Trajectory Optimization for Mobile Access Point

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***Abstract*—We consider the problem of trajectory optimization of an Unmanned Air Vehicle (UAV) that is equipped with an wireless access point (AP) to collect data from the ground users. The goal is to find constant altitude path and velocity of the UAV during the flight time such that the weighted sum-rate of the users is maximized. Two approaches are used in formulating this problem, one involves functional optimization while the other is based on the optimal control approach. Non-convex nature of the objective function makes it difficult to obtain optimal solutions in general. However, we provide some analytical properties of the optimal trajectories in the large flying time regime, which will provide the validation for the obtained numerical results.**

I. Introduction

While the primary use of Unmanned Air Vehicles (UAVs) originated from military applications, there is now considerable interest in civilian areas thanks to recent progress in terms of performance, cost and weight, etc. Typical commercial applications include, rescue missions, aerial surveying, delivery of goods, etc. According to Federal Aviation Administration, sales of UAVs for commercial purposes are expected to grow from 600,000 in 2016 to 2.7 million by 2020 [1].

Mounting access points (APs) or base stations on UAVs in a wireless network provides an additional degree of freedom in terms of mobility in the system design. The advantages include, dynamic network deployment, fast response to geographically varying traffic demands, etc. See [2] for an extensive overview.

There is a range of interesting issues arising from the study of flying APs. This notably includes static positioning and path planning problems. In the static positioning problem, a fixed location is determined for the UAV that provides optimal data service or coverage to a population of ground users [3]–[6]. Note that the static positioning problem ignores constraints on the flight time and also ignores the potential service which can be offered to users while in-flight towards the chosen location or while returning from it to the UAV base. As an alternative, a path planning problem arises when a total flight time constraint is considered and when connectivity service is enabled at any point on the trajectory followed by the UAV.

Few works in the literature have considered the interplay between UAV kinematics and data collection from ground users in a wireless system. In [7], path planning of a constant velocity UAV is considered where the objective is to minimize the total mission time subject to constraints on the minimum amount of data collected from each user. Necessary conditions

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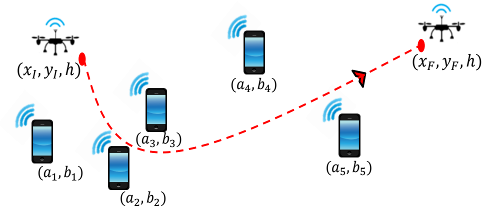

Figure 1. Information collection using a mobile AP.

for optimal paths and a numerical method to generate them are provided. In [8], both path and the velocity of an UAV are optimized in order to maximize the minimum rate of the users in a multiple access setting. An iterative algorithm that converges to a local minimum is proposed.

In this paper, we aim to find a path and velocity of the UAV such that the weighted sum-rate of the users is maximized. Different from previous works [7], [8], analysis of the optimal trajectory when the flying time is infinite is provided and also dynamic programming (DP) is used to obtain the optimal trajectories. We start by introducing the UAV and communication system models.

II. System model

A wireless communication system where an AP serves K users is considered. The AP is mobile as it is mounted on an UAV, while the users are static and are located on the ground, as shown in Figure 1. We first present the UAV model and then introduce the communication system model between AP and the ground users.

A. UAV Model

We assume that the flight time of UAV lasts for a duration of time T. During the flight time, $t \in [0, T]$, UAV flies at a constant altitude of h and its position on the ground plane is given by the Cartesian coordinates $(x(t), y(t))$. The UAV starts at an initial location (x_I, y_I) at time $t = 0$ and has to reach the destination (x_F, y_F) by time T. Moreover, the maximum velocity at which UAV can travel is given by V. Therefore, we have

$$\sqrt{\dot{x}^2(t) + \dot{y}^2(t)} \leq V, \; t \in [0, T],$$

where $\dot{x}(t)$ and $\dot{y}(t)$ represent the time-derivatives.

