New Outer Bounds for the Interference Channel with Unilateral Source Cooperation

Martina Cardone[†], Daniela Tuninetti[*], Raymond Knopp[†], Umer Salim[‡]
[†]Eurecom, Biot, 06410, France, Email: {cardone, knopp}@eurecom.fr
[*] University of Illinois at Chicago, Chicago, IL 60607, USA, Email: danielat@uic.edu
[‡] Intel Mobile Communications, Sophia Antipolis, 06560, France, Email: umer.salim@intel.com

***Abstract*—This paper studies the two-user interference channel with unilateral source cooperation, which consists of two source-destination pairs that share the same channel and where one full-duplex source can overhear the other source through a noisy in-band link. Novel outer bounds of the type $2R_1 + R_2$ and $R_1 + 2R_2$ are developed for the class of injective semi-deterministic channels with independent noises at the different source-destination pairs. The bounds are then specialized to the Gaussian noise case. Interesting insights are provided about when these types of bounds are active, or in other words, when unilateral cooperation is too weak and leaves some system resources underutilized.**

I. Introduction

A major limitation of current wireless networks is interference. In today's systems, interference is either avoided or treated as noise. Interference avoidance is accomplished by splitting the available time / frequency / space / code resources among the users in such a way that their transmissions are "orthogonalized". In practice, perfect user orthogonalization is not possible leading to a residual interference, usually treated as noise. This approach may severely limit the system capacity.

Cooperation among wireless nodes has emerged as a potential technique to enhance performance. Cooperation leverages the broadcast nature of the wireless medium, i.e., the same transmission can be heard by multiple nodes, thus opening up the possibility that nodes help one another by relaying their message to their intended destination.

Motivated by the potential impact of cooperation in future wireless networks, this paper studies a system consisting of two source-destination pairs that share the same channel. One source, Tx2, overhears the other, Tx1, through a noisy in-band link. Therefore, Tx2, which is here assumed to operate in full-duplex, besides communicating with Rx2, may also allocate some of its resources to boost the rate of Tx1. This channel model is referred to as the *interference channel with unilateral source cooperation*. This system would model a scenario where, for example, a base station can overhear another base station and consequently help serving this other base station's associated mobile users.

A. Related Work

Lately, cooperation has received significant attention as summarized in what follows.

The Interference Channel (IC) with unilateral source cooperation is a special case of the IC with generalized feedback, or bilateral source cooperation. For this network, several outer bounds on the capacity have been derived [1], [2]. A number of schemes have been developed as well. For example, [3] proposed a strategy that exploits rate splitting, superposition coding, partial-decode-and-forward relaying, and Gelfand-Pinsker binning. This scheme, specialized to the Gaussian noise channel, turned out to match the sum-rate outer bounds of [2], [1] to within 19 bits with equally strong cooperation links and with arbitrary direct and interfering links [1] and to within 4 bits in the 'strong cooperation regime' with symmetric direct links and symmetric interfering links [4].

The IC with unilateral source cooperation was studied in [5], where it was assumed that, at any given time instant, the cooperating source has an a-priori access to $L \geq 0$ future channel outputs. For the case $L = 0$ studied in this paper, [5] derived potentially tighter outer bounds than those in [1], [2] specialized to unilateral source cooperation. However, these bounds involve several auxiliary random variables and it is not clear how to evaluate them for the Gaussian noise channel. The authors of [5] also proposed an achievable scheme, whose rate region, as pointed out in [5, Rem. 2, point 6], is no smaller than that in [3, Sec. V] specialized to unilateral source cooperation. However, in [5], no performance guarantees in terms of constant gap were given. In [6], the capacity of the IC with unilateral source cooperation was characterized to within 2 bits (per user) for a set of channel parameters that, roughly speaking, excludes the case of weak interference at both receivers. In [6], it was pointed out that in weak interference, outer bounds of the type $2R_1 + R_2$ and $R_1 + 2R_2$ might be necessary to (approximately) characterize the capacity. The derivation of such upper bounds is, to the best of our knowledge, not even available for the general memoryless non-cooperative IC. For the non-cooperative IC, such bounds were derived in [7] for the Injective Semi-Deterministic IC (ISD-IC) and showed to be achievable to within a constant gap by the Han-Kobayashi's scheme. In [1], the model of [7] was extended so as to include bilateral source cooperation and a new sum-rate outer bound was derived. Here we specialize the ISD model of [1] to unilateral source cooperation and derive bounds on $2R_1 + R_2$ and $R_1 + 2R_2$.

