

Pilot decontamination using combined angular and amplitude based projections in massive MIMO systems

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Abstract—We address the problem of noise and interference corrupted channel estimation in massive MIMO systems. Interference, which originates from pilot reuse (or contamination), can in principle be discriminated from the desired channels upon observing the distributions of path angles and amplitudes. In this paper we propose novel robust channel estimation algorithms exploiting path diversity in both angle and amplitude domains, relying on a suitable combination of a subspace projection and MMSE estimation. The proposed estimator improves on past methods in a wide range of system and topology scenarios.

I. INTRODUCTION

Massive MIMO networks, introduced in [1], are widely believed to be one of the key enablers of the future 5th generation (5G) wireless systems thanks to their potential to substantially enhance spectral and energy efficiencies in ideal setups [1], [2]. However, already in [1], [2], the residual error in channel estimation due to the unavoidable reuse of identical training sequences by user terminals in different cells was identified as a limiting factor to cancel interference in massive MIMO networks. This effect, called *pilot contamination* [3], [4], has a significant detrimental impact on the actual achievable spectral and energy efficiencies in real systems. The challenge of significantly reducing the gap between ideal and practical systems triggered considerable research activities in this area.

Techniques to avoid and/or mitigate channel contamination span from design of pilot reuse schemes (e.g. [5], [6]) to channel estimation techniques based on coordinated training sequence allocation (e.g. [7], [8]), to multi-cell joint processing (e.g. [9]), to nonlinear channel estimation techniques leveraging on some fundamental features of massive MIMO systems (e.g. [7], [10], [11]).

Without seeking to provide a comprehensive overview of these manifold research directions, we capitalize on the key properties of massive MIMO systems to substantially mitigate pilot contamination by channel estimation techniques, while keeping in mind that further improvements could be attained via overlaying coordinated pilot sequence allocation.

The key features of massive MIMO channels are the facts that channels of different users tend to be pairwise orthogonal

when the number of antennas increases and the low-rankness of the channel covariance matrices pointed out in [7], [12]. The blind signal subspace estimation in [11] capitalizes on the former property. The latter property, has been utilized in [7], [12]–[15] assuming the complete knowledge of the channel covariance matrices.

More specifically, in [11], [16] power control and power controlled handover guarantee that the signals of the users of interest in a cell are received with higher powers compared to the interference. This property along with the pairwise channel orthogonality allow to blindly estimate the user-of-interest channel subspace and discriminate between user-of-interest signals and interference based on the channel powers. Thus, it is possible to remove the pilot contamination effects based on a projection driven by the channel amplitudes. A limitation of this approach however is met for edge-of-cell users when it is hard to distinguish within-cell from out-of-cell users based on received amplitude alone and in the face of finite-sample constraints.

In a way completely independent from [11], [16], another approach based on a linear minimum mean square error (MMSE) estimator is adopted in [7] to estimate the channel of interest via projection of the received signals onto the user-of-interest subspace. This subspace, identified by a channel covariance matrix (a long-term one, as opposed to the instantaneous signal correlation matrix of [11], [16]), is related to the angular spread of the signal of interest [7] and enables to annihilate the interference from users with non-overlapping domains of multipath angles-of-arrival (AoA). Interestingly, this latter approach makes no assumption on received signal amplitudes and can also discriminate users that are received with similar powers. Yet, the approach fails to decontaminate pilots when propagation scattering creates large angle spread, causing spatial overlap among desired and interference channels.

In summary the former method [11], [16] annihilates the interference based on projections driven by the channel amplitudes while the latter [7] performs projections driven by the angular spread of the channel of interest. In this paper, we point out that the strengths of these two previously unrelated

estimation methods are strongly complementary, offering a unique opportunity for developing a robust channel estimation scheme.

Thus, we aim to properly merge the two projections in complementary domains keeping the individual benefits while exploiting potential synergies. This is done in this paper by proposing two novel schemes exploiting the asymptotic properties of massive MIMO arrays. Thus, we propose a first scheme that effectively combines projections in the angular and amplitude domains and outperforms considerably over known schemes. On the light of the previous observations, it is apparent the interest of exploiting the low-rankness of user channels also without knowledge of the channel covariance matrices and possibly based on observations of a single or few coherence time intervals. Then, we propose a second method suitable for terminals moving at vehicular speed. Numerical analysis shows that high performance at relatively high speed is achieved at the price of a larger number of antennas.