B. Communication System Model

We consider an uplink transmission scenario where the communication links between the users and the AP are modeled as orthogonal point-to-point additive white Gaussian noise (AWGN) channels. The information rate for the k-th user, $k \in \{1, \ldots, K\}$ is

$$R_k(t) = \log_2\left(1 + \mathrm{SNR}_k(t)\right),$$

where $\mathrm{SNR}_k(t)$ denotes the signal to noise ratio of k-th user at time t. Based on the distance based path loss model, for the k-th user located at $(a_k, b_k) \in \mathbb{R}^2$,

$$\mathrm{SNR}_k(t) = \frac{P}{\sigma^2} d_k(t)^{-\alpha}$$

where the distance from the UAV

$$d_k(t) = \sqrt{h^2 + (x(t) - a_k)^2 + (y(t) - b_k)^2},$$

P is the transmission power of the k-th user, $\alpha \geq 2$ is the path loss exponent and σ^2 denotes the noise power.

III. Problem Formulation

Using the above described models, our goal is to find UAV trajectories that maximize the weighted sum-rate of the users

$$C(t) \triangleq \sum_{k=1}^{K} w_k R_k(t),$$

with weights $w_k \geq 0$.

A. Optimization Problem

The optimization problem is given by

$$\max_{x(t), y(t)} \quad \int_{t=0}^{T} C(t)\, dt \tag{1a}$$

$$\text{s.t.} \quad \sqrt{\dot{x}^2(t) + \dot{y}^2(t)} \leq V, \; t \in [0, T], \tag{1b}$$

$$x(0) = x_I, \; y(0) = y_I, \tag{1c}$$

$$x(T) = x_F, \; y(T) = y_F, \tag{1d}$$

where (1b) guarantees that velocity of UAV does not exceed the maximum velocity limit, while starting and final positions of the UAV are reflected in (1c) and (1d), respectively.

We assume that there exists at least one feasible solution satisfying the constraints (1b) - (1d). This can be guaranteed by choosing T and V such that the UAV can at least travel from starting position to the destination along the minimum distance path i.e., $VT \geq \sqrt{(x_F - x_I)^2 + (y_F - y_I)^2}$. Since $R_k(t)$ is not a concave function of $x(t)$ and $y(t)$, (1) is a non-convex functional optimization problem which is difficult to solve in general.

B. Control Approach

In this subsection, we reformulate (1) as an optimal control problem. The UAV mobility is modeled as a deterministic dynamical system and we aim to find a control law i.e., trajectory such that the weighted sum-rate of the users is maximized. The optimal control problem is given by

$$\max_{v(t), \phi(t)} \int_{t=0}^{T} C(\boldsymbol{s}(t))\, dt \tag{2}$$

subjected to

$$\begin{aligned}
&\dot{\boldsymbol{s}}(t) = \boldsymbol{f}(t, \boldsymbol{s}, \boldsymbol{u}) \text{ (state equation)}\\
&v(t) \leq V \text{ (Input constraints)}\\
&\boldsymbol{s}(0) = [x_I \; y_I]^{\mathrm{T}},\\
&\boldsymbol{s}(T) = [x_F \; y_F]^{\mathrm{T}} \text{ (Boundary conditions)}
\end{aligned}$$

where

$$\boldsymbol{f}(t, \boldsymbol{s}, \boldsymbol{u}) = v(t) \begin{bmatrix} \cos\phi(t) \\ \sin\phi(t) \end{bmatrix},$$

the state $\boldsymbol{s}(t) = [x(t) \; y(t)]^{\mathrm{T}}$, control inputs $\boldsymbol{u}(t) = [v(t) \; \phi(t)]^{\mathrm{T}}$ with $v(t), \phi(t)$ being the velocity and heading angle (in azimuth), respectively.

This formulation allows us to obtain necessary conditions on the optimal trajectory and also allows us to use dynamic programming (DP) tool which can be used to obtain optimal trajectories.

In either problem formulation, the non-convex nature of the objective function and optimization of functionals makes it difficult to obtain analytic solutions. Therefore, we use numerical methods to obtain approximations of the optimal trajectories. However, before resorting to these methods, we provide some analytical properties of the optimal trajectories.

IV. Analytical Properties

In this section, first, we obtain properties of the optimal trajectory using the formulation in (1) and when the flying time $T \to \infty$, and then provide necessary conditions for the optimality by using the formulation in (2). These properties will be later useful in validating the numerical results.

