# Robust Location-Aided Beam Alignment in Millimeter Wave Massive MIMO

Flavio Maschietti†, David Gesbert†, Paul de Kerret†, Henk Wymeersch‡
†Communication Systems Department, EURECOM, Sophia-Antipolis, France
‡Department of Signals and Systems, Chalmers University of Technology, Sweden
Email: {maschiet, gesbert, dekerret}@eurecom.fr, henkw@chalmers.se

***Abstract*—Location-aided beam alignment has been proposed recently as a potential approach for fast link establishment in millimeter wave (mmWave) massive MIMO (mMIMO) communications. However, due to mobility and other imperfections in the estimation process, the spatial information obtained at the base station (BS) and the user (UE) is likely to be noisy, degrading beam alignment performance. In this paper, we introduce a robust beam alignment framework in order to exhibit resilience with respect to this problem. We first recast beam alignment as a decentralized coordination problem where BS and UE seek coordination on the basis of correlated yet individual position information. We formulate the optimum beam alignment solution as the solution of a Bayesian team decision problem. We then propose a suite of algorithms to approach optimality with reduced complexity. The effectiveness of the robust beam alignment procedure, compared with classical designs, is then verified on simulation settings with varying location information accuracies.**

## I. Introduction

Millimeter wave communications are receiving significant attention in 5G-related research, in the hope of unlocking the capacity bottleneck existing at sub-6 GHz bands [1]. The use of higher frequencies poses new implementation challenges, as for example in terms of hardware constraints or architectural features. Moreover, the propagation environment is adverse for smaller wavelength signals: compared with lower bands characteristics, diffraction tends to be lower while penetration or blockage losses can be much greater [2]. Therefore, mmWave signals experience a severe path loss which hinders the establishment of a reliable communication link and requires the adoption of high-gain directional antennas.

On the upside, millimeter wavelengths allow to stack a high number of antenna elements in a modest space [3], thus making it possible to exploit the superior beamforming performance stemming from mMIMO arrays [4].

Rather than adopting complex digital beamforming – which might require unfeasible CSI-training in mMIMO settings [4] – low cost mmWave architectures are suggested [5] where beam design is selected from discrete beam sets and then implemented in analog fashion. Another trend lies in the so-called hybrid architectures which reduce the effective dimension of the antenna space by concatenating a low-dimensional digital precoder with an RF analog beamformer [6], [7]. In mMIMO settings, all of these solutions require significant pilot and time resources to search for the beam combinations at transmitter and receiver which offer the best channel path, a problem referred to as *beam alignment* in the literature [8].

One approach for reducing alignment overhead – without compromising performance – has been proposed in [9]. It consists in exploiting device location side information so as to reduce the effective beam search space in the presence of line of sight (LoS) propagation. Similar approaches are found in [10], [11], where localization information has been confirmed as a useful source of side information, capable of assisting link establishment in mmWave communications.

In this paper, we consider important limitation factors for location-aided beam alignment. First, user terminal and infrastructure side equipment are unlikely to acquire location information with the same degree of accuracy. Moreover, even in the presence of LoS, practical propagation scenarios might include significant reflected multipaths that could be exploited.

We propose a framework for utilizing location side-information in a dual mMIMO setup while accounting for unequal degrees of uncertainties on this information at the BS and at the UE sides. Our contributions are multi-fold:

- Based on a probabilistic location information setting, we formulate a robust (Bayesian style) beam pre-selection problem. Because there are two devices (the BS and the UE) involved in making a beam pre-selection decision, we recast the problem as a decentralized team decision framework. The strength of the proposed approach lies in the fact that each device makes a beam pre-selection that is weighed upon the quality of location information it has at its disposal *and* simultaneously on the quality level of location information expected at the other end.
- We propose a family of algorithms, exploring various complexity-performance trade-off levels. We show how the devices decide to keep or drop path directions as a function of angle uncertainties (both locally and at the other link end) and average path energy.

## II. System Model

### A. Scenario

Consider the scenario in Fig. 1. A transmitter (TX) with $N_{TX} \gg 1$ antennas seeks to establish communication with a single receiver (RX) with $N_{RX} \gg 1$ antennas[1]. In order to extract the best possible combined TX-RX beamforming gain,

[1]In the rest of this paper and for notation clarification only, we will assume a downlink transmission, although all concepts and algorithms are readily applicable to the uplink as well.

the TX and the RX respectively aim to select a precoding vector $\mathbf{g} = [g_1, \ldots, g_{N_{TX}}]^{\mathrm{T}}$, and a receive-side combining vector $\mathbf{w} = [w_1, \ldots, w_{N_{RX}}]^{\mathrm{T}}$ from predefined codebooks. The codebooks include $M_{TX}$ and $M_{RX}$ beamforming vectors – i.e. beams – for the TX and the RX, respectively.

