# FULL DIVERSITY SPATIAL MULTIPLEXING BASED ON SISO CHANNEL CODING, SPATIAL SPREADING AND DELAY DIVERSITY

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## ABSTRACT

The use of multiple transmitter and receiver antennas allows to transmit multiple signal streams in parallel and hence to increase communication capacity. To distribute the multiple signal streams over the MIMO channel, linear space-time codes have been shown to be a convenient way to reach high capacity gains with a reasonable complexity. The space-time codes that have been introduced so far are block codes, leading to the manipulation of possibly large matrices. To reduce complexity, we propose an approach based on spatial spreading and delay diversity. The approach allows full symbol rate transmission in the sense that the number of symbols transmitted per sample equals the number of transmit antennas. The approach allows furthermore for full diversity in the sense that each transmitted symbol passes through all channel elements in a uniform fashion. Some optimal and suboptimal receivers schemes are discussed also.

## 1. INTRODUCTION

Spatial multiplexing has been introduced independently in a 1994 Stanford University patent by A. Paulraj and by Foschini [1] at Bell Labs. Spatial multiplexing can be viewed as a limiting case of Spatial Division Multiple Access (SDMA) in which the various mobile users are colocated in one single user multi antenna mobile terminal. In that case, the various users are no longer distinguishable on the basis of their (main) direction (DOA) since all antennas are essentially colocated. Nevertheless, if the scattering environment is sufficiently rich, the antenna arrays at TX and RX can see the different DOAs of the multiple paths. One can then imagine transmitting multiple data streams, one stream per path. For this, the set of paths to be used should be resolvable in angle at both TX and RX. Without channel knowledge at the TX, the multiple streams to be transmitted just get mixed over the multiple paths in the matrix channel. They can generally be linearly recovered at the RX if the channel matrix rank equals or exceeds the number of streams. This rank equals the number of paths that are simultaneously resolvable at TX and RX. The assumptions we shall adopt for the proposed approach are no channel knowledge at TX, perfect channel knowledge at RX, frequency-flat channels for most of the paper.

## 2. LINEAR PREFILTERING APPROACH

We consider here the case of full rate transmission ($N_s = N_{tx}$), when $N_{rx} \geq N_{tx}$ such that the rank of the channel possibly equals the number of streams $N_s$. A general ST coding setup is sketched in Fig. 1. The incoming stream of bits gets transformed to $N_s$ symbol streams through a combination of channel coding, interleaving, symbol mapping and demultiplexing. The result is a vector stream of symbols $\boldsymbol{b}_k$ containing $N_s$ symbols per symbol period. The

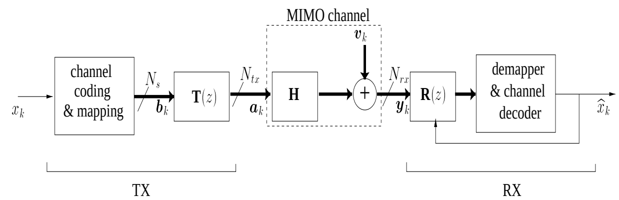


Figure 1: General ST coding setup.

$N_s$ streams then get mapped linearly to the $N_{tx}$ transmit antennas and this part of the transmission is called linear ST precoding. The output is a vector stream of symbols $\boldsymbol{a}_k$ containing $N_{tx}$ symbols per symbol period. The linear precoding is spatiotemporal since an element of $\boldsymbol{b}_k$ may appear in multiple components (space) and multiple time instances (time) of $\boldsymbol{a}_k$. The vector sequence $\boldsymbol{a}_k$ gets transmitted over a MIMO channel $\mathbf{H}$ with $N_{rx}$ receive antennas, leading to the symbol rate vector received signal $\boldsymbol{y}_k$ after sampling. The linear precoding can be considered to be an inner code, while the nonlinear channel coding etc. can be considered to be an outer code. As the number of streams is a factor in the overall bitrate, we shall call the case $N_s = N_{tx}$ the full rate case, while $N_s = 1$ corresponds to the single rate case. Instead of multiple antennas, more general multiple channels can be considered by oversampling, by using polarization diversity or other EM component variations, by working in beamspace, or by considering in phase and in quadrature (or equivalently complex and complex conjugate) components. In the case of oversampling, some excess bandwidth should be introduced at the transmitter, possibly involving spreading which would then be part of the linear precoding. As we shall see below, channel capacity can be attained by a full rate system without precoding ($\mathbf{T}(z) = I$). In that case, the channel coding has to be fairly intense since it has to spread the information contained in each transmitted bit over space (across TX antennas) and time, see the left part in Fig. 2 and [2]. The goal of introducing the linear precoding is to simplify (possibly going as far as eliminating) the channel coding part [3]. In the case of linear dispersion codes [4],[5], transmission is not continuous but packet-wise (block-wise). In that case, a packet of $T$ vector symbols $\boldsymbol{a}_k$ (hence a $N_{tx} \times T$ matrix) gets constructed as a linear combination of fixed matrices in which the combination coefficients are symbols $b_k$. A particular case is the Alamouti code which is a full diversity single rate code corresponding to block length $T = N_{tx} = 2$, $N_s = 1$. In this paper we shall focus

