

# On the Gaussian Interference Channel with Unilateral Generalized Feedback

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**Abstract**—This paper studies the Gaussian interference channel with *unilateral generalized feedback*, a system where two source-destination pairs share the same channel and where one full-duplex source overhears the other through a noisy in-band link. A superposition coding scheme is shown to achieve a known outer bound to within a small number of bits for a subset of the weak interference regime, outside which more sophisticated coding techniques based on binning are conjectured to be needed. By using the generalized Degrees of Freedom (gDoF) as performance metric, unilateral generalized feedback is shown to strictly increase the gDoF region compared to the non-cooperative case only when the strength of the cooperation link is larger than a threshold, thus providing an indication on when cooperation among users is beneficial in practical wireless systems.

## I. INTRODUCTION

We study the system depicted in Fig. 1 consisting of two source-destination pairs that share the same wireless Gaussian channel and where one full-duplex source, Tx2, has generalized feedback from the other, Tx1, through a noisy in-band link. We shall refer to this model as the Gaussian Interference Channel with Unilateral Generalized Feedback (G-IC-UGF). UGF is the key to enable source cooperation.

Source cooperation has been the focus of much research in the past few years. Several sum-rate outer bounds exist for bilateral source cooperation [1], [2]. In Gaussian noise, the sum-rate outer bound is shown to be achievable to within a constant gap by a strategy that combines rate splitting, superposition coding, partial-decode-and-forward relaying, and Gelfand-Pinsker binning [3]. In particular, for equally strong cooperation links (and general direct and interfering links) the *sum-capacity* was characterized to within 10 bits/user in [1]; the gap was reduced to 2 bits/user in [4] when the direct links and the interfering links have the same strength.

The G-IC-UGF has also received attention lately as it represents a more practically relevant model for cognitive radio than [5]<sup>1</sup>. In [6], we characterized the *capacity region* of the G-IC-UGF to within 2 bits/user for a large set of channel parameters that, roughly speaking, excludes the case of weak interference at both receivers and for which we conjectured the necessity of outer bounds of the type  $2R_1 + R_2/R_1 + 2R_2$ .

<sup>1</sup>The Cognitive IC (CIC) provides an outer bound to the IC-UGF of interest here. The CIC is obtained by giving *for free* to Tx2 the message of Tx1. The capacity region of the G-CIC is exactly known for some parameter regimes and to within 1 bit otherwise [5]. The G-IC-UGF is expected to behave like its G-CIC counterpart when C in Fig. 1 satisfies  $C \gg 1$ .

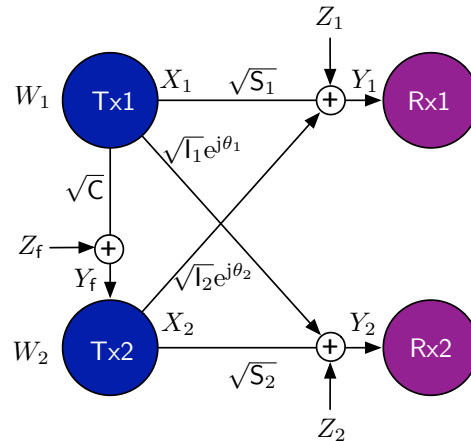


Fig. 1: The Gaussian IC with unilateral generalized feedback.

This kind of bounds were needed for the non-cooperative G-IC in weak interference [7], thus they seem to be also needed in the same regime when the cooperation link is “weak”. On the other hand, when the cooperation link is “strong”, the G-IC-UGF should tend to the non-causal cognitive model of [5] for which these bounds are known to be redundant. Recently, we developed bounds on  $2R_1 + R_2/R_1 + 2R_2$  for the case of *injective semi-deterministic IC-UGF* and then specialized them to the Gaussian noise case [8]. In this work we show that these novel bounds suffice to characterize, to within a constant gap, the capacity region of the *symmetric G-IC-UGF* in some parts of the regime left open in [6].

The constant gap result implies the exact characterization of the generalized Degrees of Freedom (gDoF) region of the channel in the corresponding regimes. For the considered set of channel parameters, we show that the gDoF region of the G-IC-UGF is strictly smaller than that of the non-causal G-CIC, since the cooperation link is not strong enough. However, despite a small parameter region, the gDoF region of the G-IC-UGF is strictly greater than that of the non-cooperative G-IC, implying that in this regime cooperation is indeed useful.

The rest of the paper is organized as follows. Section II describes the channel model. Section III contains our constant gap and gDoF region results. Section IV concludes the paper.

