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Transmitter and Receiver Design for Multi-Antenna Interfering Systems

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Abstract

In modern wireless communication systems, the per-user data rate demand is constantly growing. In addition the rapid evolution of mobile services accentuates this evolution. To sustain the heavy user data rate demand network operators try to deploy cellular system with more cells and applying more efficient spectrum reuse techniques. One possible solution to increase system throughput is to get the user closer to the transmitting base station and hence deploy very dense network infrastructure. A possible way to reduce the costs of deploying such a dense network is to allow users to deploy their own base station with the objective to achieve high data rate for in-home coverage. Those kind of unregulated small cells are usually called *femto cell*, refer to [1] and references therein. Such small cells will not be isolated, their coverage areas will overlap in many situations and in addition they will be deployed under the macro base station coverage area that has the objective of serving high mobility users. In this setup strong interference situations will result. Interference has been commonly identified as the main bottleneck of the modern wireless cellular communications systems. With small dense cells this is more the case. This consideration has led to intense research activities that has recently pushed network operators and manufacturers to include coordination and interference management techniques in new communication standards for a more proactive and efficient way to suppress/control interference. From an information theoretic point of view this problem can be mathematically studied as, what is called, an interference channel.

In the first part of this thesis, we focus our attention on the beamforming design for the interference channel. We first study the MISO case where a new algorithm for beamforming design under quality of service (QoS) constraints is introduced. Then we study the case where each user in the network is equipped with multiple antennas. We introduce the concept of Interference Alignment (IA) focusing in particular on the problem of existence of an interference alignment solution. Then we move to the joint optimization of transmitter and receiver to maximize the weighted sum rate (WSR) for a MIMO interference channel proposing a new algorithm based on the relationship between the minimization of the weighted sum

mean squared error (WSMSE) and WSR maximization. Then a robust version of the proposed algorithm is introduced where the expected WSMSE is minimized when the channel uncertainties are modeled with channel mean and covariance, also called stochastic channel state information (CSI). Finally the problem of CSI acquisition is handled proposing a transmission strategy that allows each user to get the necessary CSI in a distributed or centralized way.

Since femtocells are user deployed, without any planning from the operators differently from *picocells*, these devices should be characterized by a high level of *cognition* that should allow them to learn the environment and determine the best transmission configuration to maximize the femtocell throughput while reducing the interference generated to surrounding macro cell communications. For the reasons described above the problem of femto/macro interaction has been recently studied under the cognitive radio framework.

The second part of the thesis is devoted to the beamforming design problem in cognitive radio settings. We start by considering an underlay scenario where the the secondary network is modeled as a MISO interference channel. The secondary beamformers are optimized such that the total transmitted power is minimized controlling the interference generated to the primary receivers. An iterative algorithm, based on uplink downlink duality, is introduced to solve the problem. Then we move to the interweave cognitive radio setting where now all the devices are multi-antenna terminals. There the objective is to design the transmitters and receivers, at the secondary network, such that the interference, generated at each primary receiver, is zero. This is done by exploiting the spatial dimensions available at each terminal. We first show that time division duplexing (TDD) is the key to realize such a scenario in practice without requiring cooperation between primary and secondary users. To exploit the channel reciprocity of a TDD communication system, calibration is required but we show that this can be done without any cooperation between primary and secondary users. Extensions to multiple primary and/or secondary users are also studied. Finally we consider the problem of interference alignment in a spatial interweave cognitive setting where the interference constraints are given as rank constraints. An iterative algorithm for transmit and receive filters design for the secondary network is introduced. Finally we study the existence of such an interference alignment solution in the given cognitive radio setting.

Résumé

Dans des systèmes de communication sans fil modernes, la demande de débit de transmission des données par utilisateur est en croissance constante. Pour soutenir la forte demande de débit de données des utilisateurs, les opérateurs de réseaux essaient de déployer un système cellulaire avec davantage de cellules et appliquent des techniques plus efficaces de réutilisation du spectre. Une solution possible à l augmentation du débit du système est de rapprocher l'utilisateur de la station de base émettrice et donc déployer une infrastructure réseau très dense. Dans cette configuration nous obtenons de fortes interférences. Linterférence a été souvent identifiée comme le principal obstacle des systèmes modernes de communications sans fil cellulaires. Avec des petites cellules denses ceci nest plus le cas. Cette considération a conduit à d'intenses activités de recherche qui a récemment poussé les opérateurs de réseaux et les fabricants à inclure de manière plus proactive et efficace pour supprimer/contrler les interférences. D'un point de vue théorie de l'information, ce problème peut tre mathématiquement étudié comme, ce qui est appelé, un canal d'interférence.

Dans la première partie de cette thèse, nous concentrons notre attention à la conception de l'émetteur pour le canal d'interférence avec un accent particulier sur le cas MIMO. Nous proposons l'optimisation conjointe de l'émetteur et du récepteur en fonction de deux critères: l'alignement des interférences et la maximisation la somme pondérée des débits. La deuxième partie de la thèse est consacrée au problème de conception de l'émetteur dans le scénario de la radio cognitive. Nous commenons à considérer un scénario Underlay o le réseau secondaire est modelé comme un canal d'interférence MISO. Ensuite, nous passons au scénario Interweave radio cognitive o maintenant tous les appareils sont des terminaux avec antennes multiples. L'objectif est de concevoir les émetteurs et les récepteurs, au niveau du réseau secondaire, telle que l'interférence, générée à chaque récepteur principal, est égal à zéro.

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Contents

List of Figures

Acronyms

Here are the main acronyms used in this document. The meaning of an acronym is usually indicated when it first occurs in the text.

AWGN	Additive White Gaussian Noise
BC	Broadcast Channel
BF	Beamformer
BS	Base station
CR	Cognitive Radio
CSI	Channel State Information
CSIR	Channel State Information at Receiver
CSIT	Channel State Information at Transmitter
DL	Downlink
DoF	Degrees of Freedom
FDD	Frequency Division Duplex
IA	Interference Alignment
IFC	Interference channel
iid	independent identically distributed
MAC	Multiple Access Channel
MSE	Mean Square Error
MMSE	Minimum Mean Square Error
MIMO	Multi-Input Multi-Output
MISO	Multi-Input Single-Output
MU	Mobile User
PBS	Primary Base Station
PMU	Primary Mobile User
PN	Primary Network
PU	Primary User
Rx	Receiver
SIMO	Single-Input Multi-Output
SINR	Signal-to-Interference-Noise Ratio

SISO	Single-Input Single-Output
SN	Secondary Network
SNR	Signal-to-Noise Ratio
SR	Sum Rate
SBS	Secondary Base Station
SMU	Secondary Mobile User
SVD	Singular Value Decomposition
TDD	Time Division Duplex
Tx	Transmitter
UL	Uplink
WSMSE	Weighted Sum Mean Square Error
WSR	Weighted Sum Rate
ZF	Zero Forcing

Notations

Boldface upper-case letters denote matrices, boldface lower case letters denote column vectors and lower-case denote scalars.

$\mathbb{E}_x$	Expectation operator over the r.v. x
$\lvert\mathbf{H}\rvert$	Determinant of the matrix $\mathbf{H}$
$\lfloor x \rfloor$	Floor operation, rounds the elements of x to the nearest integers towards minus infinity
$\lceil x \rceil$	Ceiling operation, rounds the elements of x to the nearest integers towards infinity
$\lvert x\rvert$	Absolute value of x
$\mathbf{H}^*$	Conjugate operation
$\mathbf{H}^H$	Hermitian operation
$\mathbf{H}^\mathrm{T}$	Transpose operation
$\mathbf{H}^{-1}$	Inverse operation
$\mathbb{R}$	Set of real numbers
$\mathbf{H}$	matrix
$\mathbf{h}$	vector
h	scalar
$\mathrm{Tr}\{\mathbf{H}\}$	trace of matrix $\mathbf{H}$
$\mathcal{CN}$	Complex Normal Distribution
$\mathrm{Re}\{\mathrm{a}\}$	Real part of the complex number a
$\mathrm{Im}\{\mathrm{a}\}$	Imaginary part of the complex number a
K	Number of users in the network

Chapter 1

Introduction

Traditional wireless communication systems are designed such that the coverage area is divided in zones called *cells*. In each cell a base station (BS) handles the communication for the users that are in the corresponding cell. To avoid or reduce interference generated from communication in neighboring cells a frequency reuse pattern has been introduced [2]. This approach to handle interference prevents the reuse of a spectral resource within a set of cells called *cluster*. The interference reduction obtained with a frequency reuse factor comes at the price of a reduced spectral efficiency. For this reason in next generation cellular wireless communication standards, e.g. Code Division Multiple Access (CDMA), a frequency reuse factor of one has been used.

Frequency reuse factor of one causes, on the other hand, a drastic reduction of the network capacity due to the increase of the out of cell interference. The performances of the cell-edge users are seriously affected by this aggressive frequency reuse pattern due to the increment of the inter-cell interference that these users experience. To handle this problem current communication systems include different interference management solutions. Even if interference coming from out-of-cell transmission can be reduced using careful planning or introducing little cooperation among neighboring cells, such as smart user scheduling or soft handover, these techniques are sometimes not enough to guarantee high throughput to cell-edge user. For that reason major standardization bodies are now including explicit interference coordination strategies in next generation cellular communication standards. For example, in future releases of cellular communication standard called

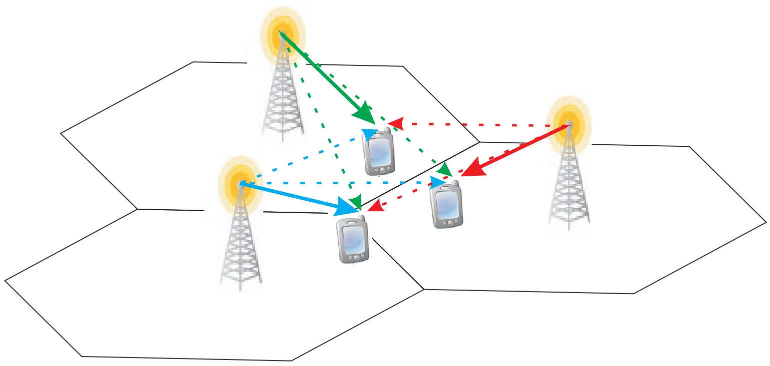

Figure 1.1: Cell-edge users problem representation

Long Term Evolution Advanced (LTE-A) those techniques are grouped in what is called *Coordinated Multipoint* transmission and reception techniques [3]. These techniques are based on more interference-aware base station cooperation. The most recent evolutions of this particular cooperative communication technique is the so called *Network* or *virtual* MIMO (Multiple-Input-Multiple-Output), where the main concept is to introduce a deeper collaboration between neighboring BSs such that each user is served by multiple BSs. This scenario can be thought as a distributed MIMO broadcast (BC) channel. For comprehensive introduction on recent results on this topic please refer to [4]. To achieve this result all the BSs should be connected to a centralized processing/control unit because full cooperation at the signal-level is required, in particular all the BSs should be aware of all the messages destined to all users in the network. These cooperative techniques, in their implementation in LTE-A, have been shown to bring a significant improvement in spectral efficiency for cell-edge users, while the resulting gain for full cell coverage is almost negligible [5]. Although very useful, at least to improve the cell-edge users performance, these techniques introduce some challenges in real systems. Realizing the required cooperation and coordination among different BSs poses different problems in real systems with limited backhaul capacity and finite latency.

1.1 Interference channel: Overview

A different way of looking at the cell-edge users problem is to mathematically describe the setting as a K-user interference channel. In this system model K pairs of transmit and receive devices transmit in the same frequency resource. Each transmitter wishes to communicate only to the corresponding receiver, then each communication generates interference to the $K-1$ non intended receivers. This system model differs from the Network MIMO approach because the level of cooperation between transmitters stops at the channel state information (CSI). Hence less signaling is required between BSs. In particular, according to the used transmission technique, different degrees of channel knowledge are exchanged between transmitters.

Interference channel has been the focus of intense research over the past few decades, starting from the celebrated paper by Carleial [6]. From an information-theoretic point of view its capacity region, intended as all the possible rate tuples that can be simultaneously achieved by all users, in general remains an open problem and is not well understood even for simple cases. In [7] the counterintuitive result that if the interference is strong enough (called strong interference regime) the interference does not limit the performances of a two user interference channel is reported. This shows that exploiting the interference instead of treating it as noise is the optimal strategy. The other known results is that treating the interference as noise is optimal in the weak interference regime. In [8] the authors show that even for the $2-$users system, the most studied case, to achieve the system capacity within one bit very complicated transmission schemes are required, that should be adapted to the particular interference regime of the system. To achieve this result the author use Han-Kobayashi [9] type scheme. This coding scheme is based on splitting the transmitted information of both users in a *private message*, that can be decoded only at the intended receiver, and a *common message*, that can be decoded at both receivers. The key innovation here is modulating the power of the private message such that the corresponding received signal is at the level of noise. In this way the interference generated at the non intended receiver can be neglected.

1.1.1 MISO Interference Channel

With the introduction of multiple antennas at the receiver, the so called single-input-multiple-output (SIMO) systems, it is possible to increase the achieved capacity [2], if the receiver has proper channel knowledge (CSIR). This results is due to the power gain achieved by combining all the received signal from all the receiving antennas. A similar result can be obtained if the transmitter is equipped with multiple antennas, system called multiple-input-single-output (MISO). In this case if the transmitter has channel state information (CSIT) then a power gain is achieved also for MISO systems. These simple results can be also extended to more complex systems where a transmitter wants to communicate with multiple receivers at the same time [10]. Also the capacity of an interference channel has been investigated when either the transmitter or the receiver is equipped with multiple antennas. For example in [11] the capacity of a two user MISO/SIMO interference channel is studied providing the capacity region for a class of MISO IFC in the strong interference regime. A new outer bound is also provided for a general MISO IFC, but the capacity for a more general interference channels, with arbitrary number of users, is still an open problem. Then, more practical approaches were introduced for system performance optimization using linear transmitters and receivers. In [12, 13] the beamformers for a K-user MISO IFC are determined to minimize the total transmit power imposing a set of per user Quality of Service (QoS) requirements at each receiver. [14, 15] address the similar problem of maximizing the minimum Signal to Interference plus Noise ratio (*max min SINR*) for the MISO IFC.

In [16, 17, 18] distributed solutions for the BF design problem are investigated, where the main objective is to reduce the signaling exchange between pairs of users. Some of the techniques use concepts from game theory to describe the proposed algorithms.

A different line of research can be found in [19, 20, 21, 22] where the objective is the characterization of the rate region of a MISO IFC where linear processing is used at the transmit side. The region under investigation is defined as the set of rate tuples that can be simultaneously achieved by the transmitting pairs. The main focus of this analysis is the definition of the Pareto boundary of the capacity region, defined as the set of points where the performance of one user can not be incremented without reducing the performance of other users.

1.1.2 MIMO Interference Channel

With the discovery that using multiple antennas at both transmitter and receiver can bring a significant increase of the system throughput [23], multiple-input-multiple-output (MIMO) communication has been widely applied to all communication systems, including the interference channel.

Interference Alignment

As we have already seen the problem of finding the capacity of an interference channel is a difficult problem that has not been completely solved yet. The problem becomes even more complicated with the introduction of MIMO pairs in the interfering network. To simplify the problem a different approach has been recently introduced. The focus now becomes the rate approximation at high signal-to-noise-ratio (SNR). In that regime the sum rate rate curve can be completely described using the prelog factor, also called degrees of freedom (DoF):

$$C(\rho) = d \log(\rho) + o(\log(\rho))$$

where $C(\rho)$ represents the sum capacity, ρ is the SNR and d is the pre-log factor. It can be interpreted as the number of interference free dimensions available in the system. It can also be defined as:

$$d = \lim_{\rho\to\infty} \frac{C(\rho)}{\log(\rho)}$$

It was introduced in [24] for a single user MIMO link and it became immediately instrumental also for more complex systems. For a 2-user MIMO IFC the achievable DoF was studied in [25], for interference channel with more users the use of *Interference Alignment* (IA) becomes instrumental [26, 27]. In [28] the authors have demonstrated the achievability of a capacity prelog factor of $K/2$ in a K-user SISO interference channel, then half the DoF of an interference free network can be achieved. The key idea behind interference alignment is to process the transmit signal (data streams) at each transmitter, so as to align all the undesired signals at each receiver in a subspace of suitable dimensions. The MIMO interference channel is more difficult to handle and some recent results on DoF for this case are reported in [29, 30]. Even though IA has the promising property to maximize the DoF, a closed form expression for the beamforming filters is not known in general. In [31, 32] a solution is proposed for K-user MIMO IFC where each pair of users is equipped with $N = K - 1$ antennas. To find an IA solution for more general system configurations iterative algorithms should be used [33, 34, 35, 36], where different cost functions are used to determine a set of IA beamformers using numerical solutions. These algorithms can be also used to evaluate the existence of an IA solution through simulations. The existence of an IA solution for MIMO IFC has been studied in several papers [37, 38, 39] where different sets of conditions are to be satisfied by a K-user MIMO IFC to admit an IA solution.

Sum Rate Maximization

The objective of IA transmission is to maximize the DoF that represents a good approximation of the rate curve at high SNR. The same concept cannot be applied at medium and low SNR regimes, for this reason IA manifests poor performances in these SNR regimes. Hence different approaches have been proposed to design the transmit and receive filters in a K-user MIMO IFC. One possible approach is the maximization of the sum rate. In the seminal work [40] the authors noted that the network capacity in general is neither a convex nor concave function of the transmit covariance matrices and hence its optimization is a difficult problem. The game theoretic approach was used in [41] to study the MIMO IFC modeling the problem as a non-cooperative game. The proposed solution is proved to achieve a Nash equilibrium but this point can be very far from the optimal sum rate point.

The weighted sum rate (WSR) maximization problem has been studied in some recent papers [42, 43, 44]. In [42] the single stream MIMO IFC is studied, proposing an iterative algorithm for the maximization of the WSR. A different approach is used in [43] where the problem is solved using second-order-cone-programming (SOCP). Finally in [44] the WSR maximization is achieved, in an interfering broadcast channel, extending the results proposed for a BC in [45]. The solution relies on the connection between the WSR maximization and the minimization of the weighted sum mean squared error (WSMSE).

Channel State Information Acquisition

To determine a set of beamformers that maximizes the DoF at high SNR, using IA, or to maximize the sum rate, using the approaches described above, different forms of channel state information (CSI) are required. In most of the cases CSI at both terminals, transmitter and receiver, is needed to achieve the proper joint design of the transmit and receive filters. This is usually acquired using some training and feedback phases between transmitters and receivers. The problem of how feedback influences the IA beamforming design was studied in [46, 47, 48]. In [46, 47], using quantized channel feedback, it is shown that full multiplexing gain can be achieved if the feedback bit rate scales sufficiently fast with the SNR. The authors of [48] introduce analog feedback for the acquisition of full CSIT. They show that using analog feedback, for acquisition of CSIT and IA beamforming design, incurs no loss of multiplexing gain if the feedback power scales with the SNR. In [49] a staggered block fading channel model is the only assumption required to achieve IA. The resulting multiplexing gain is much lower however than for the case of full CSI. These techniques are now known by the terms *delayed CSIT* (DCSIT) or *retrospective IA*. The problem of studying the maximum DoF achievable using DCSIT has recently attracted a lot of research effort. [50, 51] introduced a new transmission protocol that maximizes the achievable DoF in a BC channel. In [52] the authors extend the results from [50] to the two user MISO IFC.

1.2 Cognitive Radio

Spectrum regulatory bodies, since their foundation at the beginning of the 20th century, have allocated portions of the frequency spectrum to different wireless services in a fixed and static way. This has been done with the objective of avoiding/reducing the possibility to generate interference. With the rapid growth of wireless services the fixed frequency allocation policy, used until now, has been demonstrated to be very inefficient in term of spectral utilization. In addition al-

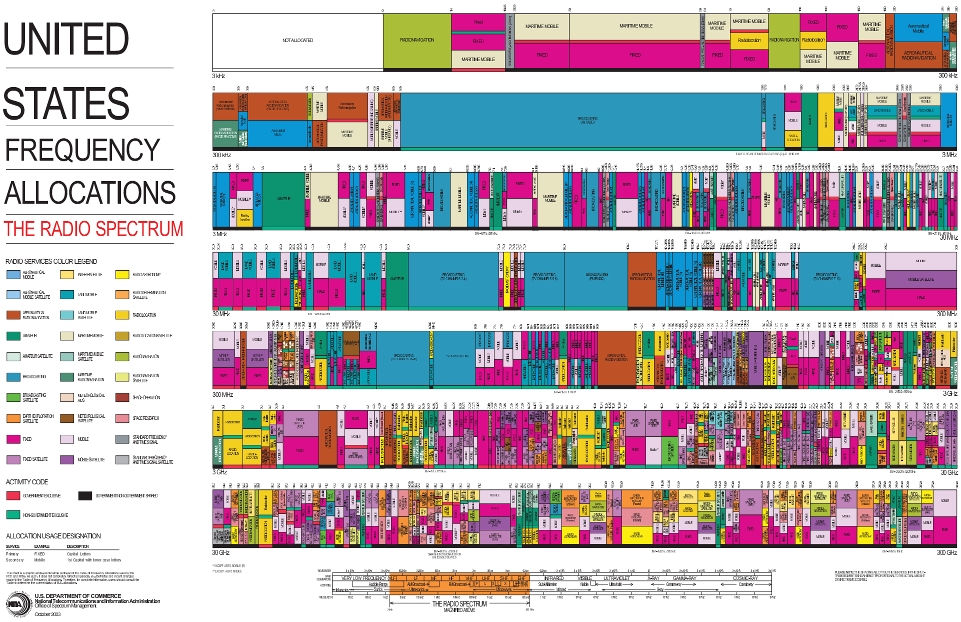

Figure 1.2: US frequency allocation chart, *www.ntia.doc.gov/osmhome/allochrt.pdf.*

most all of the frequency bands have been already assigned, Fig. 11.2. The consequent spectrum scarcity has a significant effect on wireless communication service providers since nowadays the frequency bands are assigned to the highest bidder in public auctions, then the frequency acquisition represents one of the most important cost for operators. In a recent measurement campaign [53], undertaken by Federal Communications Commission (FCC) in the US, has shown that the spectrum usage is typically concentrated over certain frequency bands, while a significant amount of the licensed bands remains unused or underutilized for 90% of time. This problem has inspired the seminal work in [54] where the concept of *Cognitive radio* (CR) has been introduced. According to this new communication paradigm, further developed in [55], a cognitive radio system is defined as a set of intelligent devices that are aware of the surrounding environment adapting their communication parameters with the objective of a reliable communication and a more efficient spectrum utilization. The most common CR scenario comprises of a set of secondary users, that represent the cognitive users, that want to coexist with a set of primary users, the legacy spectrum holders. The most important feature of the cognitive devices, as the name suggests, is the ability to learn the environment and react properly. This problem gave birth to an intense line of research where the main objective is to study how it is possible to understand if in a given frequency band a transmission is taking place or not. This goes under the name of spectrum sensing, refer to [56] and reference therein for a comprehensive review of major contributions. One of the first attempt to make the CR principles a reality was the IEEE standard 802.22, that had the objective to use TV white spaces to develop a communicating system for wireless regional area networks (WRANs). In 2009 a new standard proposal, IEEE 802.11af, considered to modify both the 802.11 PHY and MAC layers to use TV white space. For additional information on CR standards refer to [57].

From an information theoretic point of view, different cognitive radio communication paradigms have been introduced according to the amount of information exchanged between primary and secondary users and the constraints imposed on the secondary communication. In [58] the following cognitive radio communication settings are introduced: *Overlay*, *Underlay* and *Interweave*.

Overlay Paradigm

Overlay CR is a cooperative technique in which the secondary signals are designed to offset any degradation they may cause to primary communications, requiring a shared knowledge of the codebooks and modulation schemes. With this additional information some form of asymmetric cooperation can be established. For example the opportunistic user can split its rate dedicating part of his available transmit power to broadcast also the message designated to the incumbent receiver. With the remaining resources he transmits his message for private communication with the secondary receiver. Other encoding strategies [58] can be used to settle an overlay communication like dirty paper coding (DPC) or rate splitting. In this scenario primary communication is not harmed or could even by improved as a result of relaying gain. This CR setting can be also read as some combination of broadcast and interference channels with degraded message sets [59]. Even though the overlay CR is the most studied CR setting from an information theoretic point of view, the capacity of such a system is still not known in general. It is only known in some particular regimes. In [60] *weak interference* regime is studied, the authors showed that in this regime, where the link between the cognitive Tx and primary Rx is weak, the capacity of the overlay CR channel is achieved using a combination of DPC and superposition coding. The cognitive user exploits the knowledge of the primary message to encode its own message in such a way that it is received at the cognitive receiver interference free. At the same time using superposition coding, it uses part of its available power to convey also the primary message and the remaining power is used for cognitive transmission. In the opposite regime, *Strong interference* seen at both receivers, [61] found that the capacity of the CR channel is achieved using superposition coding at the cognitive transmitter.

Underlay Paradigm

Underlay CR allows coexistence of a primary (usually licensed) network and a secondary (cognitive) one, constraining interference caused by secondary transmitters on primary receivers to be under a certain threshold, usually called *Interference temperature constraint* [55]. In order to attain these interference constraints different techniques can be used ranging from coding methods up to the use of the spatial dimension (multiantenna systems). The problem of studying the capacity region, of different communicating systems, constraining the received power to some users was explored in [62], these constraints significantly change the structure of the problem. In [63] an underlay cognitive radio setting is studied in fading environments. It is shown that a significant capacity gain can be achieved by the opportunistic user in channels affected by severe fading, because the probability that the cross link primary-secondary to be in fade is non negligible end hence the secondary system can achieve a substantial rate increasing its transmitted power without interfering significantly with the primary communication. In the underlay CR paradigm constraining the interference at the primary receivers is the main objective of the cognitive transmitters. Providing the cognitive users with multiple antennas enhances the capability of controlling the interference generated at the primary receivers, for this reason the beamforming design problem in underlay cognitive systems has been the focus of intense research in recent years. [64] studies the problem of maximizing the secondary user's rate controlling the interference caused at the primary receivers. A different line of research focuses on satisfying a minimum quality of service requirement at the cognitive users in an underlay scenario [65, 66, 67]. There the secondary network is always modeled as a BC channel that wants to communicate in the presence of a set of primary receivers. In [68, 69] the objective was to optimize the sum rate of the secondary network, modeled as an interference channel, under received interference power constraints at primary users.

Interweave Paradigm

Finally, Interweave (IW) CR exploits the unused communication resources, called *white spaces*, of the primary system in an opportunistic fashion. In this communication paradigm, secondary transmission can take place only if it does not cause any interference to the primary user. The unused primary resources can be time, frequency or, as recently introduced, space.

The cognitive radio problem has been studied also from a game theoretic perspective in [70], there the authors proposed a decentralized algorithm, based on iterative water filling, to maximize the secondary system's performances. A deep analytical description under the game theoretic framework is also provided. In [71] a detailed overview on game theory and its application to CR problem is provided.

In this communication paradigm the use of multiple antennas is even more beneficial than the underlay setting. One of the first paper studying the spatial dimension in CR systems was [64]. Some attempts to make CR practical can be found in [72, 73]. The authors propose an initial transmission scheme where the primary communication is exploited in order to learn the environment and properly design the secondary users' beamformers. In the proposed analysis the secondary channel estimation errors are taken into account in the secondary BF design. The interference caused at the secondary receiver, due to primary communication, is reduced introducing a proper receive filter at secondary receivers.

In [74] a new approach to setup a cognitive transmission has been proposed for frequency selective channels. The authors proposed to apply a Vandermonde precoder as transmit filter at the cognitive user, for this reason it is called Vandermonde Frequency Division Multiplexing (VFDM). The Vandermonde precoder is constructed using the L roots of the L-tap channel that connects the cognitive transmitter with the primary receiver. With this transmitter the interference to the primary receiver is completely zeroforced. This approach has the advantage that no cooperation is required between primary and secondary to setup an interweave cognitive radio communication.

1.3 Thesis Outline and Contributions

This thesis is divided in two main parts. Part I focuses on the interference channel, where we first study the beamformer design problem in a MISO interference channel introducing some duality principles, that can be thought of as an extension to the IFC of the results obtained for the broadcast channel. Then the problem of $\max\min$SINR beamformer design is addressed. In the following chapters , introducing more antennas also at the receiver side we study the problem of joint design of transmit-receive filters in MIMO interference channel. We study interference alignment beamformers design, with particular focus on feasibility analysis, and on weighted sum rate maximization. Finally the problem of channel state information acquisition at both transmit and receive side, for solving the previously introduced problems, is studied using analog feedback.

Part II deals with cognitive radio scenarios. At first we study the problem of beamforming design in MISO underlay cognitive interference channel solving the problem of power minimization under per user power constraints and limiting the maximum amount of interference generated at primary users. Then in the following chapters we introduce the concept of *Spatial Interweave*. In chapter 8 we describe all the transmission phases required to opportunistically design the secondary beamformer in TDD communications. To exploit channel reciprocity, due to TDD transmission, we also consider the calibration problem and how this additional operation influences the design problem. We discover that calibration between non cooperative users is not required implying that spatial interweave CR setting is possible in practice without any cooperation between primary and secondary users. The simple setting with one primary and one secondary pair is extended to multiple secondary pairs and primary receiver in chapter 9. In this chapter the IA design problem is studied in a CR setting where the feasibility problem is also introduced and studied providing a set of feasibility conditions.

In the following paragraphs we give a brief overview of the thesis describing the content of different chapters underlining their contributions.

Chapter 2 - MISO Interference Channel

In this chapter we start introducing some Uplink-Downlink (UL-DL) duality principles, initially introduced for the BC channel, adapting them to the MISO interference channel. Then UL-DL duality is used for the solution of the weighted SINR (WSINR) balancing problem for MISO IFC with individual power constraints. We introduce a new iterative algorithm that solves the WSINR balancing problem when only one power constraint is active. Then we propose an iterative algorithm that solves the problem in a decentralized manner when nothing can be said on the number of active power constraints. The algorithm solves the problem using a sequence of power minimization problems with a proper set of QoS constraints. The research contributions of this chapter have been published in

- F. Negro, M. Cardone, I. Ghauri, and D. T. M. Slock, ”SINR balancing and beamforming for the MISO interference channel,” *in Personal Indoor and Mobile Radio Communications (PIMRC), 2011 IEEE 22st International Symposium on, Sept. 2011.*
- F. Negro, I. Ghauri, and D. T. M. Slock, ”On duality in the MISO interference channel,” *in Signals, Systems and Computers (ASILOMAR), 2010 Conference Record of the Forty Fourth Asilomar Conference on, Nov. 2010, pp. 2104 -2108.*

Chapter 3 - Interference Alignment Feasibility for MIMO interference channel

The focus of this chapter is the feasibility study of interference alignment solutions for constant coefficients MIMO interference channel. We first introduce the general system model of a K-user MIMO IFC that will also be used in following chapters. Then we provide a systematic method to check feasibility of IA solutions for an arbitrary DoF allocation. We validate the proposed approach using some numerical examples, comparing the result of our feasibility check with the convergence property of an iterative algorithm for determining IA solutions. We discuss interference alignment duality and the interpretation of IA as a constraint compressed SVD. The results presented in this chapter are also published in the following papers:

- F. Negro, S. Shenoy, D. T. M. Slock, and I. Ghauri, ”Interference alignment limits for K-User frequency-flat MIMO interference channels,” *in Proc. European Signal Proc. Conf. (Eusipco), Glasgow, Scotland, Aug. 2009.*

- F. Negro, S. P. Shenoy, I. Ghauri, and D. T. M. Slock, ”Interference alignment feasibility in constant coefficients MIMO interference channel,” *in Proc. 11th IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC 2010), June 2010.*

- F. Negro, I. Ghauri, and D. T. M. Slock, ”Deterministic annealing design and analysis of the noisy MIMO interference channel,” *in Information Theory and Applications Workshop (ITA), 2011, Feb. 2011, pp. 1 -10.*

Chapter 4 - Sum rate maximization for the noisy MIMO interference channel

In this chapter we introduce the weighted sum rate maximization (WSR) problem for a MIMO interference channel. We propose a new iterative algorithm based on the extension of the relation between WSR maximization and the minimization of the weighted sum mean squared error (WMSE). Then we specify the proposed algorithm when the WSR is maximized under a per-stream approach.The per-stream approach helps us to introduce a WSR duality for the MIMO IFC where the optimal transmit filter results to be an MMSE receiver filter in a dual UL communication with a proper transmit covariance matrix and dual noise variance. To reduce the possibility to converge to local optimal solution we introduce a novel approach based on Deterministic Annealing. Finally we describe how to optimize the WSR at high SNR. Some simulation results are provided to validate the proposed algorithm numerically.

In the following papers are reported the research contributions described in this chapter:

- F. Negro, S. Shenoy, I. Ghauri, and D. T. M. Slock, ”On the MIMO interference channel,” *in Information Theory and Applications Workshop (ITA), 2010, 31 2010-Feb. 5 2010, pp. 1 -9.*

- F. Negro, S. Shenoy, I. Ghauri, and D. T. M. Slock, ”Weighted sum rate maximization in the MIMO interference channel,” *in Personal Indoor and Mobile Radio Communications (PIMRC), 2010 IEEE 21st International Symposium on, Sept. 2010, pp. 684 -689.*
- F. Negro, I. Ghauri, and D. T. M. Slock, ”Deterministic annealing design and analysis of the noisy MIMO interference channel,” *in Information Theory and Applications Workshop (ITA), 2011, Feb. 2011, pp. 1 -10.*
- F. Negro, I. Ghauri, and D. T. M. Slock, ”Optimizing the noisy MIMO interference channel at high SNR,” *in Communication, Control, and Computing (Allerton), 2010 48th Annual Allerton Conference on, 29 2010-Oct. 1 2010, pp. 254 -261.*

Chapter 5 - Sum Rate Maximization with Partial CSIT via the Expected Weighted MSE

In this part of the thesis we focus on robust beamforming design for a MIMO interfering broadcast channel with the objective of maximizing the sum rate. We assume that each transmitter has stochastic channel state information (CSI) while the receiver has perfect CSI. The solution proposed for robust beamforming design is based on the relationship between WSR and Weighted MSE (WMSE) introduced for the MIMO interference channel in chapter 4. Here the optimal beamforming filters are obtained from the minimization of the sum of average WMSE, then an iterative algorithm is introduced to solve the problem. The performance of the proposed solution is finally validated numerically. The results described in this chapter are published in:

- F. Negro, I. Ghauri, and D. T. M. Slock, ”Sum Rate maximization in the Noisy MIMO Interfering Broadcast channel with partial CSIT via the expected weighted MSE,” *in Wireless Communication Systems (ISWCS), 2012 IEEE 4th International Symposium on 28-31 August 2012.*

Chapter 6 - CSI acquisition in the MIMO interference channel via analog feedback

All the joint transmit-receive filter design techniques introduced in this thesis, WSR maximization and IA, require some form of CSI at both terminals. In this chapter we study the problem of CSI acquisition at transmit and receive side introducing two transmission protocols that are based on channel training and analog feedback (FB). We also study the problem of optimizing the sum rate, by focusing in particular on the resulting degrees of freedom (DoF), as a function of the coherence time.

This approach helps us to optimize the system parameters, number of transmitting antennas and transmitted streams,considering the CSI acquisition overhead. In the following papers are reported the results provided in this chapter:

- F. Negro, U. Salim, I. Ghauri, and D. T. M. Slock, ”The noisy MIMO interference channel with distributed CSI acquisition and filter computation,” *in Signals, Systems and Computers (ASILOMAR), 2011 Conference Record of the Forty Fifth Asilomar Conference on, 2011.*
- F. Negro, D. T. M. Slock, I. Ghauri, ”On the noisy MIMO interference channel with CSI through analog feedback,” *in Communications Control and Signal Processing (ISCCSP), 2012 5th International Symposium on (ISCCSP), 2012 , pp. 1 - 6*

Chapter 7 - Beamforming for the Underlay Cognitive MISO Interference Channel

Here we focus on the problem of beamformer design for a CR network modeled as a MISO interference channel. Since we assume to work in an underlay setting we further impose a set of interference power constraints at the primary receivers. Extending the results on UL-DL duality to cognitive radio settings we design the beamformer at the secondary transmitters in order to minimize the total transmitted power. We propose an iterative algorithm that efficiently solves the power minimization problem, at the secondary network, while a set of interference constraints are imposed on the primary receivers. The research contributions in this chapter are reported also in the following paper:

- F. Negro, I. Ghauri, and D. T. M. Slock, ”Beamforming for the underlay cognitive MISO interference channel via UL-DL duality,” *in Cognitive Radio Oriented Wireless Networks Communications (CROWNCOM), 2010 Proceedings of the Fifth International Conference on, June 2010, pp. 1 -5.*

Chapter 8 - Spatial Interweave TDD Cognitive Radio Systems

In this chapter we study the joint optimization of the transmit-receive filters in a spatial interweave cognitive radio channel, we describe all the communication phases required to acquire the necessary information at primary and secondary users. We focus in particular on how to really exploit channel reciprocity in real TDD transmission using UL DL channel calibration studying how calibration influences transmit and receiver filter design at primary and secondary devices. An important result that comes out of our analysis is that calibration between non cooperative Tx and Rx is not needed for secondary beamformer design. We introduce an extension of the results to the case with multiple primary transmitter and receiver pairs. If the primary network designs its beamformers according to IA, thanks to IA duality, the secondary pair can blindly estimate the DL received subspace at all primary receivers from the transmitted signal subspace in the UL communication. Calibration issues are also studied in this setting proving that calibration between non cooperative users is not required also in the extended scenario. The results described in this chapter are partially published in:

- F. Negro, I. Ghauri, and D. T. M. Slock, ”Transmission techniques and channel estimation for spatial interweave TDD cognitive radio systems,” *in Proceedings of the 43rd Asilomar conference on Signals, systems and computers, Asilomar'09, 2009, pp. 523-527.*

Chapter 9 - Spatial Interweave Cognitive Radio Interference Channel with Multiple Primaries

In this part of the work we consider a secondary network modeled as a K-user MIMO IFC that wants to communicate in presence of L primary multi antenna receivers. The secondary users' beamformers are designed according to IA with the additional interweave constraints to generate an interference subspace, at each primary receiver, with given dimension. We study the feasibility of an IA solution in the cognitive radio system under investigation based on the results presented in chapter 3. Then we propose an iterative algorithm that finds the secondary users' transmit and receive IA filters satisfying the interweave constraints at the primary receivers. The contributions of this chapter can be found in the following paper:

- F. Negro, I. Ghauri, and D. T. M. Slock, ”Spatial interweave for a MIMO secondary interference channel with multiple primary users,” *in 4th International Conference on Cognitive Radio and Advanced Spectrum Management, (CogART 2011), October 2011.*

Part I

Interference Channel

Chapter 2

MISO Interference Channel

2.1 Introduction and State of the Art

In this first part of the thesis we focus on the K-user interference channel (IFC) where pairs of users want to communicate between each other without exchanging (data) information with non-intended pairs. Interference at each user is treated as additional Gaussian noise contribution and hence linear beamforming processing is optimal. This, in the information theoretic sense, is the *noisy* interference channel. In particular, in this chapter, we focus on the case where the transmitters are equipped with multiple antennas and they communicate with single antenna receivers. This setting has been labeled as MISO (multiple-input-single-output) interference channel [11]. As already discussed the IFC, and in particular the multi-antenna case, can be used to model interesting realistic problems, like cell-edge users problem or coexistence of macro-femto cells, that has attracted a lot of research attention in recent years. The first attempt to study the MISO IFC has been to port the solutions and methods applied to the broadcast or multiple access channels to the IFC, but as we will see this is not always a straightforward process. The first important tool that has been used to solve transmission problem is uplink/downlink (UL/DL) duality.