A. $T \to \infty$

If there exists a unique static position of UAV (x^*, y^*) that results in maximum weighted sum-rate C^* i.e.,

$$(x^*, y^*) = \arg\max_{(x,y) \in \mathbb{R}^2} \sum_{k=1}^{K} w_k R_k, \tag{3a}$$

$$C^* = \max_{(x,y) \in \mathbb{R}^2} \sum_{k=1}^{K} w_k R_k, \tag{3b}$$

where $R_k = \log_2(1 + \frac{P}{\sigma^2} {d_k}^{-\alpha})$ and the distance $d_k = \sqrt{h^2 + (x - a_k)^2 + (y - b_k)^2}$, and as the flight time $T \to \infty$, we have the following results.

Proposition 1: Optimal trajectory must pass through (x^*, y^*).

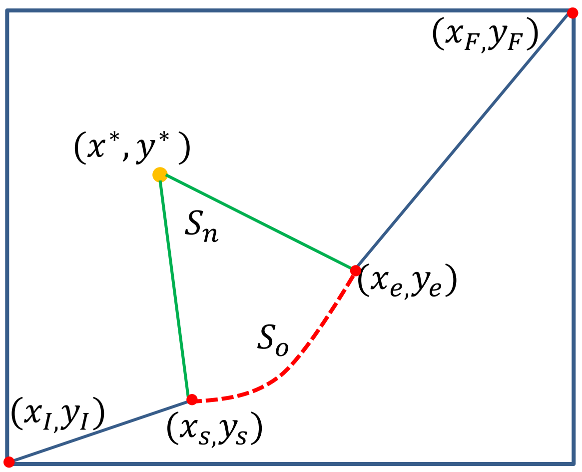

Figure 2. Replacement of segment S_0 with S_n, illustrating Proposition 1.

Proof: Assume that there exists an optimal trajectory that does not pass thorough (x^*, y^*). With out loss of generality (W.l.o.g) consider a segment of this trajectory S_o of duration $O(T)$ such that $O(T) \leq T$ and $\lim_{T\to\infty} O(T) = \infty$. Let (x_s, y_s) and (x_e, y_e) be the starting and ending coordinates of this segment. An example illustration is shown in Fig. 2. The weighted sum-rate over the segment S_o satisfies

$$\int_{S_o} C(t)dt \ \leq O(T)C^*_{s_o}, \tag{4}$$

where $C^*_{s_o}$ is the maximum value of $C(t)$ over S_o i.e., $C^*_{s_o} = \max_{S_o} C(t)$. We now show that by replacing S_o by another segment S_n and keeping the rest of the trajectory same will improve the objective, and hence conclude the proof.

The new segment S_n is such that the UAV flies from (x_s, y_s) to (x^*, y^*) in a straight line with maximum velocity, hovers there, and comes back to (x_e, y_e) in a straight line with maximum velocity. Let d_s and d_e denote the distance from coordinates (x_s, y_s) and (x_e, y_e) to (x^*, y^*), respectively. Then the total travel time to and from (x^*, y^*) is $t_{tr} = (d_s + d_e)/V$, and hovering time at (x^*, y^*) is $O(T) - t_{tr}$. The weighted sum-rate over the segment S_n is

$$\int_{S_n} C(t)\ dt = C_{tr} + (O(T) - t_{tr})C^*, \tag{5}$$

where C_{tr} is the weighted sum-rate obtained while UAV is traveling to and from (x^*, y^*). From (4) and (5), we have

$$\begin{aligned}
\lim_{T\to\infty} \frac{\int_{S_n} C(t)\ dt}{\int_{S_o} C(t)\ dt} &\geq \lim_{T\to\infty} \frac{\int_{S_n} C(t)\ dt}{O(T)C^*_{s_o}} \\
&= \lim_{T\to\infty} \frac{C_{tr}}{O(T)C^*_{s_o}} + \left(1 - \frac{t_{tr}}{O(T)}\right) \frac{C^*}{C^*_{s_o}} \\
&\overset{(a)}{=} \frac{C^*}{C^*_{s_o}} \\
&\overset{(b)}{>} 1,
\end{aligned}$$

where

(a) follows from the fact that t_{tr} is a constant and $\lim_{T\to\infty} O(T) = \infty$,
(b) follows from (3).

■

Proposition 2: Except at (x^*, y^*), UAV travels with with maximum velocity V.

