A Closed-Form Precoder for Spatial Multiplexing over Correlated MIMO Channels

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***Abstract*— This paper addresses the problem of MIMO spatial-multiplexing (SM) systems in the presence of antenna fading correlation. Existing SM (V-BLAST and related) schemes rely on the linear independence of transmit antenna channel responses for stream separation and suffer considerably from high levels of fading correlation. As a result such algorithms simply fail to extract the non-zero capacity present in highly correlated spatial channels. We make the simple but key point that just one transmit antenna is needed to send several independent streams if those streams are appropriately superposed to form a high-order modulation (such as QAM)! We build on this idea to present a new transmission scheme based on a precoder adjusting the phase and power of the input constellations in closed-form as a function of the antenna correlation. This yields a rate-preserving MIMO multiplexing scheme that can operate smoothly at any degree of correlation.**

I. Introduction

Multiple input and multiple output (MIMO) systems, employing several transmit and receive antennas at both ends, are capable of providing a large increase in capacity compared to traditional single antenna systems [1], [2]. This increase in capacity is however dependent upon the fact that the channels from a transmitter to a receiver follow independent paths. The capacity of MIMO systems can be shown to degrade if there are for example severe correlations present at the transmitter and/or receiver side [3], [4]. At worst, the capacity falls back to that of a SIMO/MISO with additional array gain. However the impact on actual *transmission algorithms* such as spatial multiplexing [2] can be dramatic. Indeed any correlation present at the transmitter effectively increases the linear dependence of the input streams' response and makes stream separation and decoding a difficult task. For example current schemes like V-BLAST literally break down in the presence of correlation levels close to one. Designing appropriate transmission techniques that can adjust smoothly to any level of correlation is therefore an important and practical issue. Although correlated scenarios have previously been considered [5], [6] the focus has mainly been on capacity issues rather than on robust practical algorithms. In order to take advantage of correlation knowledge, [6] and [7] discuss using the eigen-decomposition of the average MIMO channel and thereupon implementing a waterfilling approach across the eigenmodes of the correlation matrix. This results in widely unbalanced error-rates across streams unless some form of adaptive coding/modulation is implemented as well. To minimize the BER in the presence of transmit correlation, a transmit precoding scheme based on power allocation and per-antenna phase shifting was introduced in [8] for a 2×2 MIMO system, while [9] investigates a phase-shifting only strategy. However interesting, both of these concepts rely upon the use of numerical optimization in order to find appropriate solutions and exhaustive search-based maximum likelihood (ML) decoding techniques. In this article we revisit the issue of reliable transmission over correlated MIMO channels when only (long term) correlation properties are known to the transmitter while the receiver has full channel knowledge. We take on a new perspective to solve the problem in a simple and insightful manner. We make the following simple observations:

- MIMO channel input signal vectors can be viewed as multidimensional constellations.
- Having N transmitters and M $(\geq N)$ receivers allows transmission of multidimensional signal constellations with dimension up to $2N$ (N real and N imaginary).
- Fading correlation acts as *continuous* dimension reduction factor, as seen by the receiver. In the extreme correlation one situation, the dimension offered by the channel for transmission simply falls down to 2, i.e. one complex scalar per channel use.
- Finally, the same approach used to design complex (2D) signal constellations can be used to find a suitable precoding scheme that generates multidimensional constellations designed to match arbitrary MIMO correlation levels.

In this paper we simplify the design of the input multidimensional constellation by assuming a structure where each transmit antenna carries an independent signal drawn from a single fixed modulation (such as p-PAM or p-QAM). Each constellation is adjusted in power and phase according to the transmit correlation knowledge. The unique features of our approach include:

1) The optimized transmitter is determined in closed form from the (transmit) correlation coefficients.
2) The transmission rate Nb (where b is the modulation's efficiency in Bits/Symb) of the spatial multiplexing system is preserved regardless of correlation level.
3) When the correlation approaches 1, the signal seen at a certain stage of the receiver is equivalent to that of a

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regular (2D) constellation with an alphabet size of 2^{Nb} symbols.

