ANALYSIS OF QUANTIZATION NOISE FEEDBACK IN CAUSAL TRANSFORM CODING

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ABSTRACT

The performances of the LDU (Lower-Diagonal-Upper (LDU) factorization) transform were recently shown to be equivalent to those of the Karhunen-Loève Transform (KLT, which is optimal for Gaussian sources) in the limit of high rates [1, 2, 3]. In this paper, we further investigate the performances of the LDU for actual transform coding (TC) schemes. The results of [2] showed that the LDU should be implemented in closed loop around the quantizers, though this leads to a noise feedback effect, similar to that occuring in DPCM systems. We develop in this paper novel analyses of these effects on the distortion-rate functions and coding gains. The proposed analyses compare the results of [2] obtained for an *hypothetical* TC system for which the bit allocation is optimal and the rate is high, to those obtained for *practical* TC systems whose bit allocation is nearly optimal. By means of a theorem and numerical results, evidence is given that ordering the subsignals in the source vector by order of decreasing variance minimizes the quantization noise feedback. For the investigated practical systems, we show that deviations from the high rate assumptions arise below $\approx$ 3 b/s. The effects of the noise feedback become non negligible below $\approx$ 2 b/s. The LDU competes with the KLT above ≈ 2.5 b/s.

1. INTRODUCTION

Transform codes are popular because they provide an attractive compromise between computational complexity and performance. They allow to code with relatively low complexity long data blocks at the cost of being suboptimal in the rate-distortion sense. A pervasive use is made of orthogonal transforms, since they guarantee that the quantization noise will not be amplified. Among them, the KLT has become a benchmark, since it has been proven to be optimal for Gaussian sources [4]. In the framework of classical tranform coding (TC), where the rate is high, the sources are stationary Gaussian and the bit allocation optimal, another transform was recently shown to be optimal: the causal transform [1, 2, 3]. This transform can be shown to be a particular case of a general causal coding framework [5].

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For practical TC systems however, one or several of the assumptions above may not be verified. It seems therefore interesting to investigate how the performances of the causal transform are affected in practical cases. We shall nevertheless restrict the scope of this work to the case of Gaussian sources, essentially because this assumption renders the analytical evaluations tractable.

As a consequence of the non-orthogonality of the LDU, efficient causal coding structures should be implemented in closed loop around the quantizers, as in (A)DPCM systems [2]. As a consequence of the closed loop implementation, a quantization noise feedback increases the resulting distortion. Since the noise feedback arises in actual implementable causal coding structures, a realistic analysis of the coding performance of the causal transform should evaluate in which range of rates this noise feedback becomes important, and how the corresponding coding performances are actually deteriorated. No such analyses were proposed in [1] nor in [3]. The theoretical analysis of this particular problem in [2] assumed stringent assumptions, namely an optimal bit assignment, and a sufficiently high rate for the quantizer's performance factor to be constant. Hence, these results have remained difficult to corroborate. The main goal of the present paper is to describe quantitatively how the noise feedback impacts the coding performances of actual causal TC coding systems, and to evaluate the preciseness of the analysis of [2] in this case. A theorem showing that the signals within the source data blocks should be processed by order of decreasing variances will be proposed. This theorem completes the analysis of [2], and evidence of its validity will be given for the practical systems investigated.

Paper Outline : Some results regarding the causal transform in the classical TC framework are recalled in the second Section. In the third part, the analysis of [2] is summarized and completed. A practical causal TC system is then investigated in Section 4. The last part summarizes the main results.

2. CAUSAL TRANSFORM CODING WITH NEGLIGIBLE QUANTIZATION NOISE FEEDBACK

In the classical TC framework, a matrix transform T is applied to each source vector $\underline{x}_k$. This vector may be composed by N consecutive samples of the same scalar signal x, in which case $\underline{x}_k = [x_k \ x_{k-1} \cdots x_{k-N+1}]^T$, or by the samples of N scalar subsignals $\{x_i\}$, in which case $\underline{x}_k = [x_{1,k} \ x_{2,k} \cdots x_{N,k}]^T$ is a sample of a vectorial signal $\underline{x}$. The causality refers to the ordering of the samples $\{x_k\}$, or of the scalar signals $\{x_i\}$ which compose $\underline{x}$. The influence of a component permutation on the coding gains will be investigated in this Section and in Section 3.