UL-DL duality is a well-established tool for the study of the traditional Broadcast (BC) channel. For example it has been recently used [75] [76] to solve the BC beamforming and power allocation problem. Using this duality, the BF designed in the virtual (dual) uplink communication can be used in the actual downlink problem to achieve the same SINR values by choosing appropriate downlink power allocations. The authors give a set of conditions for duality in a BC channel and relate feasibility of the DL problem with the one of the corresponding UL that is normally easier to be solved. In the seminal work [77] a duality between the achievable rate region for the MIMO BC and the capacity region of the MIMO multiple-access channel (MAC), which is easy to compute, has been introduced. The authors showed that the dirty paper region [78] of a MIMO BC is exactly equal to the capacity region of the dual MIMO MAC, with all the transmitters having the same sum power constraint as the MIMO BC. With this new duality theory the computational complexity to compute the rate region for a MIMO BC channel has been reduced.

The multicell problem, that we call the interference channel, is more complex to handle due to the per-user (per BS) power constraints. [12] addresses duality in a similar setting, which the authors call the multicell setting, where previous results on interpretation of UL-DL duality as Lagrangian duality are exploited. [12] then solves the power minimization problem subject to Quality of Service (QoS) constraints and per base station power constraints formulated as weighted total transmit power. In [79] the authors establish the uplink-downlink beamforming throughput duality with per-base station (BS) power constraints for a multi-cell system. The objective is to provide a more solvable form for optimal downlink beamforming in the multi-cell environment. They found that the optimal downlink beamforming reveals to be the minimum mean squared error (MMSE) beamforming in the dual uplink. Even though the results are given for MISO system the extension to the MIMO case is not provided.

The $\max\min$ SINR beamforming problem formulation satisfies a fairness requirement because at the optimum all the SINRs are equal, for this reason it is also called SINR balancing problem. Balancing the SINR implies that the system performances are limited by the weak users causing a reduction of the overall sum rate. This problem has been extensively studied initially in the single cell broadcast channel. In the original works [80] and [81] the problem of signal-to-interference ratio (SIR) balancing problem, for MISO BC channel is studied. More recently the same problem is studied in [82] within the general framework of invariant interference functions studying the condition for existence and uniqueness of the solution. The more general SINR balancing problem has been studied in [76] for single cell Broadcast (BC) channel under the sum power constraint using the well-established tool of UL-DL duality [77].

In [83] the authors study the weighted SINR balancing problem for a MIMO BC channel. They show that the problem can be solved efficiently and optimally for rank one channel. An extension of the proposed algorithm, that converges to a local optimal solution, for general channel matrices, is studied. In this paper the

MIMO problem is solved working with a per-stream SINR approach.
[15] investigates the max min SINR problem for multicell multi user MISO system based on long term channel state information where only one user is scheduled in each cell. This makes the system essentially an interference channel, the only difference is that the authors consider a sum power constraint instead of the more realistic per BS power constraints. They propose an iterative algorithm to solve the problem based on UL-DL Lagrange duality. The SINR balancing problem in the MISO IFC has been studied, under general power constraints, in a recent paper [84] where only power optimization has been considered. In [14] the authors studied the beamforming design problem for SINR balancing for the multiuser multicell scenario under per base station power constraints. Their solution is based on the equivalence between the SINR balancing problem and the power minimization problem. The iterative algorithm that they derive solves the problem in a centralized fashion.
Similar problems have been studied in some recent papers with the objective of Pareto rate region characterization. The Pareto boundary is defined as the set of points where the performance of one user can not be incremented without decrementing the performance of other users. From the definition we see the importance of solutions that fall on the Pareto boundary because they are the solutions that efficiently exploit the transmission resources. In particular in [20] the two user MISO interference channel Pareto boundary characterization is considered. They propose a method based on solving the problem as a sequence of second order cone programming feasibility problems. A recent paper [22] applies a similar parameterization of the Pareto boundary for the more general setting of the multicell DL system. The system setup introduced in that paper can model, as extreme cases, the MISO IFC and a network MIMO systems.
In the seminal work [19] the problem of the Pareto characterization is described for a K-user MISO IFC. The main result is that the linear transmitters that allow to achieve the Pareto boundary are described as a linear combinations of channel vectors and they should carry all the possible transmit power. This parametrization requires a total of $K(K-1)$ complex parameters. This result has a more intuitive explanation if particularized for a 2-users IFC. In this case the BF vector should be a linear combination of zero-forcing (ZF) and maximum ratio transmitter (MRT). This means that the Pareto optimal points are obtained with BFs that represent a good compromise between a selfish transmit strategy, represented by the MRT solution, and a more altruistic solution, the ZF BF. In [85] the problem of Pareto region characterization is more completely studied under the Game Theoretic framework but specialized for the two users case. Recently in [21] a new parametrization has been introduced, based on the introduction of interference temperature constraint. This concept has been borrowed from cognitive radio [58] that

essentially describes the level of interference received at each receiver. In this new parametrization of the Pareto boundary $K(K-1)$ real parameters are required. In [22] this value has been reduced to $2K-1$ optimizing a proper function of the SINRs.

In [86] the author consider the Pareto characterization for the multi-cell multi-user setting introducing the concept of power gain region defined as the region of all jointly achievable power gains at the receivers. They show that the Pareto optimal points are achieved with rank-1 covariance matrices and the corresponding BF vectors can be parameterized using $T(K-1)$ real-valued parameters, where T is the number of transmitters. The power gain region results to be a convex region and the point on the boundary can be achieved using simple BF vectors. The same authors in [87] have introduced a new characterization for the Pareto boundary of the 2-user MISO IFC using game theory concepts coming from economic theory determining a single real-valued parametrization. The novelty is that they cast the problem as a pure exchange economic problem where each link is seen as a consumer that can exchange goods to maximize their utility. In the MISO IFC goods are the beamforming vectors and the utility the SINRs. In the recent paper [88] the authors study the Pareto boundary of the rate region of a 2 user MIMO interference channel with single beam transmission proposing an efficient iterative method for its numerical computation.

2.2 Contributions

In this chapter we first revisit some of the UL-DL duality principles, introduced for the BC channel in [75], for the MISO interference channel. We show that the results that are valid in the BC channel can be easily extended to the IFC for the case of total sum power constraint. A short introduction of the UL-DL Lagrange duality for the MISO IFC under per BS power constraint is provided based on the recent results in [12]. This theory will then be used for the solution of the weighted SINR (WSINR) balancing problem for MISO IFC. We then study the WSINR balancing problem for a MISO IFC with individual power constraints introducing some interesting consideration on power allocation at the optimal solution. Since in the MISO IFC with per BS power constraints, a subset of users always transmits with full power according to the antenna distribution and number of users in the system, we propose a new iterative algorithm that solves the WSINR balancing problem when only one power constraints is active. Subsequently we study the problem in a more general setting and we propose an iterative algorithm that solves the problem in a decentralized manner. This solution is based on the relation between the SINR balancing problem and the power minimization problem underlined in [14]. We solve the $\max \min WSINR$ problem using a sequence of power minimization problems where the QoS constraints in the beamforming problem are increased gradually until an infeasible point is found. Then, using bisection method, the optimal solution is determined.

Finally we show that is possible to characterize the entire Pareto boundary of the SINR region for a general K-user MISO IFC solving a sequence of *Weighted SINR* problems. Thanks to the one to one logarithmic relation between SINR and Rate we can then characterize the Pareto boundary of the Rate region for a general K-user MISO IFC. The basis of this characterization has been studied in [89] for a single-input single-output (SISO) IFC. Here we extend their results to the MISO IFC. A similar results has been also recently introduced in [22].

2.3 System model of MISO interference channel

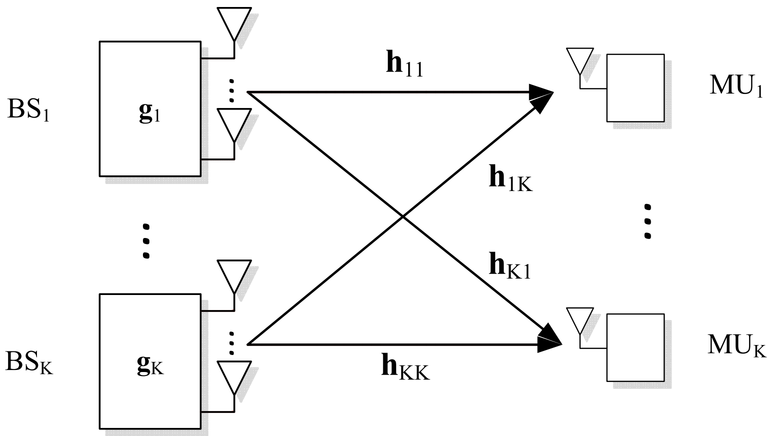

Figure 2.1: MISO Interference Channel

Fig. 2.1 depicts a K-user MISO IFC with K transmitter-receiver (Tx-Rx) pairs.

The k-th BS is equipped with M_k transmit antennas and k-th mobile user (MU) is a single antenna node. The k-th transmitter generates interference at all $l \neq k$ receivers. Assuming the communication channel to be frequency-flat, the received signal y_k at the k-th receiver can be represented as

$$y_k = \mathbf{h}_{kk}\mathbf{x}_k + \sum_{\substack{l=1 \\ l\neq k}}^{K} \mathbf{h}_{kl}\mathbf{x}_l + n_k \tag{2.1}$$

where $\mathbf{h}_{kl} \in \mathbb{C}^{1\times M_l}$ represents the channel vector between the l-th transmitter and k-th receiver, $\mathbf{x}_k$ is the $\mathbb{C}^{M_k\times 1}$ transmit signal vector of the k-th transmitter and n_k represents (temporally white) AWGN with zero mean and variance σ_k^2. Each entry of the channel matrix is a complex random variable drawn from a continuous distribution.

We denote by $\mathbf{g}_k$, the $\mathbb{C}^{M_k\times 1}$ beamforming (BF) vector of the k-th transmitter. Thus $\mathbf{x}_k = \mathbf{g}_k s_k$, where s_k represents the independent symbol for the k-th user pair. We assume s_k to have a temporally white Gaussian distribution with zero mean and unit variance. In the SIMO UL channel the k-th BS applies a receiver $\overline{\mathbf{f}}_k$ to suppress interference and retrieves its desired symbol. The output of such a receive filter is then given by

$$\overline{r}_k = \overline{\mathbf{f}}_k^H \overline{\mathbf{h}}_{kk} \overline{s}_k + \sum_{\substack{l=1\\ l\neq k}}^{K} \overline{\mathbf{f}}_k^H \overline{\mathbf{h}}_{kl} \overline{s}_l + \overline{\mathbf{f}}_k^H \overline{n}_k \tag{2.2}$$

where we denoted with $\bar{(.)}$ all the quantities that appear in the UL in order to differentiate with the same quantities in the DL.

2.4 UL-DL duality in MISO/SIMO Interference Channel Under Sum Power Constraint

In this section we derive UL-DL duality for a MISO IFC under a total power constraint. To simplify the following analysis henceforth we assume that each receiver is characterized by the same noise variance, so $\sigma_k^2 = \sigma^2$, $\forall k$ and the beamforming vectors, $\mathbf{g}_k$ $\forall k$, are unit norm. The received signal for the MISO DL IFC at the k-th mobile station is reported in (2.1) and from there we can write the corresponding SINR as:

$$SINR_k^{DL} = \frac{p_k \mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\sum_{l\neq k} p_l \mathbf{g}_l^H \mathbf{h}_{kl}^H \mathbf{h}_{kl} \mathbf{g}_l + \sigma^2} \tag{2.3}$$

where p_k is the Tx power at the BS for the stream intended to the k-th user. Imposing a set of DL SINR constraints at each mobile station: $SINR_k^{DL} = \gamma_k$ it is possible to rewrite equation (2.3) in matrix notation:

$$\mathbf{\Phi}\mathbf{p} + \boldsymbol{\sigma} = \mathbf{D}^{-1}\mathbf{p} \tag{2.4}$$

where the two matrices $\mathbf{\Phi}$ and $\mathbf{D}$ are defined in (2.5) and (2.6), $\mathbf{p} = [p_1, \ldots, p_K]^T$ and $\boldsymbol{\sigma} = \sigma^2\mathbf{1}$ are two vectors that contain all the Tx powers and and the noise variances respectively. Vector $\mathbf{1}$ is a column vector of dimensions $K \times 1$ that contains all ones. In addition we define:

$$[\mathbf{\Phi}]_{ij} = \begin{cases} \mathbf{g}_j^H \mathbf{h}_{ij}^H \mathbf{h}_{ij} \mathbf{g}_j, & j \neq i \\ 0, & j = i \end{cases} \tag{2.5}$$

$$\mathbf{D} = \text{diag}\{\frac{\gamma_1}{\mathbf{g}_1^H \mathbf{h}_{11}^H \mathbf{h}_{11} \mathbf{g}_1}, \ldots, \frac{\gamma_K}{\mathbf{g}_K^H \mathbf{h}_{KK}^H \mathbf{h}_{KK} \mathbf{g}_K}\}. \tag{2.6}$$

We can determine the Tx power solving (2.4) w.r.t. $\mathbf{p}$ obtaining:

$$\mathbf{p} = (\mathbf{D}^{-1} - \mathbf{\Phi})^{-1} \boldsymbol{\sigma} \tag{2.7}$$

Now we study the SIMO UL IFC focusing in particular on the corresponding SINR. Due to channel reciprocity we have that $\overline{\mathbf{h}}_{kl} = \mathbf{h}_{lk}^H \; \forall k, l$ and the receive filter in the UL is the reciprocal of the transmit filter of the DL $\overline{\mathbf{f}}_k = \mathbf{g}_k, \; \forall k$. Using the received signal in the UL channel in (2.2), the SINR for the UL channel can be written as:

$$SINR_k^{UL} = \frac{q_k \mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\mathbf{g}_k^H (\sum_{l \neq k} q_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \sigma^2 \mathbf{I}) \mathbf{g}_k} \tag{2.8}$$

where q_k represents the UL Tx power from the k-th MU. Imposing the same set of SINR constraints in the DL also in the UL: $SINR_k^{UL} = \gamma_k$ it is possible to rewrite (2.8) as:

$$\overline{\mathbf{\Phi}} \mathbf{q} + \boldsymbol{\sigma} = \mathbf{D}^{-1} \mathbf{q} \tag{2.9}$$

where $\mathbf{D}$ is defined as in (2.6), $\mathbf{q} = [q_1, \ldots, q_K]^T$ and

$$[\overline{\mathbf{\Phi}}]_{ij} = \begin{cases} \mathbf{g}_i^H \mathbf{h}_{ji}^H \mathbf{h}_{ji} \mathbf{g}_i, & j \neq i \\ 0, & j = i \end{cases} \tag{2.10}$$

the power vector can be found as:

$$\mathbf{q} = (\mathbf{D}^{-1} - \overline{\mathbf{\Phi}})^{-1} \boldsymbol{\sigma} \tag{2.11}$$

Comparing the definition in (2.5) and (2.10), we can see that $\overline{\mathbf{\Phi}} = \mathbf{\Phi}^T$. This implies that there exists a duality relationship between the DL MISO and UL SIMO interference channels.

It is also interesting to note that there is a strong parallel between the equations reported above, to show the duality in the MISO interference channel, and the ones used to prove duality in a BC-MAC in [75].

If we stack all the beamformers and the channel vectors in a matrix form, the cascade of channel and BF can be written as:

$$
\begin{aligned}
\mathbf{HG} &= \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} & \cdots & \mathbf{h}_{1K} \\ \mathbf{h}_{21} & \mathbf{h}_{22} & \cdots & \mathbf{h}_{2K} \\ \vdots & & \ddots & \vdots \\ \mathbf{h}_{K1} & \mathbf{h}_{K2} & \cdots & \mathbf{h}_{KK} \end{bmatrix} \begin{bmatrix} \mathbf{g}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{g}_2 & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{g}_K \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{h}_{11}\mathbf{g}_1 & \mathbf{h}_{12}\mathbf{g}_2 & \cdots & \mathbf{h}_{1K}\mathbf{g}_K \\ \mathbf{h}_{21}\mathbf{g}_1 & \mathbf{h}_{22}\mathbf{g}_2 & & \vdots \\ \vdots & & \ddots & \mathbf{h}_{K-1K}\mathbf{g}_K \\ \mathbf{h}_{K1}\mathbf{g}_1 & \cdots & \cdots & \mathbf{h}_{KK}\mathbf{g}_K \end{bmatrix}
\end{aligned} \tag{2.12}
$$

where $\mathbf{G}$ is a block diagonal matrix and the diagonal blocks are the BF column vectors at different BSs. Making the notation in (2.12) more compact, denoting the i-th row of $\mathbf{H}$ as $\mathbf{H}_i$ and the j-th column of the BF matrix as $\mathbf{G}_j$, we have:

$$
\begin{aligned}
\mathbf{HG} &= \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} \begin{bmatrix} \mathbf{G}_1 & \mathbf{G}_2 & \cdots & \mathbf{G}_K \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{H}_1\mathbf{G}_1 & \mathbf{H}_1\mathbf{G}_2 & \cdots & \mathbf{H}_1\mathbf{G}_K \\ \mathbf{H}_2\mathbf{G}_1 & \mathbf{H}_2\mathbf{G}_2 & & \vdots \\ \vdots & & \ddots & \mathbf{H}_{K-1}\mathbf{G}_K \\ \mathbf{H}_K\mathbf{G}_1 & \cdots & \cdots & \mathbf{H}_K\mathbf{G}_K \end{bmatrix}
\end{aligned} \tag{2.13}
$$

If we assume that vector $\mathbf{H}_i$ represents the channel between all the BSs and the i-th MU and vector $\mathbf{G}_i$ is the corresponding BF, equation (2.13) can be used to represent the BC. The difference is that the BF used for an IFC has a block structure where some blocks are zero while in the BC the BF matrix is full. This makes the parallel between a BC and an interference channel more clear. In a similar fashion it is possible to describe the same parallelism between the BC and the interference channel for the UL SIMO IFC using similar matrix notation.

With the previous observations it is possible to extend the results obtained for the UL-DL duality in the BC-MAC to the IFC under a sum power constraint.

A set of SINRs $\gamma_1, \ldots, \gamma_K$ is feasible whenever there exists a positive power allocation such that (2.4) for the DL ((2.9) for the UL) is fulfilled. In [75] the following is proved for the BC-MAC duality but it is also valid for the IFC:

Targets $\gamma_1, \dots, \gamma_K$ are jointly feasible in UL and DL if and only if the spectral radius ρ of the weighted coupling matrix satisfies $\rho(\mathbf{D\Phi}) < 1$.

Because $\rho(\mathbf{D\Phi}) = \rho(\mathbf{D\Phi}^T)$ target SINRs are feasible in the UL if and only if the same targets are feasible in the DL. The power allocation vectors that satisfy those constraints can be found using (2.7), for the DL, and (2.11), for the UL.

In addition the total required UL power $q_{tot} = \sum_i q_i$ is the same as the DL power $p_{tot} = \sum_i p_i$, this can be simply shown as follows:

$$\begin{aligned} \textstyle\sum_i q_i = \quad & \mathbf{1}^T\mathbf{q} = \sigma\mathbf{1}^T(\mathbf{D}^{-1} - \mathbf{\Phi}^T)^{-T}\mathbf{1} \\ & = \sigma\mathbf{1}^T(\mathbf{D}^{-1} - \mathbf{\Phi})^{-1}\mathbf{1} = \textstyle\sum_i p_i \end{aligned} \tag{2.14}$$

According to the relationship (2.14) it is possible to state that both UL and DL have the same SINR feasible region under the same sum-power constraint, i.e., target SINRs are feasible in the DL if and only if the same targets are feasible in the UL.
Using the results obtained before it is possible to extend some beamforming design techniques that use the BC-MAC duality to the beamforming design for a MISO IFC.

2.5 UL-DL duality in MISO/SIMO Interference Channel Under per User Power Constraint

In the MISO interference channel if the problem of BF design is formulated under the sum power constraint we have shown that there exist an UL-DL duality that can be used to solve the problem. Even though the sum power constraint is analytically attractive such constraint is not enough in a practical interference channel. In reality each user is subject to a per user power constraint that the transmit power can not violate. For this reason in this section we briefly describe an alternative BF design problem that still minimizes the total Tx power but imposing also per user power constraints. This problem has been studied for a multicell case, of which the IFC is a special case in [12]. There an UL-DL relation, based on Lagrangian duality for BC channel [90], has been extended to the multicell case to solve the beamformer optimization problem.

Assuming that the SINR constraints are such that there exist at least a feasible solution to the problem, the beamformer optimization problem now becomes:

$$\begin{aligned} & \min_{\mathbf{g}_1,\dots,\mathbf{g}_K} \textstyle\sum_{k=1}^K \mathbf{g}_k^H\mathbf{g}_k \\ & \text{s.t.} \quad \mathbf{g}_k^H\mathbf{g}_k \le P_k;\ k = 1,\dots,K \\ SINR_k^{DL} = & \frac{\mathbf{g}_k^H\mathbf{h}_{kk}^H\mathbf{h}_{kk}\mathbf{g}_k}{\sum_{l\neq k}\mathbf{g}_l^H\mathbf{h}_{kl}^H\mathbf{h}_{kl}\mathbf{g}_l + \sigma_k^2} \ge \gamma_k;\ k = 1,\dots,K \end{aligned} \tag{2.15}$$

where P_k represents the maximum Tx power for user k. This problem at first sight seems to be non convex due to the SINR constraints. For a downlink BC channel it has been shown in [91] that this set of constraints can be transformed into a second order cone constraint that allows for simple convex optimization solutions [92].

The Lagrange dual of the DL beamforming problem (2.15) can be rewritten as an equivalent UL optimization problem [12], Fig.2.2, for the Rx filter:

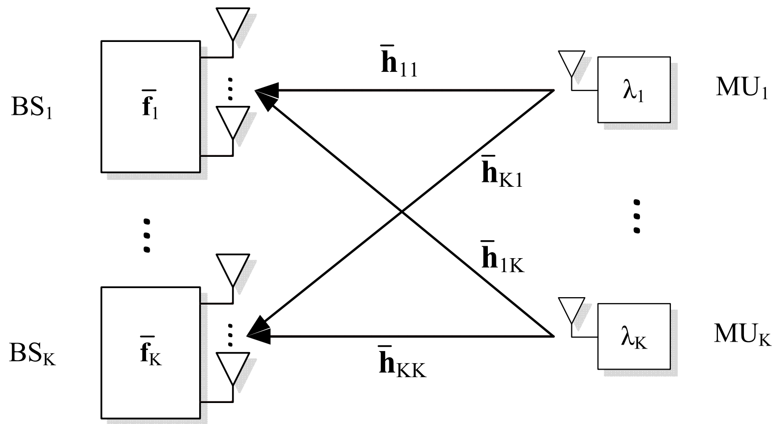

Figure 2.2: MISO Interference Channel

$$\bar{\mathbf{f}}_k = (\sum_{l \neq k} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I})^{-1} \mathbf{h}_{kk}^H \tag{2.16}$$

in which the Tx power λ_k and the noise power $\eta_k = 1 + \mu_k$ are to be optimized. In the UL problem, in (2.17), each user transmits with power $\lambda_k, \ \forall k$, and the value of the dual UL noise at the receiver is represented by $\eta_k, \ \forall k$:

$$\begin{aligned} &\max_{\lambda_1,\ldots,\lambda_K,\mu_1,\ldots,\mu_K,} \sum_{k=1}^K \lambda_k \sigma_k^2 - \sum_{k=1}^K \mu_k P_k \\ SINR_k^{UL} = &\frac{\lambda_k \bar{\mathbf{f}}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \bar{\mathbf{f}}_k}{\bar{\mathbf{f}}_k^H (\sum_{l\neq k} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I}) \bar{\mathbf{f}}_k} \leq \gamma_k; \ k = 1, \ldots, K \\ &\lambda_k \geq 0; \ k = 1, \ldots, K \\ &\mu_k \geq 0; \ k = 1, \ldots, K \end{aligned} \tag{2.17}$$

Using this UL-DL results an iterative algorithm is derived in [12] that allows also for distributed solution in TDD systems.

2.6 Max-Min SINR in the MISO IFC with per-user power constraints

In this section we consider a MISO IFC in which each receiver has an individual SINR priority $\gamma_i, \forall i = 1, \dots, K$. Fairness then leads to a max min weighted SINR (WSINR) cost function.

$$\begin{aligned} &\max_{\mathbf{g}_1,\dots,\mathbf{g}_K} \min_{k=1,\dots,K} \frac{SINR_k}{\gamma_k} \\ \text{s.t.} \quad &\mathbf{g}_k^H \mathbf{g}_k \leq P_k, \quad \forall k = 1, \dots, K \end{aligned} \tag{2.18}$$

where P_k represents the maximum available power at transmitter number k. This problem, under a sum power constraint, was already discussed in [93].

The optimal solution to SINR balancing occurs when all the weighted SINRs are equal, thus the commonly used term SINR balancing. As stated also in [84] and [89] we can claim that for fixed beamforming direction at the balanced point in the MISO IFC, at least one user transmits with full power, i.e., at least one power constraint is satisfied with equality. This is easy to verified for a SISO IFC or the MISO case with fixed BF vectors because the user with the worse equivalent channel coefficient, cascade of channel vector and BF, to maximize its SINR tends to use all its available power while the other users will adjust their power in order to equate all the SINRs.
Different is the situation when the beamforming design comes into the problem.

When the MISO IFC is separable, meaning that each user has a number of antenna greater than or equal to the number of users, $M_k \geq K$, the following proposition describes the SINR balancing behavior.

Proposition 1 *At the balanced point, in separable MISO IFC, all users transmit with full power*

Proof: To prove the above statement consider, without loss of generality, a $K = 2$ user MISO IFC with $M_k \geq 2$. Assume that the optimal solution of the SINR balancing problem is given for $\mathbf{g}_1^\star$ and $\mathbf{g}_2^\star$ where only transmitter 1 transmits with full power, $\|\mathbf{g}_1^\star\|^2 = P_1,\ \|\mathbf{g}_2^\star\|^2 < P_2$. Because Tx_2 has an excess of power the BF of user 1 can be modified s.t.:

$$\|\mathbf{g}_1^{'}\|^2 = \|\mathbf{g}_1^\star\|^2 \ ; \quad |\mathbf{h}_{11}\mathbf{g}_1^{'}|^2 > |\mathbf{h}_{11}\mathbf{g}_1^\star|^2.$$

This new choice of BF for user 1 increases its SINR but at the same time causes a reduction of the SINR of the other user: $SINR_2(\mathbf{g}_1^{'}, \mathbf{g}_2^\star) < SINR_{1,2}(\mathbf{g}_1^\star, \mathbf{g}_2^\star) < SINR_1(\mathbf{g}_1^{'}, \mathbf{g}_2^\star)$. Tx_2 to compensate for the additional interference caused by the new BF $\bar{\mathbf{g}}_1$ has to increase the transmitted power using a BF of the form:

$$\mathbf{g}_2^{'} = \mathbf{g}_2^{\star} + \delta \mathbf{h}_{12}^{\perp}$$
$$\|\mathbf{g}_2^{'}\|^2 > \|\mathbf{g}_2^{\star}\|^2 \; ; \quad |\mathbf{h}_{22}\mathbf{g}_2^{'}|^2 > |\mathbf{h}_{22}\mathbf{g}_2^{\star}|^2$$

where $\mathbf{h}_{12}^{\perp}$ is any vector that belongs to the orthogonal complement of $\mathbf{h}_{12}$ and δ is a complex scaling factor. The choice of δ should be s.t. $SINR_1(\mathbf{g}_1^{'}, \mathbf{g}_2^{'}) = SINR_2(\mathbf{g}_1^{'}, \mathbf{g}_2^{'})$. With this choice of $\mathbf{g}_2^{'}$ we can rise the useful signal power for user 2 without increasing the interference caused to the non intended receiver.

With the new set of beamformers both the SINRs are increased $SINR_{1,2}(\mathbf{g}_1^{'}, \mathbf{g}_2^{'}) > SINR_{1,2}(\mathbf{g}_1^{\star}, \mathbf{g}_2^{\star})$. This means that the original BF vectors were not optimal hence both users should transmit with full power.

This result is in line with what has been previously proposed in literature. As we will see WSINR balancing problem is a possible way to characterize the Pareto boundary of the SINR/Rate region. In [85, 86] it has been shown that the Pareto optimal solutions are achieved transmitting with full power if $M_k \geq K$, as we can see our result also confirm this approach.

Different is the situation in low SNR regime. Here we can state that the optimal transmission strategy for each user is to maximize the useful signal component. No matter how strong interference becomes, noise remains the dominant impairment. Hence the optimum transmission strategy is to beamform to match the direct link (maximum ratio BF) at each Tx. In this case the user with the worse direct link channel transmits with full power to maximize its SINR, which is also the systemwide worst SINR. This is true also for separable MISO channel, regardless the number of transmitting antennas.

It may be argued that different optimal points to the SINR balancing problem may exist at low SNR. In this situation each user can be thought as decoupled from the others, due to the high noise power. Once the system worst SINR is maximized the max min problem is solved. The remaining users can now decide to use their power in many different ways. For example they can use the minimum power such that all the SINRs are balanced or they can use full power to maximize all the SINRs. Which solution should be used depends on a possible secondary objectives. If the total power should be minimized then the balanced SINR solution is the optimal one.

2.6.1 DL power allocation optimization

For cases where a zero forcing solution is not possible ($M_k < K, \; \forall k$) only one user has its power constraint active. In this case for fixed BF vectors the corresponding power allocation vector can be found solving an eigenvalue problem [76] imposing only one power constraint to be active. At the optimum all the weighted

SINRs are equal. Denoting with τ the optimal value of the ratio SINR over target QoS we can write:

$$\frac{1}{\tau}\mathbf{p} = \mathbf{D\Phi p} + \mathbf{D}\boldsymbol{\sigma} \tag{2.19}$$

where matrices $\mathbf{\Phi}$ and $\mathbf{D}$ are defined as in equation (2.5) and (2.6) respectively. Assuming now that the $j-$th power constraint is the only one satisfied with equality and multiplying both sides of the previous equation by $\mathbf{x}_j^T = \frac{1}{P_j}\mathbf{e}_j$, where $\mathbf{e}_j$ is a vector with 1 only in position j, we get:

$$\frac{1}{\tau} = \mathbf{x}_j^T\mathbf{D\Phi p} + \mathbf{x}_j^T\mathbf{D}\boldsymbol{\sigma} \tag{2.20}$$

Introducing the compound matrix:

$$\mathbf{\Delta}_j = \begin{bmatrix} \mathbf{D\Phi} & \mathbf{D}\boldsymbol{\sigma} \\ \mathbf{x}_j^T\mathbf{D\Phi} & \mathbf{x}_j^T\mathbf{D}\boldsymbol{\sigma} \end{bmatrix} \tag{2.21}$$

the extended vector $\overline{\mathbf{p}} = [\mathbf{p} \;\; 1]^T$, and using the results from the nonnegative matrix framework [94] the solution of the WSINR balancing problem w.r.t. the power optimization is given by: $\tau = \frac{1}{\lambda_{max}(\mathbf{\Delta}_j)}$. The power vector is the corresponding positive eigenvector with the $(K+1)-$th entry normalized to one. This approach, that allows to extend the known result from SIR balancing to SINR balancing, is called *Bordering Method*, it was introduced by [94] and then used in [76] for BC channel. A different approach to handle noise in the SINR balancing problem is to transform (2.19) into an homogeneous system of linear equations. This method is based on considering a rank one modification of the matrix $\mathbf{D\Phi}$ that leads to the same solution obtained using the bordering method. The fact that the $j-$th power constraint is active: $\mathbf{x}_j^T\mathbf{p} = 1$ allows us to modify WSINR balancing problem in order to obtain an unconstrained optimization problem in terms of powers. Introducing a reparametrization of the Tx power vector:

$$\mathbf{p} = \frac{1}{\mathbf{x}_j^T\tilde{\mathbf{p}}}\tilde{\mathbf{p}} \tag{2.22}$$

we can rewrite (2.19) as

$$\frac{1}{\tau}\tilde{\mathbf{p}} = \underbrace{\left(\mathbf{D\Phi} + \mathbf{D}\boldsymbol{\sigma}\mathbf{x}_j^T\right)}_{\mathbf{\Psi}_j}\tilde{\mathbf{p}}. \tag{2.23}$$

Also in this case the solution of the problem is given by the positive eigenvalue $\tau = \frac{1}{\lambda_{max}(\mathbf{\Psi}_j)}$ and the associated positive eigenvector is the optimal power vector.

At this point a question arises: Which power constraint is the only one satisfied with equality? It is possible to show that the only feasible constraint is given by $\mathbf{x}_{j^\star} = \arg\max_{\mathbf{x}_j} \lambda_{max}(\mathbf{B})$ [95], where $\mathbf{B}$ can be the rank 1 modified matrix $\mathbf{\Psi}_j$ or matrix $\mathbf{\Delta}_j$ in (2.21).

To solve the problem when only one power constraint is active and none of the users can do ZF BF we can determine the following algorithm which solves K different optimization problems, imposing only one power constraint to be active, and finally we choose the optimal solution. The problem can be mathematically expressed as:

$$\begin{aligned} &\max_{\{p_i\},\tau_j} \tau_j \\ &\text{s.t.} \quad \mathbf{e}_j\mathbf{p} \le P_j \qquad\qquad \forall j \\ &\quad SINR_k^{DL} = \frac{1}{\gamma_k}\frac{p_k \mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk}\mathbf{g}_k}{\sum_{l\neq k} p_l \mathbf{g}_l^H \mathbf{h}_{kl}^H \mathbf{h}_{kl}\mathbf{g}_l + \sigma_k^2} \ge \tau_j \quad \forall k \end{aligned} \tag{2.24}$$

where we assume that the BFs are unit norm and for the moment they are not optimization variables, they are fixed. The Lagrange dual of the optimization problem can be transformed into an equivalent dual UL problem:

$$\begin{aligned} &\min_{\mu} \max_{\{\lambda_i\},\tau_j} \tau_j \\ &\text{s.t.} \quad \textstyle\sum_i \lambda_i \sigma_i^2 \le P_j, \ \mu \le 1 \\ &\quad SINR_k^{UL} = \frac{1}{\gamma_k}\frac{\lambda_k \mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk}\mathbf{g}_k}{\sum_{l\neq k} \lambda_l \mathbf{g}_k^H \mathbf{h}_{lk}^H \mathbf{h}_{lk}\mathbf{g}_k + \mu e_{j,k}} \ge \tau_j \quad \forall k \end{aligned} \tag{2.25}$$

where λ_i represents the Lagrange multiplier associated to the i-th SINR constraint and μ is introduced to handle the power constraint. $e_{j,k}$ represents the k-th element of $\mathbf{e}_j$. Those quantities represent the dual UL Tx power and the UL dual noise power respectively. Because we need to minimize the SINRs w.r.t. μ this variable should be large so it will assume its maximum value at the optimum: $\mu = 1$. The UL max min WSINR problem can be solved w.r.t. the UL power using one of the method described before, for example solving the following:

$$\frac{1}{\tau_l}\tilde{\boldsymbol{\lambda}} = (\mathbf{D}\mathbf{\Phi}^T + \mathbf{D}\mathbf{e}_j\boldsymbol{\sigma}^T)\tilde{\boldsymbol{\lambda}}; \quad \boldsymbol{\lambda} = \frac{P_j}{\boldsymbol{\sigma}^T\tilde{\boldsymbol{\lambda}}}\tilde{\boldsymbol{\lambda}} \tag{2.26}$$

From the SINR constraints in the UL problem (2.25) we can see that the BF vector plays the role of Rx filter. The optimal $\mathbf{g}_k$ is the one that maximizes the SINR in UL and the solution for this problem is the well known generalized eigenvector solution that for rank one channels has the following closed form solution:

$$\mathbf{g}_k = (\sum_{l\neq k} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I})^{-1}\mathbf{h}_{kk}^H \tag{2.27}$$

where η_k represents the dual noise power, in this case $\eta_k = e_{j,k}$. Finally the DL power allocation can be determined using equation (2.23). Once the K optimization problems have been solved the optimal solution that satisfies all the power constraints is the one with index $l^\star = \arg\min_j \tau_j$. In the corresponding DL power vector the $l^\star$-th user transmits with full power and at the same time all the other power constraints are inactive.

For a more general system configuration the max min WSINR problem below:

$$\begin{aligned} &\max_{\mathbf{g}_1,\ldots,\mathbf{g}_K} \ \tau \\ &\text{s.t.} \quad \mathbf{g}_k^H \mathbf{g}_k \le P_k \qquad \forall k \\ &SINR_k^{DL} = \frac{1}{\gamma_k} \frac{\mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\sum_{l \ne k} \mathbf{g}_l^H \mathbf{h}_{kl}^H \mathbf{h}_{kl} \mathbf{g}_l + \sigma_k^2} \ge \tau \quad \forall k \end{aligned} \tag{2.28}$$

can be solved as in [14] using UL-DL duality.

