On the Achievable Rates of Ultra-Wideband PPM with Non-Coherent Detection in Multipath Environments

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***Abstract*— In this work we investigate the achievable rates of ultra-wideband (UWB) systems using a *m*-ary pulse position modulation (PPM) with non-coherent receivers in multipath fading environments. We derive a random coding bound on the achievable information rates and highlight the influence of system parameters (bandwidth, delay spread). We also investigate the effect of the use of hard decisions prior to channel decoding and characterize its impact on system performance.**

I. Introduction

In this work, we consider transmission strategies for *Ultra-wideband (UWB)* systems based on *m*-ary Pulse-Position Modulation (*m*-ary PPM) and focus on the achievable data rates for non-coherent receivers (i.e. those which do not perform channel estimation). Here we take a UWB system to be loosely defined as any wireless transmission scheme that occupies a bandwidth of at least several hundreds of megahertz and more than 25% of it's carrier frequency in the case of a passband system.

The most common UWB transmission scheme is based on transmitting information through the use of short-term impulses, whose positions are modulated by a binary information source [1]. This can be seen as a generalized form of binary-PPM. Similar to direct-sequence spread-spectrum, the positions can further be modulated by an *m*-ary sequence (known as a *time-hopping sequence*) for mitigating inter-user interference in a multiuser setting [2]. This type of UWB modulation is a promising candidate for military imaging systems as well as other non-commercial sensor network applications because of its robustness to interference from signals (potentially from other non-UWB systems) occupying the same bandwidth. Based on recent documentation from the FCC [8], it is also being considered for commercial ad-hoc networking applications based on peer-to-peer communications, with the goal be to provide low-cost high-bandwidth connections to the internet from small handheld terminals in both indoor and outdoor settings. Proposals for indoor wireless LAN/PAN systems in the 3-5 GHz band (802.15.3) are also considering this type of transmission scheme as a high bit-rate alternative to existing signaling methods.

In this work, we focus on the case of non-coherent detection since it is well known [3], [4] that coherent detection is not required to approach the wideband AWGN channel capacity, $C_\infty = \frac{P_R}{N_0 \ln 2}$ bits/s, where P_R is the received signal power in watts, and N_0 is the noise power spectral density. In [4] Telatar and Tse showed this to be the case for arbitrary channel statistics in the limit of infinite bandwidth and infinite carrier frequency. Their transmission model was based on frequency-shift keying (FSK) and it was shown that channel capacity is achieved using very impulsive signals. The main goal of this work is to examine under what conditions *m*-ary PPM with non-coherent detection can approach the wideband channel capacity subject to a large but finite bandwidth constraint and different multipath delay-spread values.

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In [5] Verdu addresses the spectral efficiency of signaling strategies in the wideband regime under different assumptions regarding channel knowledge at the transmitter and receiver. The characterization is in terms of the minimum energy-per-bit to noise spectral density ratio $(E_b/N_0)_{\min}$ and the wideband slope $\mathcal{S}_0$. The latter quantity is measured in bits/s/Hz/3dB and represents growth of spectral efficiency at the origin as a function of E_b/N_0. Verdu's work is fundamental to our problem since it shows that approaching C_∞ with non-coherent detection is impossible for practical data rates (>100 kbit/s) even for the vanishing spectral efficiency of UWB systems. This is due to the fact that $\mathcal{S}_0$ is zero at the origin for non-coherent detection. To get an idea of the loss incurred, consider a system with a 2GHz bandwidth and data rate of 20 Mbit/s (this would correspond to a memoryless transmission strategy for channels with a 50ns delay-spread) yielding a spectral-efficiency of .01 bits/s/Hz. For Rayleigh statistics the loss in energy efficiency is on the order of 3dB, which translates into a factor 2 loss in data rate compared to a system with perfect channel state information at the receiver. The loss becomes less significant for lower data rates and/or higher bandwidths.

