Abstract—This paper considers a bidirectional full-duplex Multi-Input Multi-Output (MIMO) OFDM system. We consider the more realistic noise model called limited dynamic range (LDR) model which takes into account the hardware impairments in the analog sections of the transceiver chains. At the transmit side, we introduce a two stage beamformer (BF) with an inner BF of lower dimension and an outer BF of higher dimension, both BFs being at the digital (baseband) side. The inner BF in OFDM domain handles transmission, while the outer BF in time domain handles self interference (SI). At the receive side, we propose a hybrid combiner which involves an analog phase shifter based BF, with fewer RF chains compared to the number of receive antennas and a digital (baseband) BF in OFDM domain. The analog BF helps reduce SI before analog-to-digital conversion (ADC). All the BFs are optimized using maximization of weighted sum rate (WSR) which is solved using an alternating minorization approach. The proposed multi-stage BF architecture allows to reduce the coupling between classical transceiver design and MIMO SI nulling, and guarantee SI reduction during OFDM cyclic prefixes, with UL/DL possibly using different numerology or being asynchronous, allowing proper ADC operation.

I. INTRODUCTION

1In this paper, Tx and Rx may denote transmit/transmitter/transmission and receive/receiver/reception. In-band full-duplex (FD) wireless, which allows each node to transmit and receive simultaneously has the potential to almost double the spectral efficiency and is one of the prominent candidates for 5G. It avoids the use of two independent channels for bi-directional communication, by allowing more flexibility in spectrum utilization, improving data security and reduces the air interface latency and delay issue. Unfortunately, it suffers from severe self-interference (SI) which could be 110 dB higher than the receive signal and canceling it is not a trivial task due to non-linearities and imperfections in the transmit chain, as identified in [1].

In self interference cancellation (SIC) techniques, the objective is to reduce the SI to near the noise floor which makes signal reception possible. The first design and implementation of FD WiFi radio was introduced in [2]. In [3], SIC in FD is investigated experimentally and a practical FD system is proposed. The authors in [4] combine analog and digital SIC techniques and the effect of residual SI together with clipping plus-quantization noise due to the limited dynamic range (LDR) of ADCs is studied. With massive MIMO, the analog cancellation stage may become infeasible due to the large complexity associated. Also the cost of hardware components required to mimic the SI signal may become unattractive.

The use of separate Tx/Rx antenna arrays combined with various spatial precoding techniques has also been proposed to mitigate SI. Recent studies on fully digital precoder schemes under LDR using weighted sum rate (WSR) criteria for FD systems can be found in [5], [6].

Hybrid BF (HBF) is a two-stage architecture which provides BF gains by the use of a phase shifting network in the analog domain and spatial multiplexing by digital precoder at the baseband. HBF designs for single user systems can be found in [7], [8].

A. Contributions of this paper

• We propose a two stage BF design for a bidirectional FD MIMO OFDM system based on the WSR criterion which is solved using the alternating minorization approach the main advantage of which compared to the weighted sum mean square error (WSMSE) approach is it’s faster convergence. Minorization approach also involves user stream power optimization which also implicitly select the number of supportable streams for a user.

• At the Tx side we propose to use a two stage BF at the baseband where the higher dimensional precoder is applied to the time domain signal which aim to mitigates the SI and the lower dimensional precoder in the OFDM domain provides spatial multiplexing gain. At the Rx side, we introduce a HBF design. The objective of the time domain phase shifter analog BF stage is to suppress the SI before the ADC while preserving the dimension of the desired signal space.

• Compared to the only existing state of the art design on HBF for FD systems [9], we consider a more realistic LDR noise model at both the Tx and Rx.

• Through Monte Carlo simulations, we validate the performance of our proposed multi-stage/HBF design. Simulations demonstrate that using an analog combiner stage at Rx (which operates before the Rx side LDR noise) has better sum rate performance compared to using a two-stage BF at Tx side for SI nulling.

II. FULL-DUPLEX BIDIRECTIONAL MIMO SYSTEM MODEL

In this paper we shall consider a multi-stream approach with \(d_j\) streams transmitted from base station (BS) \(j\). So,
consider a bidirectional full-duplex system as depicted in Figure 1, with $N_s^1$ or $N_s^2$ transmit antennas at the node 1 or node 2 respectively. This represents a backhaul link between two BSs. Furthermore we consider an OFDM system with $N_r$ subcarriers. Node 1 or node 2 is equipped with $N_r$ or $N_r^2$ receive antennas respectively, $H_{i,j}$, $i \neq j$ represents the $N_r^1 \times N_r^2$ MIMO direct channel between node $i$ and node $j$. Let $H_{i,i}$ represent the self interference (SI) channel from the Tx of node $i$ to the receiver of node $i$. At the baseband side, user $i$ receives

$$y_i[n] = H_{RF,i} H_{i,j}[n] (V^j G_j[n] d_j[n] + e_j[n]) + H_{RF,i} H_{i,i}[n] (V^j G_j[n] d_j[n] + c_i[n]) + e_i[n] + F_{RF,i} n_i[n],$$

where $d_j[n]$, of size $d_j \times 1$, is the intended signal stream vector (all entries are white, unit variance) to node $i$. At the Tx side, we have a two stage beamformer (inner BF, $G_j$ of lower dimension and an outer BF, $V^j$ of higher dimension), both the beamformers being at the digital (baseband) side. The outer BF will be applied to the time domain signal at the Tx side, so after the IFFT it will be common to all the subcarriers. The inner BF will be different for different subcarriers. We are considering a noise whitened signal representation so that we get for the noise $n_i \sim CN(0, I_{N_r})$. We consider a two stage BF at the Tx of both nodes. With the inner precoder $G_j$ representing the frequency selective low dimension matrix and the outer precoder being $V^j$. The higher dimensional outer precoder $V^j$ at node $j$ is of dimension $M^j_r \times M^j_r$. The $M^j_r \times d_j$ digital beamformer is $G_j$, where $G_j = [g_{j}^{(1)} ... g_{j}^{(d_j)}]$ and $g_{j}^{(s)}$ represents the beamformer for stream $s$. $c_i$, $e_i$ represents the noise at the transmitter or receiver antennas of node $i$ respectively which models the effect of LDR, LDR noise at Tx or Rx closely approximates the effects of non-ideal amplifiers, oscillators and ADCs/DACs. The covariance matrix of $c_i$ is given by $k_i (k_i << 1)$ times the energy of the transmitted signal at each antenna. $c_i$ is approximated as the Gaussian model, $c_i[n] \sim CN(0, \frac{1}{N_r} \text{diag}(\sum_{n=1}^{N_r} Q_i[n]))$, where $Q_i[n]$ is the transmit signal covariance matrix at subcarrier $n$ of node $i$ and can be written as $Q_i[n] = V^i G_i[n] G_i^H[n] V^i H$ and $c_i[n]$ is statistically independent of $x_i[n]$. $e_i[n]$ is the LDR noise at the receive side and can be approximated as $e_i[n] \sim CN(0, \frac{1}{N_r} \text{diag}(Z))$, where $Z$ is sum of the covariance matrix of the undistorted receive signal across all subcarriers [10] assuming the subcarrier signals are decorrelated, $Z = \sum_{n=1}^{N_r} E(z_i[n] z_i^H[n]), z_i[n] = y_i[n] - e_i[n]$ and $e_i[n]$ is statistically independent of $z_i[n]$. The transmit power (sum of all subcarrier powers) constraint at node $j$ can be written as $\sum_{n=1}^{N_r} \text{tr}(V^j H G_j[n] G_j^H[n]) \leq P_j$. We introduce a digital self interference canceller at the base band which subtracts the residual interference signal $H_{i,i} x_i$ from the received signal. Assuming that $H_{i,i}$ is perfectly estimated at the baseband and since $x_i$ is already known to node $i$, we can rewrite the received signal at the base band as,

$$y_i'[n] = y_i[n] - F_{RF,i} H_{i,i}[n] x_i[n] = F_{RF,i} H_{i,j}[n] x_j[n] + v_i[n],$$

where $v_i[n] = F_{RF,i} (H_{i,j}[n] c_j[n] + H_{i,i}[n] e_j[n]) + e_i[n] + F_{RF,i} n_i[n]$ is the unknown interference plus noise component after SI cancellation. In this paper, for our BF design, we assume that all the channel matrices and scaling factors in (1) are known.

### A. Channel Model

Note that our HBF design which follows, is applicable for general MIMO channel models. Considering the SI channel, as the distance between the transmit and receive array doesn’t satisfy the far-field range condition, we need to employ the near-field model which has spherical wavefront. In such a case, the SI channel coefficients highly depend on the placement of the transmit and receive arrays and can be written as,

$$(H_{i,i})_{m,n} = \frac{\rho}{r_{m,n}} \exp(-j2\pi \frac{r_{m,n}}{\lambda}),$$

where $r_{m,n}$ is the distance between $m$-th element of the receive array and $n$-th element of the transmit array and $\rho$ being the SI channel power normalization factor. Note that, (3) is a simple model which doesn’t take into account the mutual antenna coupling or signal reflections in the SI channel.

### III. WSR MAXIMIZATION THROUGH ALTERNATING MINORIZATION

Consider the optimization of the two-stage BF/hybrid combiner design using WSR maximization of the Multi-cell MIMO system:

$$[V \ G \ F_{RF} \ F_{BB}] = \arg \max_{V,G,F_{RF},F_{BB}} \ WSR(G,V,F_{RF},F_{BB})$$

$$= \arg \max_{V,G} \sum_{i=1}^{N} \sum_{n=1}^{N_r} u_i \ln \det(R_i^{-1}[n] R_i[n]),$$

where the $u_i$ are the rate weights, $G$ represents the collection of digital BF's $G_i[n]$, $V$ the collection of analog BF's $V^j$. At the receiver, we apply a hybrid combiner with analog BF denoted by $F_{RF,i}$ of size $M^j_r \times N_r^i$, where $M^j_r$ represents the number of RF chains at the Rx side. $F_{BB,i}$ represent the baseband digital combiner of size $d_j \times M^j_r$. The covariance matrix of $v_i[n]$, $\Sigma_v[n]$ can be approximated under $k_i \ll 1, l_i \ll 1$ as follows [11].
\[
R_i[n] = k_i F_{RF,i} H_{i,j}[n] \text{diag} (Q_i[n]) H^H_{i,j}[n] F^H_{RF,i} + k_i F_{RF,i} H_{i,j}[n] \text{diag} (Q_i[n]) H^H_{i,j}[n] F^H_{RF,i} + l_i \text{diag} (F_{RF,i} H_{i,j}[n] Q_i[n] H^H_{i,j}[n] F^H_{RF,i}) + F_{RF,i} F^H_{RF,i},
\]
where \( R_i[n] \) is the signal plus interference plus noise covariance matrix. Further after the receive combining, we obtain \( \Sigma[n] = F_{BB,i}[n] R_i[n] F^H_{BB,i}[n] \) and \( \Sigma[n] = F_{BB,i}[n] R_i[n] F^H_{BB,i}[n] \). Direct maximization of (4), however, requires a joint optimization over the four matrix variables \((V, G, F_{RF}, F_{BB})\). Unfortunately, finding a global optimum solution for similar constrained optimization is found to be intractable. So we decouple the transmit-receiver optimization and focus on the design of the Rx combiners first. We assume that the node \( i \) applies the frequency selective hybrid combiner \( F_{BB,i}[n] \) at the output of the Rx RF chains and after the IFFT, to estimate the signal transmitted from node \( j \). The analog combiner \( F_{RF,i} \) serves to separate the SI component from the received signal, while the digital combiner \( F_{BB,i} \) decouples the streams \( d_i[n] \) intended for user \( i \) from \( j \).

\[
d_i[n] = F_{BB,i}[n] y_i[n] + F_{RF,i}[n] F_{RF,i} y_i[n].
\]

At the Rx side, maximizing the WSR is equivalent to minimizing the weighted MSE with the MSE weights being chosen as \( W_i[n] = \frac{1}{\sum_i R_i^{-1}[n]} \) [5], [12]. Further we can obtain the error covariance matrix for the detection of \( d_j \) at node \( i \) as,

\[
\Sigma_i[n] = E((d_j[n] - d_j[n]) (d_j[n] - d_j[n])^H) = (F_i[n] H_{i,j}[n] V^H G_j[n] - I)(F_i[n] H_{i,j}[n] V^H G_j[n] - I)^H + \Sigma_i[n].
\]

The MMSE receive combiner at the baseband side can be alternatively optimized as follows,

\[
F_{BB,i}[n], \forall n = \arg \min_{F_{BB,i}[n]} \text{tr} (R_{d_{i,j},d_{i,j}}[n]),
\]

\[
F_{BB,i}[n] = G_{i,j}[n] V^H H_{i,j}[n] F^H_{RF,i} R_i^{-1}[n].
\]

For optimizing the digital BF in the above equation (8), it can be done independently across different subcarriers as is evident. Further to optimize the analog combiner, we directly optimize the WSR. We make use of certain results on matrix differentiation here. It was shown in [13] that \( \frac{\partial \ln \det(A + BXC)}{\partial X} = [C(A + BXC)^{-1} B]^T \). For the notational convenience we define \( \Theta_{i,j}[n] = H_{i,j}[n] Q_i[n] H^H_{i,j}[n], \Theta_i,[n] = H_{i,i}[n] Q_i[n] H^H_{i,i}[n] \) which can be interpreted as the effective Rx signal covariance matrix before the analog combiner. Similarly we define \( \Psi_{i,j}[n] = H_{i,j}[n] \text{diag} (Q_i[n]) H^H_{i,j}[n], \Psi_i,[n] = H_{i,i}[n] \text{diag} (Q_i[n]) H^H_{i,i}[n] \) Taking the gradient of (4) w.r.t. \( F_{RF,i} \),

\[
\sum_{n=1}^{N_i} R_i^{-1}[n] F_{RF,i} \Theta_{i,j}[n] = \sum_{n=1}^{N_i} (R_i^{-1}[n] - R_i^{-1}[n]) F_{RF,i} \left( k_i \Psi_{i,j}[n] + k_i \Psi_i,[n] \right) + \left( \sum_{n=1}^{N_i} R_i^{-1}[n] - R_i^{-1}[n] \right) F_{RF,i} \left( \Theta_{i,j}[n] + \Theta_i,[n] \right) = 0.
\]

In (a), we use the result \( \text{vec}(AXB) = (B^T \otimes A) \text{vec}(X) \) from [14]. Further this leads to a generalized eigen vector solution for the analog combiner,

\[
\vec{F}_{RF,i} = V_{max} (\hat{B}_i, A_i), \hat{B}_i = \sum_{n=1}^{N_i} (\Theta_{i,j}[n])^T \otimes (R_i^{-1}[n] - R_i^{-1}[n]) + l_i (\Theta_{i,j}[n] + \Theta_i,[n])^T \otimes (R_i^{-1}[n] - R_i^{-1}[n])
\]

A. Two stage transmit BF design

We define the following Lemma below which proves the concavity of a part of the WSR (4).

Lemma 1. For each \( i \in 1, 2, n \in 1, \ldots, N_t, f_i(Q_i[n], Q_j[n]) = \ln(R_i^{-1}[n] R_{ij}[n]) \) is concave w.r.t \( Q_i[n] \), where \( Q_i[n] \) is a positive semidefinite matrix.

Proof: Using the technique from [13, Th. 2], the concavity of \( f_i(Q_i[n], Q_j[n]) \) w.r.t \( Q_i[n] \) can be proved by showing that \( f_i(t) = f_i(X_i + t Y_i, Q_i[n]) \) is concave w.r.t \( t \in [0, 1] \), where \( X_i \) is positive semidefinite and \( Y_i \) being Hermitian. The derivative of \( f_i(t) \) w.r.t \( t \) can be written as,

\[
\frac{\partial}{\partial t} f_i(t) = \left\{ \sum_{n=1}^{N_i} R_i^{-1}[n] \right\} \frac{\partial R_i^{-1}[n]}{\partial t} + F_{RF,i} H_{i,j}[n] Y_j H^H_{i,j}[n] F^H_{RF,i}
\]

\[
= k_i F_{RF,i} H_{i,j}[n] \text{diag} (Q_j[n]) H^H_{i,j}[n] F^H_{RF,i} + l_i \text{diag} (F_{RF,i} H_{i,j}[n] Y_j H^H_{i,j}[n] F^H_{RF,i})
\]

where \( \frac{\partial R_i^{-1}[n]}{\partial t} = k_i F_{RF,i} H_{i,j}[n] \text{diag} (Q_j[n]) H^H_{i,j}[n] F^H_{RF,i} \) does not depend on \( t \). Further,

\[
\frac{\partial}{\partial t} f_i(t) = \left\{ \sum_{n=1}^{N_i} R_i^{-1}[n] \right\} \frac{\partial R_i^{-1}[n]}{\partial t} + N_i R_i^{-1}[n] \frac{\partial R_i^{-1}[n]}{\partial t} + R_i^{-1}[n] \frac{\partial R_i^{-1}[n]}{\partial t}
\]

where \( N_i = F_{RF,i} H_{i,j}[n] Y_j H^H_{i,j}[n] F^H_{RF,i} \). Since we assume that \( k_i, l_i \ll 1 \), the second term inside the trace in (13) will contain quadratic terms in \( k_i \) or \( l_i \) and thus becomes negligible. Further we can show similar as in [13, Th. 2] that the first term in (13) is negative and thus we can conclude that \( f_i(t) \) is concave. This completes the proof.

Consider the dependence of WSR on \( Q_i[n] \) alone.

\[
WSR = u_i \ln \det(R_i^{-1}[n] R_{ij}[n]) + WSR_{RF,i}[n] + \sum_{m=1, m \neq i}^{N_t} WSR_{RF,i}[n]
\]

From Lemma 1, we can see that the first term in the above summation is a concave function in \( Q_i[n] \). However, the rest of terms are convex due to the dependency of \( Q_i[n] \) through the interference terms. In order to solve this non-convex problem,
we further consider a difference of convex function approach [15] which linearizes the convex part first and then Taylor series expansion as below,
\[ WSR^2(Q_j[n], \hat{Q}_j[n]) = WSR^2(Q_j[n], \hat{Q}_j[n]) - \operatorname{tr}(Q_j[n] - \hat{Q}_j[n])\hat{A}_j[n], \hat{A}_j[n] = \frac{1}{\sigma_{Q_j[n]}[Q_j[n]]} |Q_j[n]| = u_jk_j \operatorname{diag}(HF_{RF,j}^H(\hat{R}_j^{-1} - \hat{R}_j^{-1})[n]HF_{RF,j}H_j[n]) + l_jHF_{RF,j}^H(\hat{R}_j^{-1} - \hat{R}_j^{-1})[n]HF_{RF,j}H_j[n]. \] (15)

The Taylor series expansion is done around the point \( Q_j[n] \) (which represents the computed previous iteration values) and the corresponding \( R_i[n] \) is \( R_i[n] \). Then, dropping constant terms, reparameterizing the \( Q_j[n] \) as in (5), performing this linearization for all users, and augmenting the WSR cost function with the Tx power constraints, we get the Lagrangian (16) which gets maximized alternatively [16] between digital and analog BF.

\[
\mathcal{L}(V, G, A) = \sum_{i=1}^{\mathcal{N}} \lambda_i P_i + \sum_{i=1}^{\mathcal{N}} u_i \ln \det(\hat{R}_i^{-1}[n]R_i[n]) - \operatorname{tr}(G^H_i[V^H(A_i[n] + \lambda_n I)V]^iG_i[n]) . \] (16)

In the Appendix A, we derive the gradient expressions when there are terms of the form \( \ln \det(Y + F(X)) \) where \( Y = A \operatorname{diag}(C X D)B + F(X) \). Using this result, we take the derivative of (16) w.r.t the digital BF \( G_j \) which leads to,

\[
F_{RF,i}V^H_{j,i}G_j + k_j V^H_{j,i} \operatorname{diag}(H_{j,i}[n]HF_{RF,i}^H) (R_i^{-1} - \hat{R}_i^{-1}) \] (17)

\[
G_j = V_{1:d_j}(V^H_{j,i}B_{j,i}V^H_{j,i}, V^H_{j,i}(\hat{A}_j + \hat{C}_j + \lambda_j I)V_{j,i}). \] (18)

where \( B_{j,i} = H_{i,j}[n]HF_{RF,i}^H(\hat{R}_i^{-1} - \hat{R}_i^{-1})HF_{RF,i}H_{i,j}. \) \( \hat{C}_j = -H_{j,i}[n]HF_{RF,i}^H(\hat{R}_i^{-1} - \hat{R}_i^{-1})HF_{RF,i}H_{j,i} + k_j \operatorname{diag}(H_{j,i}[n]HF_{RF,i}^H(\hat{R}_i^{-1} - \hat{R}_i^{-1})HF_{RF,i}H_{j,i}). \) Further considering the derivative of (16) w.r.t the analog BF \( V \), we get,

\[
(V_{j,i}^H - \hat{C}_j)V^H_{j,i}G_j = (\hat{A}_j + \lambda_j I)V^H_{j,i}G_j. \] (19)

Further utilizing the result \( vec(AXB) = (B^T \otimes A)vec(X) \) [14], we get

\[
\left((G_j^H_{j,i})^T \otimes \hat{E}_j\right)vec(V^j) = \left((G_j^H_{j,i})^T \otimes \hat{E}_j\right)vec(V^j). \] (20)

where we define \( \hat{E}_j = \hat{A}_j + \hat{C}_j + \lambda_j I \). This leads to the generalized eigen vector solution and can be written as \( vec(V^j) = V_{max}(\left((G_j^H_{j,i})^T \otimes \hat{B}_j\right),\left((G_j^H_{j,i})^T \otimes \hat{E}_j\right)) \).

**B. Optimization of stream powers**

One advantage of the Lagrangian formulation (16) is that it allows to introduce stream powers for each BS, so \( G_j = G_j^1P_j^{1/2} \), where the diagonal matrix \( P_j \) represents the power allocated to an unknown number of supportable streams for BS \( j \). In order to render a feasible solution for the stream powers, we approximate the concave part of the WSR by a first order local minimizer function

\[
\ln\det(I + G_j^H_{j,i}V^jH_{i,j}^HHF_{RF,i}^H(\hat{R}_i^{-1} - \hat{R}_i^{-1})HF_{RF,i}H_{i,j}V^jG_j) = \ln\det(I + P_j\hat{S}_j) + \operatorname{tr}(\{P_j - \hat{P}_j\})\hat{T}_j, \] where, \( \hat{T}_j = G_j^H_{j,i}V^jH_{i,j}^HHF_{RF,i}^H(\hat{R}_i^{-1} - \hat{R}_i^{-1})HF_{RF,i}H_{i,j} \)

\[
\hat{S}_j = G_j^H_{j,i}V^jH_{i,j}^HHF_{RF,i}^H(\hat{R}_i^{-1} - \hat{R}_i^{-1})HF_{RF,i}H_{i,j}V^jG_j. \] (21)

For the concave local minimization considered above, this works well as long as the next optimum is within the minimization range. Note that \( \hat{S}_j^2, \hat{T}_j \) are diagonal since \( G_j \) diagonalizes the matrices \( V^jH_{i,j}V^j \) and \( V^jH \) \( \hat{A}_j + \hat{C}_j + \lambda_j I \). Further optimizing w.r.t \( P_j \) leads to the self interference and LDR aware water filling (SILA-WF) solution for the stream powers as,

\[
P_j = (u_j\hat{T}_j^{-1} - \hat{S}_j^{-1})^+ \] (22)

where \( (x)^+ = \max(0, x) \) is applied to all diagonal elements and the Lagrange multipliers are adjusted to satisfy the power constraints. This can be done by bisection and gets executed per BS.

1) Analog Phase Shifter Design: For the constrained analog BF case, where the BF coefficients are chosen to be phasors, we utilize the DA based approach proposed earlier in our own work [17]. We refer the reader for a more detailed discussion on this to our own paper [17, Algorithm 3].

**Algorithm 1 Minorization based multi-stage/HBF design**

Given: \( P_j, H_{i,j}[n], u_i, H_{i,j}[n] \forall i, j, n. \)

**Initialization:** \( V^j \) is selected as the eigen vectors of the direct channel covariance matrix, The \( G_j^0[n] \)s are initialized to be ZF precoders for the effective channels \( H_{i,j}[n]V^j \) with uniform power distribution across the streams. Iteration (i) :

1) Compute the Rx side digital combiner \( P_j[n] \) from (8).
2) Update the Rx side analog combiner \( F_j^H \) using (11).
3) Compute \( \hat{B}_j[n], \hat{A}_j[n] \) \& \( \hat{C}_j[n] \) from (15) and \( \hat{C}_j[n] \forall j, n. \)
4) Update \( G_j^0[n] \) from (18), and \( P_j[n] \) from (22), \( \forall k, n. \) Compute \( \lambda_j \) using bisection.
5) Update \( V^j/\lambda_j \), using DA (phasor constrained) or from (20) (unconstrained).
6) If the algorithm is converged, exit the loop, otherwise go to step 1.

**IV. SIMULATION RESULTS**

Simulations to validate the performance of the proposed hybrid BF algorithms are presented for a bidirectional FD system under LDR noise model. We follow the pathwise channel model \( H_{i,j} \) as in Section II.A, where the complex path gains are assumed to be Gaussian with variance distributed according to an exponential profile. For the SI channel, we ignore the near field effect of amplitude variation with distance and the near field effects in the phase variation. In the Uniform Linear Array (ULA), the AoD or AoA \( \phi, \theta \) are assumed to be uniformly distributed in the interval \( [0^\circ, 30^\circ] \).

![Fig. 2. Sum Rate comparisons for, Single Carrier, \( N_0^1 = N_0^1 = 8, M_0^1 = M_0^1 = 4, d_o = 1, \forall i, L = 4 \) paths.](image-url)
Fig. 3. Sum Rate comparisons for OFDM, $N_s = 4, N_i^t = N_i^r = 8, M_i^t = M_i^r = 4, d_i = 1, \forall i, L = 4$ paths.

The dimensions of the two-stage BF and hybrid BF are such that the zero forcing capabilities at both sides are comparable. However, the number of LDR noises is the number of antennas at the Tx side, whereas for the analog Rx stage, the number of LDR noises is the number of analog BF outputs, which is less. We conjecture that this would explain the better performance of the analog stage at Rx (in both figures) compared to the two-stage architecture at Tx for SI nulling. In Figure 2, we compare against the eigen beamforming (where the left and right singular vectors of the corresponding channels are used as the Combiner/BF and fully digital) and shows that its performance is inferior compared to our proposed design.

V. CONCLUSION

In this paper, we looked at beamforming solutions to null the SI power under a more practical noise model called as limited dynamic range. We proposed a multi-stage beamforming design (whose performance is validated through simulations), with a frequency flat analog or time domain combiner/BF stage and a frequency dependent baseband precoder/combiner. We optimized the WSR using an alternating minorization approach which converges to a local optimum.

APPENDIX A

GRADIENT DERIVATION

In this section we derive an expression for the gradient for the terms of the form

$$Y = A \text{diag}(CXD)B + F(X), \quad R = CXD,$$

where $F(X)$ represents any matrix function in $X$. Each element of $Y$ can be written as,

$$Y_{i,j} = \sum_{m,n} A_{i,m} R_{m,n} B_{n,j} \delta_{m-n} + F(X),$$

$$R_{m,n} = \sum_{p,q} C_{m,p} X_{p,q} D_{q,n},$$

$$X_{i,j} = \sum_{m,n} A_{i,m} \left( \sum_{p,q} C_{m,p} X_{p,q} D_{q,n} \right) B_{n,j} \delta_{m-n} + F(X),$$

where $\delta_k$ represents the Kronecker delta function. We define $V_{r,s}$ as zero-valued matrix except for a unity element at row $r$ and column $s$ and we obtain,

$$\frac{\partial \det(Y)}{\partial X} = \sum_{r,s} V_{r,s} \frac{\partial \det(Y)}{\partial Y_{r,s}} = \sum_{r,s} \left( \sum_{i,j} C_{i,m} R_{m,n} B_{n,j} \delta_{m-n} + F(X) \right) \frac{\partial \det(Y)}{\partial Y_{r,s}}$$

$$= \sum_{r,s} \frac{\partial \det(Y)}{\partial Y_{r,s}} \left( \sum_{i,j} C_{i,m} R_{m,n} B_{n,j} \right) \delta_{m-n} + \frac{\partial \det(Y)}{\partial Y_{r,s}} F(X)$$

$$= \sum_{r,s} \frac{\partial \det(Y)}{\partial Y_{r,s}} \left( \sum_{i,j} C_{i,m} R_{m,n} B_{n,j} \right) \delta_{m-n} + \frac{\partial \det(Y)}{\partial Y_{r,s}} F(X).$$

For simplicity we call the second term in the summation $F'(X)$ since that is not of interest here or the required gradients (needed forms of $F(X)$) are derived in [11]. Further using the result, $\frac{\partial \det(Y)}{\partial X} = \det(Y)(Y^{-1})^T$, we can simplify it as,

$$\frac{\partial \det(Y)}{\partial X} = \det(Y)\left[ \text{diag}(BY^{-1}A) \right]^T + F'.$$

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REFERENCES