The ISD-IC with source cooperation includes classical feedback as a special case. [8] determined the capacity to within 2 bits of the IC where each source has perfect output feedback from the intended destination; it showed that $2R_1 + R_2$ and

R_1+2R_2-type of bounds are not needed because output feedback eliminates "resource holes", or system underutilization due to distributed processing captured by the $2R_1+R_2$ and R_1+2R_2 bounds. In [9], the authors studied the symmetric Gaussian IC with all possible output feedback configurations. They showed that the bounds developed in [8] suffice for constant gap characterization except in the case of 'single direct feedback link / model (1000)'. [9, Theorem IV.1] proposed a novel outer bound on $2R_1+R_2$ for the ISD-IC, to capture the fact that the second source does not receive feedback. [10] characterized the capacity of the 'symmetric linear deterministic IC with degraded output feedback' by developing bounds on $2R_1+R_2$ and R_1+2R_2, whose extension to the Gaussian noise case was left open. In this work we extend the results of [10] and [9] to all *ISD-ICs with unilateral source cooperation for which the noises at the different source-destination pairs are independent.*

B. Contributions and paper organization

The rest of the paper is organized as follows. Section II describes the channel model and summarizes known outer bounds. Section III presents the derivation of novel outer bounds for the ISD-IC with unilateral source cooperation. Section IV specializes the new bounds to the Gaussian noise channel and, by using the generalized Degrees-of-Freedom (gDoF) metric, sheds light on when unilateral cooperation enables sufficient coordination among the sources such that the new bounds on $2R_1+R_2$ and R_1+2R_2 are not active.

II. System model and known outer bounds

A *general memoryless IC with unilateral generalized feedback*, or source cooperation, consists of two input alphabets $(\mathcal{X}_1,\mathcal{X}_2)$, three output alphabets $(\mathcal{Y}_{\mathsf{F2}},\mathcal{Y}_1,\mathcal{Y}_2)$ and a memoryless transition probability $P_{Y_{\mathsf{F2}},Y_1,Y_2|X_1,X_2}$. Each $\mathsf{Tx}u$, $u\in[1:2]$, has a message $W_u\in[1:2^{NR_u}]$ for $\mathsf{Rx}u$, where N is the codeword length and R_u is the transmission rate for user u in bits per channel use. The messages are independent and uniformly distributed on their respective domains. At time $i\in[1:N]$, $\mathsf{Tx1}$ encodes the message W_1 into $X_{1i}(W_1)$ and $\mathsf{Tx2}$ maps its message W_2 and its past channel observations into $X_{2i}(W_2,Y_{\mathsf{F2}}^{i-1})$. At time N, $\mathsf{Rx1}$ estimates its intended message W_1 based on all its channel observations as $\widehat{W_1}({Y_1}^N)$, and similarly $\mathsf{Rx2}$ outputs $\widehat{W_2}({Y_2}^N)$. A rate pair (R_1,R_2) is said to be ϵ-achievable if there exists a sequence of codes indexed by the block length N such that $\max_{u\in[1:2]}\mathbb{P}[\widehat{W}_u\neq W_u]\le\epsilon$ for some $\epsilon\in[0,1]$. The capacity is the largest non-negative rate region that is ϵ-achievable for any $\epsilon>0$.

The ISD model, introduced in [7] for the IC without cooperation, assumes that the input X_1, resp. X_2, before reaching the destinations, is first passed through a memoryless channel to obtain T_1, resp. T_2. The channel outputs are then given by $Y_1=f_1(X_1,T_2)$ and $Y_2=f_2(X_2,T_1)$ where f_u, $u\in[1:2]$, is a deterministic function which is invertible given X_u, or in other words, T_1, resp. T_2, is a deterministic function of (Y_2,X_2), resp. (Y_1,X_1).

In the case of unilateral source cooperation, the generalized feedback signal at $\mathsf{Tx2}$ satisfies $Y_{\mathsf{F2}}=f_3(X_2,Y_{\mathsf{f}})$, for some deterministic function f_3 that is invertible given X_2 and where Y_{f} is obtained by passing X_1 through a noisy channel [1].