The components of the transform vector $\underline{y}_k = T\underline{x}_k$ form a set of transform coefficients which are independently quantized using scalar quantizers. In what follows, we will consider (jointly) Gaussian sources with known covariance matrix $R_{\underline{x}\underline{x}}$.

In the causal case, $\underline{y}_k = L\underline{x}_k = \underline{x}_k - \overline{L}\underline{x}_k$, where $\overline{L}\underline{x}_k$ is the reference vector. The output $\underline{x}_k^q$ is $\underline{y}_k^q + \overline{L}\underline{x}_k$. As detailed in [1, 2], the components y_i are the prediction errors of x_i with respect to the previous components of $\underline{x}$, the $\underline{x}_{1:i-1}$, and the optimal coefficients $-L_{i,1:i-1}$ are the optimal prediction coefficients. It follows that

$$LR_{\underline{x}\underline{x}}L^T = R_{\underline{y}\underline{y}} = \overline{\text{diag}}\,\{\sigma_{y_1}^2 \cdots \sigma_{y_N}^2\} \Leftrightarrow R_{\underline{x}\underline{x}} = L^{-1}R_{\underline{y}\underline{y}}L^{-T} \quad (1)$$

where $\overline{\text{diag}}\,\{\underline{a}\}$ represent the diagonal matrix with diagonal $\underline{a}$. Exp. (1) represents the LDU factorization of $R_{\underline{x}\underline{x}}$. The distortion is given by $\mathbb{E}\,\|\underline{y}-\underline{y}^q\|_L^2 = \mathbb{E}\,\|\underline{\widetilde{y}}\|_L^2 = c\;2^{-2r}\,(\det R_{\underline{x}\underline{x}})^{\frac{1}{N}} = \sigma_q^2$, where $r = 1/N\sum_{i=1}^N r_i$. The coding gain [6] for L is

$$G_L^{(0)} = \left(\frac{\det\left[\,\text{diag}\,(R_{\underline{x}\underline{x}})\right]}{\det\left[\,\text{diag}\,(LR_{\underline{x}\underline{x}}L^T)\right]}\right)^{\frac{1}{N}} = \left(\frac{\det\left[\,\text{diag}\,(R_{\underline{x}\underline{x}})\right]}{\det\Lambda}\right)^{\frac{1}{N}} = G_V^{(0)}, \quad (2)$$

where the superscript $^{(0)}$ refers to the ideal case where the rate is sufficiently high (for the prediction to be based on unquantized data, and the quantizers'performance factor to be constant) and the bit assignment is optimal. The notation $\text{diag}\,\{A\}$ denotes the diagonal matrix with same diagonal as A, V denotes a KLT of $R_{\underline{x}\underline{x}}$ and Λ the corresponding matrix of eigenvalues. This is the best coding gain achievable among all unimodular transforms. Moreover, this gain is invariant by permutation: consider the vector $\mathcal{P}\underline{x}_k$, where $\mathcal{P}$ is a permutation matrix. Let us denote by $L_{\mathcal{P}}$ the corresponding LDU transform such that $\mathbb{E}\,(L_{\mathcal{P}}\mathcal{P}\underline{x}_k)(L_{\mathcal{P}}\mathcal{P}\underline{x}_k)^T = L_{\mathcal{P}}R_{\underline{x}\underline{x},\mathcal{P}}L_{\mathcal{P}}^T = R_{\underline{y}\underline{y},\mathcal{P}}$. Since permutation matrices are unimodular, $\det R_{\underline{y}\underline{y},\mathcal{P}} = \det R_{\underline{x}\underline{x},\mathcal{P}} = \det R_{\underline{x}\underline{x}}\det\mathcal{P}\mathcal{P}^T = \det R_{\underline{x}\underline{x}}$. Hence, the coding gain is still given by (2). We shall see in Section 3 that this invariance is broken when the effects of quantization noise are not negligible in the closed loop structure.

The causal transform presents aditionally several advantages w.r.t. the KLT [1, 2], among which : lower design and implementation complexity, or "robustness" as the transform coefficients are quantized. Moreover, the LDU can be naturally used either for lossy or lossless compression and for *on-line* transform coding [5].

