

# Linear Space-Time Coding at Full Rate and Full Diversity

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*Abstract* — The use of multiple transmitter and receiver antennas allows to transmit multiple signal substreams in parallel and hence to increase communication capacity. In order to distribute the multiple signal substreams over the MIMO channel, linear space-time codes have been shown to be a convenient way to reach high capacity gains with a reasonable complexity. The space-time codes that have been introduced so far are block codes, leading to the manipulation of possibly large matrices. To reduce complexity, this work introduces convolutive codes, associated with MIMO filters. The proposed approach also constitutes an alternative to OFDM based approaches for frequency fading channels.

## I. LINEAR PREFILTERING APPROACH

The assumptions we shall adopt for the proposed approach are: no channel knowledge at TX, perfect channel knowledge at RX, and a matrix channel impulse response spanning  $L$  symbol periods. We consider the case of full rate transmission ( $N_s = N_{tx}$ ) ( $N_s, N_{tx}$  and  $N_{rx}$  are the number of substreams, transmit and receive antennas). A general ST coding setup is sketched in Fig. 1. The incoming stream of bits gets transformed to  $N_s$  symbol substreams through a combination of channel coding, interleaving, symbol mapping and demultiplexing. The result is a vector stream of symbols  $\mathbf{b}_k$  containing  $N_s$  symbols per symbol period. The  $N_s$  substreams then get

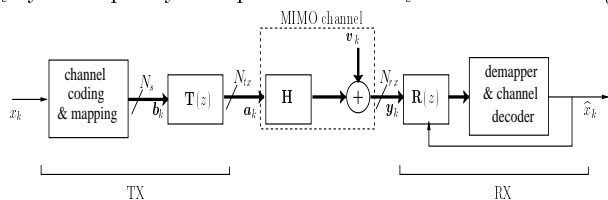


Figure 1: General ST coding setup.

mapped linearly to the  $N_{tx}$  transmit antennas and this part of the transmission is called linear ST precoding. The output is a vector stream of symbols  $\mathbf{a}_k$  containing  $N_{tx}$  symbols per symbol period. The linear precoding is spatiotemporal since an element of  $\mathbf{b}_k$  may appear in multiple components (space) and multiple time instances (time) of  $\mathbf{a}_k$ . The goal of the linear precoding is to exploit all diversity sources so that the fading channel gets transformed essentially into non-fading channel, for which simpler (scalar) channel coding can be used. The

vector sequence  $\mathbf{a}_k$  gets transmitted over a MIMO channel  $\mathbf{H}$  with  $N_{rx}$  receive antennas, leading to the symbol rate vector received signal  $\mathbf{y}_k$  after sampling.

As in [1], we assume the entries of the channel impulse response samples to be mutually independent zero mean complex Gaussian variables with unit variance (Rayleigh flat fading MIMO channel model). Then to avoid ergodic capacity loss, the prefilter  $\mathbf{T}(z)$  is required to be paraunitary ( $\mathbf{T}(z)\mathbf{T}^H(z) = \mathbf{I}$ ). The solution proposed for this problem is the following (spatial spreading + delay diversity):

$$\begin{aligned} \mathbf{T}(z) &= \mathbf{D}(z) \mathbf{Q} \\ \mathbf{D}(z) &= \text{diag}\{1, z^{-L}, \dots, z^{-(N_{tx}-1)L}\}, \mathbf{Q}^H \mathbf{Q} = \mathbf{I} \\ \mathbf{Q} &= \frac{1}{\sqrt{N_{tx}}} \begin{bmatrix} 1 & \theta_1 & \dots & \theta_1^{N_{tx}-1} \\ 1 & \theta_2 & \dots & \theta_2^{N_{tx}-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \theta_{N_{tx}} & \dots & \theta_{N_{tx}}^{N_{tx}-1} \end{bmatrix} \end{aligned} \quad (1)$$

where the  $\theta_j$  are the roots of  $\theta^{N_{tx}} - j = 0$ ,  $j = \sqrt{-1}$ .

The Matched Filter Bound (MFB) given by this precoding for substream  $n$  ( $1 \leq n \leq N_s$ ) is :

$$\text{MFB}_n = \rho \frac{1}{N_{tx}} \|\mathbf{H}\|_F^2, \rho = \frac{\sigma_b^2}{\sigma_v^2} = \frac{SNR}{N_{tx}}, \quad (2)$$

hence this  $\mathbf{T}(z)$  provides the same full diversity  $N_{tx}N_{rx}L$  for all substreams. A larger diversity order leads to a larger outage capacity. It was shown in [2] that the proposed precoding maximizes the diversity gain, and when  $N_{tx} = 2^{n_t}$  ( $n_t \in \mathbb{N}$ ) and for a finite QAM constellation with  $(2M)^2$  points, the matrix  $\mathbf{Q}$  maximizes the coding gain ( $G_C$ ) among all matrix with normalized columns, and achieves:  $G_C = \left(\frac{d^2}{N_{tx}\sigma_b^2}\right)^{N_{tx}}$ , where  $d$  is the minimum distance between two points in the constellation. For the receiver, SIC approaches have been shown in [2] to correspond to a capacity decomposition and to benefit from the  $\mathbf{T}(z)$  in the sense that substreams can be treated in any order (as opposed to VBLAST). However, to exploit the full diversity, PIC-based turbo detection should be performed.

## REFERENCES

- [1] I.E. Telatar. "Capacity of Multi-antenna Gaussian Channels". *Eur. Trans. Telecom.*, vol. 10, no. 6, pp. 585-595, Nov/Dec 1999.
- [2] A. Medles and D.T.M. Slock. "Linear Convolutive Space-Time Precoding for Spatial Multiplexing MIMO Systems". *Proc. 39th Annual Allerton Conference on Communication, Control, and Computing*, Monticello, Illinois, USA, Oct. 2001.

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