Cramer Rao Lower Bound for Multi-Source Localization in Spatial Correlated Environment

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***Abstract*—This extended abstract paper provides the modeling approach and some indicative results on the expected performance of received power-based multiple source localization in spatially-correlated log-normal propagation environment. By proper modeling approximation of the received signal strength we are able to evaluate the Crammer-Rao Lower Bound (CRLB) given the positions of the sources and sensors. Probabilistic models are used for both the sensor network as well as the multiple sources and a semi-analytic approach is taken to compute the average performance lower bound. The results are indicative on the expected localization accuracy in a multi-source localization scenario, when the correlation of the propagation environment is exploited.**

***Index Terms*—Receive signal strength; Multi-Source; localization; log-normal; spatial correlation, Cramer-Rao Lower Bound;**

I. INTRODUCTION

While the topic of source localization has been addressed extensively, multi-source localization is less popular due to its inherent difficulty. This is because: There is a limited number of scenarios where multiple non-orthogonal sources may overlap in the same band. For multi-source scenario, the amount of requiring sensors should be at least equal to problem's degrees of freedom, thus, for each unknown transmitter, including power estimation, $3N$ sensors are needed (x_i, y_i, P_i) and in order to make our estimation more resilient to noise we have to use even more. Therefore, the cost of a large number of sensors, a condition necessary for accurate localization, has been thus far prohibitive. However, the potential for cognitive-radio applications (even in 5G), the decreasing cost of sensors, and the availability of databases with localized channel strength measurements are changing this picture.

Theoretical accuracy limits are valuable in order to design a localization system. The most common theoretical bound for power-based localization (Received Signal Strength, RSS) localization, is Crammer-Rao Lower Bound (CRLB) which gives closed form expression about the minimum achievable variance of an estimator. There is extensive literature on the CRLB for single in non-correlated and correlated environment [1] , [2]. Approximate CRLB and localization algorithms for the multi-source problem was derived in [3]. Our work herein addresses the performance evaluation via the Crammer-Rao Lower Bound (CRLB) of multi-source, power-based localization in spatially-correlated log-normal propagation environment.

The rest of the paper is organized as follows, Section II presents propagation model, basic assumptions and generally clarify all necessary components of our model. Section III presents only some indicative results of the performance analysis due to space limitations.

II. OUR MODEL

Power measurements are drawn either from a set of sensors. For each set of i-th sensor and j-th transmitter we adopt the classic log-normal propagation model

$$R_{i,j} = P_j - L_0 - 10\alpha log(d_{i,j}/d_0) - n^s_{i,j} - n^f_{i,j} \ , \quad (1)$$

where $R_{i,j}$ is the j-th source power, measured by i-th sensor, $d_{i,j} = \|\mathbf{x}_i - \mathbf{s}_j\|$ is their respective distance ($\mathbf{x}_i$,$\mathbf{s}_j$ are the coordinates of i-th sensor and j-th source, respectively), P_j is j-th emitter power, d_0 is a reference distance and L_0 is the power loss in that reference distance, α is the path-loss exponent, $n^f_{i,j}$ is the noise due to fast fading, which is hereby modeled as zero-mean Gaussian (in linear scale) and ($n^s_{i,j}$) is the shadow-fading rv. We follow common practice to assume that the fast fading component can be averaged out and the shadow fading follows log-normal distribution: a Gaussian rv in the log domain with zero mean and variance σ_s^2.

The model for the correlation factor of Shadow Fading (autocorrelation) in respect to distance is given in [4] :

$$\rho(\Delta x, \Delta y) = \rho(d) = e^{-\frac{\ln(2)d}{d_c}} \ , \quad (2)$$

where d_c is the de-correlation distance, meaning that, is the distance where the correlation of tow variables became 0.5. In respect to the reciprocity of the propagation model, we have two types of correlation. One between different sensors and same transmitter, and one by different transmitters and the same sensor.

Due to the eq.1 the receiving power of i-th sensor from j-th source is following log-normal distribution at the linear domain. So, the total received power of i-th sensor is $R_i = \sum_{j=0}^{N} R_{i,j}$, where N is the total number of transmitters. But

$$\sigma_i^2 = c^{-2} \ln\left(1 + \frac{\sum_{k=1}^{N}\left(e^{c^2\bar{\sigma}^2}-1\right)e^{2cm_{i,k}} + 2\sum_{j}^{N-1}\sum_{j+1}^{N} e^{c(m_j+m_k)}\left(e^{\rho_{i,j}^{tx}c^2\bar{\sigma}^2}-1\right)}{\left(\sum_{k=1}^{N} e^{cm_{i,k}}\right)^2}\right) \quad (6)$$

the sum of log-normal is not trivial and approximation needed. As we mention at previews paragraph the shadow fading is spatially correlated, so, in order to capture the correlation from transmitters side, we use the correlated approximation for the sum of log-normal [5], which give as again a log-normal distribution with mean and variance eq.6

$$\mu_i = \frac{1}{c}\left(\ln\left(\sum_{j=0}^{N} e^{cm_{i,j}}\right) + \frac{c^2\sigma_i^2}{2} - \frac{c^2\bar{\sigma}^2}{2}\right) , \quad (3)$$

where $c = \frac{\ln(10)}{10}$ is a constant which is originated from the transformations between natural and log domains, $\rho_{i,j}^{tx}$ is the correlation factor between i-th and the j-th transmitter and calculated from 2.