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on continuous transmission in which linear precoding corresponds to MIMO prefiltering. This linear convolutive precoding can be considered as a special case of linear dispersion codes (making abstraction of the packet boundaries) in which the fixed matrices are time-shifted versions of the impulse responses of the columns of $\mathbf{T}(z)$, see Fig. 1. Whereas in the absence of linear precoding, the

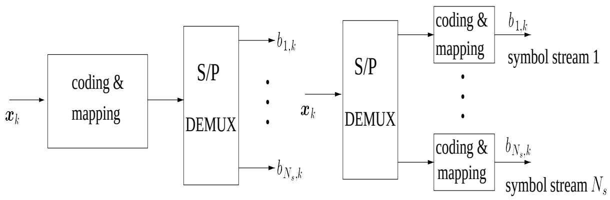


Figure 2: Two channel coding, interleaving, symbol mapping and demultiplexing choices.

last operation of the encoding part is spatial demultiplexing (serial-to-parallel (S/P) conversion) (see left part of Fig. 2), this S/P conversion is the first operation in the case of linear precoding, see the right part of Fig. 2. After the S/P conversion, we have a mixture of channel coding, interleaving and symbol mapping, separately per stream. The existing BLAST systems are special cases of this approach. VBLAST is a full rate system with $\mathbf{T}(z) = \mathbf{I}_{N_{tx}}$ which leads to quite limited diversity. DBLAST is a single rate system with $\mathbf{T}(z) = [1 \ z^{-1}, \ldots, z^{-(N_{tx}-1)}]^T$ which leads to full diversity (delay diversity). We would like to introduce a prefiltering matrix $\mathbf{T}(z)$ without taking a hit in capacity, while achieving full (spatial) diversity. The MIMO prefiltering will allow us to capture all diversity (spatial, and frequential for channels with delay spread) and will provide some coding gain. The optional channel coding per stream then serves to provide additional coding gain and possibly (with interleaving) to capture the temporal diversity (Doppler spread) if there is any. Finally, though time-invariant filtering may evoke continuous transmission, the prefiltering approach is also immediately applicable to block transmission by replacing convolution by circular convolution.

### 2.1. Capacity

Consider the MIMO AWGN channel

$$\boldsymbol{y}_k = \mathbf{H}\,\boldsymbol{a}_k + \boldsymbol{v}_k = \mathbf{H}\,\mathbf{T}(q)\,\boldsymbol{b}_k + \boldsymbol{v}_k \tag{1}$$

where the noise power spectral density matrix is $S_{\boldsymbol{vv}}(z) = \sigma_v^2\, I$, $q^{-1}\,\boldsymbol{b}_k = \boldsymbol{b}_{k-1}$. The **ergodic capacity** when channel knowledge is absent at the TX and perfect at the RX is given by:

$$\begin{aligned} C(S_{\boldsymbol{aa}}) &= \mathrm{E}_H \tfrac{1}{2\pi j} \oint \tfrac{dz}{z} \log_2 \det(I + \tfrac{1}{\sigma_v^2}\, \mathbf{H}\, S_{\boldsymbol{aa}}(z)\, \mathbf{H}^H) \\ &= \mathrm{E}_H \tfrac{1}{2\pi j} \oint \tfrac{dz}{z} \log_2 \det(I + \tfrac{1}{\sigma_v^2}\, \mathbf{H}\,\mathbf{T}(z)\, S_{\boldsymbol{bb}}(z)\, \mathbf{T}^\dagger(z)\, \mathbf{H}^H) \\ &= \mathrm{E}_H \tfrac{1}{2\pi j} \oint \tfrac{dz}{z} \log_2 \det(I + \rho\, \mathbf{H}\,\mathbf{T}(z)\, \mathbf{T}^\dagger(z)\, \mathbf{H}^H) \end{aligned} \tag{2}$$