Optimal beam alignment consists in pilot-training every combination of TX and RX beams (out of $M_{TX}M_{RX}$) and selecting the pair which exhibits the highest signal to noise ratio. In the mMIMO regime, this requires prohibitive pilot, power and time resources. As a result, a method for pruning out unlikely beam combinations is desirable. To this end, we assume that the TX (resp. the RX) pre-selects a subset of $D_{TX} \ll M_{TX}$ (resp. $D_{RX} \ll M_{RX}$) beams for subsequent pilot training. When the pre-selection phase is over, the TX trains the pre-selected beams by sending pilots of each one of the $D_{TX}$ beams, while the RX is allowed to make SNR measurements over each of its $D_{RX}$ beams. Classically, communications can then take place over one (or more) of the pre-selected TX-RX beam combinations. In this paper, we are interested in deriving beam subset pre-selection strategies that do not require any active channel sounding but can be carried out on the basis of long term statistical information including location-dependent information for the TX and the RX as introduced in [9]. In contrast with [9], we consider potential reflector location information and, in particular, we place the emphasis on robustness with respect to location uncertainties in a high-mobility scenario. Models for channels, long term location-dependent information, and corresponding uncertainties are introduced in the following sections.

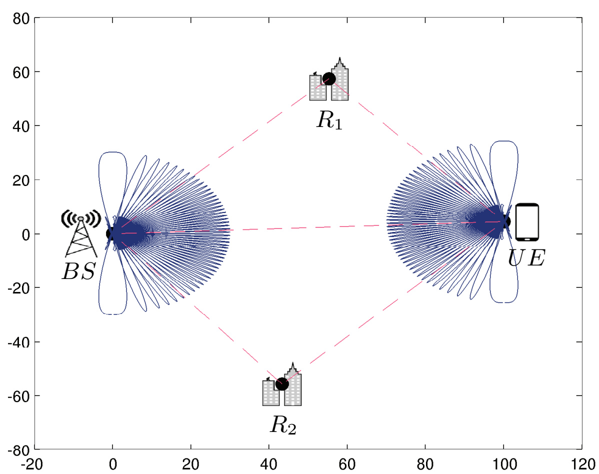


Fig. 1: Scenario example for a given realization with $L = 3$ channel paths.

### B. Channel Model

Based on recently reported data regarding the specular behavior of mmWave propagation channels [2], we model the space-time channel with a limited number $L$ of dominant propagation paths, consisting of one LoS path and $L - 1$ reflected paths.

The power-normalized $N_{RX} \times N_{TX}$ channel matrix $\mathbf{H}$ can thus be expressed as the sum of $L$ components or contributions [7]:

$$\mathbf{H} = \sqrt{N_{TX}N_{RX}}\Big(\sum_{\ell=1}^{L} \alpha_\ell \mathbf{a}_{RX}(\theta_\ell)\mathbf{a}_{TX}^{\mathrm{H}}(\phi_\ell)\Big) \tag{1}$$

where $\alpha_\ell \sim \mathcal{CN}(0, \sigma_\ell^2)$ denotes the instantaneous random complex gain for the $\ell$-th path, having an average power $\sigma_\ell^2, \ell = 1, \ldots, L$ such as $\sum \sigma_\ell^2 = 1$.

The variables $\phi_\ell \in [0, \pi)$ and $\theta_\ell \in [0, \pi)$ are the angles of departure (AoDs) and arrival (AoAs) for each path $\ell$, where one angle pair corresponds to the LoS direction while other might account for the presence of strong reflectors (buildings, hills) in the environment. The reflectors are denoted by $R_i, i = 1, \ldots, L-1$ in the rest of the paper.

The vectors $\mathbf{a}_{TX}(\phi_\ell) \in \mathbb{C}^{N_{TX}\times 1}$ and $\mathbf{a}_{RX}(\theta_\ell) \in \mathbb{C}^{N_{RX}\times 1}$ denote the antenna response at the TX and the RX, respectively. We will consider the popular example of critically-spaced uniform linear arrays (ULAs), so that $\forall \ell = 1, \ldots, L$:

$$\mathbf{a}_{TX}(\phi_\ell) = \frac{1}{\sqrt{N_{TX}}}\big[1, e^{-i\pi\cos\phi_\ell}, \ldots, e^{-i\pi(N_{TX}-1)\cos\phi_\ell}\big]^{\mathrm{T}} \tag{2}$$

$$\mathbf{a}_{RX}(\theta_\ell) = \frac{1}{\sqrt{N_{RX}}}\big[1, e^{-i\pi\cos\theta_\ell}, \ldots, e^{-i\pi(N_{RX}-1)\cos\theta_\ell}\big]^{\mathrm{T}} \tag{3}$$