2.7 Decentralized Iterative algorithm

In this section we describe an iterative algorithm that solves the weighted SINR balancing problem. It is essentially based on the link between the SINR balancing problem and the power minimization under QoS constraints underlined in [14]. The idea behind the proposed algorithm is to solve a sequence of power minimization problems with per base station power constraints incrementing at each step of the algorithm the QoS requirements imposed on the system. When the QoS constraints become not feasible then using bisection method we determine the optimal value of the max min WSINR problem. The advantage of this algorithm is that there exist a distributed solution for the power minimization problem [12] in TDD systems where UL and DL channel are reciprocal of each other.

The power minimization problem is written as:

$$\begin{aligned} &\min_{\mathbf{g}_1,\ldots,\mathbf{g}_K} \ \sum_{k=1}^K \mathbf{g}_k^H \mathbf{g}_k \\ &\text{s.t.} \quad \mathbf{g}_k^H \mathbf{g}_k \le P_k; \ k = 1, \ldots, K \\ &SINR_k^{DL} = \frac{\mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\sum_{l \ne k} \mathbf{g}_l^H \mathbf{h}_{kl}^H \mathbf{h}_{kl} \mathbf{g}_l + \sigma_k^2} \ge \gamma_k; \ k = 1, \ldots, K \end{aligned} \tag{2.29}$$

where P_k represents the maximum Tx power for user k.

The Lagrange dual of the DL beamforming problem (2.29) can be rewritten as an equivalent UL optimization problem for the Rx filter (2.27) where the dual noise is $\eta_k = \mu_k + 1$. The dual UL problem can be mathematically expressed as:

$$\begin{aligned} &\max_{\{\mu_i\}} \min_{\{\lambda_i\}} \sum_{k=1}^K \lambda_k \sigma_k^2 - \sum_{k=1}^K \mu_k P_k \\ &SINR_k^{UL} = \frac{\lambda_k \tilde{\mathbf{f}}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \tilde{\mathbf{f}}_k}{\tilde{\mathbf{f}}_k^H (\sum_{l \ne k} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I}) \tilde{\mathbf{f}}_k} \ge \gamma_k; \ k = 1, \ldots, K \\ &\lambda_k \ge 0; \ \mu_k \ge 0; \ \forall k. \end{aligned} \tag{2.30}$$

At the optimum the SINR constraints in the UL and DL problems must be satisfied with equality [12]. Using this property it is possible to derive the UL and DL Tx powers. The UL Tx power is determined using the following:

$$\lambda_k = \gamma_k \frac{\overline{\mathbf{f}}_k^H (\sum_{l \neq k} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I}) \tilde{\mathbf{g}}_k}{\overline{\mathbf{f}}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \overline{\mathbf{f}}_k} \stackrel{a}{=} \frac{\gamma_k}{\mathbf{h}_{kk} \overline{\mathbf{f}}_k} \tag{2.31}$$

where a is obtained using (2.27). Because a scaling factor in the receiver filter at the BS does not affect the UL SINR, the optimal DL BF is $\mathbf{g}_k = \sqrt{p_k}\overline{\mathbf{f}}_k$ and p_k is such that the WSINR in DL for user k is satisfied with equality. The last quantity that remains to be optimized is the Lagrange multiplier μ_k. On this purpose we use a subgradient method:

$$\mu_k^{(n)} = [\mu_k^{(n-1)} + t(\mathbf{g}_k^H \mathbf{g}_k - P_k)]_+ \tag{2.32}$$

where t represents the step size.

As stated at the beginning of this section the most important feature of the proposed algorithm is the possibility of distributed implementation that relies on channel reciprocity and few feedback of scalar quantities.

Algorithm 1 Iterative Algorithm for max min WSINR

```
Initialize: i= 0 and a feasible γ^0 = [γ_1^(0), ..., γ_K^(0)]
repeat
    i=i+1
    Find g_k^(i) solving Power min for γ^(i)
    Set γ_min = γ^(i)
    Increase γ^(i+1) = αγ^(i)
until γ^(i) is feasible
repeat
    Set γ_max = γ^(i)
    i=i+1
    Set γ^(i) = (γ_max + γ_min)/2
    Find g_k^(i) solving Power min for γ^(i)
    if γ^(i) is feasible then
        Set γ_min = γ^(i)
    else
        Set γ_max = γ^(i)
    end if
until |γ_max − γ_min| < ε
```

2.8 SINR Region Characterization

The beamforming problem in terms of max min WSINR described in (2.18) and further refined in (2.28) can be interpreted as exploring the SINR region along the ray with direction $\boldsymbol{\gamma} = [\gamma_1, \dots, \gamma_K]$. Solving the max min WSINR problem allows us to find the maximum values of SINR on the direction given by $\boldsymbol{\gamma}$. Then the optimal point is given by the intersection of the straight line described by $\boldsymbol{\gamma}$ and the Pareto boundary of the SINR region. This result was claimed for a SISO IFC in [89] and in [20] for the two user MISO IFC, here is extended to the more general K-user MISO case. The Pareto boundary of the SINR region is commonly defined as follows:
A SINR tuple $(S_1, \dots, S_K)$ *belongs to the Pareto boundary if there is no other tuple* $(\hat{S}_1, \dots, \hat{S}_K)$ *with* $(\hat{S}_1, \dots, \hat{S}_K) \geq (S_1, \dots, S_K)$ *and* $(\hat{S}_1, \dots, \hat{S}_K) \neq (S_1, \dots, S_K)$.
This result is important from an information theoretic point of view because solving the simple max min WSINR problem allows us to draw the entire Pareto boundary of the rate region, thanks to the logarithmic relation between SINRs and rates. This result is valid for a general K-user MISO IFC regardless of system parameters. In a recent paper [19] the authors provide a characterization of the Pareto boundary of the Rate region where the BF at each base station is a linear combination of the cross channels directly connected to it. This representation requires $K(K-1)$ complex parameters while the use of max min WSINR only requires $(K-1)$ real values, the fairness constraints γ_k. In [21] the authors propose a similar characterization of the Pareto boundary of the rate region using what they call *rate profile*. That problem can be thought as a rate balancing problem imposing different priority constraints and they state that to solve the problem a centralized solution is necessary.
On the other hand for max min WSINR it is possible to develop a distributed algorithm to solve the problem, as shown in the previous section, that represents a preferable solution compares to a centralized approach. In a recent paper [22] a similar result has been introduced. The authors study the Pareto characterization for a multicell DL system, where the IFC is a special case, introducing also hardware impairments. The solution proposed requires the same number of parameters, compare to the one proposed here, for the complete characterization of the Pareto boundary.

2.9 Numerical Examples

In this section we present some numerical results in which we study the behavior of the proposed algorithm to solve the max min WSINR. In Fig. 2.3 we report the Rate region of a 2-user MISO IFC where each base station has $M_k = 2, \ \forall k$ transmitting antennas for a single channel realization. We plot on the same figure the rate obtained optimizing the max min $WSINR$ for different priority constraints γ_k. The rate region reported, the region in the figures represented with the blue markers, is obtained using the BF parametrization proposed in [19] for the 2-user MISO IFC that allows to draw the entire rate region, and hence also the Pareto boundary. As we can see the rates obtained optimizing the max min WSINR (red

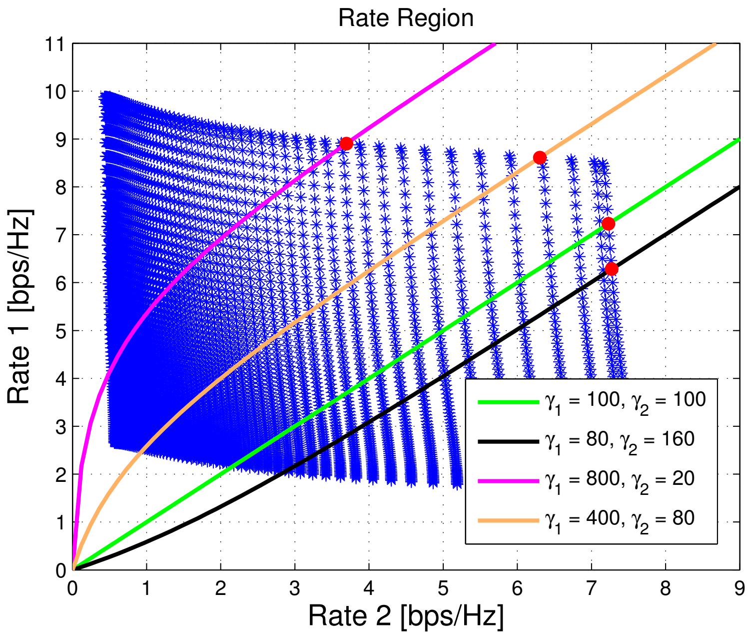

Figure 2.3: Rate region for a 2-user MISO IFC for $\sigma_k^2 = 30$ dB

points in the figure) lie always on the boundary of the region. In addition we can see that varying the priority constraint γ_k it is possible to explore different points on the boundary. This figure sustain our statement on the possibility to characterize the entire Pareto boundary of the rate region using max min WSINR. The solid lines drawn on the figure represent the rays with direction given by γ. Those curves are straight lines in the SINR region, Fig. 2.4, but due to the log relation between SINR and Rate they have a logarithmic behavior.

2.10 Conclusions

In this chapter we introduced the MISO interference channel, and we studied the problem of $\max\min$ SINR with minimum QoS constraints and per-user power

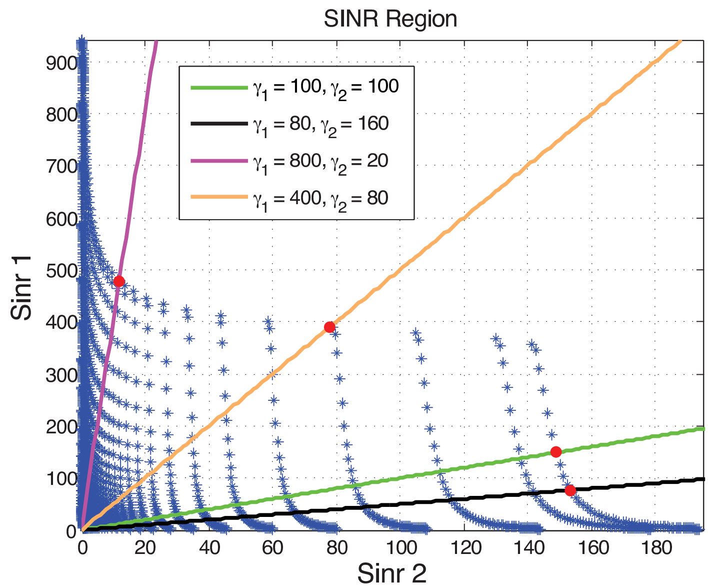

Figure 2.4: SINR region for a 2-user MISO IFC for $\sigma_k^2 = 30$ dB

constraints. We show that SINR balancing in the MISO IFC leads to a balanced state where at least one user transmits with full power. When the IFC is separable (number of antennas sufficient to zero force), the SINR balanced state is where all users transmit with full powers. We derive an iterative algorithm to solve the given optimization problem based on the equivalence between SINR balancing problem and the power minimization problem with QoS constraints that allows distributed implementation. Finally we show that WSINR balancing problem can be used to characterize the complete Pareto boundary of the SINR (Rate) region.

Chapter 3

Interference Alignment Feasibility for MIMO Interference Channel

3.1 Introduction and state of art

The capacity of an interference channel in general has been an open problem for long time. The best known result is given for the 2-users SISO Gaussian IFC in [8], for $K > 2$ the problem becomes even more complicated. This has lead to an alternative line of attack; that of characterizing the capacity region in terms of sum-capacity pre-log factor, the so called total *degrees of freedom* (DoF), that gives a good approximation of the sum capacity at high SNR. A first study on the DoF for a MIMO IFC has been done in [25] where the DoF for a 2-user MIMO IFC has been found. There it has been shown that for a system with M_1, M_2 transmitting antennas and N_1, N_2 receiving antennas a total $d = \min\{M_1 + M_2, N_1 + N_2, \max\{M_1, N_2\}, \max\{M_2, N_1\}\}$ interference free streams can be achieved if perfect channel state information is available at both transmitters and receivers. For system with more users a different approach is necessary. In [26] the concept of interference alignment (IA) has been introduced, then in [27] the same concept has been applied to the MIMO X channel where the authors showed that an higher total DoF can be achieved compared to previously known results. Then in the recent paper [28] the authors have demonstrated the achievability of a capacity prelog factor of $K/2$ in a K-user interference channel. This result has been achieved for time-varying channel where the alignment is obtained using infinite symbol extensions. In addition they also give an example on how to apply IA in constant coefficient MIMO IFC where the preliminary result of $\frac{3M}{2}$ total DoF for a $K = 3$ users constant coefficient MIMO IFC is determined. This result can be generalized for time-varying channel to $\frac{KM}{2}$. These remarkable results are achieved using simple linear transmit-receiver filters. This has resulted in a renewed interest in joint linear processing at transmitter and receiver with the aim of maximizing the capacity prelog of a multiuser MIMO interference channel. In particular IA exploits the availability of channel state information at the transmitter (CSIT) to compute appropriate beamforming matrices such that, at each receiver, all the interference is confined within a subspace of dimension complementary to the receiver's desired signal subspace dimension. Thus, simple zero-forcing (ZF) receivers are enough to separate the desired signal from the interferers. The alignment of the interference contributions can be done in different ways. IA over the *signal scale* encodes the transmitted signal, using for examples lattice codes, such that the alignment is done at the signal level [96],[97]. On the other hand alignment over the *signal space* takes advantages of aligning the interferers along different transmits directions [28] like space, time or frequency. In [98] it has been shown that a DoF of $\frac{K}{2}$ can be achieved almost surely in a K-user real IFC with constant coefficients. This can be achieved using a new alignment scheme, called *real alignment*, based on proprieties of rational and irrational numbers. New achievable DoF for K-users symmetric $M \times N$ MIMO IFC has been introduced in [29]. In particular a total number of DoF of $K \min(M, N)$ if $K \leq R$, and for $K > R$ it is upper-bounded by $\frac{\max\{M,N\}}{R+1}$, with $R = \left\lfloor \frac{\max\{M,N\}}{\min\{M,N\}} \right\rfloor$, for both constant or time varying channels. For only time varying channel coefficients they have shown that $\frac{R}{R+1} \min\{M, N\}$ total DoF can be achieved. Similar results have been extended to constant channels in [30] where the method proposed in [98] is extended to the MIMO case.

While it is known that interference alignment is the optimal scheme (in the high SNR regime) among approaches that use linear transmit/receive processing and treat interference as Gaussian noise, the existence of solutions in many cases is not known in general. In [31] a close form IA solution is provided for square symmetric K-user MIMO IFC where each pair of users is equipped with $N = K - 1$ antennas and wants to transmit one stream each. The same solution has been extended to the case of d transmitted streams in [32]. Iterative algorithms, for finding numerically an IA solution have been proposed [33, 34, 35, 36], they can be also used to evaluate the existence of an IA solution through simulations. To achieve IA different approaches are used. In [33] [34] the interference leakage at each

receiver is minimized, the authors of [35] introduce an algorithm that solves a least squares problem in an iterative fashion. [36] explores the minimization of the mean squared error (MSE) as cost function for the IA problem in MIMO IFC. The feasibility of IA solutions for a constant coefficient MIMO IFC was studied in [37]. There, when $d_k = 1\ \forall k$, a MIMO IFC with a given distribution of Tx/Rx antennas is classified as *proper* or *improper*. All proper systems are almost surely (a.s) feasible. For a system to be proper, it is required that, for *every subset* of equations that arise due to the IA constraints, the number of variables be at least equal to the number of equations in that subset. This condition (that the system be proper) is sufficient but may not be necessary. Moreover, such a classification can be computationally expensive even for systems with relatively small number of transmit and receive antennas. Furthermore, for an arbitrary DoF allocation amongst users (d_k not constrained to be 1), additional outerbounds need to be satisfied for a system to be feasible. It turns out however, that for multi-stream transmission, conformance with the outerbounds do not necessarily provide insight into the feasibility of an IA solution. In other words, an IA solution is not guaranteed if the outerbounds are satisfied. An example follows: For a $K = 3$ user MIMO IFC where $d_k = 2\ \forall k$, $M_1 = N_1 = 4$, $M_2 = 5$, $N_2 = 3$, and $M_3 = 6$, $N_3 = 2$, the outerbounds (cf. (21) in [37]) are satisfied. However, the system does not admit an IA solution.
Similar approach of [37] was proposed in [99] to study feasibility of IA for a DL multiuser cellular network. In [38] the approach of counting the number of variables and the number of constraints in the IA problem has been studied under the more rigorous approach of algebraic geometry. The authors showed that the given approach represents a necessary and sufficient condition for the square symmetric MIMO IFC, i.e. equal number of transmit and receive antennas at all users. In a more recent paper [39] the same authors of [38] introduced a new feasibility study founding a condition that is both necessary and sufficient for a $K = 3$ user MIMO symmetric IFC with M transmitting and N receiving antennas at all terminals, each of them wants to transmit d streams. Another implication of their study is that the feasibility conditions based on comparing number variables and constraints does not always predict feasibility for the particular symmetric MIMO IFC under consideration. In a recent paper [100] has been shown that studying the feasibility problem for a *given* set of channel matrices is an NP-hard problem. On the other hand the problem studied in this chapter and in the other papers presented in this section consider feasibility for a *general* set of channel matrices with given dimensions, this problem is easier to be solved [101]. In [101] the authors studied the problem of IA feasibility for a general K-users IFC using results from algebraic geometry and differential topology. In [102, 103, 104, 105] IA for *Partially connected* channels has been studied. These networks are characterized by the propriety that some of the interfering links have zero gain, so each communi-

cation interferes only with a subset of user pairs. [104] studies the case of K-user MIMO IFC, where the model can be used to study arbitrary large networks, also feasibility conditions, based on the results in [37], for this particular interference channel are given. Then the results are applied to a more realistic cellular system model in [103].

3.2 Contributions

In this chapter we first introduce the system model of a K-user MIMO IFC that will also be used in the following chapters. Then we propose a systematic method to check feasibility of IA solutions for a given K-link Noisy MIMO IFC and an arbitrary DoF allocation. In particular, starting from interference alignment constraints, we introduce a recursive algorithm that allows an analytical evaluation of the existence of IA solutions (or lack thereof) for a given degrees of freedom allocation. We introduce a set of necessary conditions that if not satisfied prevents the existence of an interference alignment solution. Our approach is then validated using some numerical examples, comparing the result of our feasibility check with the convergence propriety of an iterative algorithm for determining IA solutions [34]. In addition we discuss interference alignment duality and we introduce the interpretation of IA as a constraint compressed SVD. Then an alternative IA feasibility check is introduced based on the idea that a stream can be suppressed at either the transmit or at the receive side. This suggestion is supported by the introduction of Homotopy method for IA. Finally we introduce the observation that working with real constellation, transmitted over complex channel, can be interpreted as transmission over real channel with doubled dimensions. This allows to increase the granularity for a finer adjustment of the achievable DoF.

3.3 System Model

Fig. 7.2 depicts a K-link MIMO interference channel with K transmitter-receiver pairs. To differentiate the two transmitting and receiving devices we assume that each of the K pairs is composed of a Base station (BS) and a Mobile user (MU). This is only for notational purposes. The k-th BS and its corresponding MU are equipped with M_k and N_k antennas respectively. The k-th transmitter generates interference at all $l \neq k$ receivers. The received signal in the Downlink (DL) phase

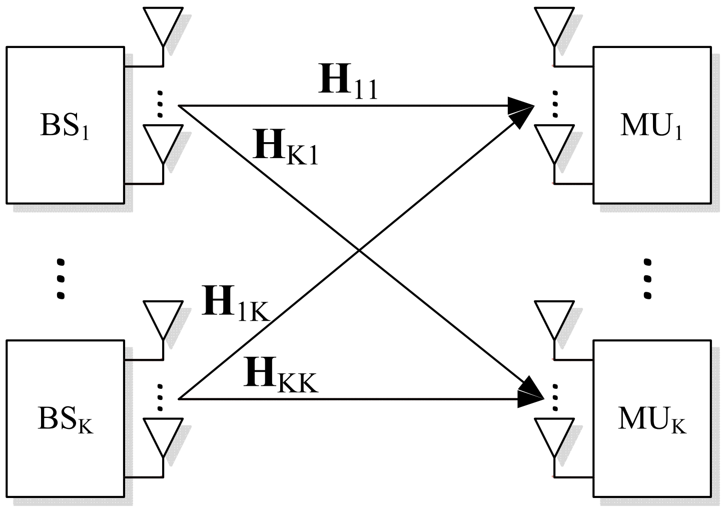

Figure 3.1: MIMO Interference channel

$\mathbf{y}_k$ at the k-th MU, can be represented as

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{x}_k + \sum_{\substack{l=1 \\ l\neq k}}^{K} \mathbf{H}_{kl}\mathbf{x}_l + \mathbf{n}_k \tag{3.1}$$

where $\mathbf{H}_{kl} \in \mathbb{C}^{N_k \times M_l}$ represents the channel matrix between the l-th BS and k-th MU, $\mathbf{x}_k$ is the $\mathbb{C}^{M_k \times 1}$ transmit signal vector of the k-th BS and the $\mathbb{C}^{N_k \times 1}$ vector $\mathbf{n}_k$ represents (temporally white) AWGN with zero mean and covariance matrix $\mathbf{R}_{n_k n_k}$. The channel is assumed to follow a block-fading model having a coherence time of T symbol intervals without channel variation. Each entry of the channel matrix is a complex random variable drawn from a continuous distribution. It is assumed that each transmitter has complete knowledge of all channel matrices corresponding to its direct link and all the other cross-links.

We denote by $\mathbf{G}_k$, the $\mathbb{C}^{M_k \times d_k}$ precoding matrix of the k-th transmitter. Thus $\mathbf{x}_k = \mathbf{G}_k\mathbf{s}_k$, where $\mathbf{s}_k$ is a $d_k \times 1$ vector representing the d_k independent symbol streams for the k-th user pair. We assume $\mathbf{s}_k$ to have a spatio-temporally white Gaussian distribution with zero mean and unit variance, $\mathbf{s}_k \sim \mathcal{N}(0, \mathbf{I}_{d_k})$. The k-th receiver applies $\mathbf{F}_k^H \in \mathbb{C}^{d_k \times N_k}$ to suppress interference and retrieve its d_k desired streams. The output of such a receive filter is then given by

$$\mathbf{r}_k = \mathbf{F}_k^H\mathbf{H}_{kk}\mathbf{G}_k\mathbf{s}_k + \sum_{\substack{l=1 \\ l\neq k}}^{K} \mathbf{F}_k^H\mathbf{H}_{kl}\mathbf{G}_l\mathbf{s}_l + \mathbf{F}_k^H\mathbf{n}_k$$

In the reverse transmission link, Fig. 7.2 Uplink (UL) phase, the received signal at the k-th BS is given by:

$$\overline{\mathbf{r}}_k = \overline{\mathbf{F}}_k^H \overline{\mathbf{H}}_{kk} \overline{\mathbf{G}}_k \overline{\mathbf{s}}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \overline{\mathbf{F}}_k^H \overline{\mathbf{H}}_{kl} \overline{\mathbf{G}}_l \overline{\mathbf{s}}_l + \overline{\mathbf{F}}_k^H \overline{\mathbf{n}}_k$$

where $\overline{\mathbf{F}}_k^H$ and $\overline{\mathbf{G}}_l$ denote respectively the $d_k \times M_k$ Rx filter at BS number k and the $N_l \times d_l$ BF matrix applied at MU l. The UL channel from the l-th MU to the k-th BS is denoted by $\overline{\mathbf{H}}_{kl}$.

3.4 Interference Alignment Feasibility

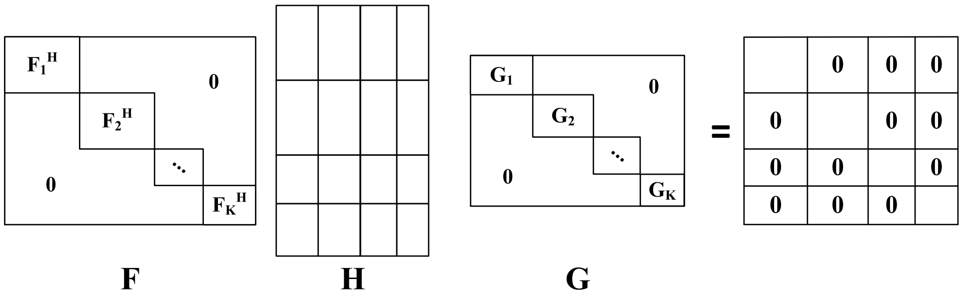

Figure 3.2: Block matrix representation of the interference alignment problem.

The objective in IA is to design aligning matrices to be applied at the transmitters such that, the interference caused by all transmitters at each non-intended Rx lies in a common *interference subspace*. Moreover, the interference subspace and the *desired signal subspace* of each Rx should be non-overlapping (linearly independent). If alignment is complete, simple ZF can be applied to suppress the interference and extract the desired signal in the high-SNR regime. Since IA is a condition for joint transmit-receive linear ZF, we need to satisfy the following conditions:

$$\mathbf{F}_k^H \mathbf{H}_{kl} \mathbf{G}_l = \mathbf{0} \quad \forall l \neq k \tag{3.2}$$

$$\mathrm{rank}(\mathbf{F}_k^H \mathbf{H}_{kk} \mathbf{G}_k) = d_k \quad \forall k \in \{1, 2, \ldots, K\} \tag{3.3}$$

The first step towards analytical evaluation of the existence of an IA solution for a given DoF allocation in a K-link MIMO IFC is the translation of the above equations into a set of conditions that need to be satisfied to admit an IA solution. To this end, the approach we adopt in this paper is of formulating the given IA problem as finding a solution to a system of equations with limited number of variables dictated by the dimensions of the overall system (the M_ks, N_ks and d_ks of the MIMO IFC). Fig. 3.2 presents a pictorial representation of such a system of equations where the block matrices **F**, **H** and **G** on the left hand side (LHS) of the equality represent respectively, the ZF Rx, overall channel matrix and beamformers. The block diagonal matrix to the right hand side (RHS) of the equality represents the total constraints in the system that need to be satisfied for an IA solution to exist. The block matrices on the diagonal of **H** represent the direct-links and the off diagonal blocks in any corresponding block row k represent the cross channels of the k-th link. The interference aligning beamformer matrix $\mathbf{G}_k$ (the diagonal blocks in **G**) aligns the transmit signal of the k-th user to the interference subspace at all $l \neq k$ users while ensuring the rank of the equivalent channel matrix $\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k$ is d_k. In

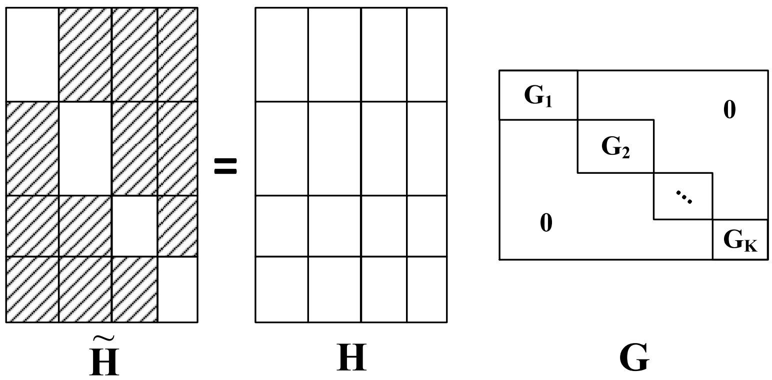

Figure 3.3: Interference alignment at all receivers.

other words, in Fig. 3.3, the $\mathbf{G}_k$ matrices are designed such that premultiplication of the overall beamformer matrix **G** with the overall channel matrix **H** results in a block matrix $\widetilde{\mathbf{H}}$ in which, all the off-diagonal blocks in any block row k (the shaded blocks of each block row) share a common column space whose dimension is at most $(N_k - d_k)$. With this accomplished, $\mathbf{F}_k$ simply projects the received signal into a subspace orthogonal to the interference subspace to retrieve the desired signal at the k-th Rx resulting in a $(d_k \times d_k)$ matrix (the rank d_k equivalent channel) for its desired streams and $(K-1)$ block-zero matrices in the k-th block row of the matrix to the right.

The only requirement on the $(d_k \times d_k)$ matrix that mixes up the desired streams is that it be of full rank. The beamforming matrix therefore, is determined up to an arbitrary $(d_k \times d_k)$ square matrix. Thus, of the total number of $(M_k \times d_k)$ variables available for the design of $\mathbf{G}_k$ matrix, transmission of d_k independent streams results in an immediate loss of d_k^2 variables thus reducing the total number

of variables available for the design of an interference aligning beamformer at each Tx to $d_k(M_k - d_k)$. The reason for evaluating the number of variables available at the Tx is the nature of the IA problem. The IA scheme essentially requires that all alignment is done at the Tx. Therefore every Tx imposes a set of constraints on the entire system (as a consequence of alignment conditions at each non intended Rx) whenever it transmits a stream to its Rx. Thus, an IA solution will be feasible only if the total number of variables available in the system is greater than or equal to the total number of constraints to be satisfied. Moreover, the variables should be distributed appropriately at each of the Tx. In the sequel, we provide a systematic method of counting the number of variables available for the design of an interference aligning beamformer at each Tx and comparing them with the constraints imposed on the system by each Tx. This method can be seen as arriving at the K-link MIMO IFC for which the existence of an IA solution is to be analyzed, by successively adding a single Tx and computing the total number of variables available for the joint design of the interference aligning beamformers at the transmitters and comparing it against the total number of alignment constraints imposed by the Tx (due to its d_k streams) at each step of this build-up.
The main idea of our approach is to convert the alignment requirements at each Rx into a rank condition of an associated interference matrix. At Rx k, the interference due to all other $(K-1)$ transmitters is grouped into a $(N_k \times \sum_{l=1;l\neq k}^{K} d_l)$ matrix

$$\mathbf{H}_I^{[k]}=[\mathbf{H}_{k1}\mathbf{G}_1, ...\mathbf{H}_{k(k-1)}\mathbf{G}_{(k-1)}, \mathbf{H}_{k(k+1)}\mathbf{G}_{(k+1)}, ...\mathbf{H}_{kk}\mathbf{G}_K],$$

that spans the interference subspace at the k-th Rx. We call this the interference matrix at user k. The total signal-space dimension at Rx k is given by the total number of receive antennas N_k, of which d_k interference-free signaling dimensions are to be reserved for the signal from the k-th Tx. This is achieved when the interference from all other transmitters lies in an independent subspace whose dimension can be at most $(N_k - d_k)$. Thus the dimension of the subspace spanned by the matrix $\mathbf{H}_I^{[k]}$ must satisfy

$$\operatorname{rank}(\mathbf{H}_I^{[k]}) = r_I^{[k]} \leq N_k - d_k \tag{3.4}$$

While the above equation prescribes an upperbound for the rank of the interference matrix, the nature of the channel matrix (full rank property) combined with the rank requirement of the beamformer at each Tx $(\operatorname{rank}(\mathbf{G}_k) = d_k)$ specifies the following lower bound on $r_I^{[k]}$

$$r_I^{[k]} \geq \max_{l\neq k}(d_l - [M_l - N_k]_+) \tag{3.5}$$

In the bound above we considered a pairwise communication where the k-th user receives interference by only one transmitter, $[x]_+ = \max(0, x)$ and $[M_l - N_k]_+$

discounts the possibility of the columns of $\mathbf{G}_l$ belonging to the orthogonal complement of $\mathbf{H}_{kl}$. Forcing the rank of $n \times m$ matrix $\mathbf{A}$ to some $r \leq \min(m, n)$ implies imposing $(n-r)(m-r)$ constraints. We explain this briefly as follows. Without loss of generality, (w.l.o.g) assume that the columns of this $n \times m$ matrix are partitioned into $\mathbf{A} = [\mathbf{A}_1; \mathbf{A}_2]$ where $\mathbf{A}_1$ is $n \times r$ and is of full column rank. Then imposing a rank r on $\mathbf{A}$ implies that $\mathbf{A}_2$ shares the same column space as $\mathbf{A}_1$ which in turn implies that $\mathbf{A}_1^{\perp T}\mathbf{A}_2 = \mathbf{0}$. Since $\mathbf{A}_1^{\perp}$ is $n \times (n-r)$, it follows that $(n-r)(m-r)$ constraints need to be satisfied for $\mathbf{A}$ to be of rank r. Thus imposing a rank $r_I^{[k]}$ on $\mathbf{H}_I^{[k]}$ implies imposing

$$(N_k - r_I^{[k]})(\sum_{\substack{l=1 \\ l \neq k}}^{K} d_l - r_I^{[k]})$$

constraints at Rx k. $r_I^{[k]}$ is maximum when the interference contribution of each interferer spans an independent subspace. Which leads us to the upper bound $r_I^{[k]} \leq \sum_{l=1;l\neq k}^{K} d_l$. However, accounting for the inequality in (3.4) we have

$$r_I^{[k]} \leq \min(d_{tot}, N_k) - d_k \tag{3.6}$$

where $d_{tot} = \sum_{k=1}^{K} d_k$, and $\min(.)$ operation appears in the above equation due to the fact that the rank of $\mathbf{H}_I^{[k]}$ cannot exceed its dimensions.

3.5 Recursive procedure to evaluate feasibility

In this section we detail a recursive method of evaluating the feasibility of an IA solution for a MIMO IFC and a corresponding DoF distribution. As mentioned earlier, the main idea here is to interpret the interference alignment requirement at each Rx as forcing a certain rank on the associated interference channel $\mathbf{H}_I^{[k]}$ which in turn imposes a certain number of constraints on the IA problem. In the earlier section we show that this rank is bounded above and below by the system parameters. The first step therefore is to ensure that the range of each r_i is non-empty. From (3.4) and (3.5), this amounts to checking if

$$(\min(d_{tot}, N_k) - d_k) - \max_{j \in \mathcal{K}-\{k\}} (d_j - [M_j - N_k]_+) \geq 0 \quad \forall k \in \mathcal{K} \tag{3.7}$$

where $\mathcal{K} = \{1, 2, ...K\}$. Indeed, an IA solution is immediately ruled out if (3.7) is not true. This is due to the fact that the full rank nature of the cross channel $\mathbf{H}_{kj}$ will ensure that the minimum rank of $\mathbf{H}_I^{[k]}$ due to $j \neq k$ will be d_j unless it possesses a null space of non zero dimension in which case it can shrink the rank

$$\sum_{i=1}^{k} d_i(M_i - d_i) \geq \sum_{i=1}^{k} (N_i - \underbrace{\min(\underline{d} - d_i, (N_i - d_i))}_{\overline{r}_I^{[i]}})(\underline{d} - d_i - \min(\underline{d} - d_i, (N_i - d_i)))$$
$$+ \sum_{i=k+1}^{K} (\ N_i - \underbrace{\min(\underline{d}, (N_i - d_i))}_{\overline{r}_I^{[i]}}\ )(\ \underline{d} - \min(\underline{d}, (N_i - d_i))\ ) \tag{3.8}$$

by a maximum of $[M_j - N_k]_+$. (3.7) can be interpreted as check for the minimum values of M_k and N_k $\forall k$ for a given DoF allocation.
Proposition: Let $\mathcal{M}_K = \{\{M_k\}, \{N_k\}, \{d_k\}\}$ *represent a* K*-link MIMO IFC where* $\{M_k\}$ *and* $\{N_k\}$ *represent the ordered set of transmit and receive antennas of each user in the system and* $\{d_k\}$ *is the ordered set of the associated DoF desired for each user (ordering is by user index). Denote by* $\mathcal{K}_o$ *the ordered set of users with decreasing* d_k *such that users with equal* d_k*s are ordered according to increasing* M_k*. Similarly, define* $\mathcal{M}_K^{'}$ *to be the MIMO IFC and the associated set* $\mathcal{K}_o^{'}$ *obtained by interchanging* $\{M_k\}$ *and* $\{N_k\}$*. Then an IA solution exists if both of the following conditions are satisfied:*

1. (3.7) holds true for $\mathcal{M}_K$ and $\mathcal{M}_K^{'}$
2. Starting from a system consisting only of the K receivers, if the complete system $\mathcal{M}_K$ (respectively $\mathcal{M}_K^{'}$) is "built" by successively adding one Tx at a time from $\mathcal{K}_o$ (respectively $\mathcal{K}_o^{'}$) and (3.8) is valid (satisfied) at each step of this "build-up".

The need to satisfy both the above conditions for $\mathcal{M}_K$ *and* $\mathcal{M}_K^{'}$ arises due to the alignment duality. From the IA conditions in (3.2) (3.3), it is clear that taking the transpose of these equations results in IA conditions for the *dual* MIMO IFC and the same existence conditions should be satisfied for this dual MIMO IFC as well.
At each step k of the recursion, (3.8) accumulates the total number of variables available for designing an IA solution in an associated sub-problem comprising of a k-link MIMO IFC where only k transmitters are transmitting non-zero streams and aligning their streams into some interference subspace of all non-intended receivers in the LHS of (3.8). The RHS accumulates the total number of constraints at all receivers that arise due to these transmitters. That the number of variables contributed by the i-th Tx is given by $d_i(M_i - d_i)$ is obvious from the discussion in the previous section. We now elaborate on the method of obtaining the constraints on the RHS of (3.8). Forcing a rank on $\mathbf{H}_I^{[k]}$ amounts to satisfying a number of

constraints that is a function of the rank and the dimensions of $\mathbf{H}_I^{[k]}$. While we do not have knowledge of the exact rank of $\mathbf{H}_I^{[k]}$ at each k (since that will be the result of the IA design whose feasibility we are evaluating in the first place) we do know the numerical *range* of $r_I^{[k]}$ for each k. Therefore, instead of using the actual rank it is useful to use its upperbound (denoted by $\overline{r}_I^{[k]}$, as specified in (3.6)). On the RHS of (3.8) the first summation reflects the total number of constraints to be satisfied for an IA solution to exist in a k-link MIMO IFC with k-links transmitting a total of $\underline{d} = \sum_{i=1}^{k} d_i$ streams. For each user i accounted for in this summation, we have to ensure that at Rx-i, $r_I^{[i]} \leq (N_i - d_i)$. The column dimension of $\mathbf{H}_I^{[i]}$ is $(\underline{d} - d_i)$. In order to minimize the total number of constraints that we impose of the system (due to the act of forcing a particular $r_I^{[i]}$ at the i-th Rx), we choose the maximum possible rank of $r_I^{[i]}$, which we know to be $\min(\text{column dimensions}, N_i - d_i)$ i.e., $\overline{r}_I^{[i]} = \min(\underline{d} - d_i, N_i - d_i)$. The second summation consists of all "un-paired" receivers in the sub-problem i.e., those receivers whose corresponding transmitters are presently not transmitting any streams but still need $\underline{d}$ streams to be aligned in their interference subspace. Therefore, the maximum allowable rank of the interference matrices for all these receivers is $\overline{r}_I^{[i]} = \min(\underline{d}, N_i - d_i)$. Thus, (3.8) when true at each step, verifies that the number of variables available for the design of IA beamformers at all k transmitters is greater than the number of constraints that are imposed by an IA solution. In fact, it verifies that its is possible to align all the interference not just in the associated k-link MIMO IFC but also in the interference subspace of all un-intended receivers that are not in the k-link MIMO IFC (the un-paired receivers accounted for in the second summation). Finally, the ordering of the users in terms of increasing d_k in $\mathcal{K}_o$ ($\mathcal{K}_o^{'}$ for $\mathcal{M}_K^{'}$) ensures early identification of in-feasibility of an IA solution since a larger DoF requirement typically results in smaller number of variables available at the Tx in order to meet the rank constraints.