II. SIGNAL AND CHANNEL MODELS

We consider a network of L time-synchronized cells, with full spectrum reuse. Each base station is equipped with M antennas. There are K single-antenna users in each cell simultaneously served by their base station. The cellular network operates in time-division duplexing (TDD) mode, and due to channel reciprocity, the downlink channel is obtained by uplink training. Each base station estimates the channels of its K users during a coherence time interval. The pilot sequences inside each cell are assumed orthogonal to each other in order to avoid intra-cell interference. However the same pilot pool is reused in other cells, giving rise to pilot contamination problem. The pilot sequence assigned to the k -th user in a certain cell is denoted by

$$\mathbf{s}_k = [s_{k1} \ s_{k2} \ \cdots \ s_{k\tau}]^T, \quad (1)$$

where τ is the length of pilot. Without loss of generality we assume unitary average power of pilot symbols, i.e.,

$$\mathbf{s}_{k_1}^T \mathbf{s}_{k_2}^* = \begin{cases} 0, & k_1 \neq k_2, \\ \tau, & k_1 = k_2, \end{cases}$$

The channel vector between the k -th user located in the l -th cell and the j -th base station is denoted by $\mathbf{h}_{lk}^{(j)}$. The following multipath channel model is adopted:

$$\mathbf{h}_{lk}^{(j)} = \beta_{lk}^{(j)} \sum_{b=1}^B \mathbf{a}(\theta_{lk}^{(j)}) e^{i\varphi_{lk}^{(j)b}}, \quad (2)$$

where B is the arbitrary number of i.i.d. paths, and $e^{i\varphi_{lk}^{(j)b}}$ is the i.i.d. random phase, which is independent over channel indices l, k, j , and path index b . $\mathbf{a}(\theta)$ is the steering (or phase response) vector by the array to a path originating from the angle of arrival θ . $\beta_{lk}^{(j)}$ is the path-loss coefficient

$$\beta_{lk}^{(j)} = \sqrt{\frac{\alpha}{d_{lk}^{(j)\gamma}}}, \quad (3)$$

in which γ is the path-loss exponent, $d_{lk}^{(j)}$ is the geographical distance between the user and the j -th base station, and α is a constant.

We define

$$\mathbf{H}_l^{(j)} \triangleq [\mathbf{h}_{l1}^{(j)} | \mathbf{h}_{l2}^{(j)} | \cdots | \mathbf{h}_{lK}^{(j)}], \quad (4)$$

and the pilot matrix

$$\mathbf{S} \triangleq [\mathbf{s}_1 | \mathbf{s}_2 | \cdots | \mathbf{s}_K]^T. \quad (5)$$

We assume that power control is adopted and the transmitted symbols of user k in cell l are amplified by a factor $\sqrt{P_{lk}}$ such that the average received power of the target base station l is $MP = P_{lk} \|\mathbf{h}_{lk}^{(l)}\|^2$, where P is a constant common to all users in all cells.

During the training phase, the received signal at the base station j is

$$\mathbf{Y}^{(j)} = \sum_{l=1}^L \mathbf{H}_l^{(j)} \mathbf{A}_l \mathbf{S} + \mathbf{N}^{(j)}, \quad (6)$$

where $\mathbf{N}^{(j)} \in \mathbb{C}^{M \times \tau}$ is the spatially and temporally white additive Gaussian noise (AWGN) with zero-mean and element-wise variance σ_n^2 , and

$$\mathbf{A}_l = \text{diag}\{\sqrt{P_{l1}}, \sqrt{P_{l2}}, \dots, \sqrt{P_{lK}}\}. \quad (7)$$

Then, during the uplink data transmission phase, each user transmits C data symbols.

$$\mathbf{W}^{(j)} = \sum_{l=1}^L \mathbf{H}_l^{(j)} \mathbf{A}_l \mathbf{X}_l + \mathbf{Z}^{(j)}, \quad (8)$$

where $\mathbf{X}_l \in \mathbb{C}^{K \times C}$ is the matrix of transmitted symbols of all users in the l -th cell. The symbols are i.i.d. with zero-mean and unit average element-wise variance. $\mathbf{Z}^{(j)} \in \mathbb{C}^{M \times C}$ is the AWGN noise with zero-mean and element-wise variance σ_n^2 . Note that the block fading channel is constant during the transmission for the τ pilot symbols and the C data symbols.