In the literature, several outer bounds are known for the IC with bilateral source cooperation [1], [2], which are here specialized to the case of unilateral cooperation. In particular, for an input distribution P_{X_1,X_2}, we have:
Case A) For *a general memoryless IC with unilateral source cooperation*, the cut-set upper bound [11] gives

$$R_1\le I(X_1;Y_1,Y_{\mathsf{F2}}|X_2),\tag{1a}$$
$$R_1\le I(X_1,X_2;Y_1),\tag{1b}$$
$$R_2\le I(X_2;Y_2|X_1),\tag{1c}$$

and from [2] we have

$$R_1+R_2\le I(X_1;Y_1,Y_{\mathsf{F2}}|Y_2,X_2)+I(X_1,X_2;Y_2),\tag{1d}$$
$$R_1+R_2\le I(X_2;Y_2|Y_1,X_1)+I(X_1,X_2;Y_1).\tag{1e}$$

In (1a)-(1e), Y_{F2} always appears conditioned on X_2; for the ISD channel this implies that Y_{F2} can be replaced with Y_{f}.
Case B) For *a memoryless ISD-IC with unilateral source cooperation that satisfies* $P_{Y_{\mathsf{F2}},Y_1,Y_2|X_1,X_2}=P_{Y_1|X_1,X_2}P_{Y_{\mathsf{F2}},Y_2|X_1,X_2}$, *i.e., the noises seen by the different source-destination pairs are independent,* we have

$$\begin{aligned}R_1+R_2\le\ & H(Y_1|T_1,Y_{\mathsf{f}})-H(Y_1|T_1,Y_{\mathsf{f}},X_1,X_2)\\&+H(Y_2|T_2,Y_{\mathsf{f}})-H(Y_2|T_2,Y_{\mathsf{f}},X_1,X_2)\\&+I(Y_{\mathsf{f}};X_1,X_2|T_2).\end{aligned}\tag{1f}$$

The bound in (1f) was originally derived in [1] for the Gaussian and the linear deterministic channels by assuming that all noises are independent, which implies $I(Y_{\mathsf{f}};X_1,X_2|T_2)\le I(Y_{\mathsf{f}};X_1)$. A straightforward generalization shows that the steps in [1] are valid even when $P_{Y_{\mathsf{F2}},Y_2|X_1,X_2}$ is not a product distribution. The key step of the proof consists in showing that, for the assumed noise structure, the following Markov chains hold for all $i\in[1:N]$:

$$(W_1,{T_1}^{i-1},{X_1}^i)-({T_2}^{i-1},{Y_{\mathsf{f}}}^{i-1})-(T_{2i}),\tag{1g}$$
$$(W_2,{T_2}^{i-1},{X_2}^i)-({T_1}^{i-1},{Y_{\mathsf{f}}}^{i-1})-(T_{1i},Y_{\mathsf{f}i}).\tag{1h}$$

The proof of these two Markov chains is not difficult and left out for sake of space.
Case C) For *a memoryless ISD-IC with output feedback* $Y_{\mathsf{F2}}=Y_2$ *and with independent noises* $P_{Y_1,Y_2|X_1,X_2}=P_{Y_1|X_1,X_2}P_{Y_2|X_1,X_2}$, from [9, model (1000)] we have

$$\begin{aligned}R_1+2R_2\le\ & H(Y_2)-H(Y_2|X_1,X_2)\\&+H(Y_2|Y_1,X_1)-H(Y_2|Y_1,X_1,X_2)\\&+H(Y_1|T_1)-H(Y_1|T_1,X_1,X_2).\end{aligned}\tag{1i}$$

To the best of our knowledge, (1i) is the only upper bound of the type R_1+2R_2 which is available in the literature, but it is only valid for output feedback. Our goal in the next section is to derive bounds of the type of (1i) for the class of ISD-ICs with unilateral source cooperation with independent noises at the different source-destination pairs.

$$\begin{aligned}&P_{Y_{\mathsf{F2}},Y_1,Y_2,Y_{\mathsf{f}},T_1,T_2|X_1,X_2}(y_f,y_1,y_2,a,b,c|x_1,x_2)\\&=P_{Y_{\mathsf{f}},T_1|X_1}(a,b|x_1)P_{T_2|X_2}(c|x_2)\delta\big(y_1-f_1(x_1,c)\big)\delta\big(y_2-f_2(x_2,b)\big)\delta\big(y_f-f_3(x_2,a)\big)\end{aligned}\tag{2}$$