### C. Beam Codebook

We define the transmit and receive beam codebooks as:

$$\mathcal{V}_{TX} = \{\mathbf{g}_1, \ldots, \mathbf{g}_{M_{TX}}\}, \quad \mathcal{V}_{RX} = \{\mathbf{w}_1, \ldots, \mathbf{w}_{M_{RX}}\}. \tag{4}$$

For ULAs, a suitable design for the fixed beam vectors in the codebook consists in selecting steering vectors over a discrete grid of angles [5], [8], [10]:

$$\mathbf{g}_p = \mathbf{a}_{TX}(\bar{\phi}_p), \quad p \in \{1, \ldots, M_{TX}\} \tag{5}$$

$$\mathbf{w}_q = \mathbf{a}_{RX}(\bar{\theta}_q), \quad q \in \{1, \ldots, M_{RX}\} \tag{6}$$

where the angles $\bar{\phi}_p$ and $\bar{\theta}_q$ can be chosen according to different strategies, including regular and non regular sampling of the $[0, \pi]$ range (see details in Section V-A).

## III. Information Model

We are interested in the exploitation of long-term statistical (including location-dependent) information, to perform beam pre-selection (i.e choosing $D_{TX}$ and $D_{RX}$). In what follows we introduce the information model emphasizing the decentralized nature of information available at TX and RX sides.

### A. Definition of the Model

In order to establish a reference case, we consider the setting where the available information lets us exactly characterize the average rate (i.e. knowing the SNR) that would be obtained under any choice of TX and RX beams. To this end, we define the average beam gain matrix.

**Definition 1.** *The average beam gain matrix* $\mathbf{G} \in \mathbb{R}^{M_{RX}\times M_{TX}}$ *contains the power level associated with each* *combined choice of transmit-receive beam pair after averaging over small scale fading. It is defined as:*

$$G_{q,p} = \mathbb{E}_{\boldsymbol{\alpha}}\left[|\mathbf{w}_q^{\mathrm{H}}\mathbf{H}\mathbf{g}_p|^2\right] \tag{7}$$

*where the expectation is carried out over the channel coefficients $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_L]$ and with $G_{q,p}$ denoting the $(q,p)$-element of $\mathbf{G}$.*

**Definition 2.** *The position matrix $\mathbf{P} \in \mathbb{R}^{2\times(L+1)}$ contains the two-dimensional location coordinates $\mathbf{p}_u = [p_{u_x} \quad p_{u_y}]^{\mathrm{T}}$ for node $u$, where $u$ indifferently refers to either the TX (or BS), the RX (or UE) or one of the reflectors $R_i, i = 1, \ldots, L-1$. It is defined as:*

$$\mathbf{P} = \begin{bmatrix}\mathbf{p}_{TX} & \mathbf{p}_{R_1} & \ldots & \mathbf{p}_{R_{L-1}} & \mathbf{p}_{RX}\end{bmatrix} \tag{8}$$

The following lemma characterizes the gain matrix $\mathbf{G}$ as a function of the position matrix $\mathbf{P}$ in the configuration considered above.

**Lemma 1.** *We can write the average beam gain matrix as follows:*

$$G_{q,p}(\mathbf{P}) = \sum_{\ell=1}^{L} \sigma_\ell^2 |L_{RX}(\Delta_{\ell,q})|^2 |L_{TX}(\Delta_{\ell,p})|^2 \tag{9}$$

*where we remind the reader that $\sigma_\ell^2$ denotes the variance of the channel coefficients $\alpha_\ell$ and we have defined:*

$$L_{TX}(\Delta_{\ell,p}) = \frac{1}{\sqrt{N_{TX}}} \frac{\sin((\pi/2)N_{TX}\Delta_{\ell,p})}{\sin((\pi/2)\Delta_{\ell,p})} \tag{10}$$

$$L_{RX}(\Delta_{\ell,q}) = \frac{1}{\sqrt{N_{RX}}} \frac{\sin((\pi/2)N_{RX}\Delta_{\ell,q})}{\sin((\pi/2)\Delta_{\ell,q})} \tag{11}$$

*and*

$$\Delta_{\ell,p} = (\cos(\bar{\phi}_p) - \cos(\phi_\ell)) \tag{12}$$

$$\Delta_{\ell,q} = (\cos(\theta_\ell) - \cos(\bar{\theta}_q)) \tag{13}$$

*with the angles $\phi_\ell$ and $\theta_\ell, \ell = 1, \ldots, L$ obtained from the position matrix $\mathbf{P}$ using simple algebra (the detailed steps are relegated to the extended version [12] due to limited space).*

### B. Distributed Noisy Information Model

In a realistic setting where both BS and UE separately acquire location information via a process of GNSS-based estimation, angle of arrival estimation (for reflector position estimation) and latency-prone BS-UE feedback, a distributed noisy position information model ensues whereby positioning accuracy is *device* dependent, i.e. different at BS and UE.