Section II deals with the underlying system model for transmission and reception as well as the block-invariant channel model. In section III we evaluate expressions for the achievable information rates using random codebooks on an *m*-ary PPM alphabet as a function of the channel delay-spread and system bandwidth. Section IV deals with the effects of hard symbol decisions. We present conclusions in Section V.

II. System Description

A. Notation

Throughout the paper, small letters $'a'$ will be used for scalars, capital letters $'A'$ for vectors, and bold capital letters

$'\mathbf{A}'$ for matrices.

B. Channel Model

We consider a general multipath fading channel, namely when the channel input waveform is $x(t)$, the channel output $y(t)$ is given by

$$y(t) = \sum_{l=1}^{L} a_l(t)x(t - d_l(t)) + z(t) \tag{1}$$

where L is the number of paths, $a_l(t)$ is the gain of path l at time t, $d_l(t)$ is the delay of the path l at time t, and $z(t)$ is white Gaussian noise with power spectral density $N_0/2$. Let T_c be the *coherence time* of the considered channel. For simplicity, we assume that the channel processes $\{a_l(t)\}$ and $\{d_l(t)\}$ are piecewise constant, with their values remaining fixed on time-intervals $[nT_c, (n+1)T_c)$, $n \in \mathbb{Z}$. We further assume that $\{a_{l,nT_c}\}$ and $\{d_{l,nT_c}\}$ are stationary and ergodic discrete-time stochastic processes, and independent of each other. In this work we focus on the case where the *delay spread*, T_d, is much less than the coherence time of the channel. The average transmitted power is constrained to P_T, and the bandwidth of the baseband input signal is constrained to be *approximately* W Hz.

The large bandwidths considered here (>1GHz) provide a high temporal resolution and enable the receiver to resolve a large number of paths of the impinging wavefront. Providing that the channel has a high diversity order (i.e. in dense multipath environments), the total channel gain $\sum_{l=1}^{L} a_l^2$ is slowly varying compared to $\{a_l(t)\}$ and $\{d_l(t)\}$. It has been shown [6], [7], [10] through measurements that in indoor environments, the UWB channel can contain several hundreds of paths of significant strength. We may assume, therefore, that for all practical purposes, the total received energy should remain constant at its average path strength, irrespective of the particular channel realization. Variations in the received signal power will typically be caused by shadowing rather than fast fading. For all the following developments we will assume that the total channel gain is constant and, without loss of generality, equal to 1.

For the sake of determining some numerical guidelines, recent FCC proposals [8] suggest a maximum of 7.5GHz of bandwidth (3.1-10.6 GHz) and maximal transmitted power spectral density of $10^{-6}P_T = -42$dBm/MHz. Worst case-path loss measurements on the 3-5GHz indoor channel for non-line-of-sight transmissions are on the order of 80dB [9] at a 10m separation between transmitter and receiver. If we consider a transmit power of $P_T = -10$dBm across a 2GHz bandwidth, which is close to the maximum transmitted power level proposed by the FCC, we may obtain a worst-case received signal power to noise-spectral density ratio of $P_R/N_0 = 84$dB yielding an AWGN channel capacity of $C_\infty = 362.3$ Mbits/s. This operating point, however, cannot really be considered as being in the "ultra-wideband regime", since the processing gain W/C_∞ is small ($<$ 10 dB). In a multiuser (ad-hoc) setting and because of interoperability with narrowband systems, the target operating point will likely be a substantially lower P_R/N_0. This operating point can nonetheless be thought of as a high-SNR benchmark for the numerical results in this paper.

C. PPM Transmitter Structure

As a practical implementation of asymptoticaly optimal flash-signaling [5], we use a base-band PPM signal to transmit the information bits. The data is encoded using a randomly generated codebook $\mathcal{C} = \{C_1, C_2, \ldots, C_M\}$ of cardinality M and codeword length N. Each codeword C_w is a sequence $C_w = (c_{1,w}, c_{2,w}, \ldots, c_{N,w})$ corresponding to the emission timeslot indexes within each of the N symbol-times used for it's transmission. With $c_{i,j} \in \{0, \ldots, m-1\}$. m is later optimized according to the target received SNR. A guard interval of length at least T_d is left at the end of each symbol time in order to avoid inter-symbol interference. The assigned signal for codeword C_w is