4) The transmitter is optimized based on the BER balancing criterion (BBC) which states that all components of the SM system should be detected with similar error rate. Interestingly, most high-order constellations based on regular complex grids also follow this criterion.

II. Signal and channel models

We consider a MIMO system consisting of N transmit antennas and M $(\geq N)$ receive antennas with correlations present at the transmitter only. In this situation the channel can be described by $\mathbf{H} = \mathbf{H}_0 \mathbf{R}_t^{\frac{1}{2}}$. The $M \times N$ channel matrix $\mathbf{H}_0$ consists of complex Gaussian zero mean unit-variance independent and identically distributed (iid) elements while $\mathbf{R}_t$ is the $N \times N$ transmitter correlation matrix. We assume that the transmitter and receivers are aware of the correlation matrix $\mathbf{R}_t$, while only the receiver has knowledge of $\mathbf{H}_0$. This is a practical situation for many wireless systems where only the correlation may change slowly enough to be fed back regularly from receiver to the transmitter.

The baseband equivalent of the N-dimensional signal vector observed at the receiver can be expressed as:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{H}_0 \mathbf{R}_t^{\frac{1}{2}} \mathbf{s} + \mathbf{n}. \tag{1}$$

Where $\mathbf{n}$ is the M-dimensional noise vector whose entries are iid complex Gaussian with zero mean and a variance of σ_n^2. Also we set

$$\mathbf{s} = [\sqrt{P}_1 s_1 \ \sqrt{P}_2 e^{j\phi_2} s_2 \ ... \ \sqrt{P}_N e^{j\phi_N} s_N]^T. \tag{2}$$

$P_1, ..., P_N$ represent power levels allocated respectively to input symbols $s_1, ..., s_N$, and are selected to satisfy $\sum_{i=1}^N P_i = 1$. $\phi_2, ..., \phi_N$ correspond to phase shifts on each transmit antenna. Notice that the first symbol does not undergo a phase change and can be regarded as a reference point for all other phase components. We therefore define $\phi_1 = 0$. Standard SM assigns equal weights $P_i = \frac{1}{N}$ and $\phi_i = 0$ for $1 \leq i \leq N$. The symbols are all expected to be selected from the same modulation with an average energy of one, $E\{|s_i|^2\} = 1$.

III. Transmitter optimization

Since the instantaneous channel properties are unknown to the transmitter, the objective is to design a set of power coefficients P_1,..., P_N and phases as functions only of the correlation matrix $\mathbf{R}_t$ and independent of $\mathbf{H}_0$. We note that, on a long term basis, $\mathbf{H}_0$ will be well-conditioned (and thus 'easy' to invert) while ill-conditioning introduced in the system will typically come from $\mathbf{R}_t$. In the fully correlated case, $\mathbf{R}_t$ is rank one and non-invertible.

A. Hybrid Zero-Forcing/MRC SIC

Following the remarks above and in the interest of deriving our closed-form precoding algorithm, we assume a particular receiver structure that we denote hybrid zero-forcing maximum-ratio-combiner successive-interference-canceler (HZM-SIC). The idea behind the HZM structure is that the well conditioned and ill-conditioned components of the channel ought to be treated differently: $\mathbf{H}_0$ is inverted out through a zero-forcing filter while $\mathbf{R}_t^{\frac{1}{2}}$, being possibly very ill-conditioned, is dealt with in a MRC SIC manner rather than matrix inversion. It is important to emphasize that the main goal for such a receiver structure is to lead to an insightful and closed-form deriving of the solution to the transmitter optimization problem that is fully independent of the instantaneous channel fading. Thus we do not claim optimality in any sense for this linear receiver, although we believe the differentiation of well-conditioned from ill-conditioned channel components is a promising approach. Finally, given the general and intuitive nature of the obtained solutions (described in section IV) we show from simulations that the resulting precoding coefficients can be used for a wider range of receiver algorithms (ML etc.).