In the next section we present numerical examples to show that our approach is able to check the feasibility (or in-feasibility) of an IA solution for a given MIMO IFC. For a $\mathcal{M}_K$ which conforms to both the conditions of our approach, we are able cross validate that an IA solution exists using an iterative algorithm proposed in [34]. Indeed, it can be shown that the algorithm in [34] will always converge to an optimum solution when our conditions are met since convergence to an optimum solution implies that the d_k minimum eigenvalues of $\sum_{i \neq k} \mathbf{H}_{ki}\mathbf{G}_i\mathbf{G}_i^H\mathbf{H}_{ki}^H$ are zero which will be true if $\mathrm{rank}(H_I^{[k]}) \leq \min(d_{tot}, N_k) - d_k$ which is a part by our systematic approach.

3.6 Numerical Examples

In this section we provide some numerical examples to validate the conditions derived in this paper. In all the examples given in this section, when the MIMO IFC that satisfied the conditions in Sec. 3.5, the numerical algorithm in [34] was able to find an IA solution whereas it failed to find one when these conditions were not satisfied. We tested our conditions extensively with varied antenna and stream distributions. In particular, all the examples in [37] we also tested.

Example 1: Consider a 2-link MIMO system with $M = 2, N = 4, d = 2$. This system satisfies the 2 conditions in Sec. 3.5 and IA solutions do exist for this system.

Example 2: Similarly, the 6 user case where $M_k = 3, N_k = 4, d_k = 1 \forall k$, both conditions in Sec. 3.5 are satisfied and an IA solution is possible for this case.

Example 3: We now look at another 2-link MIMO system with $M_1 = 4, N_1 = 7, d_1 = 3, M_2 = 10, N_2 = 4, d_2 = 2$. For this system, the rank conditions are not satisfied and indeed, there is no IA solution for this case.

Example 4: In the 4-link case characterized by $M_k = 2, N_k = 3\ k = 1, 2, 3$ and $M_4 = N_4 = 2\ d_k = 1 \forall k$. The rank conditions are satisfied but (3.8) is not satisfied. Therefore we conclude that there cannot be an interference alignment solution for this system.

3.7 Alignment Duality

There are another set of conditions that need to be considered in order to complete the existence conditions. These conditions arise from the equations

$$\overline{\mathbf{F}}_l^H \overline{\mathbf{H}}_{lk} \overline{\mathbf{G}}_k = \mathbf{0} \quad \forall k \neq l \tag{3.9}$$

$$\text{rank}(\overline{\mathbf{F}}_k^H \overline{\mathbf{H}}_{kk} \overline{\mathbf{G}}_k) = d_k \quad \forall k \in \{1, 2, \ldots, K\} \tag{3.10}$$

which corresponds to the interference alignment constraints of the dual problem where all transmitters and receivers exchange roles. In other words, when $\overline{\mathbf{F}}_l = \mathbf{G}_l^H, \overline{\mathbf{G}}_k = \mathbf{F}_k^H, \overline{\mathbf{H}}_{lk} = \mathbf{H}_{kl}^H$ in (3.9) and (3.10). The dual problem of an interference channel is again an interference channel, involving the reciprocal channel. For the ZF case (interference alignment) the conditions (3.9)-(3.10) for the dual problem are obtained immediately by simply taking the transpose of (3.2)-(3.3) for the original problem [33]. If the ZF filters are replaced by MMSE receive filters that are the optimal interference suppressing filters (c.f. Sec.4.4.1) we conjecture a sum-rate duality for the K-user MIMO interference channel for an appropriate choice of receiver noise covariance matrices and transmit power constraints. This duality for maximizing the sum rate will be analyzed in more detail in this thesis in chapter 4. In this section we focus on interference alignment duality and hence restrict ourselves to a ZF design. As a direct consequence of interference alignment duality, for an interference alignment solution to exist, the conditions in (3.7)- (3.8) should also be satisfied when the M_k and N_k are interchanged.

3.8 IA as a Constrained Compressed SVD

For IA purposes, the $\mathbf{F}_k^H$, $\mathbf{G}_i$ can be constrained to be (column) unitary, since only their column spaces matter. As a result, the matrices $\mathbf{F}^H$, $\mathbf{G}$ below are (column) unitary ($\mathbf{F}\mathbf{F}^H = \mathbf{I}$,$\mathbf{G}^H\mathbf{G} = \mathbf{I}$). Now, it is useful to think of an IA solution as a constrained compressed SVD in the following form:

$$\begin{aligned}\mathbf{F}^H\mathbf{H}\mathbf{G} &= \begin{bmatrix}\mathbf{F}_1^H & 0 & \cdots & 0\\ 0 & \mathbf{F}_2^H & \ddots & \vdots\\ \vdots & & \ddots & 0\\ 0 & \cdots & 0 & \mathbf{F}_K^H\end{bmatrix}\begin{bmatrix}\mathbf{H}_{11} & \mathbf{H}_{12} & \cdots & \mathbf{H}_{1K}\\ \mathbf{H}_{21} & \mathbf{H}_{22} & \cdots & \mathbf{H}_{2K}\\ \vdots & & \ddots & \vdots\\ \mathbf{H}_{K1} & \mathbf{H}_{K2} & \cdots & \mathbf{H}_{KK}\end{bmatrix}\begin{bmatrix}\mathbf{G}_1 & 0 & \cdots & 0\\ 0 & \mathbf{G}_2 & \ddots & \vdots\\ \vdots & & \ddots & 0\\ 0 & \cdots & 0 & \mathbf{G}_K\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{F}_1^H\mathbf{H}_{11}\mathbf{G}_1 & 0 & \cdots & 0\\ 0 & \mathbf{F}_2^H\mathbf{H}_{22}\mathbf{G}_2 & & \vdots\\ \vdots & & \ddots & 0\\ 0 & \cdots & 0 & \mathbf{F}_K^H\mathbf{H}_{KK}\mathbf{G}_K\end{bmatrix}\end{aligned} \tag{3.11}$$

where the last matrix is in general block-diagonal. This resembles a ”compressed SVD” because only rectangular unitary matrices are used in the diagonalization instead of full square unitary matrices, and the term ”constrained” refers to the block diagonal nature of the unitary matrices $\mathbf{F}$, $\mathbf{G}$.

3.9 Alternative Zero Forcing Approach to IA

Another possible approach to determine if a K-link MIMO interference channel has an IA solution can be obtained interpreting interference alignment as joint transmit-receive linear zero forcing. The idea is that a stream transmitted from Tx k and causes interference to the non intended Rx j can be suppressed at either the Tx or at the Rx. Denoting with t_{kj} the size of the subset of streams d_k, that are received at Rx j that the k-th Tx suppresses, and with r_{kj} the size of the subset of streams d_k, that are received at Rx j, that the j-th Rx suppresses, the sum of these two quantities should be: $t_{kj} + r_{kj} \geq d_k$. The total number of streams that Tx k can suppress is at most $M_k - d_k$ and the total number of streams that the j-th Rx can suppress is not greater than $N_j - d_j$. Therefore, to check the feasibility of an interference alignment solution, the following conditions should be satisfied:

$$\begin{array}{ll} \sum_{j\neq k} t_{kj} & \leq M_k - d_k \\ \sum_{k\neq j} r_{kj} & \leq N_j - d_j \end{array} \tag{3.12}$$

$$\forall t_{kj}, r_{kj} \in \{0, 1 \ldots, d_k\}, \text{ and } t_{kj} + r_{kj} = d_k$$

$$\max_{k\neq j}(d_j - [M_k - N_j]) \leq (N_j - d_j)\forall j \in \{1, \ldots, K\}$$

As before, due to alignment duality, (3.12) must be true when M_k and N_k values are interchanged (the dual channel case). One possible way to verify if all this inequalities are satisfied or not is to check all the possible $\prod_{k=1}^{K}(d_k + 1)^{K-1}$ combination of t_{kj} and r_{kj}. If there is at least a combination that satisfies the constraints, that one corresponds to the interference alignment solution.Such an alternate approach has some interesting implications.

Example 5: Consider $\mathcal{M}_3 = \{\{M_k\} = \{N_k\} = \{1, 3, 6\}, \{d_k\} = \{1, 2, 3\}\}$. w.l.o.g., order the users in terms of increasing d_k, then, the first user pair is in no position to do anything. However, $\mathbf{G}_2$ can be designed to suppress interference caused at the Rx of user-1 and $\mathbf{G}_3$ can be designed to suppress interference caused at the receivers of users 1 and 2. Similarly, $\mathbf{F}_2$ can suppress interference generated by user-1 while $\mathbf{F}_3$ can be designed to suppress interference from transmitters of user-1 and user-2. Thereby enabling reception of d_k interference free streams $\forall k$ user pairs. More interestingly, based on the structure of the above problem, we have the following conjecture that draws attention to the benefits of systems with unequal stream distributions.

Conjecture: There exists a MIMO IFC $\mathcal{M}_K^{(u)}$ with unequal antenna and stream distribution for any given network DoF d_{tot}, such that the total number of antennas in $\mathcal{M}_K^{(u)}$, $\mathcal{A}_{tot}^{(u)} = \sum_k(M_k + N_k)$, required to achieve d_{tot} is less than the total number of antennas in $\mathcal{M}_K^{(e)}$ where $M_k = M, N_k = N, d_k = d_{tot}/K\ \forall k$. $\mathcal{M}_K^{(e)}$ is the so-called identical stream and antenna configuration (ISAC) [106] or symmetric [37] system.

The conjecture is motivated by the generalization of *Example 6* to any K-link system. Consider a K-link MIMO IFC with user pairs indexed in the order of increasing d_k. Let the following relationship hold.

$$d_{(k+1)} = d_k + 1,\ k \in 2, \ldots, K.$$

Then it can be shown that an IA solution exists if each user pair has the following antenna distribution:

$$M_k = N_k = \sum_{i=1}^{k} d_i,\ k \in \{1, \ldots, K\}.$$

Let $\mathcal{A}_{tot}^{(e)}$ represent the total number of antennas in an ISAC system $\mathcal{M}_K^{(e)}$. We know from [106] [37] that, for $\mathcal{M}_K^{(e)}$ the minimum number of antennas per-user needs to satisfy

$$M + N \geq (K+1)\frac{d_{tot}}{K}.$$

It is easily verified that, for $K \geq 2$, $\mathcal{A}_{tot}^{(u)} < \mathcal{A}_{tot}^{(e)}$.
It is also possible to prove this starting from a given $\mathcal{M}_K^{(e)}$ and splitting the d_{tot} into a DoF allocation where not all users have the same DoF.

3.10 Homotopy Methods

Homotopy methods [107] are used to find the roots of a non-linear system of equations of the form $\mathcal{F}(x) = 0$. A homotopy transformation is such that it starts from a trivial system $\mathcal{G}(x)$, with known solution, and it evolves towards the target system $\mathcal{F}(x)$ via continuous deformations according to the homotopy parameter $t = 0 \to 1$:

$$\mathcal{H}(x,t) = (1-t)\ \mathcal{G}(x) + t\mathcal{F}(x)$$

Predicting the solution at the next value of $t^{(i+1)} = t^{(i)} + \Delta t$ is called an Euler prediction step; a solution at $t^{(i+1)}$ can be refined using a Newton correction step for fixed t. A property of Homotopy continuation methods for the solution of system of equation is that the number of solutions in the target system is at most equal to the number of solutions in the trivial system. The number of solutions with varying t remains constant as long as the Jacobian (w.r.t. x and t jointly) is full rank. So as t reaches 1, it can happen that the Jacobian becomes singular.

3.10.1 Homotopy Applied to IA

Homotopy method can be applied to the IA problem, in particular here it is not really suggested for computing IA solutions, but for counting number of solutions. The objective in IA is to design Tx and Rx filters that satisfy the ZF conditions

$$\mathbf{F}_k^H \mathbf{H}_{kl} \mathbf{G}_l = \mathbf{0} \quad \forall l \neq k \tag{3.13}$$

and the rank conditions

$$\text{rank}(\mathbf{F}_k^H \mathbf{H}_{kk} \mathbf{G}_k) = d_k \quad \forall k \in \{1, 2, \dots, K\} \tag{3.14}$$

which correspond to the traditional single user MIMO constraint $d_k \leq \min(M_k, N_k)$ for d_k streams to be able to pass over the k-th link. The main constraints are the n ZF conditions in (3.13). These conditions are bilinear equations in the Tx and Rx filters, hence they are of second order. As a result, the overall order of the ZF conditions jointly is 2^n, which is also the maximum number of solutions. It turns out that due to the particular structure of the ZF conditions (in a given ZF condition only one Tx and Rx filter appear), the actual number of solutions is much more limited. To analyze the number of IA solutions, the following approach has been proposed in [108]. Instead of choosing the homotopy parameter to be related to SNR, we choose it here to attenuate the MIMO channel singular values beyond the main ones:

$$H_{ji} = \sum_{k=1}^{d} \sigma_{jik}\mathbf{u}_{jik}\mathbf{v}_{jik}^H + t \sum_{k=d+1} \sigma_{jik}\mathbf{u}_{jik}\mathbf{v}_{jik}^H \ .$$

The IA Jacobian is still full rank if we reduce rank(H_{ji}) to $\max(d_j, d_i)$. Hence we can still count the same number of IA solutions when $t = 0$. The case of $d_k \equiv d = 1$ is considered here. Then finding the IA solutions at $t = 0$ becomes trivial. Indeed, IA requires

$$\mathbf{f}_j^H \mathbf{u}_{ji1}\mathbf{v}_{ji1}^H \mathbf{g}_i = 0$$

or hence either $\mathbf{f}_j^H \mathbf{u}_{ji1} = 0$ or $\mathbf{v}_{ji1}^H \mathbf{g}_i = 0$. The joint Tx-Rx ZF is achieved by either the Tx or the Rx suppressing the particular interfering stream. This analysis supports a suggestion provided in Sec. 3.9 which states that it should be possible to check IA feasibility and count the number of IA solutions by verifying if the ZF task can be properly distributed over Tx and Rx filters. So, the homotopy method allows to substantiate this approach, at least in the single stream per link case.

More generally, determining IA solutions by continuation methods can be obtained by perturbing the ZF conditions up to first order

$$(\mathbf{F}_j^H + d\mathbf{F}_j^H)(\mathbf{H}_{ji} + d\mathbf{H}_{ji})(\mathbf{G}_i + d\mathbf{G}_i) = 0$$

Assuming that an IA solution for channel $\mathbf{H}_{ji},\ \forall(i,j)$ has already been determined using filters $\mathbf{F}_j$ and $\mathbf{G}_i$ then considering only the terms up to first order in the product above we get:

$$\mathbf{F}_j^H \mathbf{H}_{ji} d\mathbf{G}_i + d\mathbf{F}_j^H \mathbf{H}_{ji}\mathbf{G}_i = -\mathbf{F}_j^H d\mathbf{H}_{ji}\mathbf{G}_i.$$

To find the IA solution for channel $(\mathbf{H}_{ji} + d\mathbf{H}_{ji})$ we determine the matrices $d\mathbf{F}_j^H$ and $d\mathbf{G}_i\ \forall i, j$ by solving linear equations.

3.11 Interference Alignment For Real Signals

The key observation we make in this section is that by using real signal constellations in place of complex constellations, transmission over a complex channel of any given dimension can be interpreted as transmission over a real channel of double the original dimensions (by treating the in-phase and quadrature components as separate channels). This doubling of dimensions provide additional flexibility in achieving the total DoF available in the network. We show this with a simple example of a 3 user symmetric MIMO interference channel where each transmitter and receiver has the same number of antennas. i.e., $M = N = 3$. From the results of the earlier section, we can show that the maximum interference-free streams available per-user obeys $d \leq 1.5$. Since the concept of transmitting 0.5 streams does not make any practical sense, any interference alignment solution that exists for this system allows reception of a maximum of 1 interference-free stream per user. However, if the complex channel is considered to be composed of two real channels and if the transmitters and receivers use real signal constellations, the dimensions involved in the above problem are doubled and hence allows for a finer adjustment of d_k

Representing the $2M_k \times 2N_k$ *real* MIMO channel as

$$\hat{\mathbf{H}} = \begin{bmatrix} \text{Re}\{\mathbf{H}\} & -\text{Im}\{\mathbf{H}\} \\ \text{Im}\{\mathbf{H}\} & \text{Re}\{\mathbf{H}\} \end{bmatrix}$$

and using $\hat{\mathbf{x}}$ to represent the $2N_k \times 1$ real signal vector, the received signal at the k-th receiver can now be expressed as

$$\hat{\mathbf{y}}_k = \hat{\mathbf{H}}_{kk}\hat{\mathbf{x}}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \hat{\mathbf{H}}_{kl}\hat{\mathbf{x}}_l + \hat{\mathbf{n}}_k \tag{3.15}$$

In our example, each user is now capable of transmitting 3 real streams thereby exploiting fully all the available per-user DoF.

3.12 Conclusions

We considered the problem of analytically evaluating the feasibility of an interference alignment solution for a given degrees of freedom allocation in a general K-link MIMO IFC. We derived a set of necessary conditions and presented a systematic method to check if these conditions are satisfied for a given MIMO IFC. We showed that, when an IA solution exists, these conditions are satisfied at every step of this systematic approach.We also show that an IA solution does not exist when these conditions are not satisfied. Exploring the fact that IA feasibility is unchanged when the MIMO crosslink channel matrices have a reduced rank, equal to the maximum of the number of streams passing through them we propose a new way to study the problem using numerical continuation method. The rank reduction simplifies IA design and feasibility analysis, and allows in particular a counting of the number of IA solutions. In this approach the parameter that defines the continuation method is a scale factor for the remaining channel singular values, the solution for reduced rank channels can be evolved into that for arbitrary channels.

Chapter 4

Sum Rate Maximization for the Noisy MIMO Interference Channel

4.1 Introduction and state of the art

In the previous chapter we studied the interference alignment transmission strategy. In particular our focus was on determining a set of feasibility conditions for the existence of an IA solution. This transmission strategy has shown the capability to maximize the prelog factor at high SNR regime but its performance at medium/low SNR ranges are suboptimal, as shown in [33]. The main reason for that resides on the fact that the IA transmission relies essentially on ZF transmit-receive filters. Then when the noise is negligible, compared to the interference contributions at each receiver the ZF solution becomes optimal. On the other hand when noise is the main impairment, or it becomes comparable to the interference contributions, ZF is well know to be suboptimal. For this reason different transmission strategies are to be investigated if the objective is to optimize the system performance at other SNR regimes.

In single user MIMO (SU-MIMO) link the problem of joint transmit-receive filter design was addressed in [109], where the optimization problem is the minimization of the mean-squared error (MSE) under average power constraint. Then in [110] the authors addressed the problem of designing jointly optimum linear precoder and decoder for a MIMO channel using a weighted minimum mean-squared error (WMMSE) criterion subject to a transmit power constraint. The optimum linear precoder and decoder results to diagonalize the MIMO channel into eigen subchannels. In the more recent work [111] the authors consider the joint design of linear processing at both ends of the link for a single user MIMO link according to a variety of criteria. As result they developed a unified framework for the optimization of transmit-receive filters that simplifies the design problem that can be formulated within the framework of convex optimization theory [92].

In the seminal work [91] the joint Tx-Rx filter design based on SINR criteria is studied. The authors focused on trying to optimize the worst SINR considering two design strategies: maximizing the worst SINR subject to an average power constraint, and minimizing the average power subject to a constraint on the worst SINR. They showed that the proposed problems can be easily reformulated in such a way that can be solved using standard optimization packages.

In [112] the authors studied the problem of transmit preprocessing design for the downlink of multiuser MIMO (MU-MIMO) systems. This problem describes the scenario where a multi-antenna base station transmits useful information to different multi-antenna receivers relying on the spatial dimensions for the separation of different streams at the receive side. The technique is based on decomposing a multiuser MIMO downlink channel into parallel independent single-user MIMO downlink channels.[113] considers the joint transmit and receive filter design for the uplink communication of a MU-MIMO channel where the objective function is the minimization of the total MSE under a per-transmitter power constraint.

A different line of research is based on optimizing the capacity of the system. In particular for multi-antenna Gaussian broadcast channel a capacity achieving strategy involves a non-linear interference pre-cancellation technique, known as dirty paper coding (DPC) [78]. This approach requires channel and users data information at the transmit side and needs high complex encoding and decoding operations. For this reason other, less complex solutions have been studied that are based on linear transmit and receive filters. A first attempt to maximize the sum rate for BC channel was made in [114], where the authors propose an iterative algorithm for the design of precoding matrix in a multi-antenna broadcast system. The precoding techniques are constrained to linear preprocessing at the transmitter. In addition also the problem of maximizing the minimum rate among all users is studied. This problem is shown to be quasiconvex and can be solved exactly. [115] studied the problem of weighted sum rate (WSR) maximization for a MIMO BC channel under a sum power constraint. The problem is solved using the framework of mean-squared error (MSE) duality [116, 117]. The power allocation problem was reformulated as a geometric program (GP) involving the geometric MSE, for which the global optimum can be found efficiently. Some early work on the MIMO IFC was reported in [40] by Ye and Blum for the asymptotic cases when the interference to noise ratio (INR) is extremely small or extremely large. It was shown there that a ”greedy approach” where each transmitter attempts to maximize its individual rate regardless of its effect on other un-intended receivers is provably suboptimal. It was also noted there that the network capacity in general is neither a convex nor concave function of the transmit covariance matrices thus making it difficult to find an analytical solution to the optimization problem. The MIMO IFC was studied in a game theoretic framework in [41] where such problem was modeled as a non-cooperative game and shown to have a unique Nash-equilibrium point subject to mild conditions on the channel matrices. However, since each link selfishly maximizes its own rate, the attained Nash equilibrium may not be socially efficient and then far from the max sum rate point. In [42] the authors proposed an algorithm for finding the beamformer in the single stream K-user MIMO IFC that attempts to maximize the weighted sum rate (WSR). The beamforming vectors can be interpreted as a balance between an egoistic approach, where the transmitter tries to maximize its own rate, and an altruistic approach where each beamformer puts its effort to minimize the interference that it causes to the non intended receivers. In [43] the joint linear transceiver design problem for the downlink multiuser MIMO systems with coordinated base stations has been studied. They consider the maximization of the weighted sum rate with per BS antenna power constraint problem. An iterative algorithm is proposed where the optimal receivers are MMSE filters while the transmit beamformers are found using second-order-cone-programming (SOCP). [69] addressed the problem of WSR maximization for a MIMO interference channel under per transmitter power constraint. A distributed algorithm is introduced that is based on the Karush-Kuhn-Tucker (KKT) conditions of a convex version of the WSR maximization problem obtained using Taylor expansion of the cost function. The application to cognitive radio channel is also considered. In [108] the authors present an iterative algorithm that finds an IA solution that maximize the average sum-rate. At each step an IA solution is found using a technique proposed in [33] and then they move the solution along the direction of the gradient of the sum-rate w.r.t. the beamformers in the Grassmann manifold. Even though this algorithm performs better than traditional IA solutions in the High SNR regime it is highly sub-optimal, in terms of sum-rate, in medium SNR ranges. [118] addressed the problem of maximizing the sum rate for a MIMO interference channel proposing an iterative algorithm based on gradient descend method that converges to a local optima solution. A different approach to maximize some utility functions in IFC is represented by pricing algorithms. There each receiver calculates interference prices that describe the relative decrease of its utility function with respect to an increase of interference. Those prices are then exchanged with all, or neighboring, transmitters that will consequently adjust the transmitting parameters.

This approach can be useful to develop decentralized algorithms. Refer to [119] for an overview of pricing algorithm for IFC. In a recent work [45] the authors elegantly exploit the connection between the maximum WSR (MWSR) problem and the weighted minimum mean squared error (WMMSE) problem to obtain locally optimum solutions for the (non-convex) MWSR problem for a MIMO BC channel. To find the solution the authors used the results from [120] where the transmit Wiener filter is derived for a BC channel under a total power constraint. The solution provided in [45] has been extended to the multi-cell multi-user MIMO in [44], where also a distributed solution is provided that relies on the iterative exchange of information between transmitters and receivers. The main problem with the maximization of the WSR is the high non convexity of the cost function. This implies that even if it is possible to prove convergence of the proposed algorithms to a local optimal point convergence to global optima can not been shown. In addition convergence to local optimal solution is not a rare event if the initialization point of the algorithm is not carefully chosen. To avoid this situation several heuristic approach can be used. In [121] the joint optimization of beamformers and linear receivers in a MIMO interference network where each transmitter transmits a single beam is considered. The solution provided maximizes the sum rate using an iterative algorithm. To improve convergence properties a technique, which tracks the local optimum as the SNR is incrementally increased, similar to a homotopy method is also introduced.

4.2 Contributions

At the beginning of this chapter we first introduce the WSR maximization problem for a MIMO interference channel where, in contrast to a BC channel, multiple per transmitter power constraints are to be introduced. Then we extend the relation between WSR maximization and the minimization of the weighted sum mean squared error, introduced for a BC channel in [45], to the MIMO IFC. This allows us to solve the maximization of the WSR, highly non convex problem, with a simpler to handle problem like MSE minimization. Then the local optima found using a WMSE approach reveals to be also a local optimal for the WSR problem. Using the given relationship we solve the problem for the MIMO IFC extending the results of [45] and [120]. The framework introduced with WMSE helps us to show how it is possible to solve the WSR problem directly without using the MSE approach, where some of the quantity appearing in the optimization process should be reinterpreted as receive filters and proper weighting factors. Subsequently we specify the algorithm when the WSR is maximized under a per-stream approach. In [45] it has been shown that working per stream instead of per user does not introduce any sub-optimality. This per-stream approach helps us to introduce a WSR duality for the MIMO IFC where the optimal transmit filter results to be an MMSE receiver filter in a dual UL communication with a proper transmit covariance matrix and dual noise. Due to the non convexity of the cost function, convergence to several local optima is possible in particular if the starting point is not chosen properly. To reduce the possibility to be trapped in such a stationary point we introduce a novel approach based on Deterministic Annealing. This approach has some similarity with the solution independently proposed in [121] but in our work the possibility to handle multi-stream transmission is introduced. The solution found with the WSR maximization via the minimization of the WMSE helps us to show the optimality of the extension of the method proposed in [114], for a BC channel, to the MIMO IFC. There the transmit beamformers are reparametrized such that the transmit power constraints are always satisfied with equality. Finally some discussion on how to optimize the WSR at high SNR are introduced followed by a simulation section where the proposed algorithm is validated numerically.

4.3 Weighted sum rate maximization for the MIMO IFC

The stated objective of our investigation is transmit beamforming design to maximize the WSR of MIMO IFC. From the perspective of a network operator, the maximization of the total throughput represents probably the most important objective. In heterogeneous networks, there are users with different priorities which could be a function of their subscription. In such networks, the throughput maximization translates as weighted sum rate maximization. In addition the weights in the WSR can also be chosen to characterize the queue buffer size in communication systems limited by packet arrival and transmission queues length. Hence it is very natural and equally insightful to use this cost function in the optimization procedure for the design of transmit and receive strategies.

It is for this reason that, in this work we consider the weighted sum rate maximization problem for a K-user frequency-flat MIMO IFC and propose an iterative algorithm for linear precoder/receiver design. With full CSIT, but only knowledge of $\mathbf{s}_k$ at transmitter k, it is expected that linear processing at the transmitter should be sufficient. On the receive side however, optimal WSR approaches may involve joint detection of the signals from multiple transmitters. In this paper we propose to limit receiver complexity by restricting the modeling of the signals arriving from interfering transmitters as colored noise (which is Gaussian if we consider Gaussian codebooks at the transmitters). The assumption on treating interference as an additional source of Gaussian noise can be underlined calling the interference channel under investigation *Noisy MIMO Interference Channel*. As a result, linear transmitters and receivers are sufficient. For the MIMO IFC, one approach to linear transmit precoder design is the joint design of precoding matrices to be applied at each transmitter based on channel state information (CSI) of all users. Such a *centralized* approach [40] requires (channel) information exchange among transmitters. Nevertheless, studying such systems can provide valuable insights into the limits of perhaps more practical *distributed* algorithms [122] [123] that do not require any information transfer among transmitters.

The WSR maximization problem can be mathematically expressed as follows.

$$\begin{aligned} \{\mathbf{G}_k^\star, \mathbf{F}_k^\star\} &= \arg \max_{\{\mathbf{G}_k,\, \mathbf{F}_k\}} \textstyle\sum_k u_k R_k \\ \text{s.\,t}\ \ &\mathrm{Tr}(\mathbf{G}_k^H \mathbf{G}_k) = P_k \ \ \forall k \end{aligned} \tag{4.1}$$

with $u_k \geq 0$ denoting the weight assigned to the k-th user's rate and P_k it's transmit power constraint. We use the notation $\{\mathbf{G}_k,\ \mathbf{F}_k\}$ to compactly represent the candidate set of transmitters $\mathbf{G}_k$ and receivers $\mathbf{F}_k$ $\forall k \in \{1, \ldots, K\}$ and the corresponding set of optimum transmitters and receivers is represented by $\{\mathbf{G}_k^\star, \mathbf{F}_k^\star\}$. Assuming Gaussian signaling, the k-th user's achievable rate is given, at the output of the receiver filter, by

$$R_k = \log |\mathbf{I}_k + \mathbf{F}_k^H \mathbf{H}_{kk} \mathbf{G}_k \mathbf{G}_k^H \mathbf{H}_{kk}^H \mathbf{F}_k (\mathbf{F}_k^H \mathbf{R}_{\overline{k}} \mathbf{F}_k)^{-1}|, \tag{4.2}$$

where the interference plus noise covariance matrix $\mathbf{R}_{\overline{k}}$ is defined as:

$$\mathbf{R}_{\overline{k}} = \mathbf{R}_{n_k n_k} + \sum_{l \neq k} \mathbf{H}_{kl} \mathbf{G}_l \mathbf{G}_l^H \mathbf{H}_{kl}^H.$$

We use here the standard notation $|\ .\ |$ to denote the determinant of a matrix. The MIMO IFC rate region is known to be non-convex. The presence of multiple local optima complicates the computation of optimum precoding matrices to be applied at the transmitter in order to maximize the weighted sum rate. What is known however, is that, for a given set of precoders, linear minimum mean squared error (LMMSE) receivers are optimal in terms of interference suppression.

4.3.1 Optimality of LMMSE interference suppression filters

We discuss here the optimality of using LMMSE interference suppressors at the receivers for a given set of linear precoders applied at the transmitters. For fixed $\mathbf{G}_k$'s, the received signal can be expressed as

$$\mathbf{y}_k = \mathbf{H}_{kk} \mathbf{x}_k + \mathbf{v}_k = \mathbf{H}_{kk} \mathbf{G}_k \mathbf{s}_k + \mathbf{v}_k \tag{4.3}$$

where $\mathbf{v}_k = \sum_{l=1;l\neq k}^{K} \mathbf{H}_{kl}\mathbf{x}_l + \mathbf{n}_k$ accounts for the total interference and noise contribution in $\mathbf{y}_k$. The achievable rate at each receiver can now be expressed as

$$R_k = \log |\mathbf{I}_k + \mathbf{R}_{\bar{k}}^{-1}\mathbf{H}_{kk}\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H|. \tag{4.4}$$

The LMMSE receiver for the k-th user is then given by

$$\begin{aligned}\mathbf{F}_k^{LMMSE} =& \arg\min_{\mathbf{F}_k} \operatorname{Tr}\{\mathbb{E}[(\mathbf{s}_k - \mathbf{F}_k^H\mathbf{y}_k)(\mathbf{s}_k - \mathbf{F}_k^H\mathbf{y}_k)^H]\} \\ =& (\mathbf{R}_{\bar{k}} + \mathbf{H}_{kk}\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H)^{-1}\mathbf{H}_{kk}\mathbf{G}_k.\end{aligned} \tag{4.5}$$

With the optimal LMMSE receive filter defined above we can write the MSE matrix as:

$$\mathbf{E}_k = \mathbb{E}[(\mathbf{s}_k - \mathbf{F}_k^H\mathbf{y}_k)(\mathbf{s}_k - \mathbf{F}_k^H\mathbf{y}_k)^H] = (\mathbf{I} + \mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{R}_{\bar{k}}^{-1}\mathbf{H}_{kk}\mathbf{G}_k)^{-1} \tag{4.6}$$

It can be shown that by substituting $\mathbf{F}_k^{LMMSE}$ in (4.2), the resulting expression for R_k^{LMMSE} is exactly the same as (4.4). The implication is that, for a given set of linear beamforming filters applied at the transmitters, the LMMSE interference-suppressing filter applied at the receiver does not lose any information of the desired signal in the process of reducing the N_k dimensional $\mathbf{y}_k$ to a d_k dimensional vector $\mathbf{r}_k$. This is of course under the assumption that all interfering signals can be treated as Gaussian noise. In other words, the linear MMSE interference suppressor filter is information lossless for the Noisy MIMO IFC.

4.3.2 Equivalence between WSR maximization and WMSE minimization

In this section we report an important result that will be used for the derivation of an iterative algorithm for beamformer design. We present here the correspondence between the WSR maximization problem and the minimization of the weighted sum mean squared error (WSMSE). This result has been introduced in the seminal work [45] for a MIMO BC channel, then it has been extended for the first time to the MIMO IFC in [124]. For the sake of completeness, we restate this relationship in this section. The WSR maximization problem in (4.1) can be simplified if we consider a (more tractable) optimization problem where MMSE processing at the receiver is implicitly assumed. The rationale for this assumption is clear from the previous section where the optimality of the LMMSE receiver has been shown. The optimization problem that we now consider is expressed as

$$\begin{aligned}\{\mathbf{G}_k^\star\} =& \arg\min_{\{\mathbf{G}_k\}} \sum_{k=1}^{K} -u_k \log |\mathbf{E}_k^{-1}| \\ & \text{s.t } \operatorname{Tr}(\mathbf{G}_k^H\mathbf{G}_k) = P_k \ \forall k\end{aligned} \tag{4.7}$$

where $\mathbf{E}_k$ is the minimum MSE matrix (MMSE) found in (4.6). In order to obtain the stationary points for the optimization problem (4.7), we define the following Lagrangian:

$$J\left(\{\mathbf{G}_k, \lambda_k\}\right) = \sum_{k=1}^{K} -u_k \log |\mathbf{E}_k^{-1}| + \lambda_k(\mathrm{Tr}\{\mathbf{G}_k^H \mathbf{G}_k\} - P_k)$$

where λ_k represents the Lagrange multiplier associated to the power constraint of the k-th user. Now setting the gradient of the Lagrangian w.r.t. the transmit filter $\mathbf{G}_k$ to zero, we have:

$$\begin{aligned} \frac{\partial J(\{\mathbf{G}_k,\lambda_k\})}{\partial \mathbf{G}_k^*} =& \sum_{l\neq k} u_l \mathbf{H}_{lk}^H \mathbf{R}_{\bar{l}}^{-1} \mathbf{H}_{ll} \mathbf{G}_l \mathbf{E}_l \mathbf{G}_l^H \mathbf{H}_{ll}^H \mathbf{R}_{\bar{l}}^{-1} \mathbf{H}_{lk} \mathbf{G}_k \\ & - u_k \mathbf{H}_{kk}^H \mathbf{R}_{\bar{k}}^{-1} \mathbf{H}_{kk} \mathbf{G}_k \mathbf{E}_k + \lambda_k \mathbf{G}_k = 0 \end{aligned} \tag{4.8}$$

Notice that finding a close form expression for the optimal transmit filter from the derivative above is complicated. This comes from the fact that each term in (4.8) has as a factor the optimization variable $\mathbf{G}_k$. In addition, direct computation of λ_k that satisfies the KKT conditions now becomes complex. For single antenna receivers in a broadcast channel, a solution for transmit filter design that minimizes the MSE at the receiver was proposed in [125]. The key idea was to allow for scalars to compensate for transmit power constraints. Our approach to the design of the WSR maximizing transmit filters for the MIMO IFC is inspired by this idea. Before we explain the computation of λ_k and the beamformer design any further, we digress in order to highlight the important connection between the WSR maximization and the weighted sum mean squared error minimization problem that we exploit in our iterative algorithm. Consider the problem where it is desired to optimize the transmit filters so as to minimize the WSMSE across all users (assuming MMSE receivers). Denote by $\mathbf{W}_k$ the weight matrix associated to the k-th user. Then this problem can be expressed as

$$\arg \min_{\{\mathbf{G}_k\}} \textstyle\sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k \mathbf{E}_k\}$$

$$\text{s.t. } \mathrm{Tr}\{\mathbf{G}_k^H \mathbf{G}_k\} = P_k \;\; \forall k$$

and the corresponding Lagrangian reads

$$L(\{\mathbf{G}_k, \lambda_k\}) = \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k \mathbf{E}_k\} + \lambda_k(\mathrm{Tr}\{\mathbf{G}_k^H \mathbf{G}_k\} - P_k)$$

Deriving $L(\{\mathbf{G}_k, \lambda_k\})$ with respect to $\mathbf{G}_k$ we have

$$\begin{aligned}\frac{\partial \mathrm{L}(\{\mathbf{G}_k,\lambda_k\})}{\partial \mathbf{G}_k^*} = & \sum_{l\neq k} \mathbf{H}_{lk}^H \mathbf{R}_{\bar{l}}^{-1} \mathbf{H}_{ll} \mathbf{G}_l \mathbf{E}_l \mathbf{W}_l \mathbf{E}_l \mathbf{G}_l^H \mathbf{H}_{ll}^H \mathbf{R}_{\bar{l}}^{-1} \mathbf{H}_{lk} \mathbf{G}_k \\ & - \mathbf{H}_{kk}^H \mathbf{R}_{\bar{k}}^{-1} \mathbf{H}_{kk} \mathbf{G}_k \mathbf{E}_k \mathbf{W}_k \mathbf{E}_k + \lambda_k \mathbf{G}_k = 0\end{aligned} \tag{4.9}$$

Comparing the gradient expressions for the two Lagrangians (4.8) and (4.9) we see that they can be made equal if

$$\mathbf{W}_k = u_k \mathbf{E}_k^{-1}$$

In other words, with a proper choice of the weighting matrices, a stationary point for the weighted sum minimum mean square error objective function is also a stationary point for the maximum WSR problem. We exploit this relationship to henceforth compute the $\mathbf{G}_k$ that minimizes the WSMSE when $\mathbf{W}_k = u_k \mathbf{E}_k^{-1}$ instead of directly maximizing the WSR. We are now ready to extend the solution in [45] and [125] to MIMO IFC problem at hand.