$$x_m(t) = \sum_{n=1}^{N} \sqrt{E_s}\phi(t - (n-1)T_s - c_{n,m}\delta) \tag{2}$$

where δ is the spacing between two consecutive emission timeslots, $E_s = PT_s$ is the pulse energy, $T_s = m\delta + T_d$ the symbol duration, and $\phi(t)$ a unit energy pulse of duration T_p. This transmission scheme can be seen as a coded-modulation strategy with an outer block code of rate R_c and inner code (modulation) of rate $\log_2(m)/m$ as shown in figure 1. It is assumed that the system bandwidth is roughly $1/T_p$.

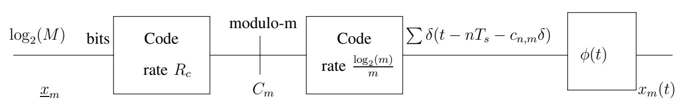

Fig. 1. Transmitter block diagram.

D. Receiver Structure

The received signal is $r(t) = (x_m * h)\mid_t + z(t)$ where $\forall t \in [(n-1)T_s, nT_s]$, $h(t) = h_n(t) = \sum_{i=1}^{L} a_{i,n}\delta(t - d_{i,n})$ is the channel impulse response and $'.*.'$ stands for the convolution operator.

At the receiver, the received signal is first projected on the subspace spanned by the set of orthonormal functions $\{\phi_{0,n}, \phi_{1,n}, \ldots, \phi_{\frac{T_s - T_d - T_p}{T_p}, n}\}$ where $\phi_{i,n}(t) = \phi(t - (n-1)T_s - iT_p)$. This allows us to reduce the number of degrees of freedom of the received signal while capturing the majority of it's information bearing part[1]. In fact, the sent signal is spread in time due to the multipath effect of the channel and by consequence the received signal lies in a signal-space of higher dimensionality than the emitted signal one. A more refined treatment of front-end receiver structures can be found in [11]. Let $< .,. >$ be the scalar product defined by the

[1]due to the finite duration of $\phi(t)$, this projection is incomplete since $\phi(t)$ does not form a complete basis for the transmitted signal space. If $\phi(t)$ were of infinite duration (e.g. for band-limited pulses) then we could use a finite-dimensional basis but suffer from interference from adjacent symbols.

following $\langle h(t), g(t)\rangle = \int_{-\infty}^{+\infty} h(t)g(t)\,dt$. $\forall n \in [1, N]$ and $\forall k \in [1, M]$ let

$$\begin{aligned} R_{n,c_{n,k}} &= \left[i \in \left[0, \frac{T_d}{T_p}\right], \langle r(t), \phi_{i,n}(t - c_{n,k}\delta)\rangle \right] \quad (3) \\ &= H_{n,c_{n,k}} + Z_{n,c_{n,k}} \end{aligned}$$

where $H_{c_{n,k},n}$ (resp. $Z_{c_{n,k},n}$) is the signal part (resp. the noise part) of $R_{c_{n,k},n}$.

$$Z_{n,c_{n,k}} = \left[i \in \left[0, \frac{T_d}{T_p}\right], \langle z(t), \phi_{i,n}(t - c_{n,k}\delta)\rangle \right] \quad (4)$$

$Z_{c_{n,k},n}$ is a Gaussian random vector with mean zero and covariance matrix $\mathbf{K}_z = \frac{N_0}{2}\mathbf{I}$. A non-coherent energy detector is then used

$$\begin{aligned} s_{k,n} &= (H_{n,c_{n,k}} + Z_{n,c_{n,k}})(H_{n,c_{n,k}} + Z_{n,c_{n,k}})^T \quad (5) \\ &= \left\| \sqrt{E_s} A_n \mathbf{R}_{\phi,n}(c_{n,k} - c_{n,m}) + Z_{n,c_{n,k}} \right\|^2 \quad (6) \end{aligned}$$

where $A_n = [a_{n,1}, a_{n,2}, \ldots, a_{n,L}]$, $\mathbf{R}_{\phi,n}^T(t)$ is given by equation 7, and $r_\phi(t)$ is the autocorrelation function of $\phi(t)$. A similar energy-based detector can be implemented using analog technology [11].