For the sake of exposition we start with describing the optimization procedure for the 2×2 case. We later extend the derivation to the case of arbitrary number of transmitter and receiver antennas.

B. HZM-SIC receiver for 2×2 case

The hermitian square-root correlation matrix for a 2×2 setup may be expressed as:

$$\mathbf{R}_t^{\frac{1}{2}} = \begin{bmatrix} \alpha & \beta e^{j\psi} \\ \beta e^{-j\psi} & \alpha \end{bmatrix} \tag{3}$$

where by construction $\alpha^2 + \beta^2 = 1$, and $\rho = 2\alpha\beta$ is the modulus of the antenna correlation coefficient ($\rho \leq 1$).

1) Zero-forcing stage: Applying a linear zero-forcing filter on (1) in order to neutralize $\mathbf{H}_0$:

$$\mathbf{z} = \mathbf{H}_0^\dagger \mathbf{y} = \mathbf{R}_t^{\frac{1}{2}} \mathbf{s} + \mathbf{H}_0^\dagger \mathbf{n}, \tag{4}$$

where $(\cdot)^\dagger$ refers to the Moore-Penrose pseudoinverse.

Equation (4) can be written out in full as:

$$\mathbf{z}_1 = \alpha \sqrt{P}_1 s_1 + \beta \sqrt{P}_2 e^{j(\phi_2 + \psi)} s_2 + n_1 \tag{5}$$

$$\mathbf{z}_2 = \beta \sqrt{P}_1 e^{-\psi} s_1 + \alpha \sqrt{P}_2 e^{j\phi_2} s_2 + n_2 \tag{6}$$

2) MRC stage with correlation coefficients: We next start with estimating s_1 by applying MRC on $\mathbf{z}$ with conjugate coefficients from the first column of $\mathbf{R}_t^{\frac{1}{2}}$:

$$\eta = (\mathbf{R}_{t\ :,1}^{\frac{1}{2}})^* \mathbf{z} = \alpha \mathbf{z}_1 + \beta e^{j\psi} \mathbf{z}_2 \tag{7}$$

$$= \sqrt{P}_1 s_1 + 2\alpha\beta e^{j(\psi+\phi_2)} \sqrt{P}_2 s_2 \tag{8}$$

$$+ \ \alpha n_1 + \beta e^{j\psi} n_2. \tag{9}$$

The notation $\mathbf{A}_{:,l}$ expresses the l'th column of the matrix $\mathbf{A}$. $\hat{s}_1$ can be estimated directly with a slicer over $\frac{1}{\sqrt{P_1}} \eta$.

3) Successive interference canceler: After obtaining $\hat{s}_1$, the symbol can be subtracted from the correlated signal observation $\mathbf{z}$. In the derivation we assume no propagation of error ($\hat{s}_1 = s_1$), such that we can define

$$\hat{\mathbf{z}} = \mathbf{z} - [\alpha \ \beta e^{-\psi}]^T \sqrt{P}_1 s_1. \tag{10}$$

A second MRC, is performed on $\hat{\mathbf{z}}$ to estimate $\hat{s}_2$:

$$\hat{\eta} = (\mathbf{R}_{t\ :,2}^{\frac{1}{2}})^* \hat{\mathbf{z}} = e^{j\phi_2} \sqrt{P}_2 s_2 + \beta \hat{n}_1 + \alpha \hat{n}_2. \tag{11}$$

Observe that the decoding structure becomes identical to the decoding of a standard r-QAM modulated symbol where successive decisions are made over each quadrant.