III. Novel outer bounds

In this section we derive two novel outer bounds on the capacity region of the ISD-IC with unilateral source cooperation with independent noises at the different source-destination pairs. Our main result is as follows:

Theorem 1. *For an ISD-IC with unilateral source cooperation satisfying* (2)*, at the top of the page, for some memoryless transition probabilities* $P_{Y_{\mathsf{f}},T_1|X_1}, P_{T_2|X_2}$ *and for some injective functions* f_1, f_2, f_3 *as discussed in Section II, the capacity region is outer bounded by*

$$\begin{aligned}2R_1+R_2 \le\ & H\left(Y_1\right)-H\left(Y_1|X_1,X_2\right)\\&+H\left(Y_1|T_1,Y_{\mathsf{f}},X_2\right)-H\left(Y_1|T_1,Y_{\mathsf{f}},X_1,X_2\right)\\&+H\left(Y_2|T_2,Y_{\mathsf{f}}\right)-H\left(Y_2|T_2,Y_{\mathsf{f}},X_1,X_2\right)\\&+I\left(Y_{\mathsf{f}};X_1,X_2|T_2\right),\end{aligned}\tag{3}$$

$$\begin{aligned}R_1+2R_2 \le\ & H\left(Y_2\right)-H\left(Y_2|X_1,X_2\right)\\&+H\left(Y_2|T_2,Y_{\mathsf{f}},X_1\right)-H\left(Y_2|T_2,Y_{\mathsf{f}},X_1,X_2\right)\\&+H\left(Y_1,Y_{\mathsf{f}}|T_1\right)-H\left(Y_1,Y_{\mathsf{f}}|X_1,X_2,T_1\right),\end{aligned}\tag{4}$$

for some input distribution P_{X_1,X_2}.

Proof: By Fano's inequality and by giving side information similarly to [1], we have

$$\begin{aligned}&N(2R_1+R_2-3\epsilon_N)\\&\le 2I\left(W_1;{Y_1}^N\right)+I\left(W_2;{Y_2}^N\right)\\&\le I\left(W_1;{Y_1}^N\right)+I\left(W_1;{Y_1}^N,{T_1}^N,{Y_{\mathsf{f}}}^N|W_2\right)\\&+I\left(W_2;{Y_2}^N,{T_2}^N,{Y_{\mathsf{f}}}^N\right)\\&=H\left({Y_1}^N\right)-H\left({Y_1}^N,{T_1}^N,{Y_{\mathsf{f}}}^N|W_1,W_2\right)\\&+H\left({Y_1}^N,{T_1}^N,{Y_{\mathsf{f}}}^N|W_2\right)-H\left({Y_2}^N,{T_2}^N,{Y_{\mathsf{f}}}^N|W_2\right)\\&+H\left({Y_2}^N,{T_2}^N,{Y_{\mathsf{f}}}^N\right)-H\left({Y_1}^N|W_1\right).\end{aligned}$$

We now analyze and bound each pair of terms. First pair:

$$\begin{aligned}&H\left({Y_1}^N\right)-H\left({Y_1}^N,{T_1}^N,{Y_{\mathsf{f}}}^N|W_1,W_2\right)\\&\le\sum_{i\in[1:N]}H\left(Y_{1i}\right)-H\left(Y_{1i},T_{1i},Y_{\mathsf{f}i}|X_{1i},X_{2i}\right)\end{aligned}$$

by using: the chain rule for the entropy, the definition of the encoding functions (for the ISD-IC with unilateral source cooperation the encoding function $X_{2i}(W_2,Y_{\mathsf{F2}}^{i-1})$ is equivalent to $X_{2i}(W_2,{Y_{\mathsf{f}}}^{i-1})$), the fact that conditioning reduces entropy, the ISD property of the channel, and the fact that the channel is memoryless. Second pair:

