THROUGHPUT ANALYSIS FOR DECENTRALIZED SLOTTED REGULAR WIRELESS NETWORKS

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Abstract— **In this work, we are interested in characterizing the performance of decentralized multiple-access and retransmission schemes for wireless ad hoc networks. We are considering a regular linear network model of sender/receiver pairs, where nodes transmit their information messages over a common radio channel. We obtain expressions for node throughput as a function of inter-node separation for different retransmission protocols while communications are done in a single-hop fashion. Then we extend our model to take into account relaying between nodes. We analyze the achievable throughput under traffic patterns where local communication predominates, as a function of transmission distance and relay distance. Our analysis focuses on static ad hoc networks where we show that coding and retransmission provide reliable communication with a completely decentralized multiple-access strategy for both single-hop and multi-hop communications.**

I. INTRODUCTION

The study of wireless ad hoc networks has recently received significant attention. An ad hoc network is a collection of wireless nodes forming a network without the use of any existing network infrastructure or centralized coordination. This lack of any centralized control gives rise to many issues at the physical layer which make the analysis of such networks complex. In [1], Gupta and Kumar determined the capacity of wireless networks under certain assumptions and point out a basic behavior of current wireless networks. They showed that given n nodes in the unit disk and an uniform traffic pattern, the aggregate capacity is of $O(\sqrt{n})$. In [2], the model in [1] was modified to take into account mobility and using only one-hop relaying, it was shown that an $O(n)$ aggregate throughput can be obtained.

In this work we are interested in decentralized ad hoc wireless networks ruling out the possibility of coordination between nodes (e.g. TDMA-based exclusion techniques) and provide a simple setting to characterize the performance of such networks. We start by analyzing a small decentralized network where communications are peer-to-peer and are done in a single-hop fashion. The nodes access the channel at random and employ simple protocols to retransmit the erroneously received packets. We consider two possible retransmission protocols: the first is *Slotted Aloha* (using the wireless setting as in [3]) where decoding considers only the most recent received block; the second is *Incremental Redundancy* where decoding takes into account all previously received signal blocks and performs soft combining until decoding is achieved successfully. Then we compute the node throughput and we carry out its optimization with respect to system parameters. We extend these results by requiring that nodes act as relays in addition to sources for packets in order to analyze the per node throughput under a traffic pattern where nodes communicate predominantly with nearby neighbors. The achievable throughput per node depends on the traffic pattern parameters and the relay distance (which determines the relay load imposed on each node), the latter being a parameter to be optimized. The outline of the paper is as follows: In section II, we describe the system model and the setting. Section III deals with the throughput expressions of different retransmission protocols for single-hop communications and shows some numerical results. Section IV analyzes multi-hop communications. Finally, in Section V we draw some conclusions and point out future research directions.

II. SYSTEM MODEL AND SETTING

A. Network and Propagation Model

We consider an 1-D regular linear network where n nodes are located on a straight line as studied in [4] [5] . The reason for considering regular linear networks is for analytical simplicity only. The propagation model is described by two effects: the signal attenuation due to the distance r between the transmitter and the receiver, proportional to $r^{-\alpha}$, where α is the power loss exponent (positive number) ; and Rayleigh fading that causes random power variations. The received power P_R from a mobile at distance r is expressed as:

$$P_R = R_a^2 r^{-\alpha} P = \gamma r^{-\alpha} P \tag{1}$$

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where γ is an exponentially distributed random variable (for simplicity we consider that γ has mean equal to 1) and P is the transmit power.

B. System Model

In the system we are considering, each node can transmit over a common wireless channel. Packets are sent from one node to another in a peer-to-peer fashion. We divide the nodes into source-destination pairs and define Γ as the set of all sources. All nodes are continuously involved in exactly one communication: $|\Gamma| = \lfloor \frac{n}{2} \rfloor$. Apart from the slotted transmission structure where nodes transmit packets within slots of defined duration, nodes are completely uncoordinated. This slot structure requires some local frame synchronization method. The signal model is given by:

$$y_{j,s} = \sum_{k \in \Gamma(s)} \sqrt{\gamma_{k,j,s} P r_{k,j}^{-\alpha}} x_{k,s} + n_{j,s} \quad (2)$$

where the index s denotes the slot, $y_{j,s}$ the received signal at node j, $x_{k,s}$ the transmitted signal from node k, $n_{j,s}$ the background noise during slot s and $\Gamma(s) \subseteq \Gamma$ is the set of active nodes over slot s.