where we assume that the channel coding and interleaving per stream leads to spatially and temporally white symbols: $S_{\boldsymbol{bb}}(z) = \sigma_b^2\, I$, and $\rho = \frac{\sigma_b^2}{\sigma_v^2} = \frac{SNR}{N_{tx}}$. The expectation $E_H$ is here w.r.t. the distribution of the channel. As in [6], we assume the entries $\mathbf{H}_{i,j}$ of the channel to be mutually independent zero mean complex Gaussian variables with unit variance (Rayleigh flat fading MIMO channel model). As stated in [7], to avoid capacity loss the prefilter $\mathbf{T}(z)$ requires to be paraunitary ( $\mathbf{T}(z)\mathbf{T}^\dagger(z) = \mathbf{I}$). Motivated by the consideration of diversity also (see below), we propose to use the following paraunitary prefilter

$$\begin{aligned} \mathbf{T}(z) &= \mathbf{D}(z)\; Q \\ \mathbf{D}(z) &= diag\{1,\ z^{-1}, \ldots, z^{-(N_{tx}-1)}\}\,,\, Q^H Q = I\,, |Q_{ij}| = \tfrac{1}{\sqrt{N_{tx}}} \end{aligned} \tag{3}$$

where $Q$ is a (constant) unitary matrix with equal magnitude elements. Note that for a channel with delay spread, the prefilter can be immediately adapted by replacing the elementary delay $z^{-1}$ by $z^{-L}$ for channel of length (delay spread) $L$. For the flat propagation channel $\mathbf{H}$ combined with the prefilter $\mathbf{T}(z)$ in (3), symbol stream $n$ ($b_{n,k}$) passes through the equivalent SIMO channel

$$\sum_{i=1}^{N_{tx}} z^{-(i-1)} \mathbf{H}_{:,i} Q_{i,n} \tag{4}$$

which now has memory due to the delay diversity introduced by $\mathbf{D}(z)$. It is important that the different columns $\mathbf{H}_{:,i}$ of the channel matrix get spread out in time to get full diversity (otherwise the streams just pass through a linear combination of the columns, as in VBLAST, which offers limited diversity). The delay diversity only becomes effective by the introduction of the mixing/rotation matrix $Q$, which has equal magnitude elements for uniform diversity spreading.

### 2.2. Matched Filter Bound and Diversity

The Matched Filter Bound (MFB) is the maximum attainable SNR for symbol-wise detection, when the interference from all other symbols has been removed. Hence the multistream MFB equals the MFB for a given stream. For VBLAST ($\mathbf{T}(z) = \mathbf{I}$), the MFB for stream $n$ is

$$\mathrm{MFB}_n = \rho ||\mathbf{H}_{:,n}||_2^2 \tag{5}$$

hence, diversity is limited to $N_{rx}$. For the proposed $\mathbf{T}(z) = \mathbf{D}(z)\,Q$ on the other hand, stream $n$ has MFB

$$\mathrm{MFB}_n = \rho \frac{1}{N_{tx}}\, ||\mathbf{H}||_F^2 \tag{6}$$

hence this $\mathbf{T}(z)$ provides the same full diversity $N_{tx} N_{rx}$ for all streams. Larger diversity order leads to larger outage capacity.