Proof: From Proposition 1, the optimal trajectory passes through (x^*, y^*). Let S be a segment of this trajectory such that $(x^*, y^*) \notin S$. Let

$$C_o = \int_{S, v(t)<V} C(t)\ dt, \tag{6}$$

$$C_n = \int_{S, v(t)=V} C(t)\ dt, \tag{7}$$

denote the weighted sum-rates obtained when the UAV traverses this segment with velocity $v(t) < V$ and $v(t) = V$, respectively. W.l.o.g let us assume that it takes $\delta > 0$ time less to traverse S when traveling with V than traveling with $v(t) < V$. If this time gained δ is used to hover at (x^*, y^*) and by (3), we have

$$C_o - C_n < \delta C^*. \tag{8}$$

Therefore, by switching to maximum velocity in S and using the time gained in hovering at (x^*, y^*) results in an increase in the weighted sum-rate. ■

Proposition 3: If there exists a trajectory where the total time spent by UAV except at the point (x^*, y^*) is given by $f(T)$ and $\lim_{T\to\infty} f(T) = \infty$, then it is strictly suboptimal.

Proof: Let's consider two UAV trajectories that passes through (x^*, y^*), and let S_o and S_n represent the part of the trajectories excluding (x^*, y^*). Suppose UAV spends $f(T)$, Δ(a positive constant) amounts of time in S_o and S_n, respectively. The weighted sum-rates for these trajectories are

$$C_o = \int_{S_o} C(t)\ dt + (T - f(T))C^*, \tag{9a}$$

$$C_n = \int_{S_n} C(t)\ dt + (T - \Delta)C^*. \tag{9b}$$

We now have

$$\begin{aligned}
\lim_{T\to\infty} C_n - C_o = \lim_{T\to\infty} &\left(C^* f(T) - \int_{S_o} C(t)\ dt\right) + \\
&\left(\int_{S_n} C(t)\ dt - \Delta C^*\right) \\
&\overset{(a)}{=} \infty,
\end{aligned}$$

where (a) follows from the fact that $\lim_{T\to\infty} f(T) = \infty$, Δ is a positive constant, and from the definition of C^* in (3). ■

B. Necessary Conditions

Using the formulation in (2) and Pontryagin's maximum principle, we derive the necessary conditions for optimality [9]. These conditions are based on maximizing the Hamiltonian $H \triangleq H(\boldsymbol{s}(t), \boldsymbol{u}(t), \boldsymbol{\lambda}(t), t),\ \forall t \in [0, T]$

$$H = C\left(\boldsymbol{s}(t)\right) + \boldsymbol{\lambda}^{\mathbf{T}}(t)\boldsymbol{f}(t, \boldsymbol{s}, \boldsymbol{u}) + \eta(t)(v(t) - V),$$

where $\boldsymbol{\lambda}(t) = [\lambda_1(t)\ \lambda_2(t)]^{\mathrm{T}}$. The optimal trajectory must satisfy the following conditions:

1) $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = v(t) \begin{bmatrix} \cos\phi(t) \\ \sin\phi(t) \end{bmatrix}$ (state equations)

2) $\dot{\lambda}_1(t) = -\dfrac{\partial H}{\partial x} = -\dfrac{\partial C}{\partial x}$

$\dot{\lambda}_2(t) = -\dfrac{\partial H}{\partial y} = -\dfrac{\partial C}{\partial y}$ (costate equations)

3) $\dfrac{\partial H}{\partial v} = 0,\ \dfrac{\partial H}{\partial \phi} = 0,$

$\eta(t)(v(t) - V) = 0$ (First order optimality conditions)

V. Discrete Approximation

In this section, we discretize the optimization problems (1) and (2) to obtain numerical approximations of the optimal trajectories. The time period $[0, T]$ is divided into N equal length intervals of duration $\delta = T/N$, indexed by $i = 0, \ldots, N-1$. The value of δ is chosen to be sufficiently small such that UAV's location, velocity, and heading angles can be considered to remain constant in an interval. In the i-th interval, (x_i, y_i), v_i and ϕ_i denote the UAV's position, velocity and heading angle, respectively. The rate of k-th user in time interval i is

$$R_{i,k} = \log_2\left(1 + \frac{P}{\sigma^2}\left[h^2 + (x_i - a_k)^2 + (y_i - b_k)^2\right]^{-\alpha/2}\right). \tag{10}$$