C. BER Balancing Criterion

Our optimization criterion is based on the idea that each substream should be have the same target error probability. The symbol error probability for s_1 is governed by the variance σ_η^2 of the additive noise term $\alpha n_1 + \beta e^{j\psi} n_2$ and the received minimum symbol distance for s_1. Assuming the symbols already follow a rigid regular format, the phase of the factor $2\alpha\beta e^{j(\psi+\phi_2)}\sqrt{P}_2 s_2$ must be selected to maximize the distance from the decision boundaries of s_1. For an arbitrary QAM modulation, this is done by setting ϕ_2 at the emitter such that

$$\phi_2 = -\psi. \tag{12}$$

This also corresponds to a transmit MRC with respect to the phase of the correlation matrix, a procedure known to be optimal capacity-wise as well [5], [6] at high correlation. The (average) minimum distance for a decision on η, obtained from (8) for e.g. $s_2 = -s_1$, becomes:

$$\delta_1 = \sqrt{P}_1 - 2\alpha\beta\sqrt{P}_2. \tag{13}$$

This minimum distance is not to be confused with the minimum distance between two symbols, $d_{min,\eta}$, which for example is given for a 4-QAM constellation by the relationship $d_{min,\eta} = \frac{2}{\sqrt{2}}\delta_1$. The minimum distance for s_2, assuming compensation of phase ϕ_2, is (11) given simply as

$$\delta_2 = \sqrt{P}_2. \tag{14}$$

The noise elements of $\mathbf{n}$ follow the same distribution, similarly all components in $\mathbf{H}_0$ also have an identical statistical structure. Thus the noise factors $\beta\hat{n}_1 + \alpha\hat{n}_2$ and $\alpha n_1 + \beta n_2$ have identical variance when averaged over $\mathbf{H}_0$. We can therefore equate the average probability of error for s_1 and s_2 simply by equating the minimum distances, for any value of the correlation:

$$\sqrt{P}_1 - \rho\sqrt{P}_2 = \sqrt{P}_2 \tag{15}$$

under constraint

$$P_1 + P_2 = 1. \tag{16}$$

The weights for this 2×2 system can easily be computed as function of the correlation to be

$$P_1 = \frac{(1+\rho)^2}{1+(1+\rho)^2},\ P_2 = \frac{1}{1+(1+\rho)^2}. \tag{17}$$

Special cases:

- **Uncorrelated:** With no correlation $\rho = 0$ which yields equal power transmission, justifying the standard V-BLAST design.
- **Fully correlated:** With full correlation $\rho = 1$ we find $P_1 = 0.8$ and $P_2 = 0.2$. Interestingly, this corresponds to the power allocation for a regular 2D constellation. For instance a 16-QAM constellation can be seen as the superposition of two 4-QAM constellations with respective powers 0.8 and 0.2 (see figure 1).

The latter case indicates that if antennas are fully correlated (as in a SIMO case), we can still preserve the pre-selected spatial multiplexing data rate by sending a higher order (e.g. QAM) constellation which corresponds to intuition. In practice the precoder adjusts the transmit constellation smoothly between those two cases and is capable of extracting a non-zero capacity for any level of correlation between the antennas.

IV. Transmit optimization for arbitrary number of antennas

We now describe the procedure for finding precoding weights in a general setting. The decoding starts by selecting the symbol corresponding to largest power P_i. Without loss of generality we assume that the power weights are set to satisfy

$$P_1 \geq P_2 \geq ... \geq P_N. \tag{18}$$

Thus s_1 is the first symbol to be decoded, followed by s_2 etc. in a chronological order.