4.3.3 WSR maximization via WSMSE

The relation between WSR maximization and minimization of the WSMSE justifies the algorithm provided here for solving the WSR maximization problem. Our approach to the design of the WSR maximizing transmit filters for the MIMO IFC is based on the minimization of the WSMSE in which some additional optimization variables appear [45, 125]. In this new augmented cost function we have as optimization variables the transmit and receive filters, the weight matrices [45] and some scalars that compensate for the power constraints [125]. The optimization problem that we consider now is

$$\begin{aligned}\arg\max - \sum_k u_k(\mathrm{Tr}(\mathbf{W}_k \boldsymbol{\mathcal{E}}_k) - \log|\mathbf{W}_k| - d_k^{max}) \\ \mathrm{s.\,t} \sum_k \mathrm{Tr}(\mathbf{G}_k \mathbf{G}_k^H) \leq P_k.\end{aligned} \tag{4.10}$$

where $d_k^{max} \leq \min\{N_k, M_k\}$ represents the maximum number of independent data streams that can be transmitted to user k. Assuming $\mathbb{E}\{\mathbf{s}_k \mathbf{s}_k^H\} = \mathbf{I}_k$, the MSE covariance matrix becomes:

$$\begin{aligned}\boldsymbol{\mathcal{E}}_k = & \ \mathbb{E}[(\mathbf{s}_k - \alpha_k^{-1} \mathbf{F}_k^H \mathbf{y}_k)(\mathbf{s}_k - \alpha_k^{-1} \mathbf{F}_k^H \mathbf{y}_k)^H]\} \\ = & \ \mathbf{I} - \alpha_k^{-1} \mathbf{G}_k^H \mathbf{H}_{kk}^H \mathbf{F}_k - \alpha_k^{-1} \mathbf{F}_k^H \mathbf{H}_{kk} \mathbf{G}_k \\ & + \alpha_k^{-2} \sum_{l=1}^{K} \mathbf{F}_k^H \mathbf{H}_{kl} \mathbf{G}_l \mathbf{G}_l^H \mathbf{H}_{kl}^H \mathbf{F}_k + \alpha_k^{-2} \mathbf{F}_k^H \mathbf{R}_{n_k n_k} \mathbf{F}_k\end{aligned} \tag{4.11}$$

This cost function is concave or even quadratic in one set of variables, keeping the others variables fixed. Hence we shall optimize it using alternating maximization. The corresponding Lagrangian can be written as:

$$\begin{aligned} J(\{\mathbf{G}_k, \mathbf{F}_k, \mathbf{W}_k, \lambda_k, \alpha_k\}) = & -\lambda_k(\mathrm{Tr}\{\mathbf{G}_k^H \mathbf{G}_k\} - P_k) \\ & -\sum_k u_k(\mathrm{Tr}(\mathbf{W}_k \boldsymbol{\mathcal{E}}_k) - \log|\mathbf{W}_k| - d_k^{max}) \end{aligned} \quad (4.12)$$

The first step in our optimization process is the calculation of the optimal Rx filters assuming fixed all the remaining optimization variables. It can easily be seen that the optimal Rx filter is an MMSE filter as derived in the previous section, equation (4.5):

$$\mathbf{F}_k = \left(\mathbf{R}_{\overline{k}} + \mathbf{H}_{kk}\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H\right)^{-1}\mathbf{H}_{kk}\mathbf{G}_k \quad (4.13)$$

The following step in the optimization procedure is the determination of the optimal expression for the weighting matrix $\mathbf{W}_k$ while keeping the other variables fixed.

Setting the derivative of the Lagrangian w.r.t $\mathbf{W}_k$ equal to zero we obtain:

$$\mathbf{W}_k = \boldsymbol{\mathcal{E}}_k^{-1} \quad (4.14)$$

From the derivative of the Lagrangian w.r.t. the scalar coefficient α_k we find

$$\begin{aligned} \frac{\partial J(\{\mathbf{G}_k,\alpha_k,\lambda_k\})}{\partial \alpha_k} = & \ \alpha_k^{-2}\mathrm{Tr}\{\mathbf{W}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{F}_k\} + \alpha_k^{-2}\mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k^H\mathbf{H}_{kk}\mathbf{G}_k\} \\ & -2\alpha_k^{-3}\sum_{l=1}^{K}\mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k^H\mathbf{H}_{kl}\mathbf{G}_l\mathbf{G}_l^H\mathbf{H}_{kl}^H\mathbf{F}_k\} \\ & -2\alpha_k^{-3}\mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k^H\mathbf{R}_{n_kn_k}\mathbf{F}_k\} \\ = & \ 0 \end{aligned}$$

now solving for α_k we have

$$\alpha_k = 2\frac{\mathrm{Tr}\{\sum_{l=1}^{K}\mathbf{W}_k\mathbf{F}_k^H\mathbf{H}_{kl}\mathbf{G}_l\mathbf{G}_l^H\mathbf{H}_{kl}^H\mathbf{F}_k + \mathbf{W}_k\mathbf{F}_k^H\mathbf{R}_{n_kn_k}\mathbf{F}_k\}}{\mathrm{Tr}\{\mathbf{W}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{F}_k\} + \mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k^H\mathbf{H}_{kk}\mathbf{G}_k\}}. \quad (4.15)$$

To determine the optimal BF matrix we solve the following:

$$\begin{aligned} \frac{\partial \mathbf{J}(\{\mathbf{G}_k,\lambda_k\})}{\partial \mathbf{G}_k^*} = & \ u_k\alpha_k^{-1}\mathbf{H}_{kk}^H\mathbf{F}_k\mathbf{W}_k - \lambda_k\mathbf{G}_k \\ & -\sum_{l=1}^{K}u_l\alpha_l^{-2}\mathbf{H}_{lk}^H\mathbf{F}_l\mathbf{W}_l\mathbf{F}_l^H\mathbf{H}_{lk}\mathbf{G}_k = 0. \end{aligned} \quad (4.16)$$

Then the expression of the optimal BF matrix is:

$$\mathbf{G}_k = \left(\sum_{l=1}^{K} u_l \alpha_l^{-2} \mathbf{H}_{lk}^H \mathbf{F}_l \mathbf{W}_l \mathbf{F}_l^H \mathbf{H}_{lk} + \lambda_k \mathbf{I} \right)^{-1} \mathbf{H}_{kk}^H \mathbf{F}_k \mathbf{W}_k u_k \alpha_k^{-1} \tag{4.17}$$

The only variable that still needs to be optimized is the Lagrange multiplier λ_k. For that we propose the following approach. First check if $\mathrm{Tr}(\mathbf{G}_k^H \mathbf{G}_k) \leq P_k$ for $\lambda_k = 0$. If yes, then $\lambda_k = 0$. If not, the Tx power equality constraint is active. In this case to determine the optimal value of the Lagrange multiplier λ_k we consider equation (4.16) that for the optimality of the BF matrix it is satisfied. In addition we pre-multiplying the derivative of the cost function w.r.t. the BF matrix by the matrix $\mathbf{G}_k^H$, taking the trace of the product then it is still equal to zero:

$$\mathrm{Tr}\left\{ \mathbf{G}_k^H \frac{\partial \mathrm{J}(\{\mathbf{G}_k, \lambda_k\})}{\partial \mathbf{G}_k^*} \right\} = 0$$

$$\begin{aligned} &\mathrm{Tr}\left\{ u_k \alpha_k^{-1} \mathbf{G}_k^H \mathbf{H}_{kk}^H \mathbf{F}_k \mathbf{W}_k \right\} - \lambda_k \mathrm{Tr}\left\{ \mathbf{G}_k^H \mathbf{G}_k \right\} \\ &- \sum_{l=1}^{K} u_l \alpha_l^{-2} \mathrm{Tr}\left\{ \mathbf{G}_k^H \mathbf{H}_{lk}^H \mathbf{F}_l \mathbf{W}_l \mathbf{F}_l^H \mathbf{H}_{lk} \mathbf{G}_k \right\} = 0. \end{aligned} \tag{4.18}$$

In equation (4.18) we impose the power constraint to be satisfied with equality, hence the contribution $\lambda_k Tr\left\{ \mathbf{G}_k^H \mathbf{G}_k \right\} = \lambda_k P_k$. Finally the optimal expression for the Lagrange multiplier λ_k is the following:

$$\lambda_k = \sum_{l=1}^{K} \frac{u_l \alpha_l^{-2}}{P_k} \mathrm{Tr}\{ \mathbf{G}_k^H \mathbf{H}_{lk}^H \mathbf{F}_l \mathbf{W}_l \mathbf{F}_l^H \mathbf{H}_{lk} \mathbf{G}_k \} - \frac{u_k \alpha_k^{-1}}{P_k} \mathrm{Tr}\{ \mathbf{G}_k^H \mathbf{H}_{kk}^H \mathbf{F}_k \mathbf{W}_k \} \tag{4.19}$$

At this point we have derived the optimal expressions of the complete set of optimization variables, so the algorithm is complete. Comparing the solution found here for MIMO IFC and the corresponding solution for a BC channel in [45] we can see how the solution for an IFC reveals a more complex structure. This is due to the intrinsic differences between the two settings. In the MIMO IFC, the transmitters can collaborate only at the level of CSI exchange, while in the BC channel the unique BS is also aware of the messages directed to all the receivers. In addition the per-transmitter power constraints that we have to impose in the IFC determine another fundamental difference between the two systems.

Interestingly, fixing the receivers to be MMSE filters leads to a simplification of the complete procedure. In particular looking at the expression of α_k in (4.15) we realize that using the definition of MMSE receiver given in (4.13) the optimal value of this scalar parameter becomes $\alpha_k = 1 \ \ \forall k$.

Therefore, assuming that MMSE receivers are the optimal choice, we can simplify the algorithm assuming a priori that the value of $\alpha_k = 1$ and hence it could be removed from the set of optimization variables. In [125], where this scalar variable has been introduced, the optimization is done only w.r.t. the transmit filters assuming a set of generic receivers. If the assumption of MMSE receiver filter is imposed also in their setting the optimization of that scalar variable becomes unnecessary. The assumption of MMSE receivers leads also to a modified expression for the Lagrange multiplier. It is possible to show that expression (4.19) can be rewritten as follows:

$$\lambda_k = -\frac{1}{P_k}\Bigg[\sum_{l\neq k} u_l \mathrm{Tr}\{\mathbf{W}_l \mathbf{F}_l^H \mathbf{H}_{lk}\mathbf{G}_k(\mathbf{F}_l^H \mathbf{H}_{lk}\mathbf{G}_k)^H\} - u_k \mathrm{Tr}\{\mathbf{W}_k \mathbf{F}_k^H \mathbf{R}_{n_k n_k}\mathbf{F}_k\} \\ - \sum_{l\neq k} u_k \mathrm{Tr}\{\mathbf{W}_k \mathbf{F}_k^H \mathbf{H}_{kl}\mathbf{G}_l(\mathbf{F}_k^H \mathbf{H}_{kl}\mathbf{G}_l)^H\}\Bigg].$$

With the optimal value of the Lagrange multiplier derived above, the final expression for the BF becomes (4.20). In the expression of λ_k used in (4.20) we can interpret the matrix $\mathbf{J}_l^{(k)}$ as the residual interference that the k-th transmitter generates at the non intended receiver l while the matrix $\mathbf{J}_k^{(l)}$ represents the residual interference that the k-th receiver receives from non intended transmitters. The algorithm proposed in [45] was developed for a MIMO broadcast channel, where only an overall Tx power constraint is applied on the system and, in addition, maximizing the WSR automatically requires to transmit with full power. On the other hand in the MIMO IFC the WSR maximization may require some links to transmit with a power less than the maximum power available at that link.

At low SNR regime the maximization of the WSR leads to activate only one stream per link, allocating full power on the best singular mode of the direct channel $\mathbf{H}_{kk}$. For SNR values sufficiently high the maximization of the sum rate converges to an IA solution. IA feasibility may imply zero streams for some links. Here we propose to determine the optimal value of $\lambda_k \geq 0$ using a linear search algorithm.

Grouping together all the optimization steps that describe our maximization procedure we have the following two-steps iterative algorithm to compute the precoders that maximize the weighted sum rate for a given MIMO IFC (c.f Table **Algorithm 2**). Introducing the augmented cost function, for the calculation of the optimal BF matrix that maximize the WSR, we are able to determine an iterative algorithm that can be easily proved to converge to a local optima that corresponds also to an extremum of the original cost function (4.7).

Each step of our iterative algorithm increases the cost function, which is bounded above (e.g. by cooperative WSR), and hence convergence is guaranteed, as also

Algorithm 2 MWSR Algorithm for MIMO IFC

Fix an arbitrary initial set of precoding matrices $\mathbf{G}_k, \quad \forall \in k = \{1, 2 \ldots K\}$
set $n = 0$
repeat
  $n = n + 1$
  Given $\mathbf{G}_k^{(n-1)}$, compute $\mathbf{F}_k^{(n)}$ and $\mathbf{W}_k^{(n)}$ from (4.13) and (4.14) respectively $\forall k$
  Given $\mathbf{F}_k^{(n)}$ and $\mathbf{W}_k^{(n)}$, compute $\mathbf{G}_k^{(n)}$ $\forall k$ using (4.17)
until convergence

$$\mathbf{G}_k = \left[\sum_{l=1}^{K} u_l \mathbf{H}_{lk}^H \mathbf{F}_l \mathbf{W}_l \mathbf{F}_l^H \mathbf{H}_{lk} - \frac{1}{P_k}\left(\sum_{l \neq k}\left(u_l \mathrm{Tr}\{\mathbf{W}_l \mathbf{J}_l^{(k)}\} - u_k \mathrm{Tr}\{\mathbf{W}_k \mathbf{J}_k^{(l)}\}\right) - u_k \mathrm{Tr}\{\mathbf{W}_k \mathbf{N}_k\}\right)\mathbf{I}\right]^{-1} \mathbf{H}_{kk}^H \mathbf{F}_k \mathbf{W}_k \tag{4.20}$$

$$\mathbf{J}_l^{(k)} = \mathbf{F}_l^H \mathbf{H}_{lk} \mathbf{G}_k \mathbf{G}_k^H \mathbf{H}_{lk}^H \mathbf{F}_l; \quad \mathbf{J}_k^{(l)} = \mathbf{F}_k^H \mathbf{H}_{kl} \mathbf{G}_l \mathbf{G}_l^H \mathbf{H}_{kl}^H \mathbf{F}_k; \quad \mathbf{N}_k = \mathbf{F}_k^H \mathbf{R}_{n_k n_k} \mathbf{F}_k$$

shown in [45] for a BC channel. The convergence behavior of this algorithm has been also shown in [44] for MIMO interfering broadcast channels.

4.3.4 Direct optimization of the WSR

In the section above we derived an iterative algorithm for the optimization of the BF filters to maximize the WSR via the minimization of the WSMSE. Introducing an augmented cost function we obtained the optimal expression of the transmit filter that also maximize the WSR. In this section we show how it is possible to optimize the BF matrices directly from the WSR expression. On this purpose we assume implicitly that MMSE receiver filters are used, then the rate expression of the k-th user is the one in (4.4).

Using matrix inversion lemma [1] we can rewrite the expression of the LMMSE receiver (4.13) as

$$\mathbf{F}_k = \mathbf{R}_{\overline{k}}^{-1} \mathbf{H}_{kk} \mathbf{G}_k \mathbf{E}_k.$$

[1] If $\mathbf{P}$ and $\mathbf{R}$ are positive definite the following relation is true [126]:

$$\mathbf{P}\mathbf{B}^T(\mathbf{B}\mathbf{P}\mathbf{B}^T + \mathbf{R})^{-1} = (\mathbf{P}^{-1} + \mathbf{B}^T\mathbf{R}^{-1}\mathbf{B})\mathbf{B}^T\mathbf{R}^{-1} \tag{4.21}$$

With this equivalent expression of the LMMSE we can interpret some quantities in the derivative (4.8) of the Lagrangian associated to the WSR maximization problem (4.7) as follows:

$$\frac{\partial J(\{\mathbf{G}_k,\lambda_k\})}{\partial \mathbf{G}_k^*} = \sum_{l\neq k} u_l \mathbf{H}_{lk}^H \underbrace{\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{ll}\mathbf{G}_l\mathbf{E}_l}_{\mathbf{F}_l} \underbrace{\mathbf{E}_l^{-1}}_{\mathbf{W}_l} \underbrace{\mathbf{E}_l\mathbf{G}_l^H\mathbf{H}_{ll}^H\mathbf{R}_{\bar{l}}^{-1}}_{\mathbf{F}_l^H} \mathbf{H}_{lk}\mathbf{G}_k$$
$$-u_k\mathbf{H}_{kk}^H \underbrace{\mathbf{R}_{\bar{k}}^{-1}\mathbf{H}_{kk}\mathbf{G}_k\mathbf{E}_k}_{\mathbf{F}_k} + \lambda_k\mathbf{G}_k = 0.$$

Now adding and subtracting the term: $u_k\mathbf{H}_{kk}^H\mathbf{F}_k\mathbf{W}_k\mathbf{F}_k^H\mathbf{H}_{kk}\mathbf{G}_k$ in the equation above, we obtain:

$$\frac{\partial \mathbf{J}(\{\mathbf{G}_k,\lambda_k\})}{\partial \mathbf{G}_k^*} = -u_k\mathbf{H}_{kk}^H\mathbf{F}_k\mathbf{W}_k + \lambda_k\mathbf{G}_k + \sum_{l=1}^{K} u_l\mathbf{H}_{lk}^H\mathbf{F}_l\mathbf{W}_l\mathbf{F}_l^H\mathbf{H}_{lk}\mathbf{G}_k = 0. \tag{4.22}$$

that is the same as (4.16), assuming $\alpha_k = 1$. From (4.22) we can derive the optimal expression of the transmit filters (4.17). This implies that a stationary point of the original cost function is also a stationary point of the augmented cost function.

4.4 Per-Stream WSR maximization

Instead of the per user approach considered so far, leading to a full matrix $\mathbf{W}_k$ in (4.14), it is possible to consider a per stream approach with diagonal weighting matrices, as already remarked in [45] for the MIMO BC problem. This can be done in a variety of ways. In the more general case of WSR maximization, we can consider absorbing a $d_k \times d_k$ unitary factor into the transmit filter $\mathbf{G}_k$. Indeed, the WSR is insensitive to multiplication of the $\mathbf{G}_k$ to the right by $d_k \times d_k$ unitary matrices, since such transformations leave the spatiotemporally white vector symbol streams $\mathbf{s}_k$ spatiotemporally white. Now, such $d_k \times d_k$ unitary matrix can be chosen to make the columns of $\mathbf{R}_{\bar{k}}^{-1/2}\mathbf{H}_{kk}\mathbf{G}_k$ orthogonal. In that case a per user LMMSE receive filter $\mathbf{F}_k$ is also at the same time a per stream LMMSE receive filter (in which case other streams of the same user would be considered as interference (which is treated here as colored noise)). In the case of full CSIT, it is indeed possible to avoid detection complexity with a proper design of the transmitter. Indeed, in the classical SU-MIMO problem with full CSIT, the optimal strategy is based on the channel SVD, which leads to a per stream treatment, avoiding any multi-stream detection at the receiver side.

The cost function proposed in this paper for the per-user approach (4.10) can be written in the per-stream case as:

$$\mathcal{O} = -\sum_{k=1}^{K} u_k \sum_{n=1}^{d_k} (-\ln(w_{kn}) - 1 + w_{kn}(1 - \mathbf{f}_{kn}^H \mathbf{H}_{kk} \mathbf{g}_{kn})(1 - \mathbf{f}_{kn}^H \mathbf{H}_{kk} \mathbf{g}_{kn})^H$$
$$+ w_{kn} \mathbf{f}_{kn}^H \underbrace{(\mathbf{R}_{n_k n_k} + \sum_{(im) \neq (kn)} \mathbf{H}_{ki} \mathbf{g}_{im} \mathbf{g}_{im}^H \mathbf{H}_{ki}^H)}_{\mathbf{R}_{\overline{kn}}} \mathbf{f}_{kn}). \tag{4.23}$$

The optimization problem when we work per stream becomes:

$$\begin{array}{cc} \max\limits_{\mathbf{f}_{kn}, \mathbf{g}_{kn}, w_{kn}} & \mathcal{O} \\ \text{s.t.} & \sum_n^{d_k} \mathbf{g}_{kn}^H \mathbf{g}_{kn} \leq P_k \ \ \forall k \end{array} \tag{4.24}$$

and the corresponding Lagrangian is:

$$J = \mathcal{O} + \sum_{k=1}^{K} \lambda_k \left(P_k - \sum_{n=1}^{d_k} \mathbf{g}_{kn}^H \mathbf{g}_{kn}\right) \tag{4.25}$$

To solve the given optimization problem we use alternating optimization. As first step we determine the Rx filter assuming all the other optimization variables to be fixed. Deriving the cost function above w.r.t. the Rx filter we obtain an MMSE receiver per stream:

$$\mathbf{f}_{kn} = (\mathbf{H}_{kk} \mathbf{g}_{kn} \mathbf{g}_{kn}^H \mathbf{H}_{kk}^H + \mathbf{R}_{\overline{kn}})^{-1} \mathbf{H}_{kk} \mathbf{g}_{kn} \tag{4.26}$$

Given the optimal Rx filter we derive (4.25) w.r.t. the scalar weight and we find:

$$w_{kn} = e_{kn}^{-1} \tag{4.27}$$

where $e_{kn} = (1 + \mathbf{g}_{kn}^H \mathbf{H}_{kk}^H \mathbf{R}_{\overline{kn}}^{-1} \mathbf{H}_{kk} \mathbf{g}_{kn})^{-1}$. The third step is the optimization of the beamforming vectors:

$$\mathbf{g}_{kn} = \left[\sum_{l=1}^{K} \sum_{j=1}^{d_l} u_l \mathbf{H}_{lk}^H \mathbf{f}_{lj} w_{lj} \mathbf{f}_{lj}^H \mathbf{H}_{lk} + \lambda_k \mathbf{I} \right]^{-1} \mathbf{H}_{kk}^H \mathbf{f}_{kn} w_{kn} u_k \tag{4.28}$$

To determine the optimal value of the Lagrange multiplier λ_k we can proceed as done for the per-user approach (section 4.3.3). To obtain the closed form expression for λ_k, when to power constraint is satisfied with equality we can multiply the derivative of the Lagrangian w.r.t $\mathbf{g}_{kn}$ by the BF vector hence the following holds true:

$$\sum_{n=1}^{d_k} \left[\mathbf{g}_{kn}^H \frac{\partial J}{\partial \mathbf{g}_{kn}^*} \right] = 0$$

solving the equation above w.r.t. the Lagrange multiplier we get:

$$\begin{aligned} \lambda_k = \quad & \tfrac{1}{P_k} \left[\sum_{n=1}^{d_k} \mathbf{g}_{kn}^H \mathbf{H}_{kk}^H \mathbf{f}_{kn} w_{kn} u_k \right] \\ & - \tfrac{1}{P_k} \left[\sum_{n=1}^{d_k} \sum_{l=1}^{K} \sum_{j=1}^{d_l} u_l \mathbf{g}_{kn}^H \mathbf{H}_{lk}^H \mathbf{f}_{lj} w_{lj} \mathbf{f}_{lj}^H \mathbf{H}_{lk} \mathbf{g}_{kn} \right] \end{aligned} \tag{4.29}$$

Introducing the compound quantities: $\mathbf{G} = [\mathbf{g}_{k1}, \ldots, \mathbf{g}_{kd_k}]$, $\mathbf{F} = [\mathbf{f}_{k1}, \ldots, \mathbf{f}_{kd_k}]$ and $\mathbf{W} = [w_{k1}, \ldots, w_{kd_k}\}$ the expression above has the same form as (4.19), obtained in the per-user approach, this because each BS has only a total power constraint and not a per-stream constraint.

The final algorithm (PS-MWSR algorithm in **Algorithm 3**) for the per-stream optimization requires the iteration of the two steps for the optimization of Rx filters, weights, Tx beamforming vectors, in the prescribed order, until convergence.

Algorithm 3 PS-MWSR Per-Stream Algorithm for MIMO IFC

Fix an arbitrary initial set of precoding matrices $\mathbf{G}_k, \quad \forall \in k = \{1, 2 \ldots K\}$
set $n = 0$
repeat
  $n = n + 1$
  for $k = 1$ to K **do**
    Given $\mathbf{g}_i^{(n-1)}$ $\forall i$, compute $\mathbf{f}_{kl}^{(n)}$ and $w_{kl}^{(n)}$ from (4.26) and (4.27) respectively for $l = 1, \ldots, d_k$
    Given $\mathbf{f}_{kl}^{(n)}$ and $w_{kl}^{(n)}$ for $l = 1, \ldots, d_k$, compute $\mathbf{g}_{kl}^{(n)}$ for $l = 1, \ldots, d_k$ using (4.28)
  end for
until convergence

4.4.1 Rate Duality in MIMO IFC

In the previous section the expressions of the beamformer (4.28) and the MMSE Rx filter (4.26) are given when we assume to work per stream. Looking deeper at the expression of the cost function (4.23) it is possible to establish a duality relationship between the DL IFC considered and a dual UL IFC:

- The DL channel matrix $\mathbf{H}_{kl}$ becomes $\overline{\mathbf{H}}_{lk}^{H}$ in the dual UL
- The Rx (Tx) filter in the DL (UL) $\mathbf{f}_{kn}$ ($\mathbf{g}_{kn}$) becomes the Tx (Rx) filter in the UL (DL) $\overline{\mathbf{g}}_{kn}$ ($\overline{\mathbf{f}}_{kn}$)
- The unit DL Tx signal variance for stream (k, n) becomes $u_k w_{kn}$ in the dual UL channel
- DL noise covariance matrix $\mathbf{R}_{n_k n_k} = \sigma_k^2 \mathbf{I}$ becomes $\lambda_k \mathbf{I}$ in the UL.

With this relationship we can interpret the BF filter in the DL as an MMSE Rx filter in the virtual UL IFC.
A similar reasoning can be naturally extended to the per-user approach discussed in section 4.3. In this case the dual MMSE Rx filter can be obtained by minimizing the dual MSE:

$$\begin{aligned}\overline{\mathcal{E}_k} =&\mathbb{E}[(\overline{\mathbf{s}}_k - \overline{\mathbf{F}}_k^H \overline{\mathbf{y}}_k)(\overline{\mathbf{s}}_k - \overline{\mathbf{F}}_k^H \overline{\mathbf{y}}_k)^H]\} \\ =&\mathbf{W}_k - \mathbf{W}_k \overline{\mathbf{G}}_k^H \overline{\mathbf{H}}_{kk}^H \overline{\mathbf{F}}_k - \overline{\mathbf{F}}_k^H \overline{\mathbf{H}}_{kk} \overline{\mathbf{G}}_k \mathbf{W}_k \\ &+ \textstyle\sum_{l=1}^K \overline{\mathbf{F}}_k^H \overline{\mathbf{H}}_{kl} \overline{\mathbf{G}}_l \mathbf{W}_l \overline{\mathbf{G}}_l^H \overline{\mathbf{H}}_{kl}^H \overline{\mathbf{F}}_k + \overline{\mathbf{F}}_k^H \overline{\mathbf{R}}_{n_k n_k} \overline{\mathbf{F}}_k\end{aligned} \tag{4.30}$$

where with $\overline{(.)}$ we denote the quantities in the dual domain and $\mathbb{E}\{\overline{\mathbf{s}}_k \overline{\mathbf{s}}_k^H\} = \mathbf{W}_k$. Optimizing w.r.t. the dual Rx filter we obtain:

$$\overline{\mathbf{F}}_k = \left(\sum_{l=1}^{K} \overline{\mathbf{H}}_{kl} \overline{\mathbf{G}}_l \mathbf{W}_l \overline{\mathbf{G}}_l^H \overline{\mathbf{H}}_{kl}^H + \overline{\mathbf{R}}_{n_k n_k} \right)^{-1} \overline{\mathbf{H}}_{kk} \overline{\mathbf{G}}_k \mathbf{W}_k \tag{4.31}$$

that, with the dual relationships described before it corresponds to the expression of the optimal BF filter in (4.17) if the dual noise covariance matrix is $\overline{\mathbf{R}}_{n_k n_k} = \lambda_k \mathbf{I}$. With this relation we can interpret the proposed algorithm for WSR maximization as a process that minimizes the WSMSE in both DL and UL communications, then MMSE Rx filters are optimal.

4.4.2 Discussion on Local Maxima

At high SNR, the number of streams per user that WSR will turn on correspond necessarily to a feasible stream distribution for IA. IA feasibility was investigated e.g. in [127]. The number of feasible streams per user is not necessarily unique for a given maximum feasible number of total streams (= sum rate prelog). If the transmit and receive filters are designed with a set of d_k such that feasible IA solutions exists with a number of streams for link k that is smaller or equal to d_k, and such that $\sum_k d_k$ exceeds the sum rate prelog, then various distributions of feasible numbers of streams within the assigned $\{d_k\}$ can exist. Each such distribution should correspond to a local maximum for the SR and hence potentially also for a WSR. If on the other hand the scenario is such that the chosen $\{d_k\}$ correspond to the unique distribution of d_k that sum up to the SR prelog, then this problem should not arise.

In the IFC problem also (as opposed to the BC) the Pareto optimal boundary of the rate region may have multiple points with the same tangent hyperplane orientation. As a result, the WSR problem, which seeks points on this boundary with a hyperplane orientation corresponding to the selected weights, may have multiple extrema.

4.5 Deterministic Annealing to Avoid Local Optima

In the previous section we have described an alternating optimization algorithm that designs BF and Rx filters in order to maximize the WSR in a K-user MIMO IFC. As already remarked, the WSR cost function is a non convex function and this makes the optimization troublesome due to the presence of many local optima. In optimization, a number of heuristic approaches exist to handle non convex optimization problems. Some examples of such methods are: genetic algorithms, ant colony optimization or simulated annealing (SA). We will describe briefly the SA approach. This method takes its name from the physical annealing process in which a system is first "melted" and then slowly cooled down in order to allow the atoms in the system to find a state with lower energy until the system is "frozen" in a globally optimum state.

In SA the problem is optimized using a sequence of random moves, the size of which reduces as a parameter called temperature decreases. The random moves would allow the optimization process to get out of local optima. In a certain sense, the randomness tend to convexify the problem. Cooling protocols have been derived to allow ending up in the global optimum with high probability. *Deterministic Annealing* (DA) is a related technique but does not involve any randomness, see e.g. [128]. In DA, an increase of the temperature parameter allows to convexify the problem: the temperature parameter transforms (deterministically) the originally non-convex cost function into a convex cost function (convex should be replaced by concave in the case of maximization). So, at high temperature, there is no problem in finding the global optimum. Then gradually the temperature gets reduced, making the problem increasingly non-convex. However, if the temperature variation is sufficiently small, the global optimum at the previous higher temperature will be in the region of attraction of the global optimum at the next lower temperature and the global optimum remains tracked in this way.

As in physical systems, also in the optimization problem it can happen that phase transitions occur as the temperature cools down [128]. A phase transition corresponds to a split of the trajectory (as a function of temperature) of the global optimum into several trajectories. From a mathematical perspective a phase transition is characterized by the Hessian of the problem becoming singular at a critical temperature (hence being positive semidefinite instead of positive definite). In our problem the cost function is the WSR, a highly non convex function, and the annealing parameter is related to the noise variance, $t \propto \sigma^2$ (or the inverse of the SNR).

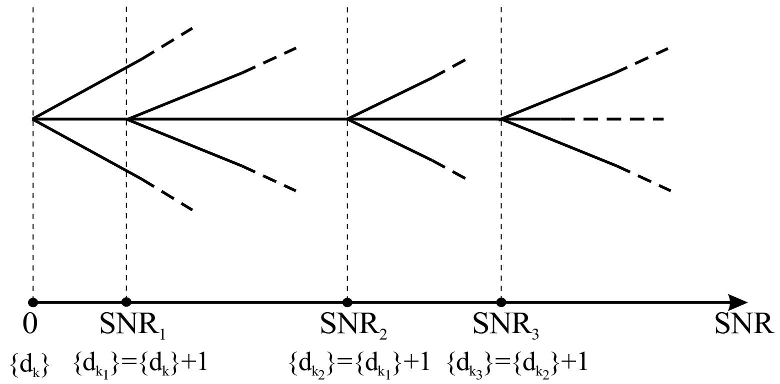

Figure 4.1: Phase transitions representation

Interestingly also in WSR maximization in a K-user MIMO IFC, phase transitions can appear. At low SNR (high noise variance), any interference is negligible compared to the noise. Hence, all links can be considered decoupled, and, like in single-user MIMO, rate maximization becomes SNR maximization for a single stream to which all transmit power is devoted. Hence in link k, the optimal Tx and Rx filters correspond to the left and right singular vectors corresponding to the largest singular value of $\mathbf{H}_{kk}$. Hence, as the SNR goes to zero, the globally optimum solution is clear. However, zero SNR itself is already a phase transition because as soon as the SNR becomes positive, a multitude of local optima may exist that we shall interpret below. As the SNR increases further, at some point another phase transition may occur, at which point a second stream needs to be introduced in one of the links. We shall see that at such a phase transition, it is possible to determine the filters corresponding to the new stream. However, as soon as the SNR increases further, many further local optima get introduced due to the appearance of the additional stream. Then, as the SNR increases further, another phase transition can occur, with the introduction of one more stream at one of the transmitters. This process goes on until a stream distribution is reached, at some higher SNR, corresponding to a maximal stream distribution for which interference alignment is feasible. Indeed, at very high SNR, the Tx and Rx filters converge to the (max WSR-)IA solution, and the sum rate prelog is maximized if the number of streams is maximized (see [129]). This whole process is depicted schematically in Fig.4.1.

Whereas DA is about tracking of a global optimum, the tracking of extrema, the zeros of the KKT conditions, is actually called a homotopy method. So in DA, going from one phase transition to the next and tracking the (appropriate) extremum, this could be considered a homotopy method.

4.6 Deterministic Annealing for WSR Maximization

What we propose in this paper is to extend the MWSR algorithm presented before in order to include DA and hence reduce the probability to be trapped in local optima. So we consider again DA for the original full rank channels, for SNR increasing from zero. To modify the algorithm proposed in Algorithm 2 to include DA we only need to run the algorithm for each SNR point initializing the algorithm with the optimal beamformers found at the previous SNR iteration. However, this does not handle phase transitions, corresponding to the introduction of a new stream. Hence, at every SNR increment, we need to try adding a stream to each of the K links (one at a time). It is possible to find the proper initialization for the Tx and Rx filters of the new stream analytically.

4.6.1 Initialization at Phase Transitions

To find the direction of the BF vector corresponding to the new stream, indexed as (k, n), we need to optimize our per-stream cost function (4.23) w.r.t. the quantities corresponding to the new allocated stream. Note that the new stream, if it should be switched on, will be switched on with very small power. Hence the new stream will barely perturb the existing streams.
For the moment we do not include in the optimization function the power constraint, so we need to find the Tx and Rx filter that minimize the MSE for stream (k, n). The derivative of the MSE w.r.t. the Rx filter is:

$$\frac{\partial \mathcal{O}}{\partial \mathbf{f}_{kn}} = -\mathbf{g}_{kn}^H \mathbf{H}_{kk}^H + \mathbf{f}_{kn}^H \mathbf{H}_{kk} \mathbf{g}_{kn} \mathbf{g}_{kn}^H \mathbf{H}_{kk}^H + \mathbf{f}_{kn}^H \mathbf{R}_{\overline{kn}} \tag{4.32}$$

considering only the terms up to first order in $\mathbf{g}_{kn}$ the expression for the receiver is $\mathbf{f}_{kn} = \mathbf{R}_{\overline{kn}}^{-1} \mathbf{H}_{kk} \mathbf{g}_{kn}$ that has an expression like matched filter (MF) in colored noise. Consider a parametrization of the BF vector in direction vector and power allocation like: $\mathbf{g}_{kn} = \overline{\mathbf{g}}_{kn} \sqrt{p_{kn}}$ and define $x_{kn} = \overline{\mathbf{g}}_{kn}^H \mathbf{H}_{kk}^H \mathbf{R}_{\overline{kn}}^{-1} \mathbf{H}_{kk} \overline{\mathbf{g}}_{kn}$. Substituting the Rx filter with its expression in function of the BF, the MSE cost function can be written as:

$$e_{kn} = 1 - p_{kn}x_{kn} + (p_{kn}x_{kn})^2$$

Considering only the contribution up to first order in x_{kn} the minimization of the MSE leads to the maximization of x_{kn} and hence the optimal BF vector direction is

$$\overline{\mathbf{g}}_{kn} = v_{max}(\mathbf{H}_{kk}^H \mathbf{R}_{\overline{kn}}^{-1} \mathbf{H}_{kk}) \tag{4.33}$$

where $v_{max}(\mathbf{A})$ represents the eigenvector corresponding to the maximum eigenvalue of matrix $\mathbf{A}$. Once we have the direction of the BF associated to the new stream we need to determine the corresponding power.
Consider $\mathbf{G}_k$ the BF matrix obtained until the current SNR point for link k and its decomposition as $\mathbf{G}_k = \overline{\mathbf{G}}_k \mathbf{P}_k^{1/2}$, where $\overline{\mathbf{G}}_k$ has normalized columns and $\mathbf{P}_k^{1/2}$ is the power allocation matrix. For the per-stream approach the MMSE is diagonal and hence:

$$\mathbf{E}_k^{-1} = \mathbf{I} + \mathbf{G}_k^H \mathbf{H}_{kk}^H \mathbf{R}_{\overline{k}}^{-1} \mathbf{H}^H \mathbf{G}_k^H = \mathbf{I} + \mathbf{D}\mathbf{P}_k$$

Introducing the additional stream we obtain the following matrix :

$$\mathbf{X} = [\mathbf{G}_k \ \mathbf{g}_{kn}]^H \mathbf{H}_{kk}^H \mathbf{R}_{\overline{k}}^{-1} \mathbf{H}_{kk}^H [\mathbf{G}_k \ \mathbf{g}_{kn}] = \begin{bmatrix} \mathbf{D}\mathbf{P}_k & \sqrt{p_{kn}}\mathbf{u} \\ \sqrt{p_{kn}}\mathbf{u}^H & ap_{kn} \end{bmatrix}$$

where $\mathbf{u} = \mathbf{G}_k^H \mathbf{H}_{kk}^H \mathbf{R}_{\overline{k}}^{-1} \mathbf{H}_{kk} \overline{\mathbf{g}}_{kn}$ and $a = \overline{\mathbf{g}}_{kn}^H \mathbf{H}_{kk}^H \mathbf{R}_{\overline{k}}^{-1} \mathbf{H}_{kk} \overline{\mathbf{g}}_{kn}$. The corresponding rate for user k is

$$\ln|\mathbf{E}_k^{-1}| = \ln|\mathbf{I} + \mathbf{X}| = \ln|\mathbf{I} + \mathbf{D}\mathbf{P}_k| + \ln(1 + p_{kn}d_{kn})$$

$d_{kn} = a - \mathbf{u}^H(\mathbf{I} + \mathbf{D}\mathbf{P}_k)^{-1}\mathbf{u}$.
Finally to find the power allocation among different streams of user k we propose the following.

