \left( \mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{PAM}}}{\sin^2(\theta)} \Phi \right)}, \quad (29)$$

$$\text{vec}(\mathbf{F}) = \mu \left[ \mathbf{F}^T \otimes \mathbf{I}_{M_t} \right] \mathbf{L} \left[ \mathbf{R}^{1/2} \otimes \left( \mathbf{R}^{1/2} \right)^* \right] \left[ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\text{vec} \left( \left[ \mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{QAM}}}{\sin^2(\theta)} \Phi^* \right]^{-1} \right) d\theta}{\sin^2(\theta) \det \left( \mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{QAM}}}{\sin^2(\theta)} \Phi \right)} + \frac{1}{\sqrt{M}} \int_0^{\frac{\pi}{4}} \frac{\text{vec} \left( \left[ \mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{QAM}}}{\sin^2(\theta)} \Phi^* \right]^{-1} \right) d\theta}{\sin^2(\theta) \det \left( \mathbf{I}_{M_t M_r} + \delta \frac{g_{\text{QAM}}}{\sin^2(\theta)} \Phi \right)} \right]. \quad (30)$$