For ease of exposition, we define an equivalent channel vector $\tilde{\mathbf{h}}_{lk}^{(j)} \triangleq \sqrt{P_{lk}} \mathbf{h}_{lk}^{(j)}$, and the matrix $\tilde{\mathbf{H}}_l^{(j)} \triangleq \mathbf{H}_l^{(j)} \mathbf{A}_l$. In the following we will study different estimation methods of such equivalent channels of interest.

III. MMSE CHANNEL ESTIMATION

We briefly recall the MMSE channel estimator in a multi-cell multi-user setting. We rewrite (6) in a vectorized form,

$$\mathbf{y}^{(j)} = \bar{\mathbf{S}} \sum_{l=1}^L \tilde{\mathbf{h}}_l^{(j)} + \mathbf{n}^{(j)}, \quad (9)$$

where $\mathbf{y}^{(j)} = \text{vec}(\mathbf{Y}^{(j)})$, $\mathbf{n}^{(j)} = \text{vec}(\mathbf{N}^{(j)})$, and $\tilde{\mathbf{h}}_l^{(j)} = \text{vec}(\tilde{\mathbf{H}}_l^{(j)})$. The pilot matrix $\bar{\mathbf{S}}$ is given by

$$\bar{\mathbf{S}} \triangleq \mathbf{S}^T \otimes \mathbf{I}_M = [\mathbf{s}_1 \otimes \mathbf{I}_M \ \cdots \ \mathbf{s}_K \otimes \mathbf{I}_M]. \quad (10)$$

We define the covariance matrices $\tilde{\mathbf{R}}_{lk}^{(j)} \triangleq \mathbb{E}\{\tilde{\mathbf{h}}_{lk}^{(j)} \tilde{\mathbf{h}}_{lk}^{(j)H}\} \in \mathbb{C}^{M \times M}$, and $\tilde{\mathbf{R}}_l^{(j)} \triangleq \mathbb{E}\{\tilde{\mathbf{h}}_l^{(j)} \tilde{\mathbf{h}}_l^{(j)H}\} \in \mathbb{C}^{MK \times MK}$. By assuming that the channel vectors of different users are mutually

uncorrelated, which is also valid for our channel model (2), we may obtain

$$\tilde{\mathbf{R}}_l^{(j)} = \text{diag}\{\tilde{\mathbf{R}}_{l1}^{(j)}, \dots, \tilde{\mathbf{R}}_{lK}^{(j)}\}. \quad (11)$$

A linear MMSE estimator for $\mathbf{h}_j^{(j)}$ is given by

$$\begin{aligned} \hat{\mathbf{h}}_j^{(j)\text{MMSE}} &= \tilde{\mathbf{R}}_j^{(j)} \bar{\mathbf{S}}^H \left(\bar{\mathbf{S}} \left(\sum_{l=1}^L \tilde{\mathbf{R}}_l^{(j)} \right) \bar{\mathbf{S}}^H + \sigma_n^2 \mathbf{I}_{\tau M} \right)^{-1} \mathbf{y}^{(j)}, \\ &= \tilde{\mathbf{R}}_j^{(j)} \left(\tau \left(\sum_{l=1}^L \tilde{\mathbf{R}}_l^{(j)} \right) + \sigma_n^2 \mathbf{I}_{KM} \right)^{-1} \bar{\mathbf{S}}^H \mathbf{y}^{(j)} \end{aligned} \quad (12)$$

As shown in our previous works [7], [13], the above MMSE estimator can fully eliminate the effects of interfering channels when $M \rightarrow \infty$, under the condition that the multipath AoAs of interference and desired channels have disjoint angular supports.