C. Setting

For the purpose of our analysis, we make the following assumptions:

- An infinite number of packets is available for each source. A packet can be seen as a separate codeword for which transmission is stopped when an acknowledgment of successful decoding is returned by the receiver. Furthermore, we assume that the ACK/NACK feedback signaling channel is error-free and delay-free. Moreover, the signaling overhead is insignificant with respect to the data channel.
- We suppose single-user decoding where each decoder treats the signals from other users as noise. Moreover, the single-user decoder for each node has perfect knowledge of the channel gain and the total interference power. This can be achieved in a real system by inserting some pilot symbols.
- We assume a block-fading channel model. The fading remains constant on the whole slot and is an i.i.d process across successive slots. In a real system, this can be achieved via frequency hopping across a large system bandwith.
- For each slot, each node transmits a packet with probability p_t and remains silent with probability $1 - p_t$.
- The system is completely symmetric with respect to any user: all users have the same transmit power, i.e., $P_k = P \ \forall k \in \Gamma$.

III. Single-Hop Communications

A. Outage Probability

The instantaneous average mutual information for a (s, d) pair conditioned on the channel gain $\gamma_{i,j,s}$ and the interference power I_Γ is:

$$\begin{aligned} I_{i,j,s} &= I(X_{i,j,s}; Y_{j,s} | \gamma_{i,j,s}, I_\Gamma) \\ &= \log \left(1 + \frac{\gamma_{i,j,s} P r_{ij}^{-\alpha}}{N_0 + I_\Gamma} \right) \text{bit/dim} \end{aligned} \quad (3)$$

where N_0 is the background noise power, P is the transmit power and $r_{ij} = |X_i - X_j|$where X_j is the position of the receiver and I_Γ is defined as:

$$I_\Gamma = \sum_{\substack{k \neq i \\ k \in \Gamma}} 1_{\{T_k = 1\}} \gamma_{k,j,s} P r_{kj}^{-\alpha} \quad (4)$$

where T_k is a Bernouilli random variable reporting that user k is transmitting with probability $\Pr(T_k = 1) = p_t$. $P_{out}(i)$ (given below) is the outage probability of the channel, the probability that the mutual information $I_{i,j,s}$ falls below some fixed spectral efficiency R_i. Expressions of the mutual information necessary for the outage probability evaluation are derived under the assumption that all user signals are Gaussian with flat power spectral density. The Gaussian assumption yields an upper-bound to the minimum achievable outage probability [4] [6].

$$\begin{aligned} P_{out}(i) = 1 - \exp\left(- \frac{(2^{R_i} - 1) N_0}{P r_i^{-\alpha}} \right) \\ \prod_{\substack{k \neq i \\ k \in \Gamma}} \left(\frac{p_t}{1 + (2^{R_i} - 1) \frac{r_i^\alpha}{r_k^\alpha}} + 1 - p_t \right) \end{aligned} \quad (5)$$

The computation is lengthy, so we omit it.

B. Slotted Aloha

The Slotted Aloha protocol can provide random multiple access to a common channel with minimal coordination between the channel users. The transmitter sends a codeword to the receiver and waits for an ACK. When the transmitter gets a NACK, it will resend the previous codeword until it gets an ACK from the receiver. We are interested in the per-node throughput, and following the analysis of [3], we define the throughput as $\eta_i = \frac{R_i}{\tau}$ where τ is the mean delay measured in slots for the transmission of an information message. In Aloha, the receiver has no memory of the past signals, and the probability of successful decoding after l transmitted slots is given by:

$$\Pr(I_{i,j,1} < R, \cdots, I_{i,j,l} > R) = P_{out}(i)^{l-1}(1 - P_{out}(i)) \tag{6}$$

and the mean delay is given by:

$$\tau = \frac{(1 - P_{out}(i)) \sum_{l=1}^{\infty} l P_{out}(i)^{l-1}}{p_t} = \frac{1}{p_t(1 - P_{out}(i))} \tag{7}$$

And by normalizing the transmit power by p_t we obtain the per node throughput for Slotted Aloha:

$$\eta(i) = R_i p_t \exp\left(-\frac{(2^{R_i} - 1)N_0 p_t}{P r_i^{-\alpha}}\right) \prod_{\substack{k \neq i \\ k \in \Gamma}} \left(\frac{p_t}{1 + (2^{R_i} - 1)\frac{r_i^{\alpha}}{r_k^{\alpha}}} + 1 - p_t\right) \tag{8}$$