*Noisy information model at the TX:* The noisy position matrix $\hat{\mathbf{P}}^{(TX)}$ available at the TX is modeled as:

$$\hat{\mathbf{P}}^{(TX)} = \mathbf{P} + \mathbf{E}^{(TX)} \tag{14}$$

where $\mathbf{E}^{(TX)}$ denotes the following matrix:

$$\mathbf{E}^{(TX)} = \begin{bmatrix}\mathbf{e}_{TX}^{(TX)} & \mathbf{e}_{R_1}^{(TX)} & \ldots & \mathbf{e}_{R_{L-1}}^{(TX)} & \mathbf{e}_{RX}^{(TX)}\end{bmatrix} \tag{15}$$

containing the random position estimation error made by TX on $\mathbf{p}_u$, with an arbitrary, yet known, probability density function $f_{\mathbf{e}_u^{(TX)}}$.

*Noisy information model at RX:* Akin to the TX side, the receiver obtains the estimate $\hat{\mathbf{P}}^{(RX)}$, where:

$$\hat{\mathbf{P}}^{(RX)} = \mathbf{P} + \mathbf{E}^{(RX)} \tag{16}$$

where $\mathbf{E}^{(RX)}$ is defined as $\mathbf{E}^{(TX)}$ in (15), but containing the random position estimation error made by RX on $\mathbf{p}_u$, with a known distribution $f_{\mathbf{e}_u^{(RX)}}$.

Note that we assume $\mathbf{e}_{TX}^{(TX)} = \mathbf{e}_{TX}^{(RX)} = 0$, which indicates that the position information of the static BS is known perfectly by all.

### C. Shared Information

In what follows the number of dominant path $L$, and their average path powers $\sigma_l^2, l = 1, \ldots, L$ are assumed to be known by both BS and UE based on prior averaged measurements. Similarly, statistical distributions $f_{\mathbf{e}_U^{(TX)}}, f_{\mathbf{e}_U^{(RX)}}$ are supposed to be quasi-static and as such are supposed to be available (or estimated) to both BS and UE. In other words, the BS (resp. the UE) is aware of the quality for position estimates which it and the UE (resp. BS) have at their disposal. For instance, typically, the BS might know less about the UE location than the UE itself, e.g. due to latency in communicating UE position to the BS in a highly mobile scenario or due to the use of different position technologies (GPS at the UE, LTE TDOA localization at the BS). In contrast, the BS might have greater capabilities to estimate the position of the reflectors accurately compared to the UE, due to a larger number of antennas at the BS or due to interactions with multiple UEs. Both the BS and UE are aware of this situation and might wish to exploit it for greater coordination performance. The central question of this paper is "how?".

## IV. Coordinated Beam Alignment Methods

In this section, we present strategies for coordinated beam alignment which aim at restoring robustness in the beam pre-selection phase in the face of an arbitrary amount of uncertainty (noise) as shown in equations (14) and (16).

Let $\mathcal{D}_{TX}$ (resp. $\mathcal{D}_{RX}$) be the set of $D_{TX} = |\mathcal{D}_{TX}|$ (resp. $D_{RX} = |\mathcal{D}_{RX}|$) pre-selected beams at the TX (resp. the RX).

To evaluate the chosen beams, we will use the following figure of merit $\mathbb{E}[R(\mathcal{D}_{TX}, \mathcal{D}_{RX}, \mathbf{P})]$, where:

$$R(\mathcal{D}_{TX}, \mathcal{D}_{RX}, \mathbf{P}) = \max_{p\in\mathcal{D}_{TX}, q\in\mathcal{D}_{RX}} \log_2\left(1 + \frac{G_{q,p}(\mathbf{P})}{N_0}\right) \tag{17}$$

where $N_0$ is the thermal noise power[2] and the average gain is obtained from the position matrix $\mathbf{P}$ as shown in Lemma 1.

### A. Beam Alignment under Perfect Information

Before introducing the distributed approaches to this problem, we focus on the idealized benchmark where both the TX and the RX obtain the perfect position matrix $\mathbf{P}$. The beam sets $(\mathcal{D}_{TX}^{\text{up}}, \mathcal{D}_{RX}^{\text{up}})$ that maximize the transmission rate are then found as follows:

$$(\mathcal{D}_{TX}^{\text{up}}, \mathcal{D}_{RX}^{\text{up}}) = \operatorname*{argmax}_{\mathcal{D}_{TX}\subset\mathcal{V}_{TX}, \mathcal{D}_{RX}\subset\mathcal{V}_{RX}} R\big(\mathcal{D}_{TX}, \mathcal{D}_{RX}, \mathbf{P}\big). \tag{18}$$

[2] Assume for simplification an interference-free network. Refer to the extended version [12] for more details concerning the multi-user case.