### 2.3. Pairwise Probability of Error $\mathbf{P}_e$

The received signal is:

$$\boldsymbol{y}_k = \mathbf{H}\;\mathbf{T}(q)\;\boldsymbol{b}_k + \boldsymbol{v}_k = \mathbf{H}\;\mathbf{D}(q)\;Q\;\boldsymbol{b}_k + \boldsymbol{v}_k = \mathbf{H}\;\mathbf{D}(q)\;\boldsymbol{c}_k + \boldsymbol{v}_k \tag{7}$$

where $\boldsymbol{c}_k = Q\,\boldsymbol{b}_k = [c_1(k)\, c_2(k)\, \ldots\, c_{N_{tx}}(k)]^T$. We consider now the transmission of the coded symbols over a duration of $T$ symbol periods. The accumulated received signal is then:

$$\mathbf{Y} = \mathbf{H}\,\mathbf{C} + \mathbf{V} \tag{8}$$

where $\mathbf{Y}$ and $\mathbf{V}$ are $N_{rx} \times T$ and $\mathbf{C}$ is $N_{tx} \times T$. The structure of $\mathbf{C}$ will become clear below. Over a Rayleigh flat fading i.i.d. MIMO channel, the probability of deciding erroneously $\mathbf{C}'$ for transmitted $\mathbf{C}$ is upper bounded by (see [3]):

$$\mathbf{P}(\mathbf{C} \to \mathbf{C}') \le (\prod_{i=1}^{r} \lambda_i)^{-N_{rx}} (\frac{\rho}{4})^{-N_{rx}\, r} \tag{9}$$

where $r$ and $\lambda_i$ are rank and eigenvalues of $(\mathbf{C} - \mathbf{C}')^H(\mathbf{C} - \mathbf{C}')$. Introduce $\boldsymbol{e}_k = \frac{1}{\sigma_b}(\boldsymbol{c}_k - \boldsymbol{c}'_k)$, then:

$$\mathbf{C}-\mathbf{C}' = \begin{bmatrix} e_1(0) & e_1(1) & \dots & \dots & \dots & \dots \\ 0 & \ddots & \ddots & \dots & \dots & \dots \\ \vdots & \ddots & \ddots & \ddots & \dots & \dots \\ 0 & \dots & 0 & e_{N_{tx}}(0) & e_{N_{tx}}(1) & \dots \end{bmatrix} \quad (10)$$

Let i be the time index of the first error:

$$\mathbf{C}-\mathbf{C}' = \begin{bmatrix} 0 & \dots & 0 & e_1(i) & \dots & \dots & \dots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \dots & \dots \\ 0 & \dots & \dots & \dots & 0 & e_{N_{tx}}(i) & \dots \end{bmatrix} . \quad (11)$$

$$\prod_{n=1}^{N_{tx}} e_n(i) \neq 0 \quad (12)$$

the upper bound on the pairwise error probability becomes (maximized for a single error event $i$):

$$\mathbf{P}(\mathbf{C}\rightarrow\mathbf{C}') \leq (\prod_{n=1}^{N_{tx}} |e_n(i)|^2)^{-N_{rx}} . (\frac{\rho}{4})^{-N_{rx}\,N_{tx}} . \quad (13)$$

Hence, full diversity $N_{rx}\,N_{tx}$ is guaranteed, and the coding gain is: $\min_{\boldsymbol{e}_i\neq 0}\prod_{n=1}^{N_{tx}}|e_n(i)|^2$. The condition (12) is well known in the design of lattice constellations (see [8], [9]), a field based on the theory of numbers. A solution that satisfies our criteria of unitary matrix and equal magnitude components of $Q$, is the Vandermonde matrix:

$$Q^s = \frac{1}{\sqrt{N_{tx}}}\begin{bmatrix} 1 & \theta_1 & \dots & {\theta_1}^{N_{tx}-1} \\ 1 & \theta_2 & \dots & {\theta_2}^{N_{tx}-1} \\ \vdots & \vdots & & \vdots \\ 1 & \theta_{N_{tx}} & \dots & {\theta_{N_{tx}}}^{N_{tx}-1} \end{bmatrix} \quad (14)$$

where the $\theta_i$ are the roots of $\theta^{N_{tx}} - j = 0$ , $j=\sqrt{-1}$.
It was shown in [7] that when $N_{tx} = 2^{n_t}$ ($n_t\in\mathbb{N}$), and for a finite QAM constellation with $(2M)^2$ points, then $Q^s$ maximizes the coding gain among all matrix $Q$ with normalized columns, and achieves: $\min_{\boldsymbol{e}_i\neq 0}\prod_{n=1}^{N_{tx}}|e_n(i)|^2 = \left(\frac{(2d)^2}{N_{tx}\sigma_b^2}\right)^{N_{tx}}$, where $2d$ is the minimum distance between two points in the constellation.