A. NLP

After discretization, the functional optimization in (1) becomes a nonlinear programming (NLP) as follows.

$$\max_{\{x_i, y_i\}_{i=1}^{N-1}} \quad \sum_{i=0}^{N-1}\sum_{k=1}^{K} w_k R_{ik} \tag{11a}$$

$$\text{s.t.} \quad (x_1 - x_I)^2 + (y_1 - y_I)^2 \le V^2, \tag{11b}$$

$$(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 \le V^2, \quad i = 1, \ldots, N-2, \tag{11c}$$

$$(x_F - x_{N-1})^2 + (y_F - y_{N-1})^2 \le V^2, \tag{11d}$$

Note that in the above problem only time is discretized, and the coordinates $(x_i, y_i) \in \mathbb{R}^2$. We then numerically solve (11) to obtain approximations to the optimal trajectories. This strategy will be called as direct method.

B. Dynamic Programming

In this section, using discrete-time approximation of (2), we formulate it as finding an optimal control of a discrete dynamical system. Then the discrete-time dynamic system is given by

$$\boldsymbol{s}_{i+1} = \boldsymbol{s}_i + f(i, \boldsymbol{s}_i, \boldsymbol{u}_i),\ i = 0, 1 \ldots, N-1 \tag{12}$$

where $\boldsymbol{s}_i = [x_i\ y_i]^{\mathrm{T}}$ describes the state i.e., the position of the AP and $\boldsymbol{u}_i = [v_i\ \phi_i]^{\mathrm{T}}$ specifies the control action i.e., velocity and heading angle, respectively, in the i-th time interval. The states are computed using

$$f(i, \boldsymbol{s}_i, \boldsymbol{u}_i) = v_i \begin{bmatrix} \cos\phi_i \\ \sin\phi_i \end{bmatrix}, \tag{13}$$

starting with the initial state $\boldsymbol{s}_0 = [x_o\ y_o]^{\mathrm{T}}$.

For a given set of control actions $\pi = \{\boldsymbol{u}_0, \boldsymbol{u}_1, \ldots, \boldsymbol{u}_{N-1}\}$ the cost function is given by

$$J_\pi(\boldsymbol{s}_0) = J(\boldsymbol{s}_N) + \sum_{i=0}^{N-1}\sum_{k=1}^{K} w_k R_{i,k}, \tag{14}$$

where the terminal cost

$$J(\boldsymbol{s}_N) = \begin{cases} -\infty, & \text{if} \ \ \boldsymbol{s}_N \ne [x_f\ y_f]^{\mathrm{T}} \\ \sum\limits_{k=1}^{K} w_k R_{N,k}, & \text{otherwise} \end{cases} \tag{15}$$

and $R_{i,k}$ is defined in (10).

An optimal policy π^* that maximizes the cost is

$$\pi^* = \max_{\pi \in \Pi} J_\pi(\boldsymbol{s}_0), \tag{16}$$

where $\Pi = \{\boldsymbol{u}_i,\ i = 0, \ldots, N-1 \mid v_i \le V, 0 \le \phi \le 360^o\}$. The optimization problem (16) can be solved by DP [10]. Given initial state $\boldsymbol{s}_0$, the optimal cost can be computed recursively using Bellman's equations by proceeding backwards in time by [10]

$$J(\boldsymbol{s}_i) = \max_{\boldsymbol{u}_i} \sum_{k=1}^{K} w_k R_{i,k} + J(\boldsymbol{s}_{i+1}),\ i = N-1, \ldots, 0, \tag{17}$$

where the terminal cost $J(\boldsymbol{s}_N)$ is given in (15). An optimal policy π^* solves (17). However, this solution is computationally expensive as the state space $\boldsymbol{s}_i \in \mathbb{R}^2$, and for each state we have to find the optimal v_i and ϕ_i.

VI. Results

In this section, numerical results are obtained for the optimal trajectories using both NLP and DP approaches. We consider the involved parameters to be dimensionless, which can be achived by proper scaling. For simulations, the UAV starts at $(0, 0)$ and has to reach the destination $(8, 8)$ in time T units. It has a maximum velocity of $V = 1$ unit, and travels at a fixed altitude of $h = 1$ unit. The transmission power and noise variance of all users are set to unity, and the path loss exponent $\alpha = 2$. There are 5 users in the system and the weights of all users $w_k = 1, \forall k$.