Jammer Water-Filling (JWF) algorithm

Include in the matrix $\mathbf{P}_k$ the power allocated to the new stream p_{kn} and in the diagonal matrix $\mathbf{D}$ include the element d_{kn} associated to the new stream. To find the power allocation matrix we take the original per-stream cost function (4.23) and optimize it with respect to (and then eliminate) the weights w_{kn} for link k. After this, the terms in the WSR affected by $\mathbf{P}_k$ are

$$\mathcal{O} = \ln|\mathbf{I} + \mathbf{D}\mathbf{P}_k| - Tr\{\mathbf{P}_k \mathbf{\Delta}\} - \lambda_k(Tr\{\mathbf{P}_k\} - P_k)$$

where $Tr\{\mathbf{P}_k\boldsymbol{\Delta}\}$ takes into account the interference power generated to the non intended receivers (for this reason we called this algorithm Jammer WF):

$$Tr\{\mathbf{P}_k\boldsymbol{\Delta}\} = \sum_i p_{ki} \underbrace{\sum_{l\neq k} \frac{u_l}{u_k} \sum_{m=1}^{d_l} w_{lm}|\mathbf{f}_{lm}^H \mathbf{H}_{lk}\overline{\mathbf{g}}_{ki}|^2}_{\boldsymbol{\Delta}_{ki}} .$$

Deriving the cost function above w.r.t. p_{ki} the expression for the power allocation is:

$$p_{ki} = \left[\frac{1}{\lambda_k + \boldsymbol{\Delta}_{ki}} - \frac{1}{d_{ki}}\right]_+ \tag{4.34}$$

where $[(.)]_+ = max((.), 0)$. To find the optimal value of λ_k we first check if the power constraint is inactive. In particular we determine the powers using (4.34) assuming $\lambda_k = 0$ and we verify if the transmitted power is less then the power constraint. If the power constraint is not satisfied we determine λ_k using a bisection method. Consider the following function of the Lagrange multiplier

$$\mathcal{T}(\lambda_k) = \sum_i \left[\frac{1}{\lambda_k + \boldsymbol{\Delta}_{ki}} - \frac{1}{d_{ki}}\right]_+ - P_k$$

as we can see $\mathcal{T}(\lambda_k)$ is a decreasing function of λ_k. In particular for $\lambda_k^0 = 0$ $\mathcal{T}(\lambda_k) > 0$ while for λ_k^1, determined as water-level of a tradition WF algorithm on $\mathcal{T}(\lambda_k)$ when $\boldsymbol{\Delta}_{ki} = 0,\ \forall i$, the function $\mathcal{T}(\lambda_k) < 0$. The optimal value $\lambda_k^\star$ can be found using a bisection algorithm to solve $\mathcal{T}(\lambda_k) = 0$. The final extended BF matrix $\mathbf{G}_k = [\mathbf{G}_k \ \ \mathbf{g}_{kn}]$ is obtained using the procedure described so far is used as initialization of the DA-WSR for the following SNR point.

Algorithm 4 DA-MWSR Algorithm for MIMO IFC

set $t = 0$
Fix the initial set of precoding matrices $\mathbf{G}_k,\ \forall \in k = \{1, 2 \ldots K\}$
repeat
  increment SNR: $t^{(i+1)} = t^{(i)} + \delta t$
  Augment $\mathbf{G}_k$ $\forall k$
  repeat
    Given $\mathbf{G}_k$ compute $\mathbf{F}_k$ and $\mathbf{W}_k$, $\forall k$
    Given $\mathbf{F}_k$, $\mathbf{W}_k$, compute $\mathbf{G}_k$ $\forall k$
  until convergence
until target SNR is reach

It turns out that an alternating optimization approach as the one considered here (or also the one used in [121]), in spite of the non-concavity of the problem, optimizes the WSR up to second order in transmit power (or SNR). Indeed, we are able to determine analytically the optimal Tx and Rx filters up to zeroth order in Tx power, the one iteration of an alternating optimization approach will provide the optimal Tx and Rx filters up to first order in Tx power, which maximize WSR up to second order in Tx power. In other words, the alternating optimization approach inherently sets course on the trajectory of the optimum.

4.7 Hassibi-style Solution

An alternative approach is the extension of [114] to the MIMO IFC and involves normalizing the transmit filter so as to always satisfy the per-user power constraint. i.e.,

$$\bar{\mathbf{G}}_k = \sqrt{P_k}\,\frac{1}{\sqrt{\mathrm{Tr}\{\mathbf{G}_k^H\mathbf{G}_k\}}}\,\mathbf{G}_k = \sqrt{P}_k\,\beta_k\,\mathbf{G}_k \tag{4.35}$$

This converts the constrained WSR optimization problem considered so far (4.1) to an unconstrained optimization problem, thereby avoiding the introduction of Lagrange multipliers. The solution proposed in [114] was for a MISO BC problem. To extend it properly to a MIMO case (here IFC), it suffices to follow thread one of the philosophy of [45], as mentioned in Section 4.3. In the case of the MISO case, the $\mathbf{F}_k$, $\mathbf{E}_k$, which are frozen during the optimization over the $\mathbf{G}_k$, are scalars. In [114], two different but equivalent sets of scalars are considered. In any case, the philosophy of [114] is to freeze the scalars during the update of the transmit filters $\mathbf{G}_k$. In the MIMO case, these scalars become square or rectangular matrices, hence a more careful reasoning is required.

The sum rate maximization problem, with the normalized beamformers, can be written as

$$\max_{\mathbf{G}_k}\sum_{k=1}^{K} u_k \log|\mathbf{I} + P_k\beta_k^2\mathbf{H}_{kk}\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{R}_{\overline{k}}^{-1}|$$

where $\mathbf{R}_{\bar{k}}$ is now given by

$$\mathbf{R}_{\overline{k}} = \mathbf{R}_{n_k n_k} + \sum_{l\neq k} P_l\beta_l^2\mathbf{H}_{kl}\mathbf{G}_l\mathbf{G}_l^H\mathbf{H}_{kl}^H.$$

To solve the optimization problem, to find the optimal transmit filter, we derive the WSR expression first w.r.t. the BF matrix $\mathbf{G}_k$. Equating the result to zero and absorbing the scalar contributions $P_k\beta_k$ of the resulting equation in $\bar{\mathbf{G}}_k$ we get:

$$\begin{aligned}\frac{\partial\mathcal{R}(\mathbf{G}_k)}{\partial\mathbf{G}_k^*} =& u_k\mathbf{H}_{kk}^H\mathbf{R}_{\bar{k}}^{-1}\mathbf{H}_{kk}\bar{\mathbf{G}}_k\mathbf{E}_k - \tfrac{u_k}{P_k}\bar{\mathbf{G}}_k\mathrm{Tr}\{\mathbf{E}_k\bar{\mathbf{G}}_k^H\mathbf{H}_{kk}^H\mathbf{R}_{\bar{k}}^{-1}\mathbf{H}_{kk}\bar{\mathbf{G}}_k\} \\ &+\sum_{l\neq k}\frac{u_l}{P_k}\bar{\mathbf{G}}_k\mathrm{Tr}\{\mathbf{E}_l\bar{\mathbf{G}}_l^H\mathbf{H}_{ll}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{lk}\bar{\mathbf{G}}_k\bar{\mathbf{G}}_k{}^H\mathbf{H}_{lk}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{ll}^H\bar{\mathbf{G}}_l\} \\ &-\sum_{l\neq k}u_l\mathbf{H}_{lk}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{ll}\bar{\mathbf{G}}_l\mathbf{E}_l\bar{\mathbf{G}}_l\tilde{\mathbf{H}}_{ll}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{lk}\bar{\mathbf{G}}_k = 0\end{aligned} \tag{4.36}$$

In contrast to a MISO system, solving the above expression for $\bar{\mathbf{G}}_k$ is not straightforward for a general MIMO IFC. In a MISO system, simply extending [114] makes it possible to fix all scalar quantities involved in the expression thereby allowing us to find the the beamformer by iterating between the beamformer vectors and the fixed scalars. However, in moving from the MISO IFC to the MIMO IFC, the scalars now become matrices ($\mathbf{E}_k$ and $\mathbf{F}_k$) and hence a more structured reasoning is required to identify which quantity should be taken fixed and which not. To this end, using the expression for the MMSE Rx filter in (4.21), we can simplify the expression above interpreting some terms as Rx filters as done in section 4.3.4. In addition adding and subtracting the terms $\frac{u_k}{P_k}\mathrm{Tr}\{\mathbf{F}_k^H\mathbf{H}_{lk}\bar{\mathbf{G}}_k\bar{\mathbf{G}}_k{}^H\mathbf{H}_{lk}^H\mathbf{F}_k\mathbf{W}_k\}$ and $u_k\mathbf{H}_{kk}^H\mathbf{F}_k\mathbf{W}_k\mathbf{F}_k^H\mathbf{H}_{kk}\mathbf{G}_k$, we get:

$$\begin{aligned}&\frac{\partial\mathcal{R}(\mathbf{G}_k)}{\partial\mathbf{G}_k^*} = -u_k\mathbf{H}_{kk}^H\mathbf{F}_k\mathbf{W}_k + \sum_{l=1}^{K}u_l\mathbf{H}_{lk}^H\mathbf{F}_l\mathbf{W}_l\mathbf{F}_l^H\mathbf{H}_{lk}\overline{\mathbf{G}}_k \\ &\underbrace{\frac{1}{P_k}\left[-u_k\mathrm{Tr}\{\mathbf{F}_k^H\mathbf{H}_{kk}\bar{\mathbf{G}}_k\mathbf{W}_k\} + \sum_{l=1}^{K}u_l\mathrm{Tr}\{\overline{\mathbf{G}}_k^H\mathbf{H}_{lk}^H\mathbf{F}_l\mathbf{W}_l\mathbf{F}_l^H\mathbf{H}_{lk}\overline{\mathbf{G}}_k\}\right]}_{\lambda_k}\overline{\mathbf{G}}_k = 0.\end{aligned} \tag{4.37}$$

This expression corresponds to equation (4.16) obtained with the extension of [45] to a MIMO IFC. Thus, the extension of [114] to the MIMO IFC as well as the extension of [45] to the MIMO IFC yield exactly the same solution. Interestingly, it was observed that extending the approach in [114] to the MIMO BC leads to the same solution as that of [45] thus proving the optimality of integrating the [125] solution in the approach proposed in [45] (i.e., iterating between transmit filters and receive filters with corresponding weights). Indeed, it can be shown that the KKT condition $\mathbf{G}_k$ is satisfied when the solution for $\mathbf{G}_k$ and λ_k are substituted thereby proving optimality of using the [125] approach both for the MIMO BC and MIMO IFC.

4.8 WSR Maximization at High SNR

In the first part of this chapter we have introduced an iterative algorithm that maximizes the WSR for all possible values of the SNR. In the following we will focus our attention only to the high SNR regime. In particular we study how it is possible to optimize the WSR only in that particular region.

In high SNR regime the behavior of the rate can be described using two quantities [130] : the *multiplexing gain* or *pre-log* or also *degrees of freedom* (DoF) and the *high SNR rate offset*. The former describes the slope of the asymptote of the rate curve in the high SNR, the latter can be interpreted as the axis intercept of the high SNR asymptote on the rate axis. The approximation can be mathematically represented as:

$$\mathcal{R}_k = r_k \log(\rho) + \alpha_k + O(\rho)$$

where α_k and r_k represent respectively the rate offset and the pre-log factor for the rate of user k. With ρ we denote the SNR. Using the approximation given before the WSR can be rewritten as:

$$\mathcal{R} = \sum_{k=1}^{K} w_k \mathcal{R}_k = r \log(\rho) + \alpha + O(\rho). \tag{4.38}$$

$r = \sum_{k=1}^{K} w_k r_k$ denotes the *weighted sum prelog factor* and $\alpha = \sum_{k=1}^{K} w_k \alpha_k$ is the *weighted sum rate offset*.

In high SNR regime also the expression of the Rx and Tx filter changes. In particular the linear receiver becomes a ZF receiver: $\mathbf{F}_k = \mathbf{F}_k^{IA} + O(\rho)$. Note that with this assumption only the row space of the Rx filter influences the rate so we can assume the Rx filter to be unitary. The interference plus noise covariance matrix $\mathbf{R}_{\overline{k}}^{-1}$ in high SNR becomes: $\mathbf{R}_{\overline{k}}^{-1} = \rho \mathbf{P}_{R_k^I}^{\perp}$, where $\mathbf{P}_{R_k^I}^{\perp}$ is the projection matrix onto orthogonal complement of the column space of the interference matrix $\mathbf{R}_k^I$ at user k.

We assume that the interference subspace at the $k-$th receiver has dimension $\text{rank}(\mathbf{R}_k^I) = i_k \leq N_k$

With this interpretation of the interference plus noise covariance matrix in high SNR the dominating term in the rate expression becomes:

$$\mathcal{R}_k = \min(d_k, N_k - i_k) \log(\rho) \tag{4.39}$$

hence to maximize the rate the Tx filters need to minimize the interference subspace dimension by interference alignment so that $i_k \leq N_k - d_k$, hence d_k should be IA-feasible. If this is the case the rate pre-log factor becomes $r_k = d_k$.

4.8.1 Maximization of the pre-log factors

From equation (4.38) the WSR maximization becomes in first instance the maximization of the weighted sum pre-log factor r:

$$\max_{\{d_k\}} \sum_{k=1}^{K} w_k d_k \tag{4.40}$$

this factor is the dominant term between the two quantities in (4.38) as SNR goes to infinity. The solution of this optimization problem will give the set of pre-log factors $\{d_k^\star\}$ that corresponds to the DoF allocation of the maximum WSR. Because each value of the pre-log factor can vary in a finite set: $d_k \in \{0, 1, \ldots, \min\{M_k, N_k\}\}$ a possible way of solving the optimization problem is using an exhaustive search among all the possible feasible DoF allocations that maximize (4.40).

A first important remark here is that for a given set of weights $\{w_k\}$ several optimal DoF allocation can be possible. This corresponds to the possibility of the WSR to have several local maxima. Using the proposed approach to determine the optimal DoF allocation can help to maximize the WSR using the iterative algorithm proposed in the first part of this paper. In particular imposing one of the possible optimal pre-log distribution in our iterative algorithm we can determine which DoF allocation effectively maximize the WSR among all the optimal distribution of streams.

A second remark arise from the observation that the determined optimal prelog-factor distribution is strictly related to the given set of weights $\{w_k\}$. If we change the weights the DoF allocation can change. This means that using the maximization procedure described above it is possible to explore the complete pre-log region varying the set of weights. We recommend that given the set of weights $\{w_k\}$, one determines an optimal choice for the prelogs $\{d_k\}$ with which one then runs the MWSR algorithm.

In the optimal stream allocation it is possible to have that one or more d_k are set to zero. In this case it corresponds to switch off the corresponding users.

4.8.2 Maximization of the high SNR rate offsets

Once the optimal multiplexing gain distribution is determined we need to optimize the weighted sum rate offset α. As described in [130] the high SNR rate offset is given by:

$$\alpha_k = \log |\mathbf{G}_k^H \mathbf{H}_{kk}^H \mathbf{P}_{R_k^I}^{\perp} \mathbf{H}_{kk} \mathbf{G}_k| \tag{4.41}$$

The beamformer can be parametrized as $\mathbf{G}_k = \overline{\mathbf{G}}_k \mathbf{U}_k \mathbf{\Delta}_k$, where $\overline{\mathbf{G}}_k$ is determined using IA and satisfies the property: $\overline{\mathbf{G}}_k^H \overline{\mathbf{G}}_k = \mathbf{I}_{d_k}$. The two matrices $\mathbf{U}_k$ and $\mathbf{\Delta}_k$

have dimensions $d_k \times d_k$. The former is a unitary matrix and the latter is a diagonal matrix.
Taking the eigendecomposition of the matrix $\overline{\mathbf{H}}_{kk}^H \overline{\mathbf{H}}_{kk} = \overline{\mathbf{G}}_k^H \mathbf{H}_{kk}^H \mathbf{P}_{R_k^I}^{\perp} \mathbf{H}_{kk} \overline{\mathbf{G}}_k = \mathbf{V}_k \mathbf{\Lambda}_k \mathbf{V}_k^H$, we can choose the unitary matrix $\mathbf{U}_k = \mathbf{V}_k$. With this parametrization the maximization problem of the the rate offset becomes:

$$\alpha_k^\star = \max_{\mathbf{\Delta}_k} \log |\mathbf{\Delta}_k^2 \mathbf{\Lambda}_k| \qquad (4.42)$$
$$\text{s.t.} \quad \text{Tr}\{\mathbf{\Delta}_k^2\} = P_k.$$

But $\log |\mathbf{\Delta}_k^2 \mathbf{\Lambda}_k| = \log |\mathbf{\Delta}_k^2| + \log |\mathbf{\Lambda}_k|$. Hence the optimum is reached for uniform power allocation $\mathbf{\Delta}_k^2 = \frac{P_k}{d_k}\mathbf{I}$. From this we can see that the expression for the BF at high SNR is:

$$\mathbf{G}_k = \sqrt{\frac{P_k}{d_k}} \overline{\mathbf{G}}_k \qquad (4.43)$$

Finally we can conclude that the high SNR rate expression is:

$$\mathcal{R}_k = d_k \log(\rho) + d_k \log(\frac{P_k}{d_k}) + \log |\overline{\mathbf{G}}_k^H \mathbf{H}_{kk}^H \mathbf{P}_{R_k^I}^{\perp} \mathbf{H}_{kk} \overline{\mathbf{G}}_k| \qquad (4.44)$$

As we said in Section 9.5 a necessary condition for the existence of a IA solution is related to the number of variables that we have in the MIMO IFC and the number of constraints that define the problem. Now we want to discuss how the variation of the rate offset can be related to this two quantities.
In particular if we assume that for the given MIMO IFC an IA solution exist we can have the following two cases:

- The number of variables is greater than the number of IA constraints. In this case an excess of variables implies continuously varying α_k (with w_k)
  Consider for example the system $K = 2, M_k = 2, N_k = 2, d = (1, 1)$, we can choose the two 2×1 Tx filters arbitrarily, and then the two 1×2 Rx filters are determined by IA.
  It is possible that subsets of equations have no excess of parameters, then the filters involved are not continuously varying

- The number of variables equals the number of IA constraints. Here no excess parameters exist but we may still get a discrete set of solutions $\{\alpha_k\}$ IA is described by a set of polynomial equations hence there are a finite number of solutions. For example in the case $K = 3$, $M_k = N_k = 2N$, 6 filters have N^2 DoF, and $6N^2$ ZF conditions. In this case an IA BF can be determined using the procedure described in [118]. In particular the first BF is determined taking the N eigenvector of a $2N \times 2N$ matrix

$\mathbf{H}_{31}^{-1}\mathbf{H}_{32}\mathbf{H}_{12}^{-1}\mathbf{H}_{13}\mathbf{H}_{23}^{-1}\mathbf{H}_{21}$, all the remaining BF can be found from $\mathbf{G}_1$. Using this way to determine the BF we have a different solution for a different choice of the N eigenvectors out of the possible $2N$.

4.9 Simulation Results

We provide here some simulation results to compare the performance of the proposed max-WSR algorithm (DA-MWSR) where we deterministic annealing is used to avoid local optimal point. i.i.d Gaussian channels (direct and cross links) are generated for each user. For a fixed channel realization transmit and receiver filters are computed based on IA algorithm and DA-MWSR algorithm over multiple SNR points. The resulting sum rate (SR) is averaged over 50 channel realizations. In Fig. 4.2 we compare the SR obtained using three different algorithms. In par-

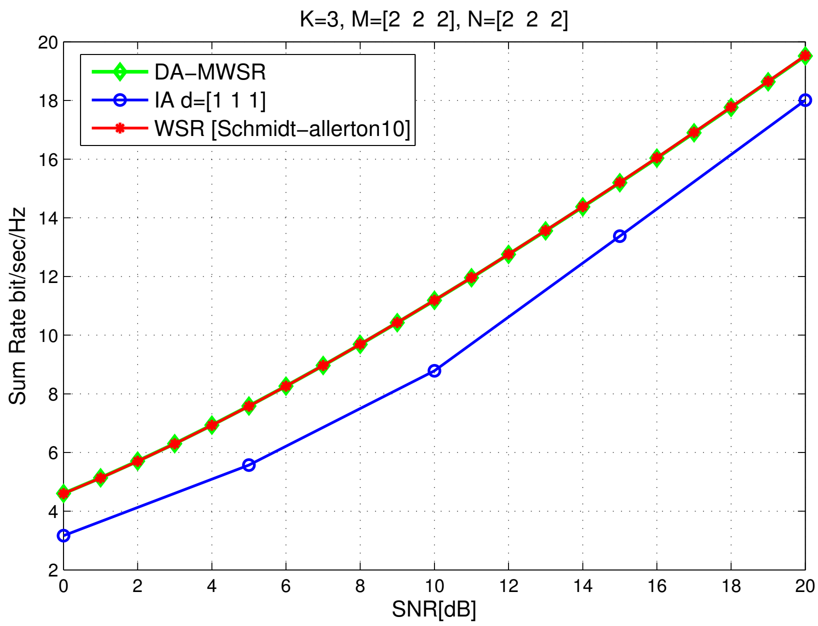

Figure 4.2: WSR for $K = 3$, $M_k = 2$, $N_k = 2$

ticular we compare our algorithm DA-MWSR with IA algorithm proposed in [33] and another WSR algorithm recently proposed in [121] where also a numerical continuation method is used to find the BF to maximize the WSR. This algorithm works only for single stream transmissions. As we can see both algorithms that maximize the WSR outperform IA in all SNR regimes. On the other hand there is no difference between the proposed algorithm and the one in [121].

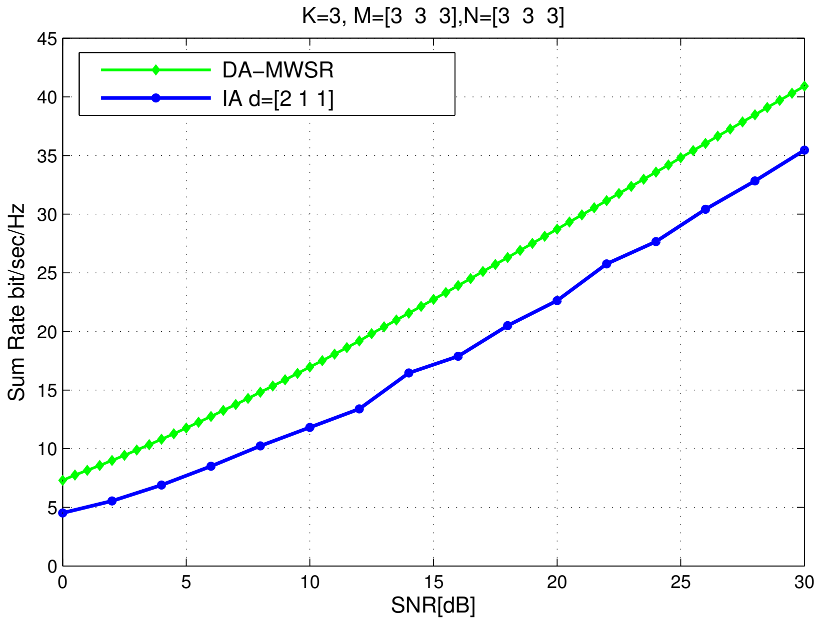

Figure 4.3: WSR for $K = 3$, $M_k = 3$, $N_k = 3$

In Fig. 4.3 we report the SR for a $K = 3$ users IFC where each Tx and Rx are equipped with $M_k = N_k = 3$ antennas. According to IA the total maximum number of streams that can be transmitted in the system is $d = 4$. We determine the IA beamformers and receiver filters using the algorithm in [33] for a stream distribution $d_1 = 2, d_2 = d_3 = 1$. We compare the performance of IA with our algorithm where the annealing parameter, noise variance, has been increased of $\delta t = 0.5$ dB. As we can see the proposed algorithm outperforms IA even at high SNR regime. The slope of the sum rate obtained using our algorithm is the same of the IA curve. This shows that the correct number of streams has been sent.

Fig. 4.4 depicts the performances of the proposed algorithm, WSR DA, in comparison with IA for a $K = 3$ user IFC with an asymmetric antennas distribution. We assume that $M_1 = N_1 = 5,\ M_i = N_i = 4\ i = 2, 3$, the stream distribution, according to IA is $d_k = 2\ \forall k$. As we can see also in this case the proposed algorithm outperform IA keeping the same slope in the high SNR regime.

Finally in Fig. 4.5 we compare the results of the algorithm proposed in this work, DA-MWSR, with a similar algorithm recently proposed for the MIMO in-

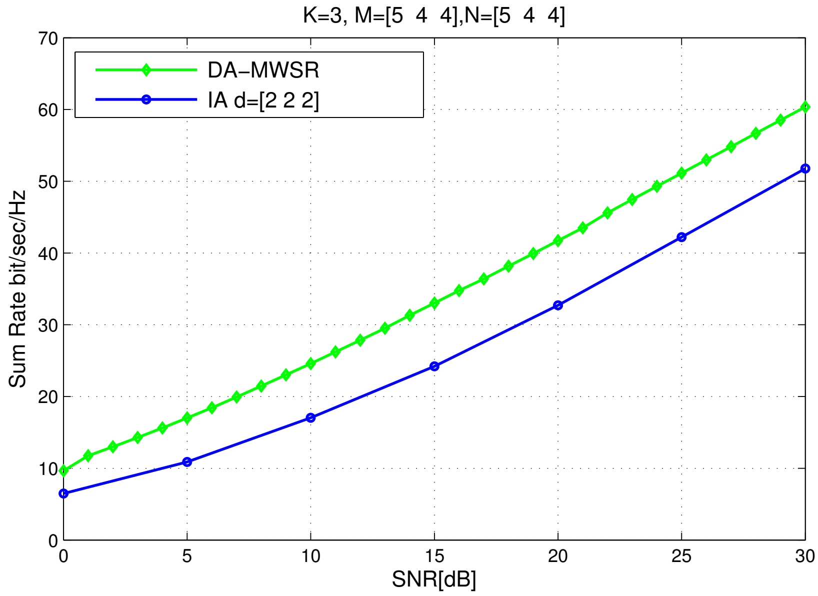

Figure 4.4: WSR for $K = 3$, $M_1 = N_1 = 5,\ M_i = N_i = 4,\ i = 2, 3$, $d_k = 2\ \forall k$

terfering broadcast channel, from which the IFC is a spacial case, described in [44], called WSR-[Luo-ITSP11] in the figure. The main steps of the algorithms are the same but in our approach we introduced deterministic annealing to reduce the probability to fall in a local optimal solution. As we can see in low SNR regime the two WSR algorithms have similar performances but in high SNR they manifest different characteristics. In particular at high SNR different local optima start to appear and, as we can see from the picture, our algorithm has better performances in term of WSR. In addition comparing the studied algorithm with an IA solution we see that our solution is able to sustain the correct number of transmitted streams.

4.10 Conclusions

We addressed maximization of the weighted sum rate for the MIMO IFC introducing an iterative algorithm to solve this optimization problem. In the high-SNR regime, this algorithm leads to an optimized Interference Alignment (IA) solution. In the finite SNR regime the performance of this algorithm is superior to that of IA and all known algorithms since it maximizes the WSR. Convergence to a local optimum was also shown experimentally. Convergence to local optima is known and is related to the non-convexity of the MIMO IFC rate region. To avoid to be

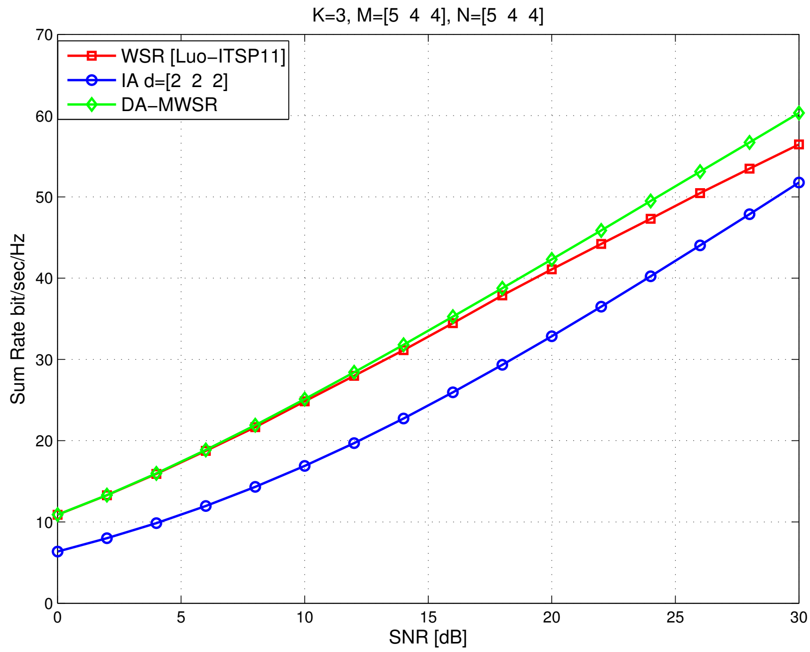

Figure 4.5: WSR for $K = 3$, $M_1 = N_1 = 5,\ M_i = N_i = 4,\ i = 2, 3$, $d_k = 2\ \forall k$

stuck in one suboptimal stationary point we propose to introduce Deterministic Annealing. This approach allows to track the variation of the known solution of one version of the problem into the unknown solution of the desired version by a controlled variation of a parameter called temperature. In our problem the temperature is related to the inverse of the SNR. The proposed algorithm include filter design for the progressive switching on of streams as the SNR increases.

Chapter 5

Sum Rate Maximization with Partial CSIT via the Expected Weighted MSE

5.1 Introduction and State of the Art

In previous chapters we described two ways for beamforming design in MIMO interference channel, interference alignment and maximum WSR approaches. Both methods require perfect channel state information at both sides, transmitter and receiver. In practical systems only imperfect (estimated and often fed back) CSI is available at each transmitter implying that more robust schemes need to be considered for beamforming design. In [131] the problem of robust beamforming design for single user MIMO with different types of CSI has been studied. There imperfect CSI is divided in two different classes: *deterministic and stochastic (statistical)*. In the deterministic case the known channel lies in an uncertainty region defined according to some norm. In stochastic channel representation the channel knowledge is given in term of mean and/or covariance. The design criterion described in the paper was the minimization of the mean squared error. The authors showed that the optimal transmit directions are the right singular vectors of the channel, channel mean, depending if perfect or erroneous CSI are considered. The beamformer design problem for a multiuser MIMO MAC channel, in presence of statistical CSIT, has been studied in [132]. The authors found that the optimal transmit directions,

when the minimization of the average sum mean squared error (MSE) is studied, were given by the eigenvectors of the channel mean or correlation matrix. [133] considered the robust joint optimization of transmitters and receivers in a MIMO broadcast setting where CSIT is modeled with deterministic uncertainty. In the paper is proposed a framework that allows to solve different problem such as: power minimization with MSE constraints, worst-case MSE minimization. The authors solved this set of non convex problems introducing an iterative algorithm based on alternating minimization optimizing transmitter and receiver in different iteration steps reformulating the problem as a semidefinite programming.
The authors in [134] studied the problem of robust beamforming design for a MISO interference channel under deterministic CSI uncertainty. The cost functions that are taken into account include worst-case sum rate maximization and worst-case minimum rate maximization. To solve the problem centralized and distributed algorithms were introduced based on semidefinite programming. [135] considered the problem of power gain region characterization in presence of perfect CSIR and deterministic CSIT uncertainty for a MISO interference channel. To find the optimal robust beamformers the authors cast the problem as a second order cone program for an efficient solution. They also observe that at high SNR zero forcing beamformers achieves full multiplexing gain if the channel uncertainty scales inversely proportional with the SNR, otherwise single user transmission is optimal. In [136] the authors studied the problem of joint transceiver design for MIMO interference channel introducing an optimization problem based on a function of the mean squared error (MSE). They considered the sum MSE and per-user MSE minimization providing two iterative algorithms that solve the problems using alternating minimization. Apart the case when perfect CSI is available they also solved the same problems assuming a stochastic CSI uncertainty. In [137] the authors propose a robust beamforming approach for a MIMO interference channel in presence of deterministic CSI uncertainty. Their iterative algorithm alternately computes transmit and receive filters with the objective of maximizing the worst-case per-stream SINR using semidefinite relaxation. [138] considered the problem of weighted sum rate maximization of a MIMO interference channel when imperfect CSIT, with bounded error, is used to design transmit and receive filters. The authors provided an iterative algorithm, based on alternating minimization, that solved the problem reformulating it in a semi-definite programming for a more efficient solution.

5.2 Contributions

In this chapter we focus on robust beamforming design for a MIMO interfering broadcast channel (IBC) with the objective of maximizing the sum rate when stochastic CSIT is available while the receiver has perfect CSI. The solution proposed for robust beamforming design is based on the relationship between WSR and Weighted MSE (WMSE) [45] and chapter 4. The main difference with respect to [131] resides in the multi user approach that makes impossible to directly use the results derived in that paper. Here the optimal expressions of the beamforming filters are obtained from the minimization of the sum of average WMSE, where the expectation is taken w.r.t. channel realizations. The optimal expression obtained for transmit and receive filters are then used to develop an iterative algorithm, based on alternating minimization, that converges to a local optimal solution. In a recent paper [139] a similar approach is considered. The main difference it that there the objective is the sum MSE minimization and not the sum rate maximization as in this chapter. Then even if the approaches look very similar the convergence points are different. In addition we show that minimizing the expected value of the WMSE corresponds to the maximization of a lower-bound of the WSR. Finally we introduce some simulation results to validate numerically the proposed algorithm. We compare, in term of achieved sum rate, the proposed solution with IA beamforming design, obtained from partial and perfect CSIT, and with the solution proposed in chapter 4. We see that if the channel uncertainty scales inversely proportionally with the SNR then there is no loss in DoF with respect to an IA solution.

5.3 Signal Model

We consider a K-cell MIMO interfering broadcast channel (IBC), schematically reported in Fig. 5.1. For ease of exposition, we denote the transmitters as Base station (BS) and the receivers as Mobile user (MU). The k-th BS is equipped with M_k transmitting antennas and wants to communicate with L_k MUs in its own cell. We denote with $N_i^{(k)}$ the number of antennas at i-th MU in cell k. Because all Tx-Rx pairs share the same frequency bands each transmission generates interference at all non intended receivers. At Rx number i in cell k the received signal vector $\mathbf{y}_i^{(k)}$ can be written as

$$\mathbf{y}_i^{(k)} = \mathbf{H}_{ik}^{(k)}\mathbf{x}_i^{(k)} + \sum_{l\neq k}\mathbf{H}_{ik}^{(k)}\mathbf{x}_l^{(k)} + \sum_{j\neq k}\sum_{l=1}^{L_j}\mathbf{H}_{ij}^{(k)}\mathbf{x}_l^{(j)} + \mathbf{n}_i^{(k)} \tag{5.1}$$

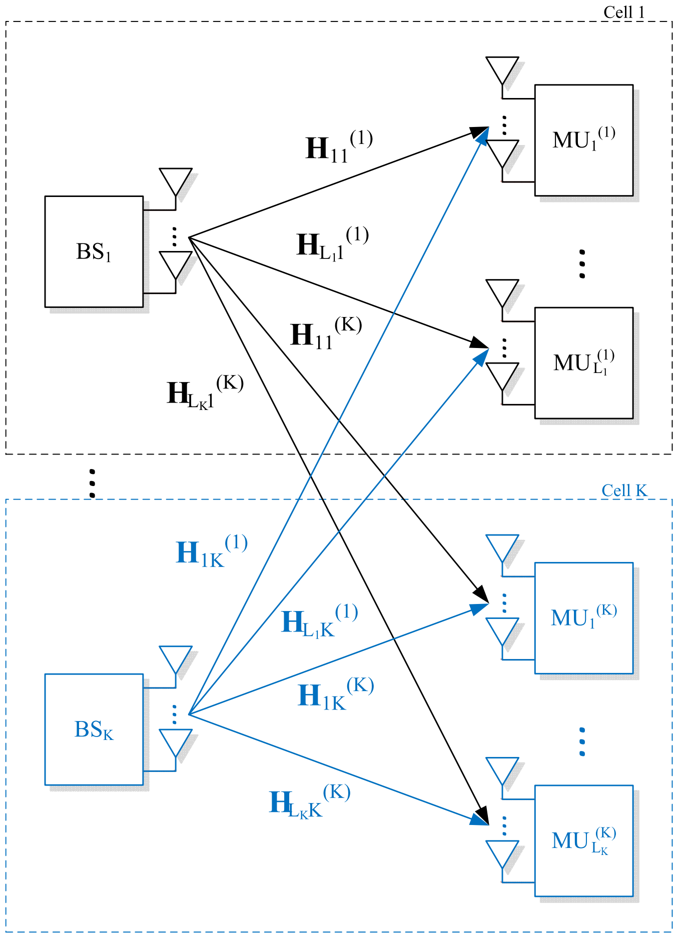

Figure 5.1: MIMO Interference Broadcast Channel

where $\mathbf{H}_{ij}^{(k)} \in \mathbb{C}^{N_i^{(k)} \times M_j}$ represents the channel matrix between the j-th BS and i-th MU in cell number k, $\mathbf{x}_i^{(k)}$ is the $\mathbb{C}^{M_k \times 1}$ transmit signal vector of the k-th BS for its i-th MU and the $\mathbb{C}^{N_i^{(k)} \times 1}$ vector $\mathbf{n}_i^{(k)}$ represents (temporally white) AWGN with zero mean and covariance matrix $\mathbf{R}_{n_k n_k}$. The channel is assumed to follow a block-fading model having a coherence time of T symbol intervals without channel variation. Each entry of the channel matrix is an independent complex random variable drawn from a Gaussian distribution $\mathcal{CN}(0, 1)$. We denote by $\mathbf{G}_i^{(k)}$, the $\mathbb{C}^{M_k \times d_i^{(k)}}$ precoding matrix of the k-th BS to the i-th receiver. Thus $\mathbf{x}_i^{(k)} = \mathbf{G}_i^{(k)} \mathbf{s}_i^{(k)}$, where $\mathbf{s}_i^{(k)}$ is a $d_i^{(k)} \times 1$ vector representing the $d_i^{(k)}$ independent symbol streams for the (k, i)-th user pair. We assume $\mathbf{s}_i^{(k)}$ to have a spatio-temporally white Gaussian distribution with zero mean and unit variance, $\mathbf{s}_i^{(k)} \sim \mathcal{N}(0, \mathbf{I}_{d_i^{(k)}})$. The i-th receiver, to suppress interference and retrieve its $d_i^{(k)}$ desired streams, applies the filter matrix $\mathbf{F}_i^{(k)H} \in \mathbb{C}^{d_i^{(k)} \times N_i^{(k)}}$.