3. HIGH RESOLUTION ANALYSIS OF A CLOSED LOOP CAUSAL TRANSFORM CODING SCHEME

The analysis of [2] can be closely related to that of the noise feedback in closed loop DPCM coding schemes [6]. In all the subsequent analyses we will assume Entropy Coded Uniform Quantization (ECUQ). For this type of quantizers, the additive quantization noise model is accurate for a wide range of rates (see [5]). The operational distortion-rate functions of the quantizers are denoted by $\sigma_{\widetilde{y}_i}^2 = c2^{-2r_i}\sigma_{y_i}^2$, where c generally depends on r. For sufficiently high rates, c tends to $\pi e/6$. This approximation (known as the Gish and Pierce approximation) will be retained in this Section, as well as the optimal bit assignment assumption.

3.1. Quantization Noise Feedback Analysis

The following development accounts only for the first order of the perturbations. In the sequel, the superscript $'$ will denote quantities obtained in the presence of noise feedback. The causal transform will be denoted by L' because its design may be different from the that of the transform L (designed for a system without, or with negligible feedback). We shall see however that as in DPCM, the optimal predictor does not essentially vary, and that $L' \approx L$.

In the case where the reference vector is based on quantized data, the output vector becomes $\underline{y}_k = \underline{x}_k - \overline{L}'\underline{x}_k^q = \underline{x}_k - \overline{L}'(\underline{x}_k - \underline{\widetilde{x}}_k) = L'\underline{x}_k + \overline{L}'\underline{\widetilde{y}}_k$. The difference vector $\underline{y}_k$ now not only contains the prediction error $L'\underline{x}_k$ of $\underline{x}_k$, but also the quantization error $\underline{\widetilde{y}}_k$ filtered by the predictor $\overline{L}'$. An alternative (and equivalent) representation of the closed loop causal coding scheme as described above can be obtained by coding the transform components without reconstructing the data, that is, by using the quantized whitened versions y_i^q instead of x_i^q to compute the prediction. In both cases, the variances of the quantization noises are, for an optimal bit assignment, equal :

$${\sigma'}_{\widetilde{y}_i}^2 = c2^{-2r}(\prod_{i=1}^N {\sigma'}_{y_i}^2)^{\frac{1}{N}} = {\sigma'}_q^2, \quad (3)$$

and the autocorrelation matrix of the noise is $R'_{\underline{\widetilde{y}}\underline{\widetilde{y}}} = {\sigma'}_q^2 I$. Comparing with Section 2, the prediction error variances are increased because the reference vector is based on quantized data, and the quantization noise variances are therefore increased to ${\sigma'}_q^2$. One shows that the problem of optimizing L' corresponds to the optimal prediction of $\underline{x}$ perturbed by a white noise. Thus, we should look for

$$\min_{L'_{i,1:i-1}} L'_i(R_{\underline{x}\underline{x}} + {\sigma'}_q^2 I){L'}_i^T\;. \quad (4)$$

The resulting prediction error variances are

$${\sigma'}_{y_i}^2 \approx (LR_{\underline{x}\underline{x}}L^T + {\sigma'}_q^2\overline{L}\overline{L}^T)_{ii} \approx (LR_{\underline{x}\underline{x}}L^T + \sigma_q^2\overline{L}\overline{L}^T)_{ii}, \quad (5)$$

where σ_q^2, L and $\overline{L}$ are non perturbed quantities (Sec. 2).

Suppose now that the transform L of Section 2, eq. (1) (*i.e.* optimized for a system with negligible noise feedback) is used to compute the reference vectors in a closed loop coding scheme. Then the variances of the transform signals will also be given by (5), which is obtained with the transform L' of eq. (4). Thus, the optimal predictor design should not essentially vary when one accounts for the first order of the perturbations in closed loop causal TC. The performances degradation come mainly, at high rates, from the filtering of the quantization noise by the predictors (rows of the matrix L). The distortion (3) is in this case

$$\begin{aligned}\tfrac{1}{N}\,\mathbb{E}\,\|\underline{\widetilde{y}}\|'^2_{L'} &\approx \tfrac{1}{N}\,\mathbb{E}\,\|\underline{\widetilde{y}}\|'^2_{L} \approx \tfrac{1}{N}\sum_{i=1}^N \frac{\pi e}{6}2^{-2r_i}{\sigma'}_{y_i}^2 \\ &\approx \tfrac{\pi e}{6}2^{-2r}\left(\prod_{i=1}^N {\sigma'}_{y_i}^2\right)^{\frac{1}{N}} \approx \sigma_q^2\left(1 + \tfrac{\sigma_q^2}{N}\sum_{i=1}^N \frac{\|\overline{L}_i\|^2}{\sigma_{y_i}^2}\right).\end{aligned} \quad (6)$$