## 3. ML RECEPTION

In principle, we can perform Maximum Likelihood reception since the delay diversity transforms the flat channel into a channel with finite memory. However, the number of states would be the product of the constellation sizes of the $N_{tx}$ streams to the power $N_{tx}-1$. Hence, if all the streams have the same constellation size $|\mathcal{A}|$, the number of states would be $|\mathcal{A}|^{N_{tx}(N_{tx}-1)}$, which will be much too large in typical applications. Suboptimal ML reception can be performed in the form of sphere decoding [10]. The complexity of this can still be too large though and therefore suboptimal receiver structures will be considered in the next section.

## 4. MIMO DFE RECEPTION

Let $\mathbf{G}(z)=\mathbf{H}\,\mathbf{T}(z)=\mathbf{H}\,\mathbf{D}(z)\,Q$ be the cascade transfer function of channel and precoding. The matched filter RX is

$$\begin{aligned} \boldsymbol{x}_k = \mathbf{G}^\dagger(q)\,\boldsymbol{y}_k &= \mathbf{G}^\dagger(q)\,\mathbf{G}(q)\,\boldsymbol{b}_k + \mathbf{G}^\dagger(q)\,\boldsymbol{v}_k \\ &= \mathbf{R}(q)\,\boldsymbol{b}_k + \mathbf{G}^\dagger(q)\,\boldsymbol{v}_k \end{aligned} \quad (15)$$

where $\mathbf{R}(z)=\mathbf{G}^\dagger(z)\,\mathbf{G}(z)$, and the psdf of $\mathbf{G}^\dagger(q)\,\boldsymbol{v}_k$ is $\sigma_v^2\,\mathbf{R}(z)$. The DFE RX is then:

$$\widehat{\mathbf{b}}_k = -\underbrace{\overline{\mathbf{L}}(q)}_{\text{feedback}}\;\mathbf{b}_k + \underbrace{\mathbf{F}(q)}_{\text{feedforward}}\;\mathbf{x}_k \quad (16)$$

where feedback $\overline{\mathbf{L}}(z)$ is strictly "causal". Two design criteria for feedforward and feedback filters are possible: MMSE ZF and MMSE, see [7], where we introduced triangular MIMO feedback structures, allowing to incorporate channel decoding in the feedback, and leading to the stripping approach of Verdu & Müller or Varanasi & Guess..

## 5. RECEIVER PROCESSING AND CAPACITY ISSUES

In this section we study the influence of simplified receivers on the capacity of the system. For this let us first note:

- $Q = IDFTdiag\{1,\theta_1,\theta_1^2,\dots,\theta_1^{N_{tx}-1}\}$, where $IDFT$ is the $N_{tx}$-point Inverse Discrete Fourier Transform.
- For random variables $\mathbf{Y}$ and $\mathbf{X}=(x_1,\dots,x_N)$, applying the chain rule of the mutual information leads to:

$$I(\mathbf{Y},\mathbf{X}) = \sum_{k=1}^{N} I(\mathbf{Y},x_k|x_1,x_2,\dots,x_{k-1}) \geq \sum_{k=1}^{N} I(\mathbf{Y},x_k) \quad (17)$$

  in the Gaussian case the latter term corresponds to the mutual information between $x_k$ and its LMMSE estimate on the basis of $Y$.
- In [7] it was shown that:

$$\begin{aligned} C &= \frac{1}{2\pi j}\oint\frac{dz}{z}\log_2\det(I_{N_{tx}}+\rho\,Q^H\,\mathbf{D}^\dagger(z)\,\mathbf{H}^H\,\mathbf{H}\,\mathbf{D}(z)\,Q) \\ &= \sum_{n=1}^{N_{tx}}\log_2 \mathrm{SNR}_n^{MMSE} = \sum_{n=1}^{N_{tx}}\log_2(1+\mathrm{SNR}_n^{UMMSE}) \end{aligned} \quad (18)$$

  where $\mathrm{SNR}_n^{MMSE}$ is the $SNR$ at the output of the stage $n$ of an MMSE DFE receiver.