A. HZM-SIC algorithm:

Let us define $\mathbf{z}$ by

$$\mathbf{z} = \mathbf{H}_0^\dagger \mathbf{y} = \mathbf{R}_t^{\frac{1}{2}}\mathbf{s} + \mathbf{H}_0^\dagger \mathbf{n}, \tag{19}$$

then we can obtain η through a MRC with coefficients taken from the first column of $\mathbf{R}_t^{\frac{1}{2}}$:

$$\begin{aligned}
\eta &= (\mathbf{R}_{t\,:,1}^{\frac{1}{2}})^*\mathbf{z} = \sum_{l=1}^{N} r_{l,1}^*\ \mathbf{z}_l && (20)\\
&= \sum_{l=1}^{N} r_{l,1}^*\ (\sum_{k=1}^{N} r_{l,k}\ \sqrt{P}_k e^{j\phi_k} s_k) + (\mathbf{R}_{t\,:,1}^{\frac{1}{2}})^*\mathbf{H}_0^\dagger\mathbf{n} && (21)\\
&= \sqrt{P}_1 s_1 + (\sum_{l=1}^{N} r_{l,1}^* r_{l,2})\sqrt{P}_2 e^{j\phi_2} s_2 + ... && (22)\\
&+ (\sum_{l=1}^{N} r_{l,1}^* r_{l,N})\sqrt{P}_N e^{j\phi_N} s_N + (\mathbf{R}_{t\,:,1}^{\frac{1}{2}})^*\mathbf{H}_0^\dagger\mathbf{n}, && (23)
\end{aligned}$$

where we use the short hand notation $r_{i,j}$ to denote element $(\mathbf{R}_t^{\frac{1}{2}})_{i,j}$ in the square-root correlation matrix. We further define $\tau_{i,j} = |r_{i,j}|$.

To minimize interference caused by other symbols, we apply the phase-wise transmit MRC shown by equation (12):

$$\phi_k = -\angle(\sum_{l=1}^{N} r_{l,1}^* r_{l,k}). \tag{24}$$

where $\angle$ denotes the phase of the expression. Equation (24) makes certain that the superimposed QAM symbols maximize their distance from the decision boundary. The error probability for s_1 is then, as previously, governed by the additive noise variance and the minimum distance. The minimum distance for s_1 is reached for $s_2 = s_3 = ... = s_N = -s_1$. Written out with weights and correlation coefficients we arrive to:

$$\delta_1 = \sqrt{P}_1 - (\sum_{l=1}^{N} \tau_{1,l}\tau_{2,l})\sqrt{P}_2 - \ ... \ - (\sum_{l=1}^{N} \tau_{1,l}\tau_{N,l})\sqrt{P}_N \tag{25}$$

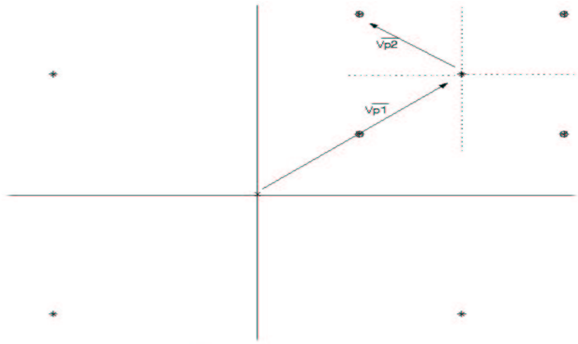

Fig. 1. Illustration of superimposed 4-QAM constellations, $\rho = 1$

where we have used the fact that $\mathbf{R}_t^{\frac{1}{2}}$ is hermitian.

Assuming no error propagation, s_1 is detected and subtracted from equation (19):

$$\hat{\mathbf{z}} = \mathbf{z} - \sqrt{P}_1 \mathbf{R}_{t\,:,1}^{\frac{1}{2}} s_1. \tag{26}$$

An additional MRC with weights from $(\mathbf{R}_{t\,:,2}^{\frac{1}{2}})^*$ can then be used to obtain estimate for $e^{j\phi_2} s_2$. Similar to (25), we can find the minimum distance for s_2 as follows:

$$\delta_2 = \sqrt{P}_2 - (\sum_{l=1}^{N} \tau_{2,l}\tau_{3,l})\sqrt{P}_3 - \ ... \ - (\sum_{l=1}^{N} \tau_{2,l}\tau_{N,l})\sqrt{P}_N. \tag{27}$$

By repeating this N times, we obtain expressions for N minimum distances, on a form analogous to (25) and (27).