III. Achievable rates

In this section we focus on calculating the achievable rates of the system described in Section II when using an optimal soft-decoding strategy. This will be achieved through the derivation of an upper-bound on the decoding error probability. For the sake of simplicity of the following developments, and without loss of generality, we assume that $\forall i, j \ \ |d_i - d_j| \geq T_p$.

A. Decoding Error Probability

The decoder forms the decision variables

$$s_k = \frac{1}{N}\sum_{n=1}^{N} s_{k,n}$$

and uses the following threshold rule to decide on a message: if s_k exceeds a certain threshold ρ for exactly one value of k, say $\hat{k}$, then it will declare that $\hat{k}$ was transmitted. Otherwise, it will declare a decoding error. This is the same sub-optimal decoding scheme considered in [4]. An upper bound of the decoding error probability is given by the following theorem:

Theorem 1: The probability of codeword error is upper bounded by

$$\Pr[\text{error}] \leq M \min_{t>0} e^{-N\left[t\rho + \frac{r}{2}\ln(1-N_0 t) - \ln\left((1-p^+) + p^+ e^{\frac{t}{1-N_0 t}\alpha E_s}\right)\right]} \quad (8)$$

with $p^+ = \frac{\min((2T_d+T_p)/\delta, m)}{m}$, $r = \frac{Td}{Tp} + 1$, $\alpha = \underset{0\leq t\leq T_p}{E}[r_\phi(t) + r_\phi(T_p - t)]$, and $\rho = (1-\epsilon)E_s\alpha + r\frac{N_0}{2}$.

Proof: The decision variable for the transmitted codeword C_w is given by

$$\frac{1}{N}\sum_{n=1}^{N} \left\| \sqrt{E_s} A_n \mathbf{R}_{\phi,n}(0) + Z_{n,c_{n,w}} \right\|^2 \quad (9)$$

by the ergodicity of the noise process, this time-average will exceed the threshold with probability arbitrarily close to 1 for any $\epsilon > 0$ as N gets large. For all $k \neq w$ We bound the probability $Pr[s_k \geq \rho]$ using a Chernoff bound

$$\begin{aligned} \Pr[s_k \geq \rho] &= \Pr[N s_k \geq N\rho] \\ &\leq \min_{t>0} e^{-tN\rho} \prod_{n=1}^{N} E[\exp(t s_{n,k})] \quad (10) \end{aligned}$$

The next lemma is needed to end the proof of Theorem 1.

Lemma 2: Let C_w be the transmitted codeword, then

$$\forall k \in [1, M], n \in [1, N] \qquad E[\exp(t s_{k,n})] \leq E[\exp(t s_{w,n})] \quad (11)$$

The proof of the lemma is derived in the Appendix. Returning to the error probability calculation, the probability p of having a *collision* at slot n between the sent codeword C_w and a candidate codeword C_k (i.e. the probability that $|c_{n,w} - c_{n,k}| \leq \lfloor \frac{T_d}{\delta} \rfloor$), is upper bounded by $p^+ = \frac{\min((2T_d+T_p)/\delta, m)}{m}$. We have now that for all $c_{n,k}$ such that $|c_{n,w} - c_{n,k}| \leq \lfloor \frac{T_d}{\delta} \rfloor$

$$\begin{aligned} E[\exp(t s_{n,k})] &\overset{(c)}{\leq} E\left[\exp\left(t \left\|H_{n,c_{n,m}} + Z_{n,c_{n,m}}\right\|^2\right)\right] \\ &= \underset{H}{E} E\left[\exp\left(t\left\|H_{n,c_{n,m}} + Z_{n,c_{n,m}}\right\|^2\right) \ / \ H\right] \\ &= \underset{H}{E}\left[(1 - N_0 t)^{\frac{-r}{2}} \exp \frac{t}{1 - N_0 t}\left\|H_{n,c_{n,m}}\right\|^2\right] \\ &\overset{(d)}{=} \exp\left(\frac{t}{1 - N_0 t}\alpha E_s\right)(1 - N_0 t)^{\frac{-r}{2}} \quad (12) \end{aligned}$$