### 5.1. SIC with stream-wise SIMO MMSE DFE

This receiver performs a successive detection of the substeams; we denote by $\{k_1,k_2,\dots,k_{N_{tx}}\}$ the order of detection of the substreams. Then at step $n$ the receiver has already detected and cancelled the substeams $\{k_1,k_2,\dots,k_{n-1}\}$. The processing is then done by first filtering by a MISO (Multiple Input Single output) MMSE filter, that corresponds to the LMMSE estimates of the substream $k_n$, followed by a ML (e.g. Viterbi) detector of a SISO channel. Let $\mathbf{R}(z)=I_{N_{tx}}+\rho Q^H\mathbf{D}^\dagger(z)\,\mathbf{H}^H\mathbf{H}\mathbf{D}(z)Q$, $\mathbf{V}_n=[e_{k_n},e_{k_{n+1}},\dots,e_{k_{N_{tx}}}]$ and $\overline{\mathbf{V}}_n=[e_{k_1},e_{k_2},\dots,e_{k_{n-1}}]$ where $e_{k_n}$ is the $N_{tx}\times 1$ vector containing 1 at position $k_n$ and 0 elsewhere. The LMMSE filtering error has a power spectrum density equal to: $MSE_n(z)=((\mathbf{V}_n^H\mathbf{R}(z)\mathbf{V}_n)^{-1})_{11}$, and the corresponding (predictive) DFE has $\mathrm{SNR}_n^{MMSE}=e^{\frac{1}{2\pi j}\oint\frac{dz}{z}\log_2 MSE_n^{-1}(z)}$, the $\log_2$ of which is the capacity $C_n^{MMSE\,DFE}$ on stream $n$. We can note that:

$$\begin{aligned} \mathbf{R}_{k_nk_n}^{-1}(z) &= (\{\mathbf{V}_n^H\mathbf{R}(z)\mathbf{V}_n - \\ &\quad \mathbf{V}_n^H\mathbf{R}(z)\overline{\mathbf{V}}_n(\overline{\mathbf{V}}_n^H\mathbf{R}(z)\overline{\mathbf{V}}_n)^{-1}(\mathbf{V}_n^H\mathbf{R}(z)\overline{\mathbf{V}}_n)^\dagger\}^{-1})_{11} \\ &\geq ((\mathbf{V}_n^H\mathbf{R}(z)\mathbf{V}_n)^{-1})_{11} = MSE_n(z) \end{aligned} \quad (19)$$

where $\mathbf{R}_{k_nk_n}^{-1}(z)=e_{k_n}^H\,Q^H\mathbf{D}^\dagger(z)(I_{N_{tx}}+\rho\,\mathbf{H}^H\,\mathbf{H})^{-1}\mathbf{D}(z)Qe_{k_n}$. If we note $v(z)=[1,z^{-1},\dots,z^{-N_{tx}+1}]^T$ and using the fact that $Q = IDFTdiag\{1, \theta_1, \theta_1^2, \ldots, \theta_1^{N_{tx}-1}\}$, then $\mathbf{R}_{k_n k_n}^{-1}(z) = \frac{1}{N_{tx}} v(z\, e^{-\frac{j2\pi(k_n-1)}{N_{tx}}})^{\dagger}(I_{N_{tx}} + \rho\, \mathbf{H}^H\, \mathbf{H})^{-1} v(z\, e^{-\frac{j2\pi(k_n-1)}{N_{tx}}}) = \mathbf{R}_{11}^{-1}(ze^{-\frac{j2\pi(k_n-1)}{N_{tx}}})$. Finally:

$$\mathrm{SNR}_n^{MMSE} \geq e^{-\frac{1}{2\pi j}\oint \frac{dz}{z} \log_2 \mathbf{R}_{11}^{-1}(z)} = \mathrm{SNR}_1^{MMSE} \qquad (20)$$

That shows also that $SNR_1^{MMSE}$ is the same for any processing order, hence the capacity of the first substream is independent of the order of processing. In the same way we can show that this property holds also for the last processed substream, and that for any order of processing and for all $1 \leq n \leq N_{tx}$ the following inequality is satisfied:

$$\begin{array}{llll} & \mathrm{SNR}_1^{MMSE} \leq & \mathrm{SNR}_n^{MMSE} & \leq \mathrm{SNR}_{N_{tx}}^{MMSE} \\ \Rightarrow & C_1^{MMSE\,DFE} \leq & C_n^{MMSE\,DFE} & \leq C_{N_{tx}}^{MMSE\,DFE} \end{array} \qquad (21)$$