C. Incremental Redundancy

The basic idea behind Incremental Redundancy is that it adjusts the code rate by incrementally transmitting redundancy information until decoding is successful. Indeed, if the receiver fails to successfully decode a packet, a NACK is sent to the transmitter. The latter will send additional new redundancy bits which are accumulated and processed by the receiver. As shown in [3], the throughput is given by:

$$\eta(i) = \frac{R_i p_t}{\sum_{l=0}^{\infty} \Pr(\sum_{s=1}^{l} I_{i,j,s} < R_i)} \tag{9}$$

One can notice that $\Pr(\sum_{s=1}^{l} I_{i,j,s} < R_i)$ is the cumulative density function of the sum of l i.i.d random variables distributed as $I_{i,j,s}$ and evaluated in R_i. This can be computed numerically by using the characteristic function and discrete Fourier transforms as we have already computed the cumulative density function of $I_{i,j,s}$ in closed form (5).

D. Numerical Results

We are interested in the throughput of a (s, d) pair where the source s is the intended node which communicates with the destination d. When the destination d is surrounded by transmitters, the communication (s, d) sees a high level of interference. This is actually the worst-case scenario, which we consider in Fig.1 and in the next section for our analysis. The throughput is expressed as a function of different system parameters: the transmit SNR $\frac{P}{N_0}$, the target information rate R_i and the transmit probability p_t. In the numerical results provided below, we consider a system that supports delay-limited applications. As a result, we are interested in optimal throughput (maximized over the transmit probability p_t) versus the mean delay (finite), where the latter is a function of the information rate R. Moreover, we compute the per node throughput

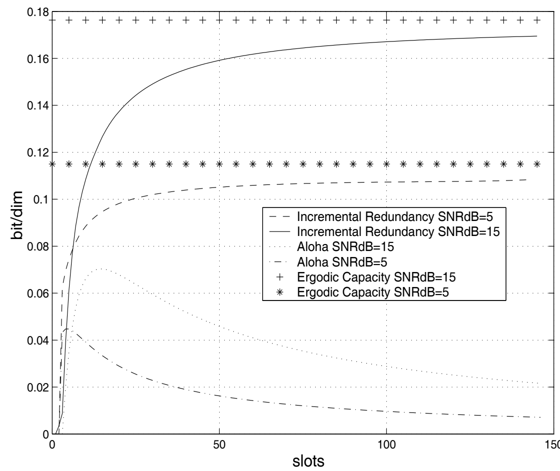

Fig. 1. Average throughput vs mean delay for different retransmission protocols for the worst-case scenario (destination surrounded by transmitters). $n = 30$ nodes.

averaged over all transmitting positions $E_i[\eta(i)]$. As opposed to Incremental Redundancy, the throughput for very high delay is zero for Slotted Aloha (actually one can say that the mean delay τ is growing faster than R which leads to a zero throughput). Since the throughput is zero for $\tau = 0$ ($R = 0$) and goes to 0 for high delay (high R), there exists an optimal delay/target information rate. Incremental redundancy is capacity achieving as shown in Fig.1 where the ergodic capacity was computed by Monte Carlo simulations, it means that $\eta(i) < p_t E[C_i]$ for finite delay, where $E[C_i]$ is the per node ergodic capacity.

IV. Multi-hop Communications

The performance limitation of an ad hoc network comes first from the long-range peer-to-peer communication (that causes excessive interference) and second the excessive amount of relayed traffic. We are interested in assessing the tradeoff between the levels of interference generated during parallel transmissions, and the average number of relays needed to transmit data between a source and its destination. Indeed, a high transmit power causes more interference and loss of packets (spatial concurrency), whereas a small transmit power increases the relay traffic. As relay communication is the bottleneck of the network, in our setting we will consider the per node throughput under non uniform traffic conditions where nodes communicate mostly with nearby nodes. In [7], traffic patterns that might allow the per node capacity to scale well with the size of the network are discussed. We consider a modified discrete exponential law for the source-destination distance distribution. That is, the probability that the distance between a source and a destination is i is given by:

$$\Pr(X = i) = p_i = (e^{\lambda} - 1)e^{-\lambda i}, \; i > 0 \tag{10}$$

A. Communication Scheme

Let us call D the relay distance. If the source destination distance is smaller than D, the communication is done in a peer-to-peer single-hop fashion, if not the message is sent through multiple hops on its way to destination. A node acts as a source, a destination or a relay. As a relay, it transmits messages to a node which is either a final destination of a (s, d) pair or an adjacent relay. Similarly, it receives messages either from a source or from an adjacent relay.