and for all $c_{n,k}$ such that $|c_{n,w} - c_{n,k}| > \lfloor \frac{T_d}{\delta} \rfloor$

$$E[\exp(t s_{n,k})] = (1 - N_0 t)^{\frac{-r}{2}}. \quad (13)$$

In the above, (c) follows from lemma 2, and (d) from the fact that $\left\|H_{n,c_{n,w}}\right\|$ is constant and independent of particular realization of the channel by assumption. Let l be the number of collisions between codewords C_w and C_k, then we have that

$$\Pr[s_k \geq \rho] \leq \min_{t>0} e^{-tN\rho} \exp\left(\frac{t}{1 - N_0 t} l\alpha E_s\right)(1 - N_0 t)^{\frac{-Nr}{2}} \quad (14)$$

Averaging over all the realizations of the randomly generated codebook we obtain

$$\begin{aligned} &\underset{code}{E}[\Pr[s_k \geq \rho]] \\ \leq\ & \underset{code}{E}\left[\min_{t_l \in \mathbb{R}^+} e^{-t_l N\rho} \exp\left(\frac{t_l}{1 - N_0 t_l} l\alpha E_s\right)(1 - N_0 t_l)^{\frac{-Nr}{2}}\right] \\ \overset{(e)}{\leq}\ & \min_{t\in\mathbb{R}^+} \underset{code}{E}\left[e^{-tN\rho} \exp\left(\frac{t}{1 - N_0 t} l\alpha E_s\right)(1 - N_0 t)^{\frac{-Nr}{2}}\right] \\ =\ & \min_{t\in\mathbb{R}^+}\left(\sum_{l=0}^{N}\begin{pmatrix} l \\ N \end{pmatrix} p^l (1-p)^{N-l}\right. \\ & \left. e^{-tN\rho}\exp\left(\frac{t}{1 - N_0 t} l\alpha E_s\right)(1 - N_0 t)^{\frac{-Nr}{2}}\right) \\ =\ & \min_{t\in\mathbb{R}^+} e^{-tN\rho}(1 - N_0 t)^{\frac{-Nr}{2}}\left((1-p) + p e^{\frac{t}{1-N_0 t}\alpha E_s}\right)^N \quad (15) \end{aligned}$$

note that in (e) we perform a less optimal minimization operation for the sake of feasibility of the analytical developments.

$$\mathbf{R}_{\phi,n}^{T}(t)=\begin{pmatrix} r_\phi(d_{1,n}-t) & r_\phi(d_{2,n}-t) & \dots & r_\phi(d_{L,n}-t) \\ r_\phi(d_{1,n}-t-T_p) & r_\phi(d_{2,n}-t-T_p) & \dots & r_\phi(d_{L,n}-t-t_p) \\ \vdots & \vdots & \vdots & \vdots \\ r_\phi(d_{1,n}-t-T_d) & r_\phi(d_{2,n}-t-T_d) & \dots & r_\phi(d_{L,n}-t-T_d) \end{pmatrix} \tag{7}$$

Noticing that expression (15) is an increasing function of p and using a union bound we obtain the desired result. ■

B. Achievable Rates

The decoding error probability depicted in (8) decays to zero exponentially in N as long as the transmission rate R satisfies

$$\begin{aligned} R &= \frac{1}{NT_s}\log(M) & (16)\\ &\leq \max_{t\in\mathbb{R}^+}\frac{1}{T_s}\Big(t\rho+\frac{r}{2}\ln(1-N_0t)- \\ &\quad \ln\Big((1-p^+)+p^+e^{\frac{t}{1-N_0t}\alpha E_s}\Big)\Big) & (17)\end{aligned}$$

Due to the finite cardinality of the symbol alphabet for a given T_d, T_p, and Ts, our information rate is bounded by

$$R\leq\frac{1}{T_s}\log_2(m)\quad\text{bits/s} \tag{18}$$

By numerically optimizing the achievable rate R over t and symbol duration T_s we obtain it's variation as function of system parameters $\frac{P_R}{N_0}$, T_d, T_p. Typical delay spreads for UWB channel are of the order of 50 ns, for indoor environments, and several hundreds of ns for outdoor environments [12], [13].