## B. Optimal Bayesian Beam Alignment

Let us now consider the core of this work whereby the TX and the RX must make beam pre-selection decisions in a decentralized manner, based on their respective location information in (14) and (16). Interestingly, this problem can be recast as a so-called team decision theoretic problem [13] where team members (here TX and RX) seek to coordinate their actions so as to maximize overall system performance, while not being able to accurately predict each other decision due to noisy observations. For instance in Fig. 1, with $D_{TX} = D_{RX} = 2$, the TX might decide to beam in the direction of the RX and Reflector 1, while the RX might decide to beam in the direction of the TX but also Reflector 2 (for example, if its information on the position of Reflector 1 is not accurate enough). As a result, a strong mismatch would be obtained for one of the pre-selected beam pairs. The goal of the robust decentralized algorithm is hence to avoid such inefficient behavior.

Beam pre-selection at the TX is equivalent to a mapping:

$$\begin{aligned} d_{TX}: \quad & \mathbb{R}^{2\times(L+1)} \to \mathcal{V}_{TX} \\ & \hat{\mathbf{P}}^{(TX)} \mapsto d_{TX}(\hat{\mathbf{P}}^{(TX)}) \end{aligned} \tag{19}$$

while at the RX, we have:

$$\begin{aligned} d_{RX}: \quad & \mathbb{R}^{2\times(L+1)} \to \mathcal{V}_{RX} \\ & \hat{\mathbf{P}}^{(RX)} \mapsto d_{RX}(\hat{\mathbf{P}}^{(RX)}) \end{aligned} \tag{20}$$

Let $\mathcal{S}$ denote the space containing all possible choices of pairs of such functions. The optimally-robust team decision strategy $(d^*_{TX}, d^*_{RX}) \in \mathcal{S}$ maximizing the expected rate is as follows:

$$\begin{aligned} &(d^*_{TX}, d^*_{RX}) = \\ &\quad \underset{(d_{TX},d_{RX})\in\mathcal{S}}{\operatorname{argmax}} \ \mathbb{E}\Big[R\big(d_{TX}(\hat{\mathbf{P}}^{(TX)}), d_{RX}(\hat{\mathbf{P}}^{(RX)}), \mathbf{P}\big)\Big] \end{aligned} \tag{21}$$

where the expectation operator is carried out over the joint probability density function $f_{\mathbf{P},\hat{\mathbf{P}}^{(TX)},\hat{\mathbf{P}}^{(RX)}}$.

The optimization in (21) is a stochastic functional optimization problem which is notoriously difficult to solve [14]. In order to circumvent this problem, we now examine strategies offering trade-offs between the optimal robustness of (21) and the implementation complexity.

## C. Naive Beam Alignment

A simple, yet naive, implementation of decentralized coordination mechanisms consists in having each side making its decision by treating (mistaking) local information as perfect and global. Thus, TX and RX solve for (18), where the TX naively assumes $\hat{\mathbf{P}}^{TX} = \mathbf{P}$ and the RX considers $\hat{\mathbf{P}}^{RX} = \mathbf{P}$. We denote the resulting mappings as $(d^{\text{naive}}_{TX}, d^{\text{naive}}_{RX}) \in \mathcal{S}$, found as follows:

$$\begin{aligned} &d^{\text{naive}}_{TX}(\hat{\mathbf{P}}^{(TX)}) = \\ &\quad \underset{\mathcal{D}_{TX}\subset\mathcal{V}_{TX}}{\operatorname{argmax}} \ \max_{\mathcal{D}_{RX}\subset\mathcal{V}_{RX}} R\big(\mathcal{D}_{TX}, \mathcal{D}_{RX}, \hat{\mathbf{P}}^{(TX)}\big) \end{aligned} \tag{22}$$

$$\begin{aligned} &d^{\text{naive}}_{RX}(\hat{\mathbf{P}}^{(RX)}) = \\ &\quad \underset{\mathcal{D}_{RX}\subset\mathcal{V}_{RX}}{\operatorname{argmax}} \ \max_{\mathcal{D}_{TX}\subset\mathcal{V}_{TX}} R\big(\mathcal{D}_{TX}, \mathcal{D}_{RX}, \hat{\mathbf{P}}^{(RX)}\big) \end{aligned} \tag{23}$$

which can be solved by exhaustive set search or a lower complexity greedy approach (see details later). The basic limitation of the naive approach in (22) and (23) is that it fails to account for either (i) the noise in the gain matrix estimate at the decision maker, or (ii) the differences in location information quality between the TX and the RX.