5.4 WSR maximization for the MIMO interfering Broadcast channel

In this chapter we focus our attention on the maximization of the WSR of K-cell MIMO IBC when only partial CSIT is available. In the first part of this section we introduce the sum rate maximization problem, studied for the MIMO IFC in chapter 4, for the K-cell MIMO IBC [44]. We limit receiver complexity by treating the interference as colored noise, from here the definition of Noisy IBC. As a result, linear receivers are sufficient.

Assuming Gaussian signaling, the WSR maximization problem can be mathematically expressed as follows:

$$\begin{aligned}
&\left\{\mathbf{G}_i^{\star(k)}\right\} = \arg \min_{\left\{\mathbf{G}_i^{(k)}\right\}} \sum_{k=1}^{K} \sum_{i=1}^{L_k} -u_i^{(k)} \log |\mathbf{E}_i^{(k)-1}| \\
&\text{s.t} \sum_{i=1}^{L_k} \mathrm{Tr}(\mathbf{G}_i^{(k)H} \mathbf{G}_i^{(k)}) \leq P_k \; \forall k
\end{aligned}$$

where $\mathbf{E}_i^{(k)} = (\mathbf{I} + \mathbf{G}_i^{(k)H} \mathbf{H}_{ik}^{(k)H} \mathbf{R}_{\bar{i}}^{(k)-1} \mathbf{H}_{ik}^{(k)} \mathbf{G}_i^{(k)})^{-1}$, $u_i^{(k)} \geq 0$ denotes the weight assigned to the (k, i)-th user's rate and P_k the corresponding transmit power constraint. The interference plus noise covariance matrix $\mathbf{R}_{\bar{i}}^{(k)}$ is:

$$\mathbf{R}_{\bar{i}}^{(k)} = \mathbf{R}_{n_k n_k} + \sum_{(l,j) \neq (k,i)} \mathbf{H}_{il}^{(k)} \mathbf{G}_j^{(l)} \mathbf{G}_j^{(l)H} \mathbf{H}_{il}^{(k)H}.$$

As described in chapter 4 the maximization of the WSR can be tackled introducing an augmented cost function in which two additional optimization variables appear, receive filters and weighting matrices $\mathbf{W}_i^{(k)}$. The optimization problem that we consider is

$$\arg \max_{\{\mathbf{G}_i^{(k)}, \mathbf{F}_i^{(k)}, \mathbf{W}_i^{(k)}\}} \sum_{(k,i)} -u_i^{(k)} (\mathrm{Tr}(\mathbf{W}_i^{(k)} \boldsymbol{\mathcal{E}}_i^{(k)}) - \log |\mathbf{W}_i^{(k)}|) \tag{5.2}$$

$$\text{s.t} \sum_{i=1}^{L_k} \mathrm{Tr}(\mathbf{G}_i^{(k)H} \mathbf{G}_i^{(k)}) \leq P_k.$$

Assuming $\mathbb{E}\{\mathbf{s}_i^{(k)}\mathbf{s}_i^{(k)H}\} = \mathbf{I}_{d_i^{(k)}}$, the MSE covariance matrix for general Tx and Rx filters is

$$\begin{aligned}\mathcal{E}_i^{(k)} =& \mathbb{E}[(\mathbf{s}_i^{(k)} - \mathbf{F}_i^{(k)H}\mathbf{y}_i^{(k)})(\mathbf{s}_i^{(k)} - \mathbf{F}_i^{(k)H}\mathbf{y}_i^{(k)H}]\} \\ =& \mathbf{I} - \mathbf{G}_i^{(k)H}\mathbf{H}_{ik}^{(k)H}\mathbf{F}_i^{(k)} - \mathbf{F}_i^{(k)H}\mathbf{H}_{ik}^{(k)}\mathbf{G}_i^{(k)} \\ &+ \sum_{(l,j)} \mathbf{F}_i^{(k)H}\mathbf{H}_{il}^{(k)}\mathbf{G}_j^{(l)}\mathbf{G}_j^{(l)H}\mathbf{H}_{il}^{(k)H}\mathbf{F}_i^{(k)} + \mathbf{F}_i^{(k)H}\mathbf{R}_{n_k n_k}\mathbf{F}_i^{(k)}\end{aligned} \tag{5.3}$$

This cost function is concave or even quadratic in one set of variables, keeping the other two fixed. Hence we shall optimize it using alternating maximization. Here we consider a channel knowledge at the transmitter side that can be modeled in term of channel mean and variance, that can represent the channel estimate and estimation error:

$$\mathbf{H}_{ij}^{(k)} = \widehat{\mathbf{H}}_{ij}^{(k)} + (\mathbf{R}_i^{(k)r})^{\frac{1}{2}}\widetilde{\mathbf{H}}_{ij}^{(k)}(\mathbf{R}_j^t)^{\frac{H}{2}} \tag{5.4}$$

where $\widehat{\mathbf{H}}_{ij}^{(k)}$ can model the channel estimate for the channel between Tx j and Rx i in cell k. $\mathbf{R}_j^t$ is the Tx side covariance matrix while $\mathbf{R}_i^{(k)r}$ represents the covariance matrix at the Rx side. $\widetilde{\mathbf{H}}_{ij}^{(k)}$ is a matrix with iid Gaussian, zero mean and unit variance, entries. We should underline here that this restrictive Kronecker covariance model is not required for the technique described in this paper to be applicable. We only assume this model to simplify some of the expressions.
With the given parametrization of the channel we can obtain the expected value of the MSE:

$$\begin{aligned}\overline{\mathcal{E}}_i^{(k)} =& \mathbb{E}_{\mathbf{H}|\widehat{\mathbf{H}}}\{\mathcal{E}_i^{(k)}\} = \mathbf{I} - \mathbf{G}_i^{(k)H}\widehat{\mathbf{H}}_{ik}^{(k)H}\mathbf{F}_i^{(k)} - \mathbf{F}_i^{(k)H}\widehat{\mathbf{H}}_{ik}^{(k)}\mathbf{G}_i^{(k)} \\ &+ \sum_{(l,j)} \mathbf{F}_i^{(k)H}\widehat{\mathbf{H}}_{il}^{(k)}\mathbf{G}_j^{(l)}\mathbf{G}_j^{(l)H}\widehat{\mathbf{H}}_{il}^{(k)H}\mathbf{F}_i^{(k)} \\ &+ \mathbf{F}_i^{(k)H}\left[\sum_{(l,j)} Tr\{\mathbf{R}_l^t\mathbf{G}_j^{(l)}\mathbf{G}_j^{(l)H}\}\mathbf{R}_i^{(k)r} + \mathbf{R}_{n_k n_k}\right]\mathbf{F}_i^{(k)}\end{aligned}$$

where the expectation above is taken w.r.t. the CSI uncertainty. We should remark that assuming a Tx side covariance matrix of the form $\mathbf{R}_l^t = \alpha_l\mathbf{I}$ then it is possible to interpret the expected MSE above as the original MSE in (5.3) with $\mathbf{H}_{ij}^{(k)} = \widehat{\mathbf{H}}_{ij}^{(k)}$ and an augmented noise covariance matrix contribution of the form $\mathbf{R}'_{n_k n_k} = \sum_l \alpha_l P_l \mathbf{R}_i^{(k)r} + \mathbf{R}_{n_k n_k}$.Hence the proposed partial CSI algorithm solves exactly the perfect CSI WSR for a system with modified channel and noise covariance matrices! The optimization problem (5.2) now becomes the following:

$$\arg \max_{\{\mathbf{G}_i^{(k)}, \mathbf{F}_i^{(k)}, \mathbf{W}_i^{(k)}\}} \sum_{(k,i)} -u_i^{(k)}(\mathrm{Tr}(\mathbf{W}_i^{(k)}\overline{\mathcal{E}}_i^{(k)}) - \log|\mathbf{W}_i^{(k)}|) \tag{5.5}$$

$$\text{s. t} \sum_{i=1}^{L_k} \mathrm{Tr}(\mathbf{G}_i^{(k)H}\mathbf{G}_i^{(k)}) \leq P_k.$$

The corresponding Lagrangian can be written as:

$$\begin{aligned} J(\{\mathbf{G}_i^{(k)}, \mathbf{F}_i^{(k)}, \mathbf{W}_i^{(k)}, \lambda_k\}) &= -\sum_k \lambda_k (\sum_{i=1}^{L_k} \mathrm{Tr}(\mathbf{G}_i^{(k)H}\mathbf{G}_i^{(k)}) - P_k) \\ -\sum_{(k,i)} u_i^{(k)} (\mathrm{Tr}(\mathbf{W}_i^{(k)} \overline{\mathcal{E}}_i^{(k)}) - \log |\mathbf{W}_i^{(k)}|) \end{aligned} \tag{5.6}$$

This new cost function will be optimized w.r.t. one set of variables, keeping the other two fixed. The first step is the calculation of the optimal Rx filters assuming fixed the matrices $\mathbf{G}_i^{(k)}$ and $\mathbf{W}_i^{(k)}$. From the derivative of J w.r.t. $\mathbf{F}_i^{(k)}$ the optimal receiver results to be an MMSE filter of the form:

$$\begin{aligned} \mathbf{F}_i^{(k)} = (\sum_{(l,j)} [\widehat{\mathbf{H}}_{il}^{(k)} \mathbf{G}_j^{(l)} \mathbf{G}_j^{(l)H} \widehat{\mathbf{H}}_{il}^{(k)H} + \mathrm{Tr}\{\mathbf{R}_l^t \mathbf{G}_j^{(l)} \mathbf{G}_j^{(l)H}\} \mathbf{R}_i^{(k)r}] \\ + \mathbf{R}_{n_k n_k})^{-1} \widehat{\mathbf{H}}_{ik}^{(k)} \mathbf{G}_i^{(k)} \end{aligned} \tag{5.7}$$

The following step is the determination of the optimal expression for the matrix $\mathbf{W}_i^{(k)}$ while keeping the other two variable sets fixed. Then, equating the derivative of the Lagrangian (5.6) w.r.t. $\mathbf{W}_i^{(k)}$ to zero, we get :

$$\mathbf{W}_i^{(k)} = \overline{\mathcal{E}}_i^{(k)-1} \tag{5.8}$$

The final step is the maximization of the given cost function w.r.t. the BF matrix. To accomplish this task we derive the Lagrangian w.r.t. the matrix $\mathbf{G}_k$ and equate it to zero:

$$\begin{aligned} \frac{\partial \mathrm{J}(\{\mathbf{G}_i^{(k)}, \lambda_k\})}{\partial \mathbf{G}_i^{(k)*}} &= -\sum_{(l,j)} u_j^{(l)} \mathrm{Tr}\{\mathbf{W}_j^{(l)} \mathbf{F}_j^{(l)H} \mathbf{R}_j^{(l)r} \mathbf{F}_j^{(l)}\} \mathbf{R}_k^t \mathbf{G}_i^{(k)} - \lambda_k \mathbf{G}_i^{(k)} \\ u_i^{(k)} \widehat{\mathbf{H}}_{ik}^{(k)H} \mathbf{F}_i^{(k)} \mathbf{W}_i^{(k)} &- \sum_{(l,j)} u_j^{(l)} \widehat{\mathbf{H}}_{jk}^{(l)H} \mathbf{F}_j^{(l)} \mathbf{W}_j^{(l)} \mathbf{F}_j^{(l)H} \widehat{\mathbf{H}}_{jk}^{(l)} \mathbf{G}_i^{(k)} = \mathbf{0}. \end{aligned}$$

This leads to the following expression for the optimal BF:

$$\begin{aligned} \mathbf{G}_i^{(k)} = \left(\sum_{(l,j)} u_j^{(l)} [\widehat{\mathbf{H}}_{jk}^{(l)H} \mathbf{Q}_j^{(l)} \widehat{\mathbf{H}}_{jk}^{(l)} + \mathrm{Tr}\{\mathbf{Q}_j^{(l)} \mathbf{R}_j^{(l)r}\} \mathbf{R}_k^t] + \lambda_k \mathbf{I} \right)^{-1} \\ \times \widehat{\mathbf{H}}_{ik}^{(k)H} \mathbf{F}_i^{(k)} \mathbf{W}_i^{(k)} u_i^{(k)} \end{aligned} \tag{5.9}$$

where $\mathbf{Q}_j^{(l)} = \mathbf{F}_j^{(l)} \mathbf{W}_j^{(l)} \mathbf{F}_j^{(l)H}$. The only variable that still needs to be optimized is the Lagrange multiplier λ_k. First check if the power constraint is satisfied for $\lambda_k = 0$. If yes, then $\lambda_k = 0$. If not, the Tx power equality constraint is active. To derive the optimal value of λ_k we can use the results derived in chapter 4 for the IFC. If we define the following compound quantities $\mathbf{F}_k = diag\{\mathbf{F}_1^{(k)}, \ldots, \mathbf{F}_{L_k}^{(k)}\}$, $\mathbf{G}_k = [\mathbf{G}_1^{(k)}, \ldots, \mathbf{G}_{L_k}^{(k)}]$, $\mathbf{H}_{ij} = [\mathbf{H}_{1j}^{(i)T}, \ldots, \mathbf{H}_{L_k j}^{(i)T}]^T$, $\mathbf{W}_k = diag\{u_1^{(k)}\mathbf{W}_1^{(k)}, \ldots, u_{L_k}^{(k)}\mathbf{W}_{L_k}^{(k)}\}$ and $\mathbf{R}_k^r = diag\{\mathbf{R}_1^{(k)r}, \ldots, \mathbf{R}_{L_k}^{(k)r}\}$ we can read the IBC studied above as a traditional IFC. Then to find the optimal value of λ_k we pre-multiply the derivative of the Lagrangian J w.r.t. the compound BF matrix by $\mathbf{G}_k^H$. Thanks to the first order optimality condition taking the trace of that product we get:

$$\mathrm{Tr}\left\{\mathbf{G}_k^H \frac{\partial \mathrm{J}(\{\mathbf{G}_k, \lambda_k\})}{\partial \mathbf{G}_k^*}\right\} = 0.$$

Imposing the power constraint to be satisfied with equality, hence the contribution $\lambda_k Tr\left\{\mathbf{G}_k^H \mathbf{G}_k\right\} = \lambda_k P_k$, we are able to derive the value of the optimal Lagrange multiplier. To overcome the convergence difficulties in non-convex optimization problems, like WSR, several heuristic approaches have been proposed. As shown in chapter 4 we can use *Deterministic Annealing* (DA) since also in the WSR maximization problem the convexity properties are driven by a scalar parameters. In our problem, the role of temperature is played by the *noise power* σ_k^2, which starting from now we assume, without losing generality (w.l.g.), equal to $\sigma_k = \sigma^2\ \forall k$.

Algorithm 5 DA-MWSR Algorithm for MIMO IFC

set $t = 0$
Fix the initial set of precoding matrices $\mathbf{G}_i^{(k)}$
repeat
  increment SNR: $t^{(i+1)} = t^{(i)} + \delta t$
  repeat
    Given $\mathbf{G}_i^{(k)}$ compute $\mathbf{F}_i^{(k)}$ and $\mathbf{W}_i^{(k)}$, $\forall (k, i)$, as in (5.7)-(5.8)
    Given $\mathbf{F}_i^{(k)}$, $\mathbf{W}_i^{(k)}$, compute $\mathbf{G}_i^{(k)}$ $\forall (k, i)$, using (5.9)
  until convergence
until target SNR is reached

In the algorithm description, in **Algorithm 5**, $t^{(i)}$ represents the value of the SNR that is incremented at each step of the annealing procedure. Here we should underline that the Rx filters calculated in the proposed algorithm are not the ones actually used at the Rx side. Those filters are based on perfect CSIR and hence they are different compare to the one in (5.7). The MMSE Rx filter used at receiver i in cell k will be of the form:

$$\mathbf{F}_i^{(k)} = (\sum_{(l,j)} [\mathbf{H}_{il}^{(k)} \mathbf{G}_j^{(l)} \mathbf{G}_j^{(l)H} \mathbf{H}_{il}^{(k)H} + \mathbf{R}_{n_k n_k})^{-1} \mathbf{H}_{ik}^{(k)} \mathbf{G}_i^{(k)} \tag{5.10}$$

5.5 WSR Lower Bound with Partial CSIT

In this section we first study how the approach presented in section 5.4 is related to the ergodic sum rate, then we introduce a new lower bound for the WSR when IA transmit and receive filter are computed using partial CSIT. To make the presentation more clear we specify the results of the previous section to a MIMO IFC. This does not reduce the validity of our results because the IBC can be interpreted as a traditional MIMO IFC using the compound quantities introduced in the previous section. To simplify the derivation of the bound we assume to work per stream instead of per user. As shown in [45] working per stream does not cause a reduction of performances.

As described in section 5.4 the rate for a K-user MIMO IFC with full CSIT can be written as:

$$\mathcal{R}_{FCSIT} = \sum_{(k,n)} \log \mathbf{E}_{kn}^{-1} \tag{5.11}$$

where $\mathbf{E}_{kn}$ indicates the MMSE of the n-th stream of user k. Then the minimum MSE for stream n of user k is $\mathbf{E}_{kn} = \min_{\mathbf{f}_{kn}} \mathcal{E}_{kn} = (1 + \mathbf{g}_{kn}^H \mathbf{H}_{kk}^H \mathbf{R}_{\overline{kn}}^{-1} \mathbf{H}_{kk} \mathbf{g}_{kn})^{-1}$. Vector $\mathbf{g}_{kn}$ represents the n-th column of the BF matrix $\mathbf{G}_k$, matrix $\mathbf{R}_{\overline{kn}}$ is the interference plus noise covariance matrix for stream (k,n). From Jensen inequality we have:

$$\mathbb{E}_{\widehat{\mathbf{H}}\widetilde{\mathbf{H}}}\{\sum_{(k,n)} \log \mathbf{E}_{kn}^{-1}\} \geq -\mathbb{E}_{\widehat{\mathbf{H}}} \sum_{(k,n)} \log \mathbb{E}_{\widetilde{\mathbf{H}}}\{\mathbf{E}_{kn}\} \tag{5.12}$$

the expectation above is taken over all channel pdfs. On the other hand the rate that we obtain once we optimize the cost function in (5.5) is

$$\mathcal{R}_{PCSIT} = \sum_{(k,n)} u_k \log(\overline{\mathcal{E}}_{kn}^{\star})^{-1} \tag{5.13}$$

where $\overline{\mathcal{E}}_{kn}^{\star} = \min_{\mathbf{f}_{kn}} \overline{\mathcal{E}}_{kn}$ and is equal to:

$$\mathbf{g}_{kn}^H \widehat{\mathbf{H}}_{kk}^H [\sum_{(l,m)} [\widehat{\mathbf{H}}_{kl}\mathbf{g}_{lm}\mathbf{g}_{lm}^H \widehat{\mathbf{H}}_{kl}^H + \mathrm{Tr}\{\mathbf{R}_{kl}^t \mathbf{g}_{lm}\mathbf{g}_{lm}^H\}\mathbf{R}_{kl}^r] + \mathbf{R}_{n_k n_k}]^{-1} \widehat{\mathbf{H}}_{kk}\mathbf{g}_{kn}$$

where $\mathbf{f}_{kn}$ is the n-th row of the Rx filter matrix $\mathbf{F}_k$. From the equation above we can see that $\mathbb{E}\{\mathbf{E}_{kn}\} \neq \overline{\mathcal{E}}_{kn}^{\star}$. Calculating the expected value over the channel uncertainty we can show that:

$$\mathbb{E}_{\widetilde{\mathbf{H}}}\{\mathbf{E}_{kn}\} \leq \overline{\mathcal{E}}_{kn}^{\star} \tag{5.14}$$

This statement can be proved easily. Assume $\mathcal{E}_{kn} = \mathcal{E}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij})$, and let $B(\mathbf{f}_{kn}) = \mathbb{E}_{\mathbf{H}} \mathcal{E}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij})$.
Now consider $B(\mathbf{f}_{kn}^o) = \min_{\mathbf{f}_{kn}} B(\mathbf{f}_{kn}) = \min_{\mathbf{f}_{kn}} \mathbb{E}_{\mathbf{H}} \mathcal{E}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij})$. Then for any $\mathbf{H}_{ij}$,

$$\min_{\mathbf{f}_{kn}} \mathcal{E}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij}) \leq \mathcal{E}_{kn}(\mathbf{f}_{kn}^o, \mathbf{H}_{ij})$$

hence

$$\mathbb{E}_{\mathbf{H}} \min_{\mathbf{f}_{kn}} \mathcal{E}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij}) \leq \mathbb{E}_{\mathbf{H}} \mathcal{E}_{kn}(\mathbf{f}^o_{kn}, \mathbf{H}_{ij}) = \min_{\mathbf{f}_{kn}} \mathbb{E}_{\mathbf{H}} \mathcal{E}_{kn}(\mathbf{f}_{kn}, \mathbf{H}_{ij})$$

From (5.14) we can conclude

$$\mathbb{E}_{\widehat{\mathbf{H}}\widetilde{\mathbf{H}}}\{\sum_{(k,n)} \log \mathbf{E}^{-1}_{kn}\} \geq -\mathbb{E}_{\widehat{\mathbf{H}}} \sum_{(k,n)} \log \mathbb{E}_{\widetilde{\mathbf{H}}}\{\mathbf{E}_{kn}\} \geq -\mathbb{E}_{\widehat{\mathbf{H}}} \sum_{(k,n)} \log \overline{\mathcal{E}}^{\star}_{kn}$$

The final relationship says that with our approach we are actually maximizing a sum rate lower-bound.

As shown in [140] for a BC system also for a K-user MIMO IFC it is possible to derive a SR lower bound in case of IA transmissions. The rate for the k-th UE can be written as:

$$\mathcal{R}_k = \log(1 + \frac{1}{\sigma^2_k}|\mathbf{f}_k \mathbf{H}_{kk} \mathbf{g}_k|^2) \tag{5.15}$$

in the following we assume that $\sigma^2_k = 1$. This is for the case where perfect CSIT are available at the BSs. Here we give only the main results for the case where each user sends only one stream $d_k = 1 \; \forall k$.

To study the case with imperfect CSIT, we use the channel model (5.4) where now $\overline{\mathbf{H}}_{ij} = (\mathbf{R}^r_i)^{\frac{1}{2}} \widetilde{\mathbf{H}}_{ij} (\mathbf{R}^t_j)^{\frac{H}{2}}$. Due to IA design $\mathbf{f}_k \, \widehat{\mathbf{H}}_{ki} \, \mathbf{g}_i = 0$, $\forall k \neq i$. Then we rewrite the Rx signal as:

$$\begin{aligned} r_k =& \mathbf{f}_k \widehat{\mathbf{H}}_{kk} \mathbf{g}_k s_k + \mathbf{f}_k \overline{\mathbf{H}}_{kk} \mathbf{g}_k s_k + \sum_{i \neq k} \mathbf{f}_k \overline{\mathbf{H}}_{ki} \mathbf{g}_i s_i + \mathbf{f}_k \mathbf{n}_k \\ =& \mathbf{f}_k \widehat{\mathbf{H}}_{kk} \mathbf{g}_k s_k + \mathbf{f}_k \mathbf{n}^{'}_k \end{aligned}$$

where the equivalent noise term $\mathbf{n}^{'}_k$ represents the residual interference plus noise contribution that can be model as a Gaussian noise zero mean and variance $\sigma^{'2}_k = 1 + \tilde{\sigma}^2$.

Now absorbing all the interference contributions in the noise (we are considering the noisy IFC) we get a rate lower bound (and close approximation) by ignoring the dependence of the term $\mathbf{f}_k \overline{\mathbf{H}}_{kk} \mathbf{g}_k s_k$ on the signal s_k and absorbing $\mathbf{f}_k \overline{\mathbf{H}}_{kk} \mathbf{g}_k s_k$ into the noise also. Hence we get the rate lower bound

$$\mathcal{R}^{LB}_k = \log(1 + \frac{1}{\sigma'^2_k}|\mathbf{f}_k \widehat{\mathbf{H}}_{kk} \mathbf{g}_k|^2) \tag{5.16}$$

In other words, this rate lower bounds corresponds to the rate of an IFC with max WSR-IA design for the case in which the overall channel is $\widehat{\mathbf{H}}$ instead to $\mathbf{H}$ and the noise variances are increased by a factor $\tilde{\sigma}^2$ in link k.

5.6 Simulation Results

We provide here some simulation results to compare the performances of the proposed algorithm for the maximization of the WSR using partial CSIT (MWSR$_{PCSI}$ in figure) for a 3-user MIMO IFC. Here DA is used to avoid to be trapped in local optimal solutions. To find the IA solution for the case with partial CSIT (IA$_{PCSI}$) we use the algorithm proposed in [33] where instead of the real channel matrix $\mathbf{H}_{ij}$ we use only the channel estimate $\widehat{\mathbf{H}}_{ij}$. The channels are generated according to model (5.4). The algorithm proposed in this paper can handle any general choice of the Tx and Rx covariance matrix but in the numerical examples proposed in this section we consider, as example, $\mathbf{R}_j^t = \mathbf{I}$ and $\mathbf{R}_i^r = \tilde{\sigma}^2\mathbf{I}$. $\tilde{\sigma}^2$ represents the estimation error variance and here is scaled inversely proportional with the SNR. In Fig. 5.2 the performances, in terms of sum rate, of the proposed robust max WSR algorithm and IA for the case of partial CSIT are depicted. Those curves are compared with sum rate obtained using the MWSR, from chapter 4, and IA, from [33], with perfect CSIT (MWSR$_{FCSI}$ and IA$_{FCSI}$ respectively). It can be noted that the max WSR solution outperforms the IA solutions for both cases, perfect and partial CSIT. On the other hand the rate curves obtained with partial CSIT show a rate offset compare to the corresponding curves obtained with perfect CSIT. In

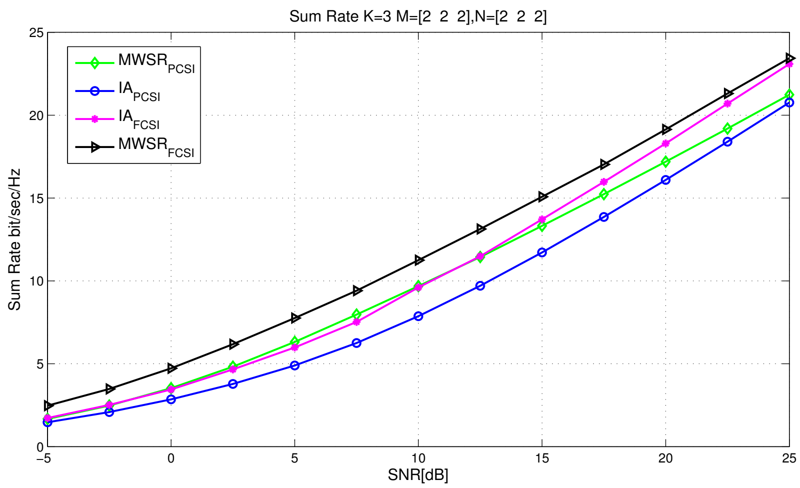

Figure 5.2: Sum Rate comparisons for $K = 3$ $M_k = N_k = 2, \ \forall k$

Fig.5.3 we report the sum rate curves for the same algorithms presented before. The main difference is that in this case the channel estimation error variance does not scale with the SNR but remains constant at -6dB. As we can see this implies that the performance obtained with algorithms based on partial CSIT are characterized by a saturation at high SNR. This is due to the fact that at high SNR to keep a finite gap with the full CSIT case the channel quality should scale with the SNR. In addition in Fig. 5.3 we report also the performance of the MWSR algorithm, derived in chapter 4, when it is implemented using only the channel estimate $\widehat{\mathbf{H}}_{ij}$ to determine the optimal BF (MWSR w channel estimate). As we can see the proposed algorithm (MWSR$_{PCSI}$) outperforms the former solution manifesting a more robust behavior.

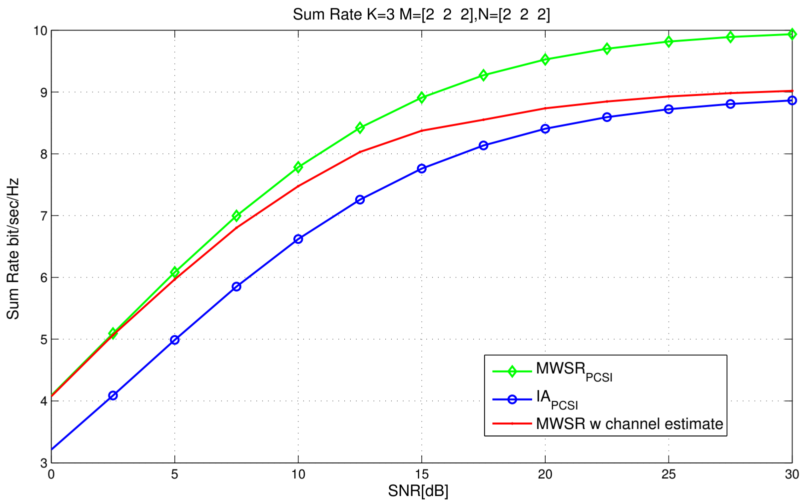

Figure 5.3: Sum Rate comparisons for $K = 3$ $M_k = N_k = 2,\ \forall k$

5.7 Conclusions

In this chapter we studied the maximization of the WSR in the case of partial CSIT for the Noisy MIMO IBC. The expectation of the WSR, w.r.t. the channel, is approximated with the expectation of the WMSE. This approximate solution gives an iterative algorithm based on alternating minimization between Tx, Rx filters and weighting matrices. The performances of the proposed algorithm, specified for a MIMO IFC, are compared with an IA solution calculated using the same partial CSIT. As we were expecting maximizing the WSR outperforms the IA solution also for the partial CSIT case. On the other hand using partial channel knowledge causes a loss in term of SNR offset but not in term of slope. So we can conclude that the proposed algorithm achieves, with partial CSIT, the same DoF of IA with perfect CSIT if the CSI quality increases with the SNR.

Chapter 6

CSI Acquisition in the MIMO Interference Channel via Analog Feedback

6.1 Introduction

With the seminal work [28] the authors have shown that interference alignment overcomes the conventional approach of orthogonalizing the resource blocks. In particular IA maximizes the degrees of freedom in a K user interference channel achieving, under some conditions, half the performance of a interference free system. In [33] and in Chapter 4 has been shown that even though IA promises maximum DoF in a MIMO IFC, it remains suboptimal at finite SNRs. To maximize the system performances, a more appropriate approach at finite SNR regimes, is to maximize the weighted sum rate. This achieves a significant improvement, in term of sum rate, compared to IA in medium to low SNR regimes. Both approaches described before, WSR maximization and IA design, require that all devices have perfect and also global channel state information (CSI). This assumption cannot always be satisfied in practical time-varying channels. For this reason different studies have been conducted for more practical situations.

The problem of CSI acquisition has been raised since the discovery that in single user MIMO systems having channel knowledge at the receive side allows to achieve very high data rates [23]. However achieving the required channel knowledge means that some part of the coherence time, the time period where the properties of the channel can be assumed constant, is used for the transmission of training symbols. This causes the consequent reduction of time left for useful data transmission. In [141] the question on how much coherence time should be used for training for a single user MIMO link is studied when only channel state information at the receiver (CSIR) is required. In [142] the capacity of a MIMO channel with M, N transmitting and receiving antennas respectively is studied when accurate estimation of the coefficients is generally not available to either the transmitter or the receiver due to fast channel variations. They computed the asymptotic capacity at high SNR in terms of the coherence time T and the number of transmit and receive antennas, showing that the optimum DoF should satisfy $d^* = \min\left\{M, N, \left\lfloor\frac{T}{2}\right\rfloor\right\}$.

[143] extended the results obtained in [141] to study the dirty paper coding (DPC) capacity region of a MIMO BC channel. The CSI acquisition scheme is based on downlink training and perfect channel feedback. A similar problem is studied in [144, 145]. There the analysis is based assuming a ZF beamformer instead of the more complex DPC. This approach leads to a rate lower bound that is optimized as function of the CSI acquisition overhead. In addition also imperfect feedback transmission is accounted in the CSI acquisition optimization. In [145] two feedback strategies are studied: digital or quantized and analog channel feedback. The authors also introduced a one-step prediction channel model with feedback delay and characterize the trade off between uplink and downlink spectral efficiencies. A similar setting of [145] is studied in [140] where the authors, to study the CSI acquisition optimization problem, introduce a new rate lower bound of BC channel when ZF beamforming is used.

The problem of feedback in interference channel has been recently studied in several papers [146, 147, 148]. Kramer in [146] studied a Gaussian interference network where a noiseless and high-rate feedback link exists and informs each transmitter of the outputs of the receiver to which it is communicating to. It is shown that for an interference channel feedback increases capacity in the case of strong interference. In [147] the interference channel with noisy feedback is studied. The authors introduced a new capacity outer bound that reveals that the noisy feedback loses its usefulness when the variance of the noise on the feedback link is larger than the noise on the forward channel. [149] focused on the two users interference channel with generalized feedback. As generalized feedback the author described the information that is gathered at each transmitter over the wireless media without any dedicated link where receivers feed back information to the transmitters. A coding strategy is proposed that allows to achieve higher data rate compared to the case without feedback. The two users Gaussian interference channel with feedback has been recently studied in [148]. The author derived achievable schemes, based on combining different coding techniques, and new capacity outer bounds that describe the capacity region for all values of channel parameters within 2 bits per user while the symmetric capacity is determined within 1 bit.

The problem of how to use feedback for the IA design, and how the corresponding rate is influenced by noisy feedback, has been treated in several recent papers [46, 47, 150, 48]. In [46] the authors consider the SISO IFC with frequency selective channels. Using quantized channel feedback they show that the full multiplexing gain can be achieved if the feedback bit rate scales sufficiently fast with the SNR. This result is extended in [47] to the MISO and MIMO IFC. In both papers the authors consider the less practical assumption that the feedback link is represented by an error-free communication. In [150] the authors study the design of the feedback link structure, called in the paper *feedback topology*, in a MIMO IFC when IA is used as transmission strategy. Using a closed form expression for IA beamformer, based on the result proposed in [31], the authors proposed two types of topologies with the objective of reducing feedback overhead. The impact of limited feedback on the system performance has also been studied. The authors of [48] proposed to use analog feedback for the acquisition of full CSIT. The channel coefficients are directly fed back to the base stations without any quantization process. This has the advantage, in contrast to digital feedback, that the complexity does not increase with SNR. In [48] CSIT processing and transmitter computation is centralized, and CSIR issues are neglected. They show that using IA with the acquisition of CSIT using analog feedback incurs no loss of multiplexing gain if the feedback power scales with the SNR.

A different approach has been studied in [49] where the author showed for different selected multiuser communication scenarios that it is possible to align the interference when the transmitters do not know the channel coefficients but they only have information about the channel autocorrelation structure of different users. In [49] a staggered block fading channel model is the only assumption required to achieve IA. The resulting multiplexing gain is much lower however than for the case of full CSI. These techniques are now known by the terms *delayed CSIT* or *retrospective IA*. A new IA approach has been introduced in [151] based on space-time alignment that employs multi-slot transmission protocol that achieves $\frac{4}{3}$ DoF without any CSIT. In [50, 51] these results are extended introducing a new transmission protocol that takes advantage of delayed CSIT and imperfect current channel estimates. This new scheme, compare to the one in [151], exploits a combination of the space-time alignment, designed for fully outdated CSIT with the use of simple zero-forcing (ZF) precoders. Another key innovation is the retransmission of the quantized version of interference generated in the previous slots instead of analog retransmission. Finally in [52] the authors considered a two users MISO IFC with time-correlated channel where each transmitter has delayed CSIT and imperfect current CSIT, obtained from prediction. They derive the DoF region extending the results in [50] to the MISO IFC. The problem of characterizing the DoF region using delayed CSIT or output feedback has been the focus of recent research works. The DoF region of the two-user MIMO interference channel with delayed CSI is studied in [152]. It is shown that, depending on the antenna dimensions, the region with delayed CSIT can be bigger than that of no CSI. In [153] the DoF region of the two-user MIMO interference channel is studied in the presence of noiseless channel output feedback and with delayed CSIT. The authors proved that output feedback and delayed CSIT can enlarge the DoF region when compared to the case in which only delayed CSIT is used. This is in contrast with the MIMO BC channel where output feedback and delayed CSIT does not increase the DoF region. The reason behind that is the fact the output feedback gives some side information to each transmitter about the signal of the concurrent transmitter. In the MIMO BC the transmitted signals come from the same transmitter then it has full knowledge of all messages. [154] investigated the two user MIMO IFC in fast fading channel under different feedback settings, such as Shannon feedback, limited Shannon feedback, and output feedback, wherein all or certain form of CSI is known at the transmitter with a finite delay. They showed that the DoF regions with Shannon and the limited Shannon feedback, are strictly bigger than the DoF region with just delayed CSIT under some conditions on the antennas distributions. These results are developed introducing a new form of retrospective interference alignment scheme that exploits transmitter cooperation made possible by output feedback in addition to delayed CSIT. This introduces a more efficient form of interference alignment than previously known schemes that use just delayed CSIT.