As can be seen in figures 2 and 3, obtained for $\delta=T_d+T_p$, the PPM based energy detector achieves data rates of the order of C_∞ for an optimaly chosen modulation size. Decreasing T_p (i.e. increasing the bandwidth) degrades the performance in the low SNR region. This can be explained by the fact that augmenting the bandwidth increases both the number of degrees of freedom of the system and the amount of noise collected by the receiver. In the low SNR region the effect of the increased receiver noise is dominant. The same kind of behavior was observed for spread-spectrum white-like signals, also called uniform in time and frequency signals, (typically standard CDMA signals) over fading channels, for which the capacity decreases to zero when the system bandwidth increases to infinity if no side information about the channel is used [4], [14], [15]. In the high SNR region the system does not take advantage of the increase in the number of degrees of freedom because T_s can not be made smaller than T_d+2T_p (guard interval).

The optimization of the modulation size, as a function of the system operating SNR, leads to a constant received peak SNR and outer code rate on the order of 1/2 irrespective of average received SNR. It should be noted, however, that the modulation size may be limited due to peak-power emission requirements [8], and thus it may not be possible to satisfy the optimality condition in practice.

IV. Effects of Hard Decision Decoding

Due to implementation constraints related to the large system bandwidth, we may be obliged to use hard decisions when decoding. We investigate the effect of such a modification on the achievable rates in the simpler case of orthogonal (at the receiver i.e. $\delta=T_d+T_p$) PPM modulation using a maximum likelihood decoding rule. Such a system can be viewed as a discrete memoryless channel (DMC) with transition probabilities $\forall i\neq j\in 1\dots m$

$$\begin{aligned} P(i|i) &= \prod_{k=1}^{m}Pr\left[s_i\geq s_k\right] \\ &= \left(Pr\left[\|H+Z_i\|^2\geq\|Z_j\|^2\right]\right)^{m-1} & (19)\\ P(i|j) &= \prod_{k=1}^{m}Pr\left[s_j\geq s_k\right] \\ &= \left(\frac{1}{2}\right)^{m-2}Pr\left[\|Z_j\|^2\leq\|H+Z_i\|^2\right] & (20)\end{aligned}$$

where $\|Z_j\|^2$ is a chi-square random variable with $r=\frac{T_d}{T_p}+1$ degrees of freedom, zero mean and variance $N_0/2$ and $\|H+Z_i\|^2$ a non-central chi-square random variable with $r=\frac{T_d}{T_p}+1$ degrees of freedom, variance $N_0/2$, and non-centrality parameter $\|H\|^2=\alpha E_s$. The capacity for the considered channel with equal probability inputs is then given by [16]

$$\forall k=1\dots m\quad C=\sum_j P(j|k)\log\frac{P(j,k)}{\frac{1}{m}\sum_i P(j|i)} \tag{21}$$

Evaluating numerically this capacity and comparing it with the achievable rate of orthogonal PPM modulation with soft decoding we obtain the results shown in Figure 4. We note that a very harsh penalty can be expected when using hard decision decoding.