## D. 1-Step Robust Beam Alignment

Making one step towards robustness requires from the TX and the RX to account for their own local information noise statistics. As a first approximation, each device might then assume that its local estimate, while not perfect, is at least globally shared, i.e. that $\hat{\mathbf{P}}^{(TX)} = \hat{\mathbf{P}}^{(RX)}$ for the purpose of algorithm derivation. We denote the resulting beam preselection as 1-step robust[3] – obtained through the following mappings $(d^{\text{1-s}}_{TX}, d^{\text{1-s}}_{RX}) \in \mathcal{S}$:

$$\begin{aligned} &d^{\text{1-s}}_{TX}(\hat{\mathbf{P}}^{(TX)}) = \\ &\quad \underset{\mathcal{D}_{TX}\subset\mathcal{V}_{TX}}{\operatorname{argmax}} \ \max_{\mathcal{D}_{RX}\subset\mathcal{V}_{RX}} \mathbb{E}_{\mathbf{P}|\hat{\mathbf{P}}^{(TX)}}\Big[R\big(\mathcal{D}_{TX}, \mathcal{D}_{RX}, \mathbf{P}\big)\Big] \end{aligned} \tag{24}$$

$$\begin{aligned} &d^{\text{1-s}}_{RX}(\hat{\mathbf{P}}^{(RX)}) = \\ &\quad \underset{\mathcal{D}_{RX}\subset\mathcal{V}_{RX}}{\operatorname{argmax}} \ \max_{\mathcal{D}_{TX}\subset\mathcal{V}_{TX}} \mathbb{E}_{\mathbf{P}|\hat{\mathbf{P}}^{(RX)}}\Big[R\big(\mathcal{D}_{TX}, \mathcal{D}_{RX}, \mathbf{P}\big)\Big] \end{aligned} \tag{25}$$

Optimization (21) is therefore replaced with a more standard stochastic optimization problem for which a vast literature is available (see [15] for a nice overview). Considering w.l.o.g. the optimization at the TX, one standard approach consists in approximating the expectation by Monte-Carlo runs according to the probability density function $f_{\mathbf{P}|\hat{\mathbf{P}}^{(TX)}}$. Once the expectation operator has been replaced by a discrete summation, the optimal solution of the discrete optimization problem can be simply again obtained by greedy search. Indeed, the nature of the problem is such that it is possible to split (24) and (25) in multiple maximizations – over the single beams in $\mathcal{V}_{TX}$ and $\mathcal{V}_{RX}$ – without loosing optimality. The greedy approach has far less complexity than the exhaustive search, which requires to search over beam sets whose size is the number of combinations resulting from picking $D_{TX}$ (resp. $D_{RX}$) beams at a time among $M_{TX}$ (resp. $M_{RX}$). The detailed algorithm is given in the extended version [12].

Note that the approach above provides robustness with respect to the local noise at the decision maker; it however fails to account for discrepancies in location information quality across TX and RX. Indeed, the true distribution of the position knowledge has been approximated by considering that both the TX and the RX share the same information.

## E. 2-Step Robust Beam Alignment

A necessary optimality condition for the optimal Bayesian beam alignment in (21) is that it is *person-by-person* optimal, i.e. each node takes the best strategy given the strategy at the other node [14].

[3] In retrospect, the naive algorithm in the previous section could be interpreted as a 0-step robust approach.

The person-by-person optimal solution $(d_{TX}^{\text{PP}}, d_{RX}^{\text{PP}}) \in \mathcal{S}$ satisfies the following system of fixed point equations:

$$d_{TX}^{\text{PP}}(\hat{\mathbf{P}}^{(TX)}) = \underset{\mathcal{D}_{TX}\subset\mathcal{V}_{TX}}{\operatorname{argmax}} \ \mathbb{E}_{\mathbf{P},\hat{\mathbf{P}}^{(RX)}|\hat{\mathbf{P}}^{(TX)}}\Big[R\big(\mathcal{D}_{TX}, d_{RX}^{\text{PP}}, \mathbf{P}\big)\Big] \quad (26)$$

$$d_{RX}^{\text{PP}}(\hat{\mathbf{P}}^{(RX)}) = \underset{\mathcal{D}_{RX}\subset\mathcal{V}_{RX}}{\operatorname{argmax}} \ \mathbb{E}_{\mathbf{P},\hat{\mathbf{P}}^{(TX)}|\hat{\mathbf{P}}^{(RX)}}\Big[R\big(d_{TX}^{\text{PP}}, \mathcal{D}_{RX}, \mathbf{P}\big)\Big] \quad (27)$$

where we used the short-hand notation $d_{TX}^{\text{PP}}$ and $d_{RX}^{\text{PP}}$ for $d_{TX}^{\text{PP}}(\hat{\mathbf{P}}^{(TX)})$ and $d_{RX}^{\text{PP}}(\hat{\mathbf{P}}^{(RX)})$, respectively. Still, the interdependence between (26) and (27) makes solving this system of equations challenging. Thus, we propose an approximate solution in which this dependence is removed by replacing the person-by-person mapping inside the expectation operator with the 1-step robust mapping described in Section IV-D.