## II. SYSTEM MODEL

The complex-valued single-antenna full-duplex G-IC-UGF, shown in Fig. 1, has Tx1 / Tx2 sending an independent

$$R_1 \leq \log \left( 1 + (\sqrt{S_1} + \sqrt{I_2})^2 \right) \quad (1a)$$

$$R_1 \leq \log (1 + C + S_1) \quad (1b)$$

$$R_2 \leq \log (1 + S_2) \quad (1c)$$

$$R_1 + R_2 \leq \log \left( 1 + \frac{S_2}{1 + I_2} \right) + \log \left( 1 + (\sqrt{S_1} + \sqrt{I_2})^2 \right) \quad (1d)$$

$$R_1 + R_2 \leq \log \left( 1 + \frac{S_1 + C}{1 + I_1} \right) + \log \left( 1 + (\sqrt{S_2} + \sqrt{I_1})^2 \right) \quad (1e)$$

$$R_1 + R_2 \leq \log \left( 1 + I_2 + \frac{S_1}{I_1 + C} \right) + \log \left( 1 + I_1 + \frac{S_2}{I_2} + C \left( 1 + \frac{S_2}{I_2 + 1} \right) \right) \quad (1f)$$

$$2R_1 + R_2 \leq \log \left( 1 + (\sqrt{S_1} + \sqrt{I_2})^2 \right) + \log \left( 1 + \frac{S_1}{1 + I_1 + C} \right) + \log \left( 1 + I_1 + \frac{S_2}{I_2} + C \left( 1 + \frac{S_2}{I_2 + 1} \right) \right) \quad (1g)$$

$$R_1 + 2R_2 \leq \log \left( 1 + (\sqrt{S_2} + \sqrt{I_1})^2 \right) + \log \left( 1 + \frac{S_2}{1 + I_2} \right) + \log \left( 1 + I_2 + \frac{S_1}{I_1 + C} \right) + \log \left( 1 + \frac{C}{1 + I_1} \right) \quad (1h)$$

message  $W_1 / W_2$  to Rx1 / Rx2, respectively; moreover, Tx2 overhears Tx1 through a noisy in-band link. Achievable rates and capacity region are defined as usual [9]. The G-IC-UGF has input/output relationship

$$\begin{bmatrix} Y_f \\ Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} \sqrt{C} & \\ \sqrt{S_1} & \star \\ \sqrt{I_1} e^{j\theta_1} & \sqrt{I_2} e^{j\theta_2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} Z_f \\ Z_1 \\ Z_2 \end{bmatrix}, \quad (2)$$

where  $\star$  indicates a channel gain that does not affect the capacity region (since Tx2 can remove its transmitted signal  $X_2$  from its generalized feedback signal  $Y_f$ ), and where some channel gains are real-valued and non-negative because a node can compensate for the phase of one of its channel gains. The channel gains are constant and therefore known to all terminals. The channel inputs are subject to the average power constraints  $\mathbb{E}[|X_i|^2] \leq 1, i \in \{1, 2\}$ , and the noises are distributed as  $Z_k \sim \mathcal{CN}(0, 1), k \in \{f, 1, 2\}$ . We focus here on the case of independent noises, but the results easily extend to the case where  $(Z_2, Z_f)$  is arbitrarily correlated and is independent of  $Z_1$ , which encompasses for example the case of (degraded) output feedback from Rx2 to Tx2.

### III. MAIN RESULTS

In this section we prove that known outer bounds on the capacity for the G-IC-UGF can be achievable to within a constant gap in some parts of the weak interference regime, which was left open in [6]. The outer bound region reported in Section III-A follows from [1], [2], [8]. The inner bound region reported in Section III-B follows from specializing the inner bound of [3]. In Section III-C, we show that the inner and outer bounds are a constant number of bits apart for the *symmetric* G-IC-UGF (extensions to the general case are omitted for sake of space). In Section III-D, we evaluate the gDoF region and compare it with that of the non-cooperative G-IC [7] and of the non-causal G-CIC [5].

#### A. Outer bound

The capacity of the G-IC-UGF is upper bounded by (1), at the top of this page. The single rate bounds in (1a)-(1c) are cut-set bounds [9]. The sum-rate bounds in (1d)-(1e) are

from [2], and that in (1f) is from [1]. The bounds in (1g)-(1h) are from [8]. Notice that the bounds from [2] and [1] were originally derived for the IC with generalized feedback, or bilateral source cooperation, and were adapted here to the case of unilateral generalized feedback.

#### B. Achievable scheme

The capacity of the G-IC-UGF is lower bounded by (3), at the top of the next page. The bound in (3) follows from [3, eq.(8)] as follows.