If the correlations between transmitters either follow a real correlation model, or the exponential correlation structure [10]; then phases derived through (24) are optimal for all iterations of the decoding algorithm. Details can be found in [11].

B. BBC-based transmit optimization

As $\mathbf{R}_t$ is an hermitian matrix, the norm of all columns in $\mathbf{R}_t^{\frac{1}{2}}$ becomes identical, and when averaged over $\mathbf{H}_0$, all symbols are affected by the same noise variance. In order to guarantee all symbols an equal error rate, it is therefore sufficient that values for $\sqrt{P}_1, \sqrt{P}_2, ..., \sqrt{P}_N$ are selected so that the minimum symbol distance observed for each symbol is identical, i.e.:

$$\delta_1 = \delta_2, \delta_2 = \delta_3, ..., \delta_{N-1} = \delta_N, \tag{28}$$

or alternatively

$$\delta_1 = \delta_N, \delta_2 = \delta_N, ..., \delta_{N-1} = \delta_N. \tag{29}$$

Based on (29) the following linear system can be set up as part of the problem to find the appropriate power levels:

$$\mathbf{\Delta p} = \mathbf{0} \tag{30}$$

where

$$\mathbf{\Delta} = \begin{bmatrix} 1 & -\sum \tau_{1,l}\tau_{2,l} & -\sum \tau_{1,l}\tau_{3,l} & ... & -\sum \tau_{1,l}\tau_{N,l} - 1 \\ 0 & 1 & -\sum \tau_{2,l}\tau_{3,l} & ... & -\sum \tau_{2,l}\tau_{N,l} - 1 \\ & & ... & & \\ 0 & 0 & 0 & 1 & -\sum \tau_{N-1,l}\tau_{N,l} - 1 \end{bmatrix}$$

$$\mathbf{p} = [\sqrt{P}_1 \ \sqrt{P}_2 \ ... \ \sqrt{P}_N]^T \tag{32}$$

and $\mathbf{0}$ is a vector with N zero elements. All sums in $\mathbf{\Delta}$ are expected to run from $l = 1$ to $l = N$. The system (31) only contains $N-1$ equations for N unknowns, however, any solution must also satisfy $\sum_{i=1}^{N} P_i = 1$. Therefore $\mathbf{p}$ can be found as the only unit-norm all-positive vector in the null space of $\mathbf{\Delta}$. Combined with (24) a full solution to the problem is obtained. Special cases:

- With no correlation, $\sum_{l=1}^{N} \tau_{m,l}\tau_{n,l} = 0$, $(1 \le m, n \le N, m \ne n)$ and the energy is distributed equally across all substreams, $P_i = \frac{1}{N}$. Again this justifies the standard V-BLAST approach.
- At the other extreme, with full transmitter correlation, $\sum_{l=1}^{N} \tau_{m,l}\tau_{n,l} = 1$ and a closed form solution can easily be found by writing out $\mathbf{\Delta}$:

$$\mathbf{\Delta}_{corr} = \begin{bmatrix} 1 & -1 & -1 & ... & -2 \\ 0 & 1 & -1 & ... & -2 \\ & & ... & & \\ & & & 1 & -2 \end{bmatrix}. \tag{33}$$

This system can directly be simplified into

$$\sqrt{P}_i = 2^{N-i}\sqrt{P}_N \tag{34}$$

for $1 \le i \le N$. As the solution must satisfy the energy requirement we arrive to $\sum_{i=1}^{N} P_i = \sum_{i=1}^{N} 2^{2(N-i)} P_N = 1$. Solving with respect to P_N gives $P_N = \frac{1}{\sum_{i=1}^{N} 2^{2(N-i)}} = \frac{3}{4^N - 1}$. Finally we obtain:

$$P_i = \frac{3 \cdot 4^N}{4^i(4^N - 1)}. \tag{35}$$

The energy for this setup decreases by one quarter from symbol s_i to s_{i+1}. If each symbol follows a p-PAM or p-QAM modulation, the final form of η, for full correlation, simply correspond to respectively a standard p^N-PAM and p^N-QAM modulations.

V. Simulations

In this section we demonstrate the effectiveness of the new weighting approach proposed in the article. We look at simulation results under quasi-static Rayleigh fading with 4-QAM symbol constellation and variable correlation at the transmitter. The transmitter is only assumed to be aware of the correlations, while the receiver has perfect channel knowledge. Figure 2 and 3 display simulation results for a 2×2 system with correlation level of $\rho = 0.9$ and full correlation at the emitter respectively. We first compare the following approaches:

- Standard ZF: a straight inversion of $\mathbf{H}$ is used as receiver
- HZM-SIC without precoding (equal symbol weights)
- Precoded HZM-SIC

The results show the increased robustness due to the proposed precoding. Interestingly in the presence of full correlation, the proposed precoding and decoding method only performs 4 dB worse off than standard ZF with no transmitter correlation (not shown) which is the loss experienced by going from a 4-QAM

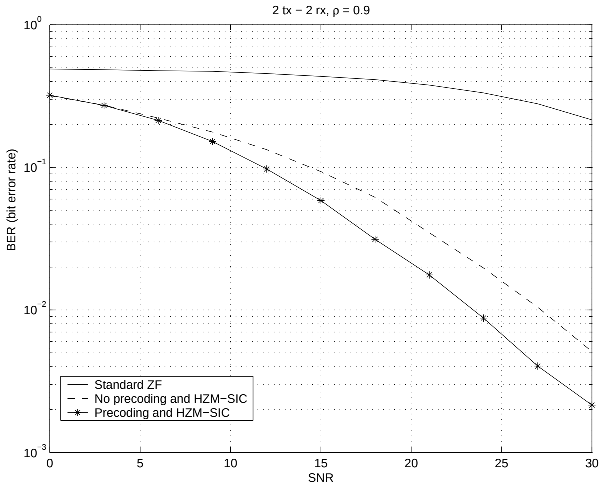

Fig. 2. Strong transmitter correlation

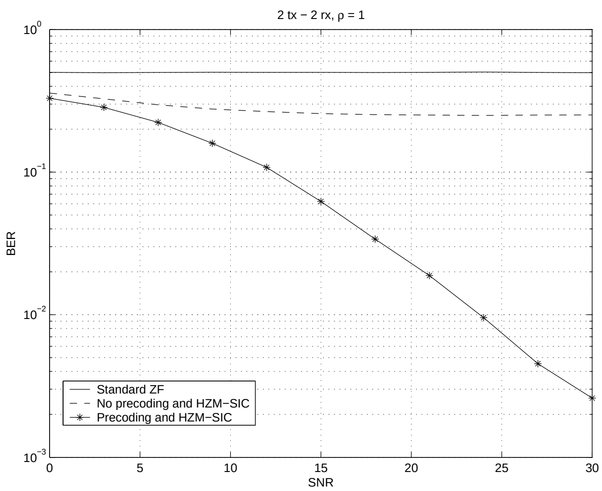

Fig. 3. Full transmitter correlation

transmission to a 16-QAM non-Gray coded transmission. Compared against a straight matrix inversion the situation is vastly improved.

ML decoding: Although the precoding approach is mainly designed with emphasis on a SIC detection, the same precoding can be used with other receiver/decoding algorithms. Figure 4 demonstrates the use of ML decoding at SNR of 15 dB for a 2×2 setup with transmitter correlation ranging from $\rho = 0$ to $\rho = 1$. The difference between ML with or without precoding is relative small at low correlation levels but becomes very substantial with higher degrees of correlation. Finally, we compare with the exhaustive search approach presented in [8] which gives optimal power weights for $\rho = 0.95$ as $P_1 = 0.78$ and $P_2 = 0.22$. The deviation from expressions of (17), $P_1 = 0.791, P_2 = 0.208$, is thus only minute and any loss incurred by the closed form algorithm is marginal, resulting in virtually equal performance under ML decoding.

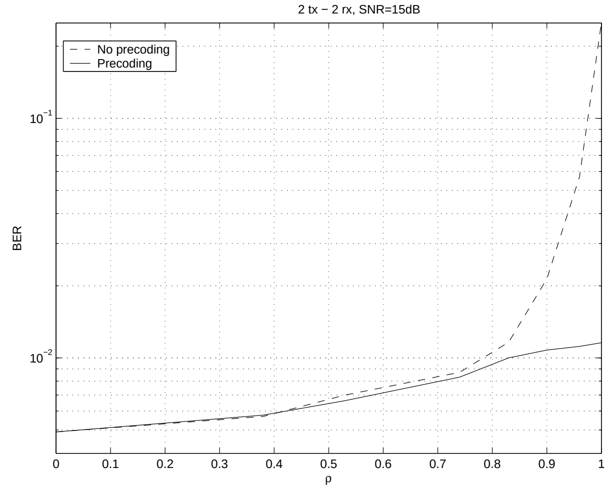

Fig. 4. ML detection

VI. Conclusions

In this article we proposed a closed-form power/phase weighting approach making use of the average channel knowledge to adapt the transmitted constellation. The algorithm assumes SIC style decoding, similar to decoding of r-QAM modulated symbols, and the resulting precoding weights may be applied on a wider range of receivers. The obtained SM scheme offers a method to preserve data rate, with smoothly degrading performance, for any correlation level.

References

[1] I. E. Telatar, “Capacity of multi-antenna Gaussian channels,” Tech. Rep. #BL0112170-950615-07TM, AT & T Bell Laboratories, 1995.

[2] G. J. Foschini, “Layered space-time architecture for wireless communication,” *Bell Labs Technical Journal*, vol. 1, pp. 41–59, Autumn. 1996.

[3] D. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, “Fading correlation and its effect on the capacity of multi-element antenna systems,” *IEEE Trans. Comm.*, vol. 48, pp. 502–513, March 2000.

[4] D. Gesbert, H. Bölcskei, D. Gore, and A. Paulraj, “Outdoor MIMO wireless channels: Models and performance prediction,” *IEEE Trans. Communications*, vol. 50, pp. 1926–1934, Dec. 2002.

[5] S. A. Jafar, S. Vishwanath, and A. Goldsmith, “Channel capacity and beamforming for multiple transmit and receive antennas with covariance feedback,” in *Proc. ICC*, vol. 7, pp. 2266–2270, 2001.

[6] M. T. Ivrlač and J. A. Nossek, “On the impact of correlated fading for MIMO-systems,” in *Proc. ISCAS*, vol. 3, pp. 655–658, 2002.

[7] M. Kiessling, J. Speidel, I. Viering, and M. Reinhardt, “A closed-form bound on correlated MIMO channel capacity,” in *Proc. IEEE Fall VTC*, vol. 2, pp. 859–863, 2002.

[8] R. U. Nabar, H. Bölcskei, and A. Paulraj, “Transmit optimization for spatial multiplexing in the presence of spatial fading correlation,” in *Proc. Globecom*, vol. 11, pp. 131–135, 2001.

[9] R. U. Nabar, H. Bölcskei, and A. Paulraj, “Cut-off rate based transmit optimization for spatial multiplexing on general MIMO channels,” in *Proc. ICASSP*, 2003.

[10] S. L. Loyka, “Channel capacity of MIMO architecture using the exponential correlation matrix,” *IEEE Communications Letters*, vol. 5, pp. 369–371, Sept. 2001.

[11] J.Akhtar and D. Gesbert, “Spatial multiplexing over correlated MIMO channels with a closed form precoder,” *IEEE Trans. Wireless Communications, submitted.*, 2003.