Intuitively, the TX (resp. the RX) finds its strategy by using the belief that the RX (resp. the TX) is using the 1-step robust strategy (which can be separately computed thanks to (24), (25)) and seeking to be (2-step) robust with respect to remaining uncertainties. In the 2-step algorithm, both local noise statistics and differences between information quality at TX and RX are thus exploited. Let us denote by $(d_{TX}^{\text{2-s}}, d_{RX}^{\text{2-s}}) \in \mathcal{S}$ the 2-step robust approach, which reads as:

$$d_{TX}^{\text{2-s}}(\hat{\mathbf{P}}^{(TX)}) = \underset{\mathcal{D}_{TX}\subset\mathcal{V}_{TX}}{\operatorname{argmax}} \ \mathbb{E}_{\mathbf{P},\hat{\mathbf{P}}^{(RX)}|\hat{\mathbf{P}}^{(TX)}}\Big[R\big(\mathcal{D}_{TX}, d_{RX}^{\text{1-s}}, \mathbf{P}\big)\Big] \quad (28)$$

$$d_{RX}^{\text{2-s}}(\hat{\mathbf{P}}^{(RX)}) = \underset{\mathcal{D}_{RX}\subset\mathcal{V}_{RX}}{\operatorname{argmax}} \ \mathbb{E}_{\mathbf{P},\hat{\mathbf{P}}^{(TX)}|\hat{\mathbf{P}}^{(RX)}}\Big[R\big(d_{TX}^{\text{1-s}}, \mathcal{D}_{RX}, \mathbf{P}\big)\Big] \quad (29)$$

where we used the short-hand notation $d_{TX}^{\text{1-s}}$ and $d_{RX}^{\text{1-s}}$ as before. This approach could then be extended by inserting the 2-step robust mapping inside the expectation, so as to get the 3-step robust approach, and so forth. Of course, it comes with an increased computational cost.

# V. Simulation Results

In this section, numerical results are presented so as to compare the performance of the proposed beam alignment algorithms. We consider the scenario in Fig. 1, with $L = 3$ multipath components. A distance of 100 m is assumed from the TX to the RX. Both TX and RX are equipped with $N_{TX} = N_{RX} = 64$ antennas (ULA). The devices have to choose $D_{TX}$, $D_{RX}$ beamforming vectors among the $M_{TX} = M_{RX} = 64$ in the codebooks. The results are averaged over 10000 independent Monte-Carlo iterations.

## A. Beam Codebook Design

Since ULAs produce unequal beamwidths according to the pointing direction – as it can be seen in Fig. 1 – we separate the grid angles $\bar{\phi}_p$ and $\bar{\theta}_q$ in equations (5) and (6) according to the inverse cosine function, as follows [9]:

$$\bar{\phi}_p = \arccos\Big(1 - \frac{2(p-1)}{M_{TX}-1}\Big), \quad p \in \{1,\dots,M_{TX}\} \quad (30)$$

$$\bar{\theta}_q = \arccos\Big(1 - \frac{2(q-1)}{M_{RX}-1}\Big), \quad q \in \{1,\dots,M_{RX}\} \quad (31)$$

As a result, and in order to guarantee equal gain losses among the adjacent angles, more of the latter are considered as the broadside direction is reached.

## B. Location Information Model

In the simulations, we use a uniform bounded error model for location information [9]. In particular, we assume that all the estimates lie somewhere inside disks centered in the actual positions $\mathbf{p}_u, u \in \{TX, RX, R_i\}, i = 1,\dots,L-1$. Let $S(r)$ be the two-dimensional closed ball centered at the origin and of radius $r$, i.e. $S(r) = \{\mathbf{p} \in \mathbb{R}^2 : \|\mathbf{p}\| \le r\}$. We model the random estimation errors as follows:

- $\mathbf{e}_u^{(TX)}$ uniformly distributed in $S(r_u^{(TX)})$
- $\mathbf{e}_u^{(RX)}$ uniformly distributed in $S(r_u^{(RX)})$

such that $r_u^{(TX)}$ and $r_u^{(RX)}$ are the maximum positioning error for node $u$ as seen from the TX and the RX, respectively.

## C. Results and Discussion

According to measurement campaigns [2], LoS propagation is the prominent propagation driver in mmWave bands. We consider as a consequence a stronger (on average) LoS path, with respect to the reflected paths. The latter are assumed to have the same average power. Moreover, we consider the following degrees of precision for localization information:

- $r_{RX}^{(TX)} = 13$ m, $r_{RX}^{(RX)} = 7$ m
- $r_{R_1}^{(TX)} = 11$ m, $r_{R_1}^{(RX)} = 18$ m
- $r_{R_2}^{(TX)} = 15$ m, $r_{R_2}^{(RX)} = 17$ m
- $r_{TX}^{(TX)} = 0$ m, $r_{TX}^{(RX)} = 0$ m

Given that 5G devices are expected to access position information with a guaranteed precision of about 1 m in open areas [16], those settings are robust with respect to the mobility of the devices or to possible discontinuous location awareness.