$$\begin{aligned}&H\left({Y_1}^N,{T_1}^N,{Y_{\mathsf{f}}}^N|W_2\right)-H\left({Y_2}^N,{T_2}^N,{Y_{\mathsf{f}}}^N|W_2\right)\\&=\sum_{i\in[1:N]}H\left(Y_{1i},T_{1i},Y_{\mathsf{f}i}|{Y_1}^{i-1},{T_1}^{i-1},{Y_{\mathsf{f}}}^{i-1},W_2,{X_2}^i\right)\\&-\sum_{i\in[1:N]}H\left(Y_{2i},T_{2i},Y_{\mathsf{f}i}|{Y_2}^{i-1},{T_2}^{i-1},{Y_{\mathsf{f}}}^{i-1},W_2,{X_2}^i\right)\\&=\sum_{i\in[1:N]}H\left(Y_{1i},T_{1i},Y_{\mathsf{f}i}|{Y_1}^{i-1},{T_1}^{i-1},{Y_{\mathsf{f}}}^{i-1},W_2,{X_2}^i\right)\\&-\sum_{i\in[1:N]}H\left(T_{1i},T_{2i},Y_{\mathsf{f}i}|{T_1}^{i-1},{T_2}^{i-1},{Y_{\mathsf{f}}}^{i-1},W_2,{X_2}^i\right)\\&\overset{(a)}{\le}\sum_{i\in[1:N]}H\left(T_{1i},Y_{\mathsf{f}i}|{T_1}^{i-1},{Y_{\mathsf{f}}}^{i-1},W_2,{X_2}^i\right)\\&-\sum_{i\in[1:N]}H\left(T_{1i},Y_{\mathsf{f}i}|{T_1}^{i-1},{T_2}^{i-1},{Y_{\mathsf{f}}}^{i-1},W_2,{X_2}^i\right)\\&+\sum_{i\in[1:N]}H\left(Y_{1i}|T_{1i},Y_{\mathsf{f}i},X_{2i}\right)\\&-\sum_{i\in[1:N]}H\left(T_{2i}|{T_1}^i,{T_2}^{i-1},{Y_{\mathsf{f}}}^i,W_2,{X_2}^i,{X_1}^i\right)\end{aligned}$$

where we used: the chain rule for the entropy, the definition of the encoding functions, the ISD property of the channel, the fact that conditioning reduces entropy, the memoryless property of the channel, and the fact that Y_2 is a deterministic function of (X_2,T_1), which is invertible given X_2; so finally, by using the Markov chain in (1h) the first two terms in the inequality in (a) cancel and we obtain

$$\begin{aligned}&H\left({Y_1}^N,{T_1}^N,{Y_{\mathsf{f}}}^N|W_2\right)-H\left({Y_2}^N,{T_2}^N,{Y_{\mathsf{f}}}^N|W_2\right)\\&\le\sum_{i\in[1:N]}H\left(Y_{1i}|T_{1i},Y_{\mathsf{f}i},X_{2i}\right)-H\left(Y_{1i}|T_{1i},Y_{\mathsf{f}i},X_{2i},X_{1i}\right).\end{aligned}$$

Third pair: since

$$\begin{aligned}&H\left({Y_1}^N|W_1\right)=\sum_{i\in[1:N]}H\left(Y_{1i}|{Y_1}^{i-1},W_1,{X_1}^i\right)\\&=\sum_{i\in[1:N]}H\left(T_{2i}|{T_2}^{i-1},W_1,{X_1}^i\right)\\&\ge\sum_{i\in[1:N]}H\left(T_{2i}|{T_2}^{i-1},W_1,{X_1}^i,{Y_{\mathsf{f}}}^{i-1}\right)\\&=\sum_{i\in[1:N]}H\left(T_{2i}|{T_2}^{i-1},{Y_{\mathsf{f}}}^{i-1}\right)-\underbrace{I\left(T_{2i};W_1,{X_1}^i|{T_2}^{i-1},{Y_{\mathsf{f}}}^{i-1}\right)}_{=0\text{ because of (1g)}}\end{aligned}$$

by using: the chain rule for the entropy, the definition of the encoding functions, the ISD property of the channel, the fact that Y_1 is a deterministic function of (X_1,T_2), which is invertible given X_1, and the fact that conditioning reduces entropy. Therefore,