This leads to the following expression for the coding gain $G_L^{(1)}$

$$G_L^{(1)} \approx G_{L'}^{(1)} \approx G_L^{(0)}\left(1 - \frac{1}{N}\sigma_q^2\sum_{i=1}^N \frac{\|\overline{L}_i\|^2}{\sigma_{y_i}^2}\right). \quad (7)$$

3.2. Influence of the Permutation on the Coding Gain

An equivalent expression of $G_L^{(1)}$ is (see Appendix 2.A in [5])

$$G_L^{(1)} \approx G_L^{(0)} \left(1 - \frac{\sigma_q^2}{N} \sum_{i=1}^{N} \left[\frac{1}{\lambda_i} - \frac{1}{\sigma_{y_i}^2}\right]\right), \qquad (8)$$

where $\{\lambda_i\}$ are the eigenvalues of $R_{\underline{x}\underline{x}}$. Thus, maximizing the coding gain entails maximizing the sum of the inverses of the prediction error variances : whereas $G_L^{(0)}$ is invariant by permutation there should be for the closed loop causal TC scheme an optimal ordering of the components $\{x_i\}$ of $\underline{x}_i$. Comparing $G_L^{(1)}$ in (8) with the infinite resolution case (2), the different prediction error variances produced by different decorrelation approaches induce now different gains. Hence, the coding gain $G_L^{(1)}$ depends on a carefull choice of the decorrelation procedure. In the case $M = 2$, maximizing the coding gain entails making the variances as different as possible. Thus, the subsignal of greater variance should be processed first, and the off-diagonal predictor should be used to decrease the variance of the subsignal of lower variance. Now under the assumptions stated above, the following theorem holds for $M > 2$.

Theorem : **Optimal ordering of the subsignals for the triangular transform**. *The optimal ordering of the subsignals in a stationary vectorial signal for maximizing the high-resolution coding gain $G_L^{(1)}$ of the causal LDU transform implemented in closed loop is obtained by processing the signals in order of decreasing variance.*

To show the theorem, consider a recursive argument. First of all, the theorem is clearly true for the case of two channels. Now consider $n-1$ channels that we have ordered in order of decreasing variance. When we add a nth channel, the question is in which position it should be put w.r.t. the other channels. Assume in a first scenario that we put the channel in a position such that all n channels are in order of decreasing variance. Assume in a second scenario that we insert the nth channel at another position. Then we can evolve from the first to the second scenario by a sequence of permutations of two consecutive channels. In one such permutation operation, assume that the channels involved in the permutation are in positions i and $i+1$. Then the channels $1, \ldots, i-1$ are unaffected in the triangular prediction approach. The channels $i+2, \ldots, n$ are also unaffected by the order in which channels i and $i+1$ are put since in any case they get orthogonalized w.r.t. the signals in those channels. So the only effect of the permutation between channels i and $i+1$ is on the prediction error variances of those channels i and $i+1$. In other words we are reduced to the two channel case, in which case we know that we should put the channels in order of decreasing variance. So, as we move from scenario one to scenario two by a succession of permutations of two consecutive channels, we decrease the coding gain. Hence, the optimal ordering is in order of decreasing variance.

This theorem is a special case of a more general theorem proposed for triangular MIMO prediction in [7].

4. ANALYSIS OF A PRACTICAL CASE

A simple mean of realizing nearly optimal bit assignment in the case of ECUQ is to quantize the signals with equal quantization stepsizes. This case allows one to check, for a practical TC system, several results. Firstly, in which range of average rates the LDU implemented in closed loop suffers from a non negligible noise feedback, and for which rates it presents similar coding performances to those of the KLT. Secondly, if the previously exposed analyses, which are subject to the assumptions of high resolution and optimal bit allocation, have some value in this practical case. Thirdly, if the claimed decorrelation strategy consisting in processing the signals by order of decreasing variance is actually the best one.