IV. AMPLITUDE BASED PROJECTION

We now briefly review the method in [11], [16]. The eigenvalue decomposition (EVD) of $\mathbf{W}^{(j)} \mathbf{W}^{(j)H}$ is written as

$$\mathbf{W}^{(j)} \mathbf{W}^{(j)H} = \mathbf{U}^{(j)} \mathbf{\Lambda}^{(j)} \mathbf{U}^{(j)H}, \quad (13)$$

where $\mathbf{U}^{(j)} \in \mathbb{C}^{M \times M} = [\mathbf{u}_1^{(j)} | \mathbf{u}_2^{(j)} | \dots | \mathbf{u}_M^{(j)}]$ is a unitary matrix and $\mathbf{\Lambda}^{(j)} = \text{diag}\{\lambda_1^{(j)}, \dots, \lambda_M^{(j)}\}$ with its diagonal entries sorted in a non-increasing order. By extracting the first K columns of $\mathbf{U}^{(j)}$, i.e., the eigenvectors corresponding to the strongest K eigenvalues, we obtain an orthogonal basis

$$\mathbf{E}^{(j)} \triangleq [\mathbf{u}_1^{(j)} | \mathbf{u}_2^{(j)} | \dots | \mathbf{u}_K^{(j)}] \in \mathbb{C}^{M \times K}. \quad (14)$$

The basic idea in [11], [16] is to use the orthogonal basis $\mathbf{E}^{(j)}$ as an estimate for a basis of the channel subspace $\hat{\mathbf{H}}_j^{(j)}$, which includes all desired user channels in cell j . Then, by projecting the received signal onto the subspace spanned by $\mathbf{E}^{(j)}$, most of the signal of interest is preserved. In contrast, the interference signal is canceled out thanks to the asymptotic property that the user channels are pairwise orthogonal as the number of antennas tends to infinity. Thus after the amplitude based projection, the estimate of $\hat{\mathbf{H}}_j^{(j)}$ is given by:

$$\hat{\mathbf{H}}_j^{(j)\text{AM}} = \frac{1}{\tau} \mathbf{E}^{(j)} \mathbf{E}^{(j)H} \mathbf{Y}^{(j)} \mathbf{S}^H, \quad (15)$$

$$\hat{\mathbf{h}}_j^{(j)\text{AM}} = \text{vec}(\hat{\mathbf{H}}_j^{(j)\text{AM}}), \quad (16)$$

where ‘‘AM’’ means amplitude. Note that this method works well when the desired channels and interference channels are separable in power domain, i.e., the instant powers of any desired channels are higher than that of any interference channels. In practice however, this assumption is idealized. In order to improve the robustness of amplitude-based projection in the presence of strong interference, we propose to take the first $\kappa^{(j)}$ eigenvectors in $\mathbf{U}^{(j)}$ to form $\mathbf{E}^{(j)}$, where $\kappa^{(j)}$ is the number of eigenvalues in $\mathbf{\Lambda}^{(j)}$ that are greater than $\mu \lambda_K^{(j)}$. μ is a design parameter that satisfies $0 \leq \mu < 1$. In section VII we let $\mu = 0.2$.

V. ANGULAR AND AMPLITUDE BASED PROJECTION

The non-overlapping angular support condition enables the MMSE estimate (12) to converge to interference-free scenario. In practice this assumption is challenged by various scattering environments. When the AoAs have overlapping support, the interference persists and the MMSE estimate suffers from pilot contamination. We hereby propose to project the estimate onto the subspace given by $\mathbf{E}^{(j)}$ obtained in section IV. The modified estimator is given by:

$$\hat{\mathbf{h}}_j^{(j)\text{AA}} = \bar{\mathbf{E}}^{(j)} \bar{\mathbf{E}}^{(j)H} \tilde{\mathbf{R}}_j^{(j)} \left(\tau \left(\sum_{l=1}^L \tilde{\mathbf{R}}_l^{(j)} \right) + \sigma_n^2 \mathbf{I}_{KM} \right)^{-1} \bar{\mathbf{S}}^H \mathbf{y}^{(j)}, \quad (17)$$