$$\begin{aligned}&H\left({Y_2}^N,{T_2}^N,{Y_{\mathsf{f}}}^N\right)-H\left({Y_1}^N|W_1\right)\\&\le\sum_{i\in[1:N]}H\left(T_{2i}|{Y_2}^{i-1},{T_2}^{i-1},{Y_{\mathsf{f}}}^{i-1}\right)-H\left(T_{2i}|{T_2}^{i-1},{Y_{\mathsf{f}}}^{i-1}\right)\\&+\sum_{i\in[1:N]}H\left(Y_{2i},Y_{\mathsf{f}i}|{Y_2}^{i-1},{T_2}^i,{Y_{\mathsf{f}}}^{i-1}\right)\\&\le\sum_{i\in[1:N]}0+H\left(Y_{2i},Y_{\mathsf{f}i},|T_{2i}\right).\end{aligned}$$

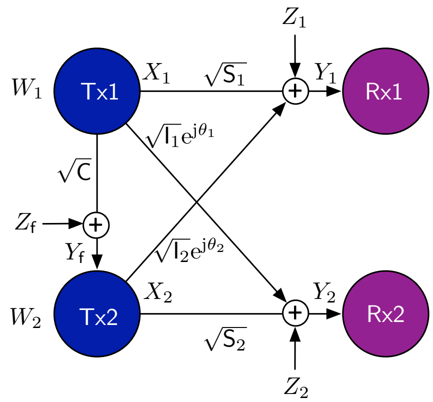

Fig. 1: The Gaussian IC with unilateral source cooperation.

By combining everything together, by introducing the time sharing random variable uniformly distributed over $[1 : N]$ and independent of everything else, by dividing both sides by N and taking the limit for $N \to \infty$ we get the bound in (3). We finally notice that by dropping the time sharing we do not decrease the bound. By following similar steps as in the derivation of (3) and by using the Markov chains in (1g) and (1h), it is straightforward to derive the upper bound in (4). ■

IV. The Gaussian noise channel

In this section we specialize the outer bounds in (3) and (4) to the practically relevant Gaussian noise channel.

A *complex-valued single-antenna full-duplex Gaussian IC with unilateral source cooperation*, shown in Fig. 1, is an ISD channel with input/output relationship

$$T_1 := \sqrt{\mathsf{I}_1}e^{j\theta_1}X_1 + Z_2,\ Z_2 \sim \mathcal{N}(0,1), \tag{5a}$$
$$T_2 := \sqrt{\mathsf{I}_2}e^{j\theta_2}X_2 + Z_1,\ Z_1 \sim \mathcal{N}(0,1), \tag{5b}$$
$$Y_1 = \sqrt{\mathsf{S}_1}X_1 + T_2 : \mathbb{E}\left[|X_1|^2\right] \le 1, \tag{5c}$$
$$Y_2 = T_1 + \sqrt{\mathsf{S}_2}X_2 : \mathbb{E}\left[|X_2|^2\right] \le 1, \tag{5d}$$
$$Y_\mathsf{f} = Y_{\mathsf{F}2} = \sqrt{\mathsf{C}}X_1 + Z_\mathsf{f},\ Z_\mathsf{f} \sim \mathcal{N}(0,1), \tag{5e}$$

where $Y_{\mathsf{F}2} = Y_\mathsf{f}$ since Tx2 can remove the contribution of X_2 from its received signal. The channel gains are constant; some are real-valued and non-negative because a node can compensate for the phase of one of its channel gains. The noises are circularly symmetric Gaussian random variables with, without loss of generality, zero mean and unit variance. For the assumption under which we derived our outer bounds, we must impose that the noise Z_1 is independent of (Z_2, Z_f) (while (Z_2, Z_f) can be arbitrarily correlated).

A. Upper bounds

The bounds in (1), (3) and (4) can be evaluated for the Gaussian noise channel in (5). We define $\mathbb{E}\left[X_1{X_2}^*\right] := \rho : |\rho| \in [0,1]$. We also assume that all the noises are independent, which represents a particular case for which our outer bounds hold. By the 'Gaussian maximizes entropy' principle, jointly Gaussian inputs exhaust the outer bounds in (1), (3) and (4). Thus, we start by evaluating each mutual information term in (1), (3) and (4) by using jointly Gaussian inputs. Then, we further upper bound (maximize) each mutual information term over the input correlation coefficient $\rho : |\rho| \in [0,1]$. By doing so we obtain: from the cut-set bounds in (1a)-(1c)