6.2 Contributions

In this chapter we introduce two transmission protocols for CSI acquisition at the BS and MU that are based on channel training and analog feedback (FB), for both TDD and FDD communication systems. The main difference between the two approaches is in the FB part: channel FB or output FB. In the channel FB solution, described also in [48] and [155], each MU feeds back to the BS the downlink channel estimates while in the output FB scheme, the MU feeds back directly the received samples of the DL training phase. In FDD communications uplink and downlink transmissions can take place at the same time. Hence with output FB, it is possible to shrink the time overhead, reducing partially the silent periods. At the end of this chapter we consider optimizing the sum rate, by focusing in particular on the resulting net degrees of freedom, as a function of the coherence time. This approach allows us to easily optimize any set of parameters to unveil the trade-off between the cost and the gains associated to CSI acquisition overhead. In particular we show that if the coherence time is shorter than a certain threshold transmitting with full DoF is no longer optimal.

6.3 Transmission Phases

We assume a block fading model, in which the channel is assumed to be constant over T channel uses, called *coherence time*. This time period T will need to be shared between the different training T_{ovrhd} and data transmission phases $T_{data} = T - T_{ovrhd}$ of the overall transmission scheme. In this section we describe all the necessary transmission phases required to set up a communication in the MIMO IFC where the beamformers and receiver filters are designed to maximize the weighted sum rate. The protocol discussed here, and also independently studied in [48], constitutes of some training (UL and DL) phases and a feedback part where each terminal disseminates the channel state information acquired through training. In this part we will focus our attention on a FB strategy where the channel coefficients are directly fed back as unquantized modulation symbols. This is usually referred to as analog transmission. This protocol can be used for both TDD and FDD communication systems but here we focus on FDD, this approach will be justified later on in the chapter.

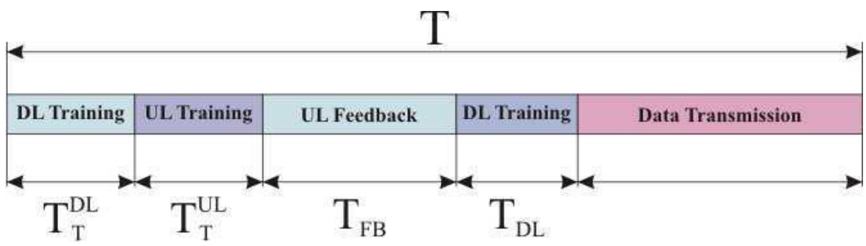

Figure 6.1: MIMO Uplink Interference Channel

6.3.1 Downlink Training Phase

During this phase each BS_k sends orthogonal pilot sequences that can be received by all the MU for a total duration of T_T^{DL}. In this way MU_i can easily estimate the DL channels $\mathbf{H}_i = [\mathbf{H}_{i1}, \ldots, \mathbf{H}_{iK}]$ directly connected to it. Because the compound channel matrix $\mathbf{H}_i$ has dimensions $N_i \times \sum_k M_k$ the minimum total duration of this training phase is

$$T_T^{DL} \geq \sum_{k=1}^{K} M_k.$$

Each BS independently transmits an orthogonal matrix $\mathbf{\Psi}_k$ of dimension $M_k \times T_T^{DL}$ with power P_T^{DL} hence the total received $N_i \times T_T^{DL}$ matrix at Rx i is:

$$\mathbf{Y}_i = \sum_{k=1} \sqrt{P_T^{DL}} \mathbf{H}_{ik} \mathbf{\Psi}_k + \mathbf{V} \tag{6.1}$$

where $\mathbf{V}$ represents the zero mean additive white Gaussian noise with variance σ_v^2. The DL Tx power can be related to the time duration of the corresponding Tx phase as

$$P_T^{DL} = \frac{T_T^{DL}}{M_k} \overline{P}_T^{DL}. \tag{6.2}$$

where $\overline{P}_T^{DL}$ represents the DL power constraint. Using an MMSE estimate on $\mathbf{Y}_i \mathbf{\Psi}_l$ each DL channel can be written as $\mathbf{H}_i = \widehat{\mathbf{H}}_i + \widetilde{\mathbf{H}}_i$ where:

$$\widehat{\mathbf{H}}_i \sim \mathcal{N}\left(0, \frac{P_T^{DL}}{\sigma_v^2 + P_T^{DL}} \mathbf{I}\right), \quad \widetilde{\mathbf{H}}_i \sim \mathcal{N}\left(0, \frac{\sigma_v^2}{\sigma_v^2 + P_T^{DL}} \mathbf{I}\right) \tag{6.3}$$

we call $\sigma^2_{\widehat{\mathbf{H}}_i}$ and $\sigma^2_{\widetilde{\mathbf{H}}_i}$ the variance of the channel estimate and channel estimation error respectively.

6.3.2 Uplink Training Phase

This phase can be seen as the dual of the DL training where now each MU sends orthogonal pilots to all BSs for the estimation of the UL channel matrices. The time duration of this phase should satisfy the following:

$$T_T^{UL} \geq \sum_{k=1}^{K} N_k.$$

Then BS_k can estimate the compound channel matrix $\overline{\mathbf{H}}_i = [\overline{\mathbf{H}}_{i1}, \ldots, \overline{\mathbf{H}}_{iK}]$ using an MMSE estimator as described for the DL training phase. Each UL channel can be represented in terms of channel estimate and channel estimation error with variance respectively $\sigma^2_{\widehat{\overline{\mathbf{H}}}}$ and $\sigma^2_{\widetilde{\overline{\mathbf{H}}}}$:

$$\widehat{\overline{\mathbf{H}}}_i \sim \mathcal{N}\left(0, \frac{P_T^{UL}}{\sigma_v^2 + P_T^{UL}} \mathbf{I}\right), \quad \widetilde{\overline{\mathbf{H}}}_i \sim \mathcal{N}\left(0, \frac{\sigma_v^2}{\sigma_v^2 + P_T^{UL}} \mathbf{I}\right). \tag{6.4}$$

The UL training power is now defined as:

$$P_T^{UL} = \frac{T_T^{UL}}{N_k} \overline{P}_T^{UL}. \tag{6.5}$$

where $\overline{P}_T^{UL}$ represents the UL power constraint. We are describing all the transmission phases for the FDD transmission scheme, hence different frequency bands are used for UL and DL communications. This separation implies that transmission and reception can take place at the same time. If we take advantage of this possibility the two training phases, UL and DL, can collapse in only one training slot that has duration $T_T = \max\{T_T^{DL}, T_T^{UL}\}$. Accounting for this new training phase implies a reduction of the total overhead T_{ovrhd}.

6.3.3 Uplink Feedback Phase

Once the UL and DL training phases are completed, each terminal knows the channel directly connected to it in the UL and DL respectively. In order to compute the IA BF matrices full DL CSI is required. In FDD case, the one under investigation, each MU has to feedback the DL channel estimate (CFB) $\widehat{\mathbf{H}}_i$ to all BS, this task can be done using analog transmission. This particular transmission phase should be designed according to the particular type of processing used for the computation of the BF matrices. We can describe two approaches: centralized and distributed. In the former a central controller acquires the necessary CSI, computes the BFs and then disseminates this information among the K BSs. In the latter approach each BS should have full CSI to compute its own beamformer. This solution can be also called *Duplicated* because each BS essentially solves the same problem and find the complete solution, the beamfomers for all users, and then it will use only its own BF.

Centralized Processing

The Rx signal vector at each BS is sent to the centralized controller that retrieves the useful channel information and computes the BF matrices. If we stack all the received vectors, from the K BSs, in $\overline{\mathbf{Y}}$ we get:

$$\overline{\mathbf{Y}} = P_{FB}^{\frac{1}{2}} \underbrace{\begin{bmatrix} \overline{\mathbf{H}}_{11} & \dots & \overline{\mathbf{H}}_{1K} \\ \vdots & \ddots & \vdots \\ \overline{\mathbf{H}}_{K1} & \dots & \overline{\mathbf{H}}_{KK} \end{bmatrix}}_{M\times N} \underbrace{\begin{bmatrix} \widehat{\mathbf{H}}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \widehat{\mathbf{H}}_2 & \dots & \mathbf{0} \\ \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \dots & \widehat{\mathbf{H}}_K \end{bmatrix}}_{N\times KM} \underbrace{\begin{bmatrix} \mathbf{\Phi}_1 \\ \vdots \\ \mathbf{\Phi}_K \end{bmatrix}}_{KM\times T_{FB}} + \underbrace{\begin{bmatrix} \mathbf{V}_1 \\ \vdots \\ \mathbf{V}_K \end{bmatrix}}_{\overline{\mathbf{V}}}$$

where $N = \sum_i N_i$ and $M = \sum_i M_i$ and

$$P_{FB} = \overline{P}_{FB} \frac{T_{FB}}{N_i} \tag{6.6}$$

with $\overline{P}_{FB}$ is the feedback power constraint. Using a centralized controller to gather all Rx data the entire system can be interpreted as a unique MIMO MAC link with a BS that is equipped with M total antennas. With this interpretation we can calculate the total amount of time necessary to satisfy the identifiability conditions. In particular we get:

$$T_{FB} \geq \frac{N \times M}{\min\{N, M\}} = \max\{N, M\} \propto K. \tag{6.7}$$

To extract the $i-$th CFB contribution we pre-multiply the received matrix $\overline{\mathbf{Y}}$ by the i-th orthonormal matrix $\mathbf{\Phi}_i^H$:

$$\overline{\mathbf{Y}}\mathbf{\Phi}_i^H = \sqrt{P_{FB}} \underbrace{\begin{bmatrix} \overline{\mathbf{H}}_{i1} \\ \vdots \\ \overline{\mathbf{H}}_{iK} \end{bmatrix}}_{\overline{\mathbf{H}}_i} \widehat{\mathbf{H}}_i + \overline{\mathbf{V}}\mathbf{\Phi}_i^H$$

then we perform a least square (LS) estimate based on the UL channel estimates $\widehat{\overline{\mathbf{H}}}_{ik}$: $\overline{\mathbf{H}}_i^{LS} = P_{FB}^{-\frac{1}{2}}(\widehat{\overline{\mathbf{H}}}_i^H \widehat{\overline{\mathbf{H}}}_i)^{-1}\widehat{\overline{\mathbf{H}}}_i^H$. Using this estimator we obtain the following CFB estimates:

$$\begin{aligned} \widehat{\widehat{\mathbf{H}}}_i &= \overline{\mathbf{H}}_i^{LS}\overline{\mathbf{Y}}\mathbf{\Phi}_i^H = \widehat{\mathbf{H}}_i + P_{FB}^{\frac{1}{2}}\overline{\mathbf{H}}_i^{LS}\widetilde{\overline{\mathbf{H}}}_i\widehat{\mathbf{H}}_i + \overline{\mathbf{H}}_i^{LS}\overline{\mathbf{V}}\mathbf{\Phi}_i^H \\ &= \mathbf{H}_i - \widetilde{\mathbf{H}}_i + P_{FB}^{\frac{1}{2}}\overline{\mathbf{H}}_i^{LS}\widetilde{\overline{\mathbf{H}}}_i\widehat{\mathbf{H}}_i + \overline{\mathbf{H}}_i^{LS}\overline{\mathbf{V}}\mathbf{\Phi}_i^H = \mathbf{H}_i - \widetilde{\widehat{\mathbf{H}}}_i \end{aligned}$$

The CFB estimate can be written as a function of the true DL channel and the CFB estimation error: $\widehat{\widehat{\mathbf{H}}}_i = \mathbf{H}_i - \widetilde{\widehat{\mathbf{H}}}_i$. The error contribution is due to the DL and UL channel estimation errors $(\widetilde{\mathbf{H}}_i, \widetilde{\overline{\mathbf{H}}}_i)$ in the DL and UL training phases respectively. The CFB estimation error $\widetilde{\widehat{\mathbf{H}}}_i$ is distributed as $\mathcal{N}(0, \sigma^2_{\widetilde{\widehat{\mathbf{H}}}_i}\mathbf{I})$ where

$$\mathrm{Cov}(\widetilde{\widehat{\mathbf{H}}}_i|\widehat{\overline{\mathbf{H}}}_i) = \sigma^2_{\widetilde{\mathbf{H}}_i}\mathbf{I} + [(\sigma^2_{\widehat{\mathbf{H}}_i}\sigma^2_{\widetilde{\overline{\mathbf{H}}}_i}) + \frac{\sigma^2}{P_{FB}}](\widehat{\overline{\mathbf{H}}}_i^H \widehat{\overline{\mathbf{H}}}_i)^{-1}.$$

Assuming that $(\widehat{\overline{\mathbf{H}}}_i^H \widehat{\overline{\mathbf{H}}}_i)^{-1}$ is distributed as an inverse Wishart matrix, then $\mathbb{E}\{(\widehat{\overline{\mathbf{H}}}_i^H \widehat{\overline{\mathbf{H}}}_i)^{-1}\} \propto \frac{1}{M-N_i}\mathbf{I}$. So we can write the covariance matrix as $\mathrm{Cov}(\widetilde{\widehat{\mathbf{H}}}_i|\widehat{\overline{\mathbf{H}}}_i) = \sigma^2_{\widetilde{\widehat{\mathbf{H}}}_i}\mathbf{I}$ where:

$$\sigma^2_{\widetilde{\widehat{\mathbf{H}}}_i} = \sigma^2_{\widetilde{\mathbf{H}}_i} + \frac{1}{M - N_i}[(\sigma^2_{\widehat{\mathbf{H}}_i}\sigma^2_{\widetilde{\overline{\mathbf{H}}}_i}) + \frac{\sigma^2}{P_{FB}}]$$

Distributed Processing

In this case the CFB transmission is organized in such a way that each BS can gather full channel knowledge from all MU. The Rx matrix at BS_k can be written as:

$$\overline{\mathbf{Y}}_k = \sqrt{P_{FB}}\underbrace{\left[\overline{\mathbf{H}}_{k1}\ldots\overline{\mathbf{H}}_{kK}\right]}_{M_k\times N}\underbrace{\begin{bmatrix}\widehat{\mathbf{H}}_1 & \mathbf{0} & \ldots & \mathbf{0}\\ \mathbf{0} & \widehat{\mathbf{H}}_2 & \ldots & \mathbf{0}\\ \vdots & & \ddots & \mathbf{0}\\ \mathbf{0} & \ldots & \ldots & \widehat{\mathbf{H}}_K\end{bmatrix}}_{N\times KM}\underbrace{\begin{bmatrix}\mathbf{\Phi}_1\\ \vdots\\ \mathbf{\Phi}_K\end{bmatrix}}_{KM\times T_{FB}} + \mathbf{V}_k$$

where

$$P_{FB} = \overline{P}_{FB}\frac{T_{FB}}{N_i} \tag{6.8}$$

with $\overline{P}_{FB}$ is the feedback power constraint. In the distributed approach to satisfy the identifiability conditions the CFB length should be:

$$T_{FB} \geq \frac{N\times M}{\min_i\{M_i, N_i\}} \propto K^2 \tag{6.9}$$

To extract the $i-$th CFB contribution we pre-multiply the received matrix $\overline{\mathbf{Y}}_k$ by the i-th orthonormal matrix $\mathbf{\Phi}_i^H$:

$$\overline{\mathbf{Y}}_k\mathbf{\Phi}_i^H = \sqrt{P_{FB}}\overline{\mathbf{H}}_{ki}\widehat{\mathbf{H}}_i + \mathbf{V}_k\mathbf{\Phi}_i^H.$$

Also in this case we use a LS estimator, based on the UL channel estimate $\widehat{\overline{\mathbf{H}}}_{ki}$, $\overline{\mathbf{H}}_{ki}^{LS} = P_{FB}^{-\frac{1}{2}}(\widehat{\overline{\mathbf{H}}}_{ki}^H\widehat{\overline{\mathbf{H}}}_{ki})^{-1}\widehat{\overline{\mathbf{H}}}_{ki}^H$. The CFB estimate can be written as function of the true DL channel and the CFB estimation error: $\widehat{\widehat{\mathbf{H}}}_i = \mathbf{H}_i - \widetilde{\widetilde{\mathbf{H}}}_i$. The error contribution is due to the DL and UL channel estimation errors ($\widetilde{\mathbf{H}}_i, \widetilde{\overline{\mathbf{H}}}_{ki}$) in the DL and UL training phases respectively:

$$\begin{aligned}\widehat{\widehat{\mathbf{H}}}_i &= \overline{\mathbf{H}}_{ki}^{LS}\overline{\mathbf{Y}}_k\mathbf{\Phi}_i^H = \widehat{\mathbf{H}}_i + P_{FB}^{\frac{1}{2}}\overline{\mathbf{H}}_{ki}^{LS}\widetilde{\overline{\mathbf{H}}}_{ki}\widehat{\mathbf{H}}_i + \overline{\mathbf{H}}_{ki}^{LS}\mathbf{V}_k\mathbf{\Phi}_i^H\\ &= \mathbf{H}_i - \widetilde{\mathbf{H}}_i + P_{FB}^{\frac{1}{2}}\overline{\mathbf{H}}_{ki}^{LS}\widetilde{\overline{\mathbf{H}}}_{ki}\widehat{\mathbf{H}}_i + \overline{\mathbf{H}}_{ki}^{LS}\mathbf{V}_k\mathbf{\Phi}_i^H = \mathbf{H}_i - \widetilde{\widetilde{\mathbf{H}}}_i\end{aligned}$$

where the estimation error is then distributed as $\mathcal{N}(0, \sigma^2_{\widetilde{\widetilde{\mathbf{H}}}_i}\mathbf{I})$, with

$$\mathrm{Cov}(\widetilde{\widetilde{\mathbf{H}}}_i|\widehat{\overline{\mathbf{H}}}_{ki}) = \sigma^2_{\widetilde{\mathbf{H}}_i}\mathbf{I} + [(\sigma^2_{\widehat{\mathbf{H}}_i}\sigma^2_{\widetilde{\overline{\mathbf{H}}}_{ki}}) + \frac{\sigma^2}{P_{FB}}](\widehat{\overline{\mathbf{H}}}_{ki}^H\widehat{\overline{\mathbf{H}}}_{ki})^{-1}$$

Assuming that $\mathbb{E}\{(\widehat{\overline{\mathbf{H}}}_{ki}^H\widehat{\overline{\mathbf{H}}}_{ki})^{-1}\} \propto \frac{1}{M_k-N_i}$, we can write the covariance matrix as $\mathrm{Cov}(\widetilde{\widetilde{\mathbf{H}}}_i|\widehat{\overline{\mathbf{H}}}_{ki}) = \sigma^2_{\widetilde{\widetilde{\mathbf{H}}}_i}\mathbf{I}$ where:

$$\sigma^2_{\widetilde{\widetilde{\mathbf{H}}}_i} = \sigma^2_{\widetilde{\mathbf{H}}_i} + \frac{1}{M_k - N_i}[(\sigma^2_{\widehat{\mathbf{H}}_i}\sigma^2_{\widetilde{\overline{\mathbf{H}}}_{ki}}) + \frac{\sigma^2}{P_{FB}}] \tag{6.10}$$

Another possible strategy to receive the analog feedback is to use linear MMSE estimate instead of the least square approach described in this section. The two solutions will be identical at high SNR but in different SNR regimes LMMSE should give better performances.

The analog FB transmission described here is based on the assumption that the number of Tx and Rx antennas satisfy the relation that $\min\{M_i\} \geq N_j, \forall j$. This is due to least squared estimated used in the FB reception. As we can see from the variance of the channel feedback estimation error in (6.10) the second contribution explodes if N_i approaches M_k. To avoid this problem, then a different transmission scheme should be applied. In particular each MU should apply a precoding matrix such that the identifiability conditions are satisfied at all BS, this requires a more careful precoding design. A possible design criterion could be to optimize the performance of the worst FB link. This solution can be also used to introduced more redundancy in the transmission that can increase the performances of the FB reception. A simple approach could be to use a Kronecker model precoder at each MU of the form:

$$\mathbf{T}_k = \mathbf{S}_k^{T_{FB} \times M \frac{N_k}{s_k}} \otimes \mathbf{B}_k^{N_k \times s_k}$$

where $\mathbf{S}_k$ and $\mathbf{B}_k$ are optimized according to the channel conditions and s_k represents the number of transmitted streams such that the identifiability conditions are satisfied at all BSs. With this model the compound channel matrix from MU_k to BS_i can be written as

$$\mathbf{G}_{ik}^{T_{FB} M_i \times M N_k} = (\mathbf{I}_{T_{FB}} \otimes \overline{\mathbf{H}}_{ik})\mathbf{T}_k = \mathbf{S}_k \otimes \overline{\mathbf{H}}_{ik}\mathbf{B}_k$$

then the equivalent channel matrix is designed for the transmission of the total number of FB $\mathbf{h}_k^{MN_k \times 1} = \mathbf{vec}\{\widehat{\mathbf{H}}_k\}$.

6.3.4 Downlink Training Phase

Once the beamformers have been computed, using a centralized or distributed approach, they can be used for the DL communications. To optimize the system performances a receive filter should be applied at each receiver. If IA is used then ZF receiver are enough, otherwise more optimal MMSE receiver are to be calculated. To compute the Rx filters each MU requires some additional information on the DL communication. On this purpose two approaches are possible: DL training or analog transmission of the entire Rx filters. In the former case BS_k sends a set of beamformed pilots that allows MU_i to estimate the cascade $\mathbf{H}_{ik}\mathbf{G}_k$. This phase lasts

$$T_{DL} \geq \sum_k d_k.$$

Then each MU can estimate the interference subspace and the signal subspace for the Rx filter design. Since the BF computation gives, in many algorithms, also the optimal Rx filter as sub-product, the other approach consists in the direct transmission to the i-th MU of the entire Rx filter matrix $\mathbf{F}_i$ using analog transmission. This solution requires a transmission duration

$$T_{DL} \geq \sum_k \frac{N_k d_k}{\min\{N_k, M_k\}}$$

The two solution proposed here are not equivalent. Which solution should be preferred depends also on the operating SNR point. For example in high SNR, where we are interested more in maximizing the total degrees of freedom the duration of this phase has a bigger impact compared to the estimation error, if the feed-forward power also scales with the SNR, then DL training is the preferable solution.

In the following we consider the approach based on training. Using a sequence of orthogonal pilots ϕ_{km} for stream (k,m) of length $1 \times T_{DL}$, the Rx signal at MU k is:

$$\mathbf{Y}_{km} = \sqrt{P_T}\mathbf{H}_{kk}\widehat{\mathbf{g}}_{km}\boldsymbol{\phi}_{km} + \sum_{(in)\neq(km)}\sqrt{P_T}\mathbf{H}_{ki}\widehat{\mathbf{g}}_{in}\boldsymbol{\phi}_{in} + \mathbf{V}_{km}$$

where the two $N_k \times T_{DL}$ matrices $\mathbf{Y}_{km} = [\mathbf{y}_{km}[1], \ldots, \mathbf{y}_{km}[T_{DL}]]$ and $\mathbf{V}_{km} = [\mathbf{v}_{km}[1], \ldots, \mathbf{v}_{km}[T_{DL}]]$ represent the signal and noise contributions. The least square estimate of the cascade channel-BF is given as:

$$\widehat{\mathbf{H}_{kl}\widehat{\mathbf{g}}_{lt}} = \frac{1}{T_{DL}\sqrt{P_T}}\mathbf{Y}_{km}\boldsymbol{\phi}_{lt}^H = \mathbf{H}_{kl}\widehat{\mathbf{g}}_{lt} + \underbrace{\frac{1}{T_{DL}\sqrt{P_T}}\mathbf{V}_{km}\boldsymbol{\phi}_{lt}^H}_{\widetilde{\mathbf{H}_{kl}\widehat{\mathbf{g}}_{lt}}} \tag{6.11}$$

the elements of the estimation error matrix are distributed according to $\mathcal{N}(0, \sigma^2_{\widetilde{\mathbf{H}_{kl}\widehat{\mathbf{g}}_{lt}}}\mathbf{I})$, where $\sigma^2_{\widetilde{\mathbf{H}_{kl}\widehat{\mathbf{g}}_{lt}}} = \frac{\sigma^2}{T_{DL}P_T}$. Using channel estimate (6.11) we can build the MMSE Rx filter as:

$$\widehat{\widehat{\mathbf{f}}}_{km}^H = P_T\left(\widehat{\mathbf{H}_{kk}\widehat{\mathbf{g}}_{km}}\right)^H\left[\sum_{in}\widehat{\mathbf{H}_{ki}\widehat{\mathbf{g}}_{in}}P_T\left(\widehat{\mathbf{H}_{ki}\widehat{\mathbf{g}}_{in}}\right)^H + \sigma^2\mathbf{I}\right]^{-1} \tag{6.12}$$

We can further develop (6.11) in order to underline the dependence of the DL channel estimate at the BS_l obtained using CFB in section 6.3.3:

$$\widehat{\mathbf{H}_{kl}\widehat{\mathbf{g}}_{lt}} = \widehat{\widehat{\mathbf{H}}}_{kl}^{(l)}\widehat{\mathbf{g}}_{lt} + \widetilde{\widehat{\mathbf{H}}}_{kl}^{(l)}\widehat{\mathbf{g}}_{lt} + \widetilde{\mathbf{H}_{kl}\widehat{\mathbf{g}}_{lt}}. \tag{6.13}$$

$\widehat{\widehat{\mathbf{H}}}_{kl}^{(l)}$ represents the DL channel estimate calculated at BS_l used for the calculation of the BF vector $\widehat{\mathbf{g}}_{lt}$. Using the expression (6.13) and the first order approximation: $(\mathbf{A}+\mathbf{\Delta A})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{\Delta A}\mathbf{A}^{-1}$ we can decompose the Rx filter (6.12) as:

$$\widehat{\widehat{\mathbf{f}}}_{km}^{(l)H} = P_T \widehat{\widehat{\mathbf{H}}}_{kk}^{(l)} \widehat{\mathbf{g}}_{km} \widehat{\mathbf{R}}_{\mathbf{yy}}^{(l)-1} + \widetilde{\widetilde{\mathbf{f}}}_{km}^{(l)H} = \widehat{\mathbf{f}}_{km}^{(l)H} + \widetilde{\widetilde{\mathbf{f}}}_{km}^{(l)H} \tag{6.14}$$

$\widehat{\mathbf{f}}_{km}^{(l)}$ corresponds to the MMSE Rx filter calculated using the DL channel estimated at BS_l. It is the same MMSE Rx filter that would have been calculated at BS_l as a sub-product of the iterative algorithm used for calculating the IA BF. Then $\widehat{\mathbf{f}}_{km}^{(l)} \widehat{\widehat{\mathbf{H}}}_{kl}^{(l)} \widehat{\mathbf{g}}_{li} = 0$ at high SNR. $\widetilde{\widetilde{\mathbf{f}}}_{km}^{(l)H}$ contains all the error contributions of (6.12) up to first order.

6.4 Output Feedback

In the previous sections we have described the transmission protocol where the necessary channel state information at each BS is acquired using analog transmission of the DL channel estimates obtained at each MU (CFB). A different approach consists to FB directly to BSs the received signal at each MU during the DL training phase instead of the DL channel estimates. This technique is called output FB (OFB). Then, once each BS accumulates enough FB samples, it estimates directly the required DL channels. The advantage of this strategy, compared to the traditional channel FB, is that the FB phase can start one time instant after the reception of the first DL training samples. In FDD transmission schemes UL and DL communications can take place at the same time. Assuming the DL frame aligned with the end of the UL training phase, the difference between the two schemes can be pictorially represented as in Fig. 6.2. At time t the received signal at MU_k during the DL training phase is

$$\mathbf{y}_k[t] = \sum_{i=1}^{K} \mathbf{H}_{ki} \boldsymbol{\psi}_i[t] + \mathbf{n}_k[t]. \tag{6.15}$$

In the next time instant [t+1] MU_k transmits back to all BSs the Rx signal at time instant [t]. So BS number l receives:

$$\begin{aligned}
\overline{\mathbf{y}}_l[t+1] &= \sum_{j=1}^{K} \overline{\mathbf{H}}_{lj} \overline{\mathbf{x}}_j[t+1] + \overline{\mathbf{n}}_l[t+1] \\
&= \sum_{j=1}^{K} \overline{\mathbf{H}}_{lj} \alpha_j \left[\sum_{i=1}^{K} \mathbf{H}_{ji} \boldsymbol{\psi}_i[t] + \mathbf{n}_k[t] \right] + \overline{\mathbf{n}}_l[t+1]
\end{aligned}$$

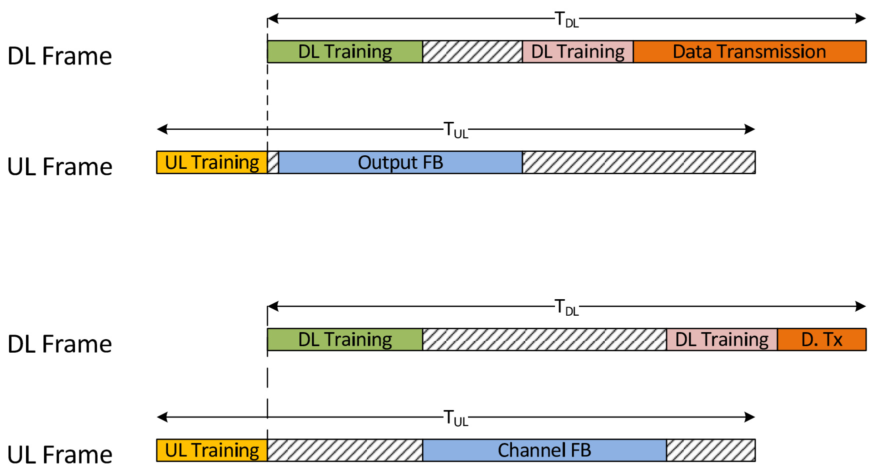

Figure 6.2: Output Feedback and Channel Feedback

where α_j denotes a scaling factor that takes into account the Tx power constraint at j-th MU. In order to being able to separate the different contributions, coming from different MUs, we assume to use time multiplexing. Each BS has to estimate all the matrices $\mathbf{H}_i = [\mathbf{H}_{i1}, \ldots, \mathbf{H}_{iK}]^{N_i \times M}$. To estimate this many coefficients the required total length of the output FB phase is:

$$T_{FB}^{o} \geq \frac{N \times M}{\min_i\{N_i, M_i\}} \tag{6.16}$$

Comparing equation (6.16) with (8.6) we can see that there is no reduction in the length of the FB phase using OFB comparing to traditional channel FB. The reduction of the overhead comes from partial elimination of silent periods, as shown in Fig.6.2. The DL overhead time due to CSI acquisition for the case of OFB can be quantified as:

$$T_{ovrhd}^{DL} = (T_{FB}^{o} + 1) + T_{DL}$$

while for CFB we have:

$$T_{ovrhd}^{DL} = T_{T}^{DL} + T_{FB}^{o} + T_{DL}.$$

From the equations above we see that using OFB we save $T_T^{DL} - 1$ time instants.

We should also study the case where the UL and DL coherence periods are aligned, Fig. 6.3. In this situation the gain obtained using OFB instead of CFB is reduced. The total time overhead in the DL transmission becomes:

$$T_{ovrhd}^{DL} = T_{T}^{UL} + T_{FB}^{o} + T_{DL}.$$

The gain of using the OFB solution is now given as: $\Delta T = T_T^{DL} - T_T^{UL} = N - M$.

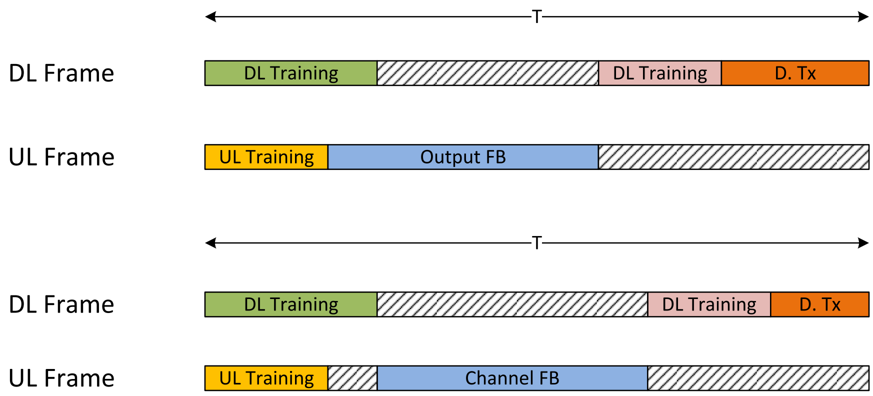

Figure 6.3: Output Feedback and Channel Feedback with aligned coherence periods

6.5 TDD Vs FDD transmission strategy

Usually TDD transmission is used arguing that thanks to reciprocity the amount of feedback required to acquire CSIT is (significantly) reduced. In this communication strategy MU_i does not need to feedback channel $\mathbf{H}_{ki}$ to BS_k because this information can be acquired using the corresponding UL channel $\overline{\mathbf{H}}_{ki}$. On the other hand this information is needed at the other base stations $BS_{j\neq k}$ for the design of their own BF matrix. From this observation we realized that for distributed BF process the organization of feedback is very complicated if we consider the possibility of reducing feedback using reciprocity, and hence we can conclude that TDD does not help in reducing the feedback overhead compared to FDD transmission scheme. On the contrary if we consider a centralized BF calculation then TDD makes feedback not required because the reduced set of CSI available at each BS using reciprocity is shared and hence the computation center can collect the total required information on the DL channels based on the UL channel estimates available at each BS. For the reasons above we developed all the transmission phases only for FDD transmission.

6.6 From Practical to more Optimal Solutions

All the different transmission phases described in section 6.3 are done one after the other but other solutions are possible to optimize the overhead. In a possible alternative approach, one does not need to wait to gather all CSIT before starting transmission. For example one user can start to transmit directly after the DL training phase as a single user MIMO link without any CSIT. Or it also is possible to start with blind/noncoherent IA first. Then, instead of going from $K = 1$ to full K immediately another possible strategy is to build intermediate IA solutions. We gradually increase the number of interfering links as soon as the corresponding transmitters acquire the required CSI to design the IA beamformers for the given interfering subsystem. Another consideration is that when the (analog) channel feedback duration is non-minimal, beamformers can be computed immediately after the minimum number of feedback coefficients has been obtained and DL transmission can start. Then the beamformers can get further updated during the remaining feedback transmission using better channel estimates. This is one advantage of analog feedback (similar to repetition coding), that ”decoding” can be done before the full ”codeword” has been received. In any case, there is a myriad of possibilities for alternative solutions, to increase the system performances using a more optimized transmission strategy.

6.7 DoF optimization as function of Coherence Time

In [48] the authors show that using analog feedback of channel estimates it is possible to achieve the same DoF as IA if the feedback power is scaled as the SNR. Here our goal is different, we want to optimize the number of transmitted streams as a function of coherence time. The rationale behind this optimization problem is the following. If the coherence time is not long enough to host the total overhead due to CSI acquisition then the transmission of $d_{tot} = \sum_k d_k$ is no longer possible regardless of the amount of power used for the feedback transmission phase. Then we should use blind IA or noncoherent transmission techniques. Another possibility is to reduce the total amount of transmitted streams. The reduction of d implies a reduction of the required number of active transmit and receive antennas as well as the number of transmitting users, so that the amount of CSI exchange is optimized as a function of the coherence time. In this section our objective is the optimization of system parameters, number of antennas and number of users, that allow us to maximize the *net DoF*, defined as the number of DoF that are actually achievable taking into account the overhead due to CSI acquisition. Our cost function then becomes:

$$\max_{n,m,k} J(n,m,k) = \max_{n,m,k} \left(1 - \frac{T_{ovrhd}(n,m,k)}{T}\right) kd(n,m,k)\log SNR \quad (6.17)$$

where $m \in [1, N],\ n \in [1, N],\ k \in [1, K]$ (to simplify we assumed an IFC of the form $(N, N, d)^K$) represent the number of active transmit and receive antennas respectively and number of active users. $T_{ovrhd} = T_T^{DL} + T_{FB} + T_{DL}$ takes into account the DL training phase, necessary for CSIT acquisition, UL feedback phase and beamformed training, sometimes also called dedicated training. To solve this problem we should be able to define a relationship between the number of transmitted streams and antennas. If we assume IA transmission we can relate antennas, users and achievable streams using the following [37]:

$$d \leq \frac{m+n}{k+1}. \quad (6.18)$$

Since our objective is the maximization of the total DoF in the network we assume that each user applies a transmit/receive technique that maximizes the achievable DoF, so transmit and receive filters are designed according to IA. This transmission strategy has the objective to maximize the achievable DoF reducing at maximum the interference subspace dimensions generated at each non intended receiver. The solution does not depend on the direct channel matrix but only on the interference matrix that each transmitter generates. This allows us to reduce partially the number of channel coefficients that need to be fed back by each MU to all BS. In particular each MU needs to feedback only the $(k-1)$ interfering channel matrices for a total of $(k-1)mn$ coefficients. In addition only the subspace spanned by the interference channel matrix is of relevance for the design of IA beamformers (that is defined up to a square $n \times n$ matrix). We can conclude that the total number of channel coefficients, that need to be fed back for IA beamforming, is :

$$N_c = kn[(k-1)m - n]_+. \quad (6.19)$$

this will have an influence on the feedback phase duration T_{FB}. According to recent results on IA feasibility [38] we know that relation (6.18) is exact only for square symmetric IFC of the form $(m, m, d)^k$ where each user is equipped with the same number of antennas m and transmits the same number of streams d. For this reason we focus on this kind of system to develop our analysis. Equation (6.18) becomes:

$$d \leq 2\frac{m}{k+1}. \quad (6.20)$$

Before studying a MIMO IFC it is also of interest to start the analysis with simpler and more intuitive cases: SIMO/MISO IFC. To study the MISO and SIMO case we assume a centralize processing of the feedback.

In a SIMO IFC only the MU is equipped with multi-antenna array then only CSIR is necessary to establish a communication. This simplifies the expression of the time overhead that becomes: $T_{ovrhd} = k$, since only one pilot per