where $\bar{\mathbf{E}}^{(j)} \triangleq \mathbf{I}_K \otimes \mathbf{E}^{(j)}$. The superscript ‘‘AA’’ denotes Angular and Amplitude domain decontamination. In the case of overlapping angular support, this approach eliminates a large amount of interference and noise, as long as there exists a non-negligible power gap between signal of interest and interference. On the other hand, when the power gap is too small (e.g., for the cell-edge users) while the angular supports are disjoint, this scheme benefits from the angular domain projection. As a result we obtain a robust channel estimate able to cope with a much wider range of topologies.

VI. AMPLITUDE AND DFT BASED PROJECTION

Note that, although the proposed method allows for robustness with respect to potential overlap between desired and interference users in the AoA and amplitude domains, it still makes use of long term covariance matrix estimates that are challenging to obtain in practice for massive MIMO systems. Additionally, it does not capitalize on the appealing feature of the amplitude based projection, i.e., the fact that the projection subspace estimation is based on the observation in a single coherence time. We now propose a low-complexity alternative of the angular and amplitude based projection method that performs the angular projections based on a small number of channel observations. Recall that for an equi-spaced linear antenna array, the steering vector has a Fourier structure. The multipath from different angles exhibit asymptotic orthogonality as the number of antennas grows [7] [12]. If we apply a discrete Fourier transform (DFT) to a channel vector which exhibits finite angular support, we will observe that the power spectrum in frequency domain is concentrated in a cluster of spatial frequencies. An example is given in Fig. 1 where two users have disjoint angular supports and their powers are concentrated in different frequencies clusters. In this case, we can remove most of the interference by filtering out the undesired frequency components.

Let us define the DFT matrix

$$\begin{aligned} \mathbf{F} &\triangleq [\mathbf{f}_0 | \mathbf{f}_1 | \dots | \mathbf{f}_{M-1}] \\ &= \begin{bmatrix} \omega^{0 \cdot 0} & \omega^{0 \cdot 1} & \dots & \omega^{0(M-1)} \\ \omega^{1 \cdot 0} & \omega^{1 \cdot 1} & \dots & \omega^{1(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^{(M-1) \cdot 0} & \omega^{(M-1) \cdot 1} & \dots & \omega^{(M-1)(M-1)} \end{bmatrix}, \end{aligned} \quad (18)$$

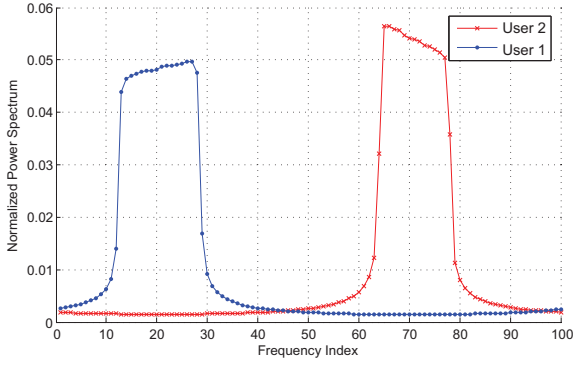


Fig. 1. Frequency spectrum of two users, with non-overlapping angular support.

where $\omega \triangleq e^{-2\pi i/M}$ and $\mathbf{f}_m \triangleq [\omega^0, \omega^{1 \cdot m}, \dots, \omega^{(M-1) \cdot m}]^T$.