$$R_1 \le \log\left(1 + \mathsf{C} + \mathsf{S}_1\right), \tag{6a}$$
$$R_1 \le \log\left(1 + (\sqrt{\mathsf{S}_1} + \sqrt{\mathsf{I}_2})^2\right), \tag{6b}$$
$$R_2 \le \log\left(1 + \mathsf{S}_2\right), \tag{6c}$$

from the bounds in (1d)-(1e)

$$R_1 + R_2 \le \log\left(1 + \frac{\mathsf{S}_1 + \mathsf{C}}{1 + \mathsf{I}_1}\right) + \log\left(1 + (\sqrt{\mathsf{S}_2} + \sqrt{\mathsf{I}_1})^2\right), \tag{6d}$$
$$R_1 + R_2 \le \log\left(1 + \frac{\mathsf{S}_2}{1 + \mathsf{I}_2}\right) + \log\left(1 + (\sqrt{\mathsf{S}_1} + \sqrt{\mathsf{I}_2})^2\right), \tag{6e}$$

and from the bound in (1f)

$$R_1 + R_2 \le \log\left(1 + \mathsf{I}_2 + \frac{\mathsf{S}_1}{\mathsf{I}_1 + \mathsf{C}}\right) + \log\left(1 + \mathsf{I}_1 + \frac{\mathsf{S}_2}{\mathsf{I}_2} + \mathsf{C}\left(1 + \frac{\mathsf{S}_2}{1 + \mathsf{I}_2}\right)\right). \tag{6f}$$

We now evaluate the new outer bounds in Theorem 1, again assuming that the channel gains are larger than one, to get

$$2R_1 + R_2 \le \log\left(1 + \left(\sqrt{\mathsf{S}_1} + \sqrt{\mathsf{I}_2}\right)^2\right) + \log\left(1 + \frac{\mathsf{S}_1}{1 + \mathsf{I}_1 + \mathsf{C}}\right) + \log\left(1 + \mathsf{I}_1 + \frac{\mathsf{S}_2}{\mathsf{I}_2} + \mathsf{C}\left(1 + \frac{\mathsf{S}_2}{1 + \mathsf{I}_2}\right)\right), \tag{6g}$$

$$R_1 + 2R_2 \le \log\left(1 + (\sqrt{\mathsf{S}_2} + \sqrt{\mathsf{I}_1})^2\right) + \log\left(1 + \frac{\mathsf{S}_2}{1 + \mathsf{I}_2}\right) + \log\left(1 + \mathsf{I}_2 + \frac{\mathsf{S}_1}{\mathsf{I}_1 + \mathsf{C}}\right) + \log\left(1 + \frac{\mathsf{C}}{1 + \mathsf{I}_1}\right). \tag{6h}$$

As we shall see in the next section, the new bounds in (6g) and (6h) are active when the system experiences weak interference and 'weak cooperation'.

B. Generalized degrees-of-freedom (gDoF) region

We now focus on the *symmetric* Gaussian IC with unilateral source cooperation for sake of space. This channel is parameterized, for some $\mathsf{S} \ge 1, \alpha \ge 0, \beta \ge 0$, as

$$\mathsf{S}_1 = \mathsf{S}_2 = \mathsf{S}^1, \quad \mathsf{I}_1 = \mathsf{I}_2 = \mathsf{I} = \mathsf{S}^\alpha, \quad \mathsf{C} = \mathsf{S}^\beta. \tag{7}$$

In particular, we derive the gDoF region, which is an exact capacity characterization in the high-SNR regime $\mathsf{S} \gg 1$. The gDoF for the i-th user, $i \in [1:2]$, is defined as

$$\mathsf{d}_i := \lim_{\mathsf{S} \to +\infty} \frac{R_i}{\log(1 + \mathsf{S})}. \tag{8}$$

This analysis reveals the channel conditions under which the novel outer bounds on $2R_1 + R_2$ and $R_1 + 2R_2$ are active, i.e., tighter than those on the single rate and on the sum-rate available in the literature [11], [2] and [1].