B. Multi-hop Throughput Expression

Let us fix a (s, d) pair. If the source destination distance ($dist(s - d)$) is greater than D, the throughput is related to the harmonic mean of the throughput at each hop:

$$\eta_{s-d} = \frac{1}{\sum_{i=1}^{k} \frac{1}{\eta_i}} \tag{11}$$

where k is the number of hops needed to reach the destination given by $k = \lceil \frac{dist(s-d)}{D} \rceil$, η_i is the throughput between two relays. Let $\eta_r'' = \min_i\{\eta_i\}$ be the minimum throughput between two relays, then:

$$\eta_{s-d} \geq \frac{\eta_r''}{k} = \frac{\eta_r'}{k(N_R + k)} \tag{12}$$

where $N_R + k$ is the maximum number of relays done by a node through the route $s - d$ (the throughput dedicated for a specific (s, d) pair is limited by the maximum number of routes through relays). The average throughput (over all (s, d) pairs) is then given by:

$$\begin{aligned} \overline{\eta_{s-d}} \geq & \frac{\eta_r'}{\overline{k}\,\overline{N_R} + \overline{k^2}} \Pr(dist(s-d) > D) \\ & + \frac{\eta_p'}{\overline{N_R} + \overline{k}} \Pr(dist(s-d) \leq D) \end{aligned} \tag{13}$$

η_r' is the per node incremental redundancy throughput between two relays and taking into account collision avoidance whereas η_p' is the peer-to-peer per node incremental redundancy throughput. $\overline{N_R}\ \overline{k^2}\ \overline{k}$ are computed in Appendix B using the exponentially decaying traffic pattern.
Above, we are assuming a routing algorithm which makes known to all relays on the route the destination's position.

C. Collision Avoidance

A collision occurs if:

- A node is receiving from two relays (left and right). To avoid this case, we schedule in time slot 1 transmissions done by the relays from right to left and in time slot 2 transmissions from left to right.
- A node is transmitting and receiving from another node at the same time. Let us define p_{t_f} as the probability that a relay transmits to a final destination or a source directly to its destination; and p_{t_r} the probability that a relay transmits to the subsequent relay or a source to a relay. A collision occurs with probability $p_t p_{t_r}$ for multi-hop communication and $p_{t_f}(p_{t_f} + 2p_{t_r})$ for single-hop communication.
- A relay R_n is receiving a message to relay from another relayR_{n-1} and a message from a source in a peer-to-peer fashion (R_n is the final destination of this source). This event occurs with probability $p_{t_r} p_{t_f}$.

Then:

$$\eta_r' = \frac{\eta_r(1 - p_{t_r} p_{t_f})(1 - p_t p_{t_r})}{T_s} \tag{14}$$

$$\eta_p' = \frac{\eta_p(1 - p_{t_r} p_{t_f})(1 - p_{t_f}(p_{t_f} + 2p_{t_r}))}{T_s} \tag{15}$$

where $T_s = 2$ and η_p, η_r are derived from (9) with
$p_t = p_{t_f} + p_{t_r}$ and p_{t_r}, p_{t_f} are parameters to be optimized.
To compute the outage probability, we make the Gaussian assumption as in (3). As we let the number of nodes n grows to infinity, we distinguish local and non local nodes. The set of strong interferers for a given receiver is limited to its nearest. The same arguments are given in [2] [8] [9]. As all the nodes are possibly transmitting, the interference is given by:

$$\begin{aligned} I &= I_L + I_\infty && (16) \\ &= \sum_{\substack{k \neq i \\ k \in \Omega_L}} 1_{\{T_k=1\}} \gamma_{k,j} P r_k^{-\alpha} + I_\infty && (17) \end{aligned}$$

where I_L is the effective interference from L surrounding nodes in the set Ω_L and I_∞ is considered as background noise and computed in Appendix A.

D. Numerical Results

We are interested in illustrating the impact of the traffic pattern and the relay load on the throughput. The relay distance influences the level of interference and the relay load: a high D reduces the incremental redundancy throughput (high level of interference) and a small D increases the relay load as shown in Fig.2. The $s - d$ distance distribution parameter λ should be high enough to reduce the relay load by imposing communications between nearby nodes. Mainly, by decreasing λ, we increase the effective range of communications, thus we need to increase the value of D to reduce the relay load but only up to the point where interference starts to become dominant. We notice that even for very localized communications (i.e. $\lambda = 1$) the optimal hop distance is greater than one. In our model, the mean source-destination distance is finite independent of the size of the network in contrast to the Gupta and Kumar model. As a result, the per-node throughput is non-vanishing even in the limit of infinite node population. This throughput increases with the available transmit power.