Without loss of generality we focus on the channel between user k in cell j and its base station, i.e., $\tilde{\mathbf{h}}_{jk}^{(j)}$. Assume we have N previous channel realizations $\tilde{\mathbf{h}}_{jk}^{(j)}(n)$, $1 \leq n \leq N$, of the current channel¹ $\tilde{\mathbf{h}}_{jk}^{(j)}$. We define a set of indices

$$\mathcal{F}_{jk} = \left\{ m \left| \sum_{n=1}^N |\mathbf{f}_m^H \tilde{\mathbf{h}}_{jk}^{(j)}(n)|^2 > \xi \sum_{n=1}^N |\tilde{\mathbf{h}}_{jk}^{(j)}(n)|^2, \right. \right. \\ \left. \left. 0 \leq m \leq M-1 \right\}. \quad (19)$$

Each element m of \mathcal{F}_{jk} corresponds to a vector \mathbf{f}_m such that the empirical mean of the channel power in the subspace spanned by \mathbf{f}_m is greater than a certain fraction ξ of the total empirical mean of the channel power. We also denote by $\mathbf{F}_{\mathcal{F}_{jk}}$ the matrix obtained from \mathbf{F} by suppressing the columns with indices that do not belong to \mathcal{F}_{jk} .

Additionally, we introduce the projection matrix

$$\mathbf{\Pi}_{jk} = \mathbf{F}_{\mathcal{F}_{jk}} \mathbf{F}_{\mathcal{F}_{jk}}^H. \quad (20)$$

Finally, we introduce the block-diagonal matrix

$$\tilde{\mathbf{F}}_j \triangleq \text{diag}\{\mathbf{\Pi}_{j1}, \mathbf{\Pi}_{j2}, \dots, \mathbf{\Pi}_{jK}\}. \quad (21)$$

In cell j , a low-complexity channel estimator that utilizes both angular and amplitude domain projections is given as

$$\hat{\mathbf{h}}_j^{(j)\text{AD}} = \tilde{\mathbf{F}}_j \hat{\mathbf{h}}_j^{(j)\text{AM}}, \quad (22)$$

The superscript ‘‘AD’’ is the acronym for amplitude and DFT based decontamination.

VII. NUMERICAL RESULTS

This section contains numerical results of our different channel estimation schemes compared with prior methods. In the simulation, we have $L = 7$ hexagonally shaped adjacent cells in the network. The radius of the cell is 1000 meters and the path loss exponent is $\gamma = 3$. In each cell, the base station

¹With a slight abuse of notation, for the current realization we drop the temporal index.

serves simultaneously $K = 4$ users, which are randomly and uniformly distributed within the cell. Define a normalized channel estimation error

$$\epsilon \triangleq 10 \log_{10} \left(\frac{1}{KL} \sum_{j=1}^L \sum_{k=1}^K \frac{\|\hat{\mathbf{h}}_{jk}^{(j)} - \tilde{\mathbf{h}}_{jk}^{(j)}\|^2}{\|\tilde{\mathbf{h}}_{jk}^{(j)}\|^2} \right), \quad (23)$$

where $\hat{\mathbf{h}}_{jk}^{(j)}$ represents the estimate of the uplink channel between the k -th user located in the j -th cell and its base station. In all simulations presented in this section, we assume that the channel covariance matrix is estimated using 1000 exact channel realizations. The multipath angle of arrival of any channel (including interference channel) follows a uniform distribution centered at the angle of line-of-sight (LOS). The angular spread is 30 degrees. The channel is assumed coherent over $C = 500$ transmitted symbols. In the implementation of the amplitude and DFT based projection we set $N = 100$ and $\xi = 10^{-2}/M$.

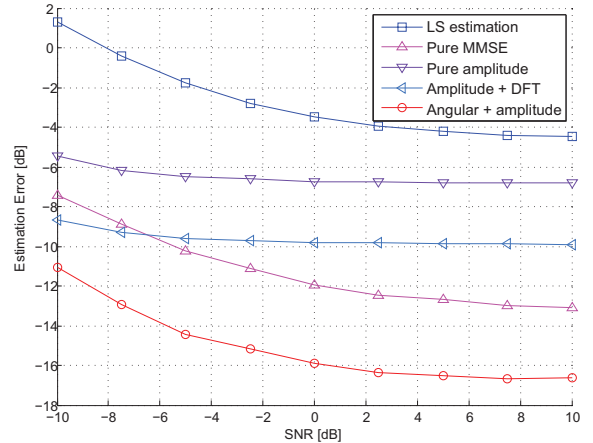


Fig. 2. Estimation performance vs. SNR, $M = 200$, 7-cell network, with 4 users per cell.