In [6] we showed that, for the Gaussian symmetric IC with unilateral source cooperation, the capacity can be achieved

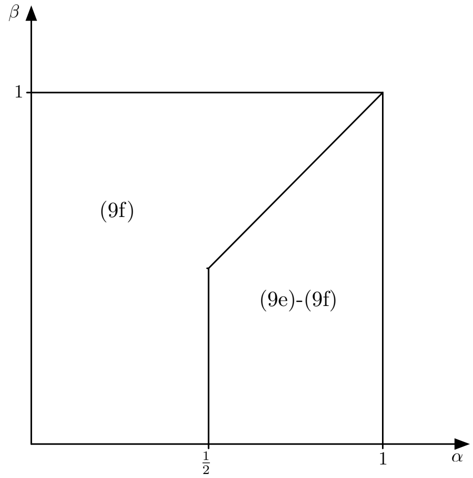

Fig. 2: Regimes where the bounds in (9e) and (9f) are active.

to within a constant gap in strong interference $\alpha \geq 1$, i.e., the interference links are stronger than the direct links, and when $\alpha < 1$ and $\beta \geq \alpha + 1$, i.e., the interference is weak and the cooperation link is 'sufficiently' strong. This constant gap result implies an exact gDoF region characterization for this set of parameters (the work in [6] is not restricted to the symmetric case). In these regimes, the capacity region of the Gaussian IC with unilateral source cooperation does not have bounds on $2R_1 + R_2$ and $R_1 + 2R_2$, similarly to the capacity region of the classical IC [12]. In other words, in these regimes the channel resources are fully utilized.

Here we focus on a sub-regime left open in [6], namely $\alpha < 1$ and $\beta \leq 1$, i.e., when the direct links are stronger than the interfering and cooperation links. By using the definition in (8), from the upper bound region in (6), we obtain that the gDoF region of the Gaussian IC with unilateral source cooperation, when $\alpha < 1$ and $\beta \leq 1$, is upper bounded by

$$\mathsf{d}_1 \leq 1, \tag{9a}$$
$$\mathsf{d}_2 \leq 1, \tag{9b}$$
$$\mathsf{d}_1 + \mathsf{d}_2 \leq 2 - \alpha, \tag{9c}$$
$$\mathsf{d}_1 + \mathsf{d}_2 \leq \max\{\alpha, 1 - \max\{\alpha, \beta\}\} + \max\{\alpha, 1 + \beta - \alpha\}, \tag{9d}$$
$$2\mathsf{d}_1 + \mathsf{d}_2 \leq 2 - \max\{\alpha, \beta\} + \max\{\alpha, 1 - \alpha + \beta\}, \tag{9e}$$
$$\mathsf{d}_1 + 2\mathsf{d}_2 \leq 2 - \alpha + \max\{\alpha, 1 - \max\{\alpha, \beta\}\} + [\beta - \alpha]^+. \tag{9f}$$

From the gDoF region above interesting insights can be drawn on when the outer bounds on $2\mathsf{d}_1 + \mathsf{d}_2$ and $\mathsf{d}_1 + 2\mathsf{d}_2$ are active. From Fig. 2 we observe that both bounds are active whenever $\alpha \geq \max\left\{\frac{1}{2}, \beta\right\}$, while in the other case only $\mathsf{d}_1 + 2\mathsf{d}_2$ is active. Moreover, we notice that, in weak interference, i.e., $\alpha < 1$, and with $\beta \leq [2\alpha - 1]^+$, the gDoF region in (9) is the same as that of the classical non-cooperative IC [12]. Moreover, for this set of parameters the outer bound region in (6) is achievable to within a constant gap (the gap computation is straightforward and not shown here for sake of space). For the other parameter regimes, designing strategies that achieve the outer bound in (6) to within a constant gap is an important open problem, which is object of current investigation.

V. CONCLUSIONS

In this work we studied the two-user IC with unilateral source cooperation where one source overhears the other source through a noisy in-band link. Our major contribution was to develop two novel outer bounds of the type $2R_1 + R_2$ and $R_1 + 2R_2$ on the capacity of this system. These bounds were first derived for the injective semi-deterministic channel and then specialized to the Gaussian case. The symmetric, i.e., equally strong interfering links and direct links, Gaussian channel was investigated in the high SNR regime, in order to highlight under which channel conditions unilateral cooperation enables sufficient coordination among the sources such that the new $2R_1 + R_2$ and $R_1 + 2R_2$ bounds are not active.

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