In the weak interference regime, following [7], each source should split its message into a *common* and a *private* part, where common denotes a message that is decoded also at the non-intended receiver, while private refers to a message that is only decoded at the intended receiver and treated as noise at the non-intended receiver. Moreover, thanks to the UGF, Tx2 overhears Tx1 and therefore can assist the communication of Tx1 to Rx1. This suggests that part of the common message of Tx1 should be *cooperative*, or decodable at Tx2. Tx2 relays this cooperative message to Rx1. The messages are superimposed to one another and sent through the channel. The proposed strategy is quite simple (compared to schemes involving also dirty paper coding / binning [3]) in the sense that only superposition coding is employed.

With reference to [3, Sec. IV], we set  $V_2 = 0$ , i.e., Tx2 does not have a cooperative message because cooperation is unilateral, and  $Q = 0$ , i.e., no time sharing and no “coherent combining”. Moreover,  $V_1$  conveys the cooperative common message of Tx1,  $U_k, k \in \{1, 2\}$  carries the common non-cooperative message of Tx $k$ , and  $T_k, k \in \{1, 2\}$  conveys the private non-cooperative message of Tx $k$ . The inputs are chosen as

$$\begin{aligned} X_1 &= V_1 + U_1 + T_1 : & P_{V_1} + P_{U_1} + P_{T_1} &= 1, \\ X_2 &= U_2 + T_2 : & P_{U_2} + P_{T_2} &= 1, \end{aligned}$$

where  $V_1, U_1, T_1, U_2, T_2$  are independent Gaussian random variables with zero mean and variance indicated by the letter  $P$  with the random variable as a subscript. The variances / powers can be chosen so as to meet the power constraints. Although the powers can be optimized so as to get the largest

$$R_1 \leq \log(1 + S_1 + l_2) - \log(2) \quad (3a)$$

$$R_1 \leq \log\left(\frac{1+C}{1+\frac{C}{1+l_1}+Cx}\right) + \log\left(1 + \frac{S_1}{1+l_1} + S_1x\right) - \log(2) \quad (3b)$$

$$R_2 \leq \log(1 + S_2) - \log(2) \quad (3c)$$

$$R_1 + R_2 \leq \log(1 + S_1 + l_2) + \log\left(1 + \frac{S_2}{1+l_2}\right) - 2\log(2) \quad (3d)$$

$$R_1 + R_2 \leq \log(1 + S_2 + l_1) + \log\left(1 + \frac{S_1}{1+l_1}\right) - 2\log(2) \quad (3e)$$

$$R_1 + R_2 \leq \log\left(\frac{1+C}{1+\frac{C}{1+l_1}+Cx}\right) + \log\left(1 + l_2 + S_1x + \frac{S_1}{1+l_1}\right) + \log\left(1 + \frac{S_2}{1+l_2}\right) - 2\log(2) \quad (3f)$$

$$R_1 + R_2 \leq \log\left(\frac{1+C}{1+\frac{C}{1+l_1}+Cx}\right) + \log\left(1 + \frac{S_1}{1+l_1}\right) + \log(1 + S_2 + l_1x) - 2\log(2) \quad (3g)$$

$$R_1 + R_2 \leq \log\left(\frac{1+C}{1+\frac{C}{1+l_1}+Cx}\right) + \log\left(1 + \frac{S_1}{1+l_1} + l_2\right) + \log\left(1 + \frac{S_2}{1+l_2} + l_1x\right) - 2\log(2) \quad (3h)$$

$$2R_1 + R_2 \leq \log\left(\frac{1+C}{1+\frac{C}{1+l_1}+Cx}\right) + \log\left(1 + \frac{S_1}{1+l_1}\right) + \log(1 + S_1 + l_2) + \log\left(1 + \frac{S_2}{1+l_2} + l_1x\right) - 3\log(2) \quad (3i)$$

$$2R_1 + R_2 \leq 2\log\left(\frac{1+C}{1+\frac{C}{1+l_1}+Cx}\right) + \log\left(1 + \frac{S_1}{1+l_1}\right) + \log\left(1 + l_2 + S_1x + \frac{S_1}{1+l_1}\right) + \log\left(1 + \frac{S_2}{1+l_2} + l_1x\right) - 3\log(2) \quad (3j)$$

$$R_1 + 2R_2 \leq \log\left(1 + \frac{S_1}{1+l_1} + l_2\right) + \log\left(1 + \frac{S_2}{1+l_2}\right) + \log(1 + S_2 + l_1) - 3\log(2) \quad (3k)$$