4.1. Optimal Bit Assignment and Equal Quantization Stepsize

The classical result of the optimal bit assignment states that given a set of variances $\sigma_{y_i}^2$, the quantization noise power $\sigma_{\tilde{y}_i}^2$ should be equal for all the components. The number of bits assigned to the ith component is $r_i = r + \frac{1}{2}\log_2[\sigma_{y_i}^2 / \left(\prod_{i=1}^{N} \sigma_{y_i}^2\right)^{\frac{1}{N}}]$. Under high resolution assumption, the quantization noise resulting from quantization with stepsize Δ_i is uniformly distributed with variance $\sigma_{\tilde{y}_i}^2 = \Delta_i^2/12$. A simple way of realizing an equal distortion is therefore to quantize all the components with an equal Δ. If the $\{y_i^q\}$ are further entropy coded, the bitrates $\{r_i\}$ are given by the Rényi relation of differential to discrete entropy $r_i = H(y_i^q) \approx \frac{1}{2}\log_2 2\pi e \sigma_{y_i}^2 - \log_2 \Delta$. It can then easily be checked that choosing $\Delta = \sqrt{2\pi e}\, 2^{-r} (\prod_{i=1}^{N} \sigma_{y_i}^2)^{\frac{1}{2N}}$ corresponds to $\frac{1}{N}\sum_{i=1}^{N} r_i = \frac{1}{N}\sum_{i=1}^{N} H(y_i^q) \approx r$ [4].

4.2. Distortion Analysis

For large quantization stepsizes (low rates), the Rényi relation above may not be accurate, be there a noise feedback or not. This renders the theoretical analysis of the system somewhat difficult at low rates. Our analysis will be guided again by that of DPCM systems. In order to describe the coding system implemented in closed loop, we shall consider perturbations w.r.t. a system using unquantized data for the prediction. For this system, the distortion of each component is $\mathbb{E}\,\tilde{y}_i^2 = d_{i,L} = c_i(\Delta) 2^{-2r_i} \sigma_{y_i}^2$, where $\sigma_{y_i}^2$ is the ith optimal prediction error variance, and the notation $c_i(\Delta)$ reflects the fact that the quantizers'performance factor may be different from each other, and may depend on Δ at low rates. The average distortion is in this case $\frac{1}{N}\mathbb{E}\,||\underline{\tilde{y}}||_L^2 = \frac{1}{N}\sum_{i=1}^{N} d_{i,L} = \frac{1}{N}\sum_{i=1}^{N} c_i(\Delta) 2^{-2r_i} \sigma_{y_i}^2$. Numerical results [5] indicate that even without noise feedback, the average distortion $\frac{1}{N}\mathbb{E}\,||\underline{\tilde{y}}||_L^2$ deviates noticeably from (and is superior to) the high rate and optimal bit allocation approximation $\frac{\pi e}{6} 2^{-2r} (\det R_{\underline{x}\underline{x}})^{\frac{1}{N}}$ at rates lower than $\approx$ 3 b/s. Now, for a closed loop causal TC system working at moderate to low rates, the average distortion may be expressed as

$$\frac{1}{N}\mathbb{E}\,||\underline{\tilde{y}}||_{L'}'^{\,2} = \frac{1}{N}\sum_{i=1}^{N} d'_{i,L} = \frac{1}{N}\sum_{i=1}^{N} c_i(\Delta) 2^{-2r_i} {\sigma'}_{y_i}^2. \qquad (9)$$