In Fig. 2 and Fig. 3 we show the channel estimation error as a function of SNR at the base station as well as the number of BS antennas. In the figures, ‘‘LS estimation’’ and ‘‘Pure MMSE’’ stand for the estimation errors of an LS estimator and an MMSE estimator (12) respectively. ‘‘Pure amplitude’’ denotes the case when we apply blind decontamination method (15) only. ‘‘Angular + amplitude’’ represents the performance of the proposed estimator (17). As we may observe, the traditional LS estimator suffers from severe pilot contamination. The pure amplitude-based method and the pure MMSE method alleviate the pilot interference, yet saturate with SNR and the number of antennas. These saturation effects come from the overlapping of interference and desired channels in power and angular domains respectively. The ‘‘Angular + amplitude’’ approach outperforms these two known methods as it discriminates against interference in both amplitude and angular domains. The low-complexity method ‘‘Amplitude +

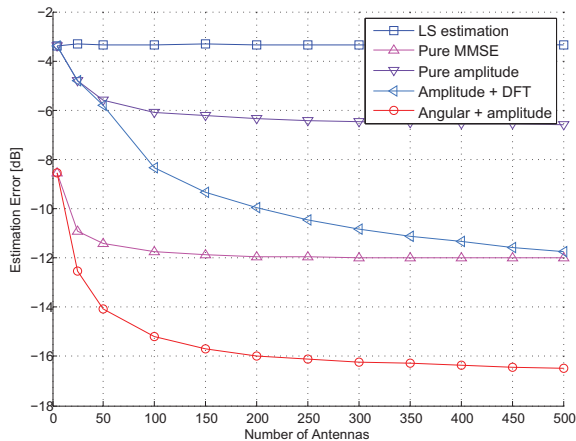


Fig. 3. Estimation performance vs. number of antennas, SNR = 0 dB, 7-cell network, with 4 users per cell.

DFT” keeps improving with the number of antennas, due to the diminishing leakage effect of DFT projection [17]. Eventually it will outperform pure MMSE in massive MIMO regime in terms of channel estimation error.

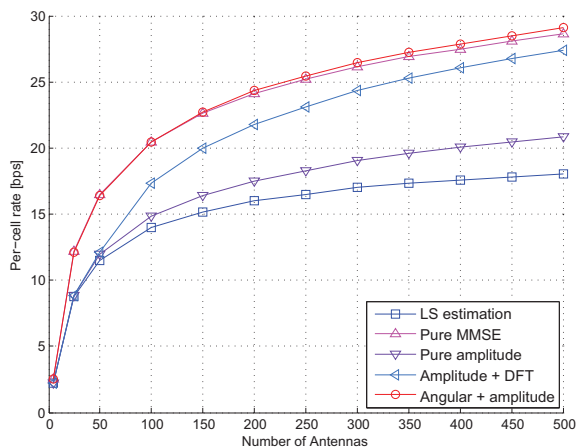


Fig. 4. Per-cell rate vs. number of antennas, SNR = 0 dB, 7-cell network, with 4 users per cell.

Fig. 4 illustrates the uplink per-cell rate as a function of BS antennas. The simulation parameters remain the same as in Fig. 3. The base station is equipped with a zero-forcing (ZF) receiver which is computed based on the acquired channel estimates. As can be seen, the “Angular + amplitude” approach has the best performance. It is interesting to note that the “Amplitude + DFT” method keeps the same slope, even though it requires no knowledge on channel covariance.

It is worth noting that the performances of all the proposed algorithms strongly depend on the accuracy of prior knowledge and channel properties such as the coherence time. The impact of these parameters will be object of further studies.

VIII. CONCLUSIONS

In this paper we proposed two novel robust channel estimation algorithms exploiting path diversity in both angle and amplitude domains. The first method called angular and amplitude based projection is robust also for a limited number of antennas but requires channel stationarity. The second estimator called amplitude and DFT based projection is suitable for higher mobility users. However, since it is based on asymptotic properties of massive MIMO channels, it requires a higher number of antennas to achieve the same performance.

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