### 5.2. SIC with stream-wise SIMO MMSE LE

In this approach, we ignore the color of the error spectrum at the output of the LMMSE filter and treat the output as an AWGN channel (as in (17)), resulting in the capacity

$$\begin{array}{lll} C_n^{MMSE\,LE} & = & \log_2 SNR_n^{MMSE\,LE} \\ & = & -\log_2 \frac{1}{2\pi j}\oint \frac{dz}{z}(\mathbf{V}_n^H \mathbf{R}(z)\mathbf{V}_n)_{11}^{-1} \end{array} \qquad (22)$$

As was done in the previous section we can show that:

$$C_1^{MMSE\,LE} \leq C_n^{MMSE\,LE} \leq C_{N_{tx}}^{MMSE\,LE} \qquad (23)$$

We can also note that $C_n^{MMSE\,LE} \leq C_n^{MMSE\,DFE}$ and that $C_1^{MMSE\,LE} = -\log_2 \frac{tr\,(I_{N_{tx}} + \rho\, \mathbf{H}^H\, \mathbf{H})^{-1}}{N_{tx}}$.

### 5.3. MIMO MMSE LE RX

In this approach we detect the different substream without any Decision Feedback, and for every substream we process by first filtering with an MMSE filter followed by detection of the transmitted symbol when modeling all interference as AWGN. The capacity of this approach is $C_1^{MMSE\,LE}$. This approach results in a substantial loss in capacity but has the advantage of a simple FIR filter receiver: the MIMO LE is in fact: $Q^H\, \mathbf{D}^{\dagger}(z)(I_{N_{tx}} + \rho\, \mathbf{H}^H\, \mathbf{H})^{-1}\mathbf{H}^H$. Also, in the case of low SNR i.e $\rho\frac{||\mathbf{H}||_2^2}{N_{tx}} = \frac{SNR\,||\mathbf{H}||_2^2}{N_{tx}^2} \approx \frac{SNR\,N_{rx}}{N_{tx}} << 1$ this approach becomes optimal.

### 5.4. Case of $N_{tx} = 2$, $N_{rx} \geq 2$

The case of two transmit antennas is important in practice. For lack of space we shall skip the details of computations and present the final results. If we note by $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2]$, $a = 1 + \rho||\mathbf{h}_1||_2^2$, $b = 1 + \rho||\mathbf{h}_2||_2^2$, $\cos^2\beta = \frac{\rho^2\, |\mathbf{h}_1^H\, \mathbf{h}_2|^2}{a\, b}$ and $\alpha^2 = \frac{4\, a\, b\, \cos^2\beta}{(a+b)^2} \leq 1$ then:

$$\begin{array}{lll} SNR_1^{MMSE\,DFE} & = & \frac{2ab(1-\cos^2\beta)}{a+b}\frac{2}{1+\sqrt{1-\alpha^2}} \\ SNR_2^{MMSE\,DFE} & = & \frac{a+b}{2}\frac{1+\sqrt{1-\alpha^2}}{2} \\ SNR_1^{MMSE\,LE} & = & \frac{2ab(1-\cos^2\beta)}{a+b} \\ SNR_2^{MMSE\,LE} & = & \frac{a+b}{2}\sqrt{1-\alpha^2} \end{array} \qquad (24)$$

For the first substream, there is a generally small and limited loss in capacity between using a DFE or a LE: $C_1^{MMSE\,DFE} - C_1^{MMSE\,LE} = \log_2 \frac{2}{1+\sqrt{1-\alpha^2}} \leq 1$. Note that alternatively applying the Viterbi algorithm for ML equalization would be complex due to the color of interference plus noise.
The loss in capacity for the second substream is $C_2^{MMSE\,DFE} - C_2^{MMSE\,LE} = \log_2 \frac{1+\sqrt{1-\alpha^2}}{2\sqrt{1-\alpha^2}}$, which is not limited and can be important for values of $\alpha^2$ close to 1. Hence we prefer to use the DFE for the detection of this substream, in which case the loss in total capacity is limited to the loss in capacity for the first substream. Alternatively, we can apply ML (Viterbi) equalization for the second (or in general last) substream since there is no more interference and the noise is white: the cleaned received signal is $Y_2(q) = \mathbf{H}[1, -e^{j\frac{\pi}{4}}q^{-1}]^T \mathbf{b}_2(q) + \mathbf{V}(q)$.

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