$$R_1 + 2R_2 \leq \log\left(\frac{1+C}{1+\frac{C}{1+l_1}+Cx}\right) + \log\left(1 + \frac{S_1}{1+l_1} + l_2\right) + \log\left(1 + \frac{S_2}{1+l_2}\right) + \log(1 + S_2 + l_1x) - 3\log(2) \quad (3l)$$

achievable region, here we set them to

$$P_{T_2} = \frac{1}{1+l_2}, \quad P_{U_2} = \frac{l_2}{1+l_2}, \quad P_{T_1} = \frac{1}{1+l_1},$$

$$P_{U_1} = x, \quad P_{V_1} = \frac{l_1}{1+l_1} - x, \quad x \in \left[0, \frac{l_1}{1+l_1}\right],$$

where the powers of the private messages (conveyed by  $T_1$  and  $T_2$ ) are chosen such that they are received below the noise level at the non-intended receiver [7].

In the next Section we will show how the free parameter  $x \in \left[0, \frac{l_1}{1+l_1}\right]$ , representing the power split among the two common messages of  $T \times 1$  carried by  $U_1$  and  $V_1$ , can be chosen in order for the outer bound in (1) and the lower bound in (3) to be at a constant number of bits apart from one another in some parameter regimes.

### C. Capacity to within a constant gap

For sake of space, we focus now on the *symmetric* G-IC-UGF defined (see Fig. 1) as

$$S_1 = S_2 = S, \quad l_1 = l_2 = l. \quad (4)$$

Our main result is

**Theorem 1.** *The outer bound in (1) and the lower bound in (3) are at most 4 bits/user apart for  $l < S$  and  $C \leq \max\{\frac{S}{l}, 1\}$ .*

The rest of the section sketches the proof steps. In order to highlight the key insights of our analysis, we will compare the gDoF region of several channel models. For some  $S \geq 1$ , let

$$l = S^\alpha : \alpha \geq 0, \quad C = S^\beta : \beta \geq 0, \quad (5)$$

where  $\alpha$  is referred as the *interference exponent* and  $\beta$  as the *cooperation exponent*. With (5) we define the gDoF for the  $k$ -th user,  $k \in \{1, 2\}$  as  $d_k := \lim_{S \rightarrow +\infty} \frac{R_k}{\log(1+S)}$ .

In [6], we characterized the capacity of the symmetric G-IC-UGF to within a constant gap, and hence its gDoF, in strong interference  $\alpha \geq 1$  and in ‘sufficiently’ strong cooperation  $\beta \geq \alpha + 1$ . In [6], we conjectured that, for the remaining cases of the weak interference regime ( $\alpha < 1$  and  $\beta < \alpha + 1$ ), outer bounds of the type  $2R_1 + R_2 / R_1 + 2R_2$  are needed. These bounds have been recently derived in [8]. Here we focus on the regime left open in [6] and pursue a characterization (to within a constant gap) of the capacity for  $\alpha < 1$  and  $\beta < \max\{1 - \alpha, \alpha\}$ . We believe that in the remaining regimes, where the cooperation link is ‘sufficiently’ strong, a superposition based scheme as in (3) is insufficient and binning is actually needed (since in this case the G-IC-UGF behaves more like the G-CIC rather than the classical G-IC).

*Regime 1:*  $l < S$ ,  $C \leq \frac{l^2}{S}$ , which implies  $\alpha < 1$ ,  $\beta \leq [2\alpha - 1]^+$ . For this set of parameters, cooperation is quite weak and we expect the G-IC-UGF to behave as the non-cooperative G-IC [7]. Therefore, we set the power of the cooperative common message  $V_1$  to zero in (3), i.e.,  $x = \frac{l}{1+l}$ . With this choice, by simple but tedious computations, it can be shown that the outer bound region in (1) and the inner bound region in (3) (which reduces to the one for the non-cooperative G-IC in [7]) are at most 2.5 bits/user far from one another. In this regime it might not be worth engaging in cooperation since cooperation can at most increase the non-cooperative capacity region by 2.5 bits/user. This implies that the G-IC-UGF in this regime

has the same gDoF region as the non-cooperative G-IC [7].