Fig. 2 compares the proposed algorithms in the settings described above. It can be seen that the 2-step robust beam alignment outperforms the other distributed solutions, being able to consider statistical information at both ends. Some additional sets of degrees of precision are considered in [12].

In Fig. 3, we consider the performance of the proposed algorithms as a function of the number of pre-selected beams – assuming a fixed SNR of 10 dB, and the same parameters as considered for Fig. 2. As expected, a higher number of pre-selectable beams leads to increased performance. Simulations show that the 2-step robust algorithm almost reaches the idealized upper bound with already $D_{TX} = D_{RX} = 5$. In addition, Fig. 3 confirms that exploiting position information allows to reduce alignment overhead while impacting only slightly on the performance if the sets of pre-selectable beams are sufficiently large with respect to the degrees of precision.

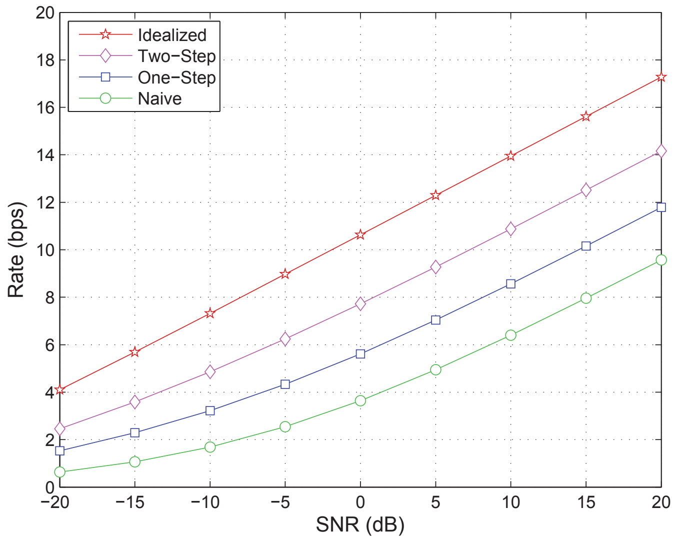


Fig. 2: Rate vs SNR, stronger LoS path, $D_{TX} = D_{RX} = 4$.

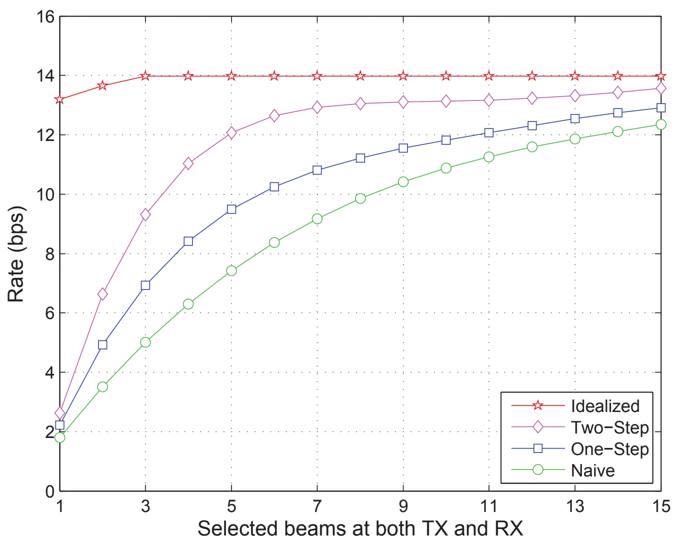


Fig. 3: Rate vs number of pre-selected beams at TX and RX (among $M_{TX} = M_{RX} = 64$), for a given SNR = 10 dB.

## VI. Conclusions

Localization information has an important role in reducing alignment overhead in mmWave communications. Dealing with the imperfect position knowledge is challenging due to the fact that the information is not shared between the TX and the RX, leading to disagreements affecting the performance. In this work, we introduced an algorithm which takes into account the imperfect information at both ends and improves the coordination between the TX and the RX by exploiting their shared statistical knowledge of localization errors.

We proposed a so-called 2-step robust approach which enforce coordination by letting one node assume a given strategy for the other one, thus strongly reducing complexity. Numerical experiments have shown that good performance can be achieved with the 2-step robust algorithm, which almost reaches the idealized upper bound – obtained with perfect information – even with small values of pre-selectable beams.

Future directions include the extension of the proposed algorithms, in order to exceed the 2-step algorithm, with the purpose of reaching the person-by-person optimum. Finding closed forms of the proposed algorithms is an interesting and challenging research problem which is still open as well.

## VII. Acknowledgment

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