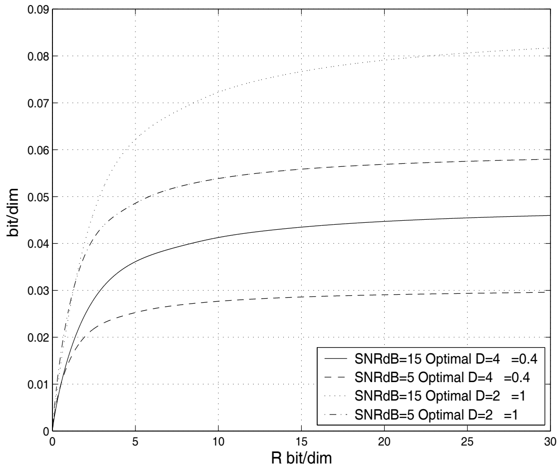

Fig. 2. The optimal relay distance D for different SNR and λ values.

V. Conclusions

We obtained throughput formulas for simple retransmission protocols for both single-hop and multi-hop ad hoc wireless networks. In the latter case, we analyzed the effect of traffic patterns, interference and relay load on the throughput with respect to system parameters (transmit SNR, transmit probability and information rate). It appears that coding and retransmission protocols are a viable and simple solution for providing fully decentralized multiple-access communications in ad hoc wireless networks despite harsh propagation characteristics (interference from nearby competing nodes).

This analysis will be extended to a hybrid network setting where relaying is used in conjunction with a fixed or wireless overlay network. Future work will also focus on more advanced strategies for cooperation, practical coding strategies and synchronization methods.

Appendix A

The power of I_∞ is given by:

$$E[I_\infty] = \sum_{k=L}^{\infty} E[I_k] \tag{A-1}$$

where $I_k = 1_{\{T_k=1\}}\gamma_{k,j}Pr_k^{-\alpha}$. Knowing that the MGF of I_k is given by $\frac{p_t}{1-tPr_k^{-\alpha}} + 1 - p_t$ and the fact that in an uniform network $r_k = k$, we have $E[I_k] = p_t P k^{-\alpha}$.

We need then to compute $\sum_{k=L}^{\infty} k^{-\alpha}$ and we are sure that this series converges as $\alpha \geq 2$. For $l \leq y \leq l+1$ we have:

$$\sum_{l=L}^{\infty}(l+1)^{-\alpha} \leq \int_{L}^{\infty} y^{-\alpha}dy \tag{A-2}$$

which leads to:

$$E[I_\infty] \leq (\frac{L^{1-\alpha}}{\alpha-1} + L^{-\alpha})Pp_t \tag{A-3}$$

and we replace in the outage probability expression (5) N_0 by $N_0 + (\frac{L^{1-\alpha}}{\alpha-1} + L^{-\alpha})Pp_t$.

Appendix B

We want to compute the average number of relaying done by a node m. Let us call Z_i a r.v that indicates if node i is using node m as relay to reach its destination (we are supposing that communications are done from left to right). Using (10):

$$\begin{aligned} \overline{N_R} &= \frac{1}{2}\sum_{i=1}^{\infty}\Pr(Z_{iD}=1) && \text{(B-1)} \\ &= \frac{1}{2}\sum_{i=1}^{\infty}\sum_{j=iD+1}^{\infty}\alpha\exp(-\lambda j) && \text{(B-2)} \\ &= \frac{e^{-\lambda D}}{2(1-e^{-\lambda})} && \text{(B-3)} \end{aligned}$$

where the factor 2 is to take into account the fact that we are transmitting from left to right. In order to compute $\overline{k}$ we need:

$$E[X \mid X > D] = \sum_{i=D+1}^{\infty} i\frac{p_i}{\Pr(X > D)} \tag{B-4}$$

where $\Pr(X > D) = \exp(-\lambda D)$. The computation are lengthy, so we omit them. We obtain :

$$\overline{k} \leq \frac{1}{D(1-\exp(-\lambda))} + 2 \tag{B-5}$$

$$\overline{k^2} \leq \frac{D(1+\exp(-\lambda))+1}{D(1-\exp(-\lambda))} + \frac{2\exp(-\lambda)}{D(1-\exp(-\lambda))^2} + D + 2 \tag{B-6}$$

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