With given mean and variance for each sensor, the general Fisher Information matrix for Gaussian rv is (see [6]):

$$\begin{aligned}[I(\theta)]_{kl} =& \frac{1}{2} tr\left(\mathbf{C}_s^{-1}(\theta)\frac{\partial \mathbf{C}_s(\theta)}{\partial\theta_k}\mathbf{C}_s^{-1}(\theta)\frac{\partial \mathbf{C}_s(\theta)}{\partial\theta_l}\right) \\ &+ \frac{\partial\mu(\theta)^T}{\partial\theta_k}\mathbf{C}_s^{-1}(\theta)\frac{\partial\mu(\theta)}{\partial\theta_l} ,\end{aligned} \quad (4)$$

where $tr()$ is the trace of the matrix and with use of σ_i from eq.6 and ρ_{ij}^s correlation factor between i-th and the j-th sensor again from eq.2 the covariance matrix of sensors

$$C = \begin{pmatrix} \rho_{11}^s\sigma_1\sigma_1 & \cdots & \rho_{1N}^s\sigma_1\sigma_N \\ \vdots & \ddots & \vdots \\ \rho_{N1}^s\sigma_N\sigma_1 & \cdots & \rho_{NN}^s\sigma_N\sigma_N \end{pmatrix} . \quad (5)$$

III. Results

Some indicative performance results are depicted in following two Figures. Fig. 1 depicts the performance degradation on localizing a source when a second one is at close distance as a function of the sensor network density. The performance for a single source is also depicted for comparative reasons. The parameters used are: pathloss equal to 3, decorrelation distance equal to 5m and shadow fading variance equal to 8dB.

Fig. 2 depicts how the CRLB is scales w.r.t sensors density, for different number of surrounding sources and the same propagation environment. The number of sources is chosen from a poison point process. The NoS parameter shows us the average number of sources within the coverage area of a source, i.e. the expected area where a sensor can detect its presence.

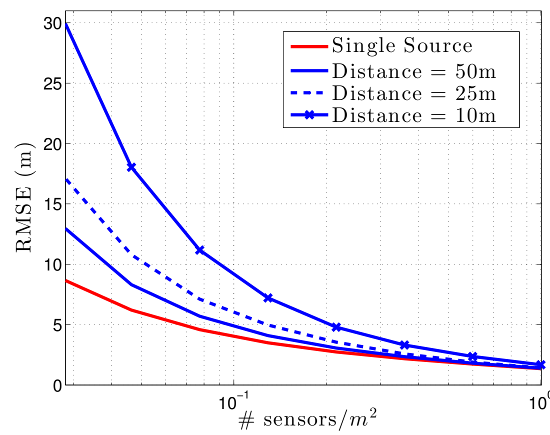

Fig. 1. Performance by adding second source

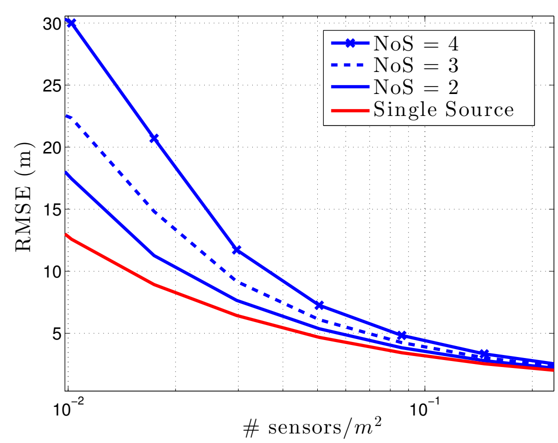

Fig. 2. Performance for different amount of sources

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References

[1] N. Patwari, A. Hero, M. Perkins, N. Correal, and R. O'Dea, "Relative location estimation in wireless sensor networks," *Signal Processing, IEEE Transactions on*, vol. 51, no. 8, pp. 2137–2148, Aug 2003.
[2] L. Mailaender, "Geolocation bounds for received signal strength (rss) in correlated shadow fading," in *Vehicular Technology Conference (VTC Fall), 2011 IEEE*, Sept 2011, pp. 1–6.
[3] I. Dagres, A. Polydoros, D. Denkovski, M. Angjelicinoski, V. Atanasovski, and L. Gavrilovska, "Algorithms and bounds for energy-based multi-source localization in log-normal fading," in *Globecom Workshops (GC Wkshps), 2012 IEEE*, Dec 2012, pp. 410–415.
[4] M. Gudmundson, "Correlation model for shadow fading in mobile radio systems," *Electronics Letters*, vol. 27, no. 23, pp. 2145–2146, Nov 1991.
[5] A. Safak and M. Safak, "Moments of the sum of correlated log-normal random variables," in *Vehicular Technology Conference, 1994 IEEE 44th*, Jun 1994, pp. 140–144 vol.1.
[6] S. Kay, *Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory*, 1993.