*Regime 2:*  $1 < S$ ,  $\frac{1^2}{S} < C \leq \max\{\frac{S}{1}, 1\}$ , which implies  $\alpha < 1$ ,  $2 - \alpha < \beta \leq \max\{1 - \alpha, \alpha\}$ . For this set of parameters, cooperation is quite strong and we expect that the G-IC-UGF benefits from cooperation. Therefore the cooperative common message carried by  $V_1$  is necessary to boost the performance. We set the power of the common non-cooperative message to  $x = \frac{1}{1 + \min\{C, 1\}}$ . This choice is motivated by the fact that, in order to approximately match the outer bound, the single rate constraint on  $R_1$  must behave gDoF-wise as an interference-free point-to-point channel, i.e.,  $d_1 \leq 1$  (i.e., in this regime  $d_1 \leq \max\{1, \min\{\alpha, \beta\}\} = 1$ ). Therefore, the fact that Tx2 can now decode part of the message of Tx1 (carried by  $V_1$ ) must not limit (up to a constant gap) the performance of the first user. In other words, since  $C$  is ‘quite large’ but not ‘huge’, the rate of  $V_1$  cannot be too large. By evaluating the rate region in (3) for  $x = \frac{1}{1 + \min\{C, 1\}}$ , it can be shown that it is at most 4 bits/user away from the outer bound region in (1).

#### D. Comparisons

Fig. 2 shows the gDoF region of the G-IC-UGF for a choice of channel parameters within the regime covered by Theorem 1. In particular we choose  $\alpha = 0.5$ , or  $1 = \sqrt{S}$ , for which time-sharing among the two users is optimal to within 1 bit/user for the non-cooperative G-IC [7], or in other words, the gDoF region is a triangle. We observe that for  $\beta = 0.3$ , or  $C \approx \sqrt[3]{S}$ , the G-IC-UGF region (blue curve) is strictly larger than the non-cooperative G-IC region (black curve), but strictly smaller than the G-CIC region (red curve) [5]. In terms of sum-gDoF we have  $(d_1 + d_2)^{G-IC} = 2(1 - \alpha) = 1.0$ ,  $(d_1 + d_2)^{G-IC-UGF} = 2(1 - \alpha) + \beta = 1.3$ ,  $(d_1 + d_2)^{G-CIC} = 2 - \alpha = 1.5$ , implying that the sum-capacity of the G-IC-UGF is about 30% larger than that of the G-IC. Notice that *both* users benefit for UGF, thus providing a strong incentive for cooperation.

From Fig. 2, we also notice that the bound in (1h) is active, but not the one in (1g). In general, in weak interference ( $\alpha < 1$ ) the bounds in (1g)-(1h) are both active when  $\beta \leq [2\alpha - 1]^+$  (regime 1 discussed above), since the G-IC-UGF gDoF-wise behaves as the G-IC, whose capacity is also characterized by this kind of bounds [7]. For the regime  $2 - \alpha < \beta \leq \max\{1 - \alpha, \alpha\}$  (regime 2 discussed above), instead, only the bound in (1h) is active. This is because the cooperation link is still not strong enough to enable sufficient coordination among the sources.

#### IV. CONCLUSIONS

In this work we studied the G-IC-UGF where, differently from the non-cooperative G-IC, one source overhears the other source through a noisy in-band link. We characterized the capacity of this channel to within a constant gap for a set of parameters which fall in the weak interference regime. For this set of parameters, the gDoF of the G-IC-UGF was compared to those of the non-cooperative G-IC and of the ideal G-CIC to highlight when cooperation might or might not be worth implementing in practical systems.

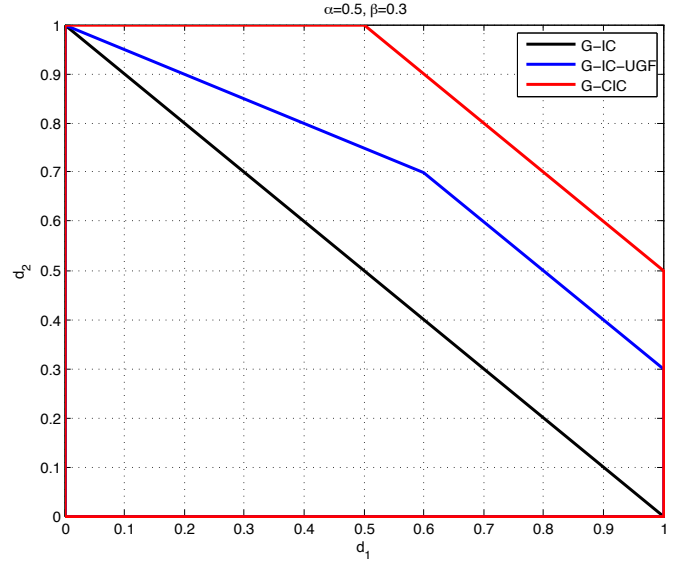


Fig. 2: gDoF regions for the G-IC, the G-IC-UGF and the G-CIC for  $\alpha = 0.5$  and  $\beta = 0.3$ .

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