For our model, we shall assume that the covariance matrix of the quantization noise is well approximated by $\frac{\Delta^2}{12} I$ at moderate to high rates. Thus, the optimal transform is again given by (4). The actual prediction error variances $\{{\sigma'}_{y_i}^2\}$ may still be approximated by expression (5). Again, for small perturbations, the optimal transform L' (minimizing (4)), or the transform designed without feedback L in (1) should be sensibly equal. Using (5), the operational distortion-rate function of the transform signals with quantization noise feedback may be evaluated as

$$\begin{aligned} d'_{i,L} &\approx c_i(\Delta) 2^{-2r_i} {\sigma'}_{y_i}^2 \\ &\approx c_i(\Delta) 2^{-2r_i} (\sigma_{y_i}^2 + d'_{i,L} (\overline{LL^T})_{ii})) \\ d'_{i,L}(1 - ||\overline{L_i}||^2 c_i(\Delta) 2^{-2r_i}) &\approx d_{i,L} \\ d'_{i,L} &\approx \frac{d_{i,L}}{1 - ||\overline{L_i}||^2 c_i(\Delta) 2^{-2r_i}} \end{aligned} \qquad (10)$$

Hence, the actual distortion in this system can be evaluated as

$$\begin{aligned}\tfrac{1}{N}\,\mathbb{E}\,\|\underline{\tilde{y}}\|'^2_L = \tfrac{1}{N}\sum_{i=1}^{N} c_i(\Delta)2^{-2r_i}\sigma'^2_{y_i} &\approx \frac{1}{N}\sum_{i=1}^{N}\frac{d_{i,L}}{1-\|\overline{L_i}\|^2 c_i(\Delta)2^{-2r_i}} \\ \approx \tfrac{1}{N}\sum_{i=1}^{N}\frac{d_{i,L}}{1-\|\overline{L_i}\|^2\frac{d_{i,L}}{\sigma^2_{y_i}}} &\approx \frac{1}{N}\sum_{i=1}^{N} d_{i,L}\left(1+\|\overline{L_i}\|^2\frac{d_{i,L}}{\sigma^2_{y_i}}\right).\end{aligned} \tag{11}$$

As the rate increases, the distortions $\{d_{i,L}\}$ tend to $\frac{\pi e}{6}2^{-2r_i}\sigma^2_{y_i} = \frac{\pi e}{6}2^{-2r}(\det R_{\underline{x}\underline{x}})^{\frac{1}{N}} = \sigma^2_q$, and the above distortion tends to expression (6), which was derived under the assumptions of optimal bit assignment and constant quantizer's performance factors.

4.3. Numerical results

The data are real Gaussian i.i.d. vectors with covariance matrix $R = HR_{AR1}H^T$. R_{AR1} is the covariance matrix of an AR(1) process with parameter $\rho = 0.9$. H is a diagonal matrix whose ith entry is $(N-i+1)^{1/3}$, $N = 3$. The signals x_i are coded

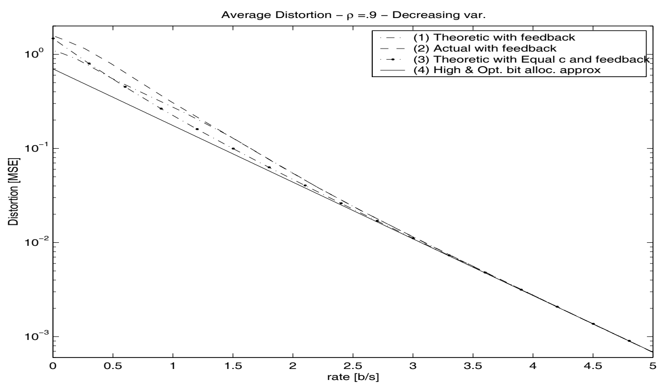

Fig. 1. Distortion for the LDU (equal Δ, decreasing variances).

by order of either decreasing, or increasing variances. For these two decorrelating strategies, sets of 10^4 vectors were transformed using the algorithm based on reconstructed data. In fig. 1 and 2, the distortion-rate functions of the closed loop causal transform are plotted for signals of decreasing and increasing variances respectively (optimal transform L of eq. (1) is used)

- *(1)* "Theoretic with feedback" refers to the analytical evaluation (11) of a system with equal stepsize,
- *(2)* "Actual with feedback" corresponds to the actual distortion-rate function of the closed loop TC system,
- *(3)* "Theoretic with Equal c and feedback" refers to the analytical evaluation (6) : optimal bit assignment algorithm and $c = \frac{\pi e}{6}$,
- *(4)* "High & Opt. bit alloc. approx" refers to the performance of an ideal system without feedback, constant quantizer performance factor, and optimal bit allocation, *i.e.* $\frac{\pi e}{6}\ 2^{-2r}\ (\det R_{\underline{x}\underline{x}})^{\frac{1}{N}}$.

It can be observed for both decorrelation strategies that :

- The performance of actual systems *(2)* deviate from their high rate approximation *(4)* for rates below approximately 3 b/s.
- These performance are accurately described by the analysis (curve *(1)*) down to approximately 1 b/s.
- The analysis of Section 3.1, which does not account for possible variations of c w.r.t. the rate underestimates the actual distortions (as discussed in Section 4.2, the actual distortion in the transform domain with equal Δ is larger than $\pi e/6\ 2^{-2r}\ (\det R_{\underline{x}\underline{x}})^{1/N}$ even without noise feedback).

Comparing now the Figures 1 and 2, better performance are clearly obtained by processing the signals by order of decreasing variance, as suggested by the high rate analysis of Section 3.1. Complementary numerical results [5] show that the performances of the LDU are inferior to those of the KLT at low rates only : ≈ 2.5 b/s; the LDU with either a decreasing- or increasing-variance decorrelation strategy is still advantageous (w.r.t. direct entropy coding the signals) at all rates. Results regarding the equivalence of the two

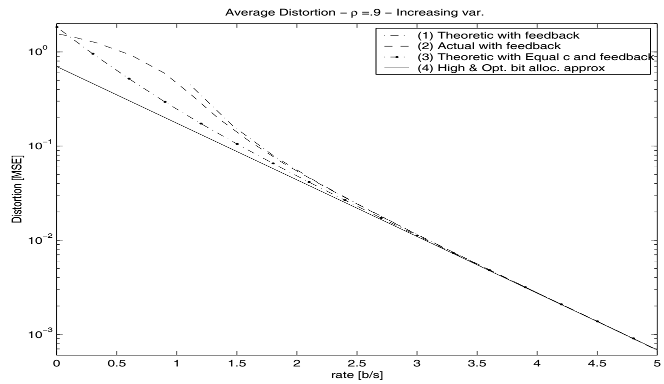

Fig. 2. Distortion for the LDU (equal Δ, increasing variances).

coding schemes discussed in Sec. 3.1, and that of the transform L and L' have also been obtained.

5. CONCLUSIONS

The analysis [2] of the noise feedback in the causal case was summarized and completed by a theorem, showing that decorrelating the signals by order of decreasing variances maximizes the coding gain. An analytical evaluation of practical TC algorithms was then presented, which use equal quantization stepsize, and entropy coded uniform quantizers. These systems allow one to corroborate the results obtained in [2]. The proposed evaluation accounts correctly for the variations of the quantizer's performance factors and the noise feedback down to ≈ 1 b/s. For these systems, the deviation from the classical TC framework are noticeable below ≈ 3 b/s for both the KLT and the LDU. In the causal case, the effects of the noise feedback become non negligible beyond ≈ 2 b/s. The decorrelation strategy suggested by the theorem was confirmed. Comparing finally the two approaches, the LDU is shown to compete with the KLT at rates higher than ≈ 2 b/s, though requiring a least computational complexity.

6. REFERENCES

[1] S.-M. Phoong and Y.-P. Lin, "Prediction-based lower triangular transform," *IEEE trans. on Sig. Proc.*, July 2000.

[2] D. Mary and D. T. M. Slock, "Vectorial DPCM coding and application to wideband coding of speech," in *ICASSP 2001*.

[3] F. Lahouti and A.K. Khandani, "Sequential vector decorrelation technique," Tech. Rep. 2001-4, Univ. of Waterloo, 2001.

[4] V. Goyal, J. Zhuang, and M. Vetterli, "Transform coding with backward adaptive updates," *IEEE Transactions on Information Theory*, vol. 46, no. 4, July 2000.

[5] D. Mary, *Causal Lossy and Lossless Coding of Vectorial Signals*, Ph.D. thesis, ENST, Paris, March 2003.

[6] N.S. Jayant and P. Noll, *Digital Coding of Waveforms*, Prentice Hall, 1984.

[7] D. Mary and D.T.M. Slock, "Causal transform coding, generalized MIMO prediction and application to vectorial DPCM coding of multichannel audio," in *WASPAA01*.