Figure 3 shows the optimal trajectories of the UAV for different values of T, which are obtained by solving NLP described in Section V-A. The optimal velocity along this trajectory is shown in Figure 4. These simulations are in match with the results in Section IV-A. As the flying time increases the UAV flies with maximum velocity to the point (x^*, y^*), hovers there, and then flies to the destination with maximum velocity.

Finally, we present the results obtained by using DP presented in Section V-B. To apply DP, we need to discretize the state space and input actions. Both x and y-coordinates are discretized with a step of 0.1, while the allowed input actions that UAV can take at any state are shown in Figure 5. In Figure 6, we present the optimal trajectory for $T = 20$ obtained by DP. This trajectory is different from the one obtained by using direct method since the input and states

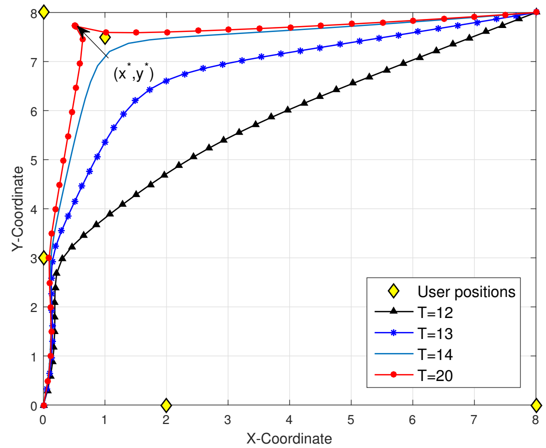

Figure 3. Optimal UAV trajectory starting from $(0,0)$ and finishing at $(8,8)$.

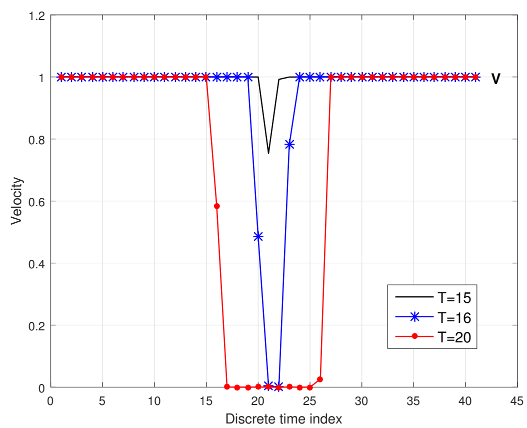

Figure 4. UAV velocity during the flight time.

are discretized. However, this result is also in match with the properties derived in Section IV-A.

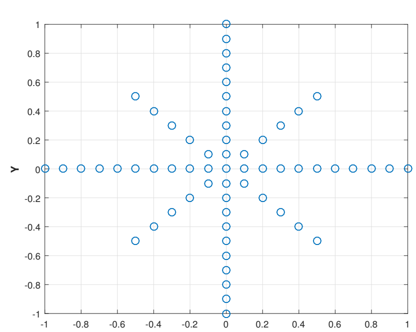

Figure 5. Allowed actions in each state for DP.

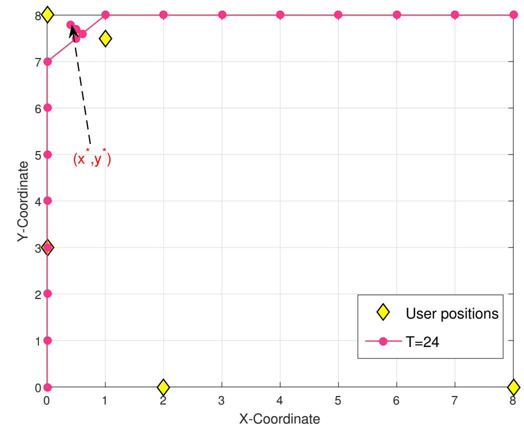

Figure 6. UAV trajectory for $T = 24$ obtained by DP.

VII. Conclusion

The problem of finding optimal trajectory of an mobile AP that maximizes the weighted sum-rate is formulated. We have presented both necessary conditions for optimality (based on control approach) and properties of the optimal trajectories in large flying time regime. Then we have numerically obtained optimal trajectories by solving NLP and DP techniques.

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