V. Conclusion

We studied the achievable rates of a general *m*-ary pulse position modulation (PPM) modulation with non-coherent detection over a time-varying multipath fading channel. We showed that with the considered modulation scheme and bandwidths in line with current UWB proposals we may achieve information rates close to C_∞ for moderate SNR. We also showed that the use of hard decisions prior to decoding leads to extremely large performance losses. These results are strengthened in [11] where average mutual information of generalized flash signaling [5] with non-coherent detection is computed for the same target system parameters. This analysis has been extended to a multiuser ad-hoc network setting. Future work will focus on practical coding schemes

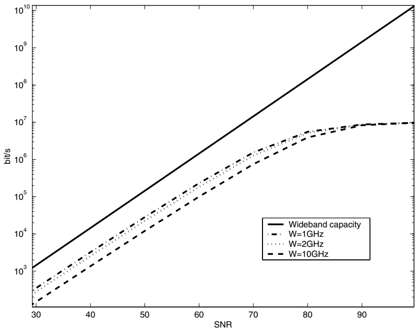

Fig. 2. Impact of the bandwidth on the achievable rate: Td=100 ns, alpha=1

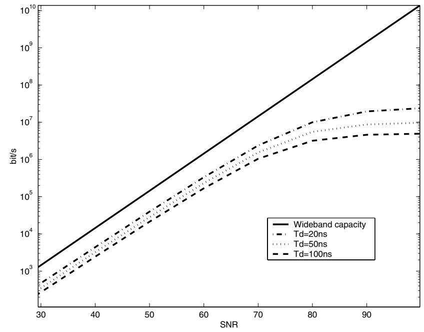

Fig. 3. Impact of the delay spread on the achievable rate: B=2 GHz, alpha=1

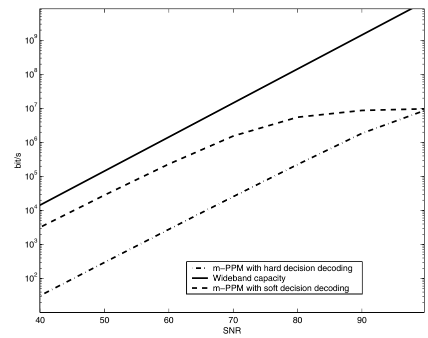

Fig. 4. Orthogonal PPM : Hard versus soft decicions decoding. B=1 GHz, alpha=1, Td=100 ns

for achieving these rates and obtaining realistic UWB system capacity for various network topologies.

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APPENDIX

Proof:

$$\begin{aligned}
E\left[\exp(ts_{k,n})\right] &= E\left[\exp\left(t\left[H_{n,c_{n,k}}H^T_{n,c_{n,k}} + H_{n,c_{n,k}}Z^T_{n,c_{n,k}} + Z_{n,c_{n,k}}H^T_{n,c_{n,k}} + Z_{n,c_{n,k}}Z^T_{n,c_{n,k}}\right]\right)\right] \\
&\overset{(a)}{\leq} E\left[\exp\left(t\left[H_{n,c_{n,m}}H^T_{n,c_{n,m}} + H_{n,c_{n,k}}Z^T_{n,c_{n,m}} + Z_{n,c_{n,m}}H^T_{n,c_{n,k}} + Z_{n,c_{n,m}}Z^T_{n,c_{n,m}}\right]\right)\right] \\
&\overset{(b)}{\leq} E\left[\exp\left(t\left[H_{n,c_{n,m}}H^T_{n,c_{n,m}} + H_{n,c_{n,m}}Z^T_{n,c_{n,m}} + Z_{n,c_{n,m}}H^T_{n,c_{n,m}} + Z_{n,c_{n,m}}Z^T_{n,c_{n,m}}\right]\right)\right] \\
&= E\left[exp(ts_{m,n})\right]
\end{aligned}$$

(a) $H_{n,c_{n,m}}H^T_{n,c_{n,m}} \geq H_{n,c_{n,m}}H^T_{n,c_{n,k}}$ because variance is the maximum value taken by an autocorrelation function, and $Z_{n,c_{n,k}}, Z_{n,c_{n,m}}$ are identically distributed. (b) because $Z_{n,c_{n,m}}H^T_{n,c_{n,k}}$ and $Z_{n,c_{n,m}}H^T_{n,c_{n,m}}$ are distributed according to the same law with the same mean and a higher variance for $ZH^T_{n,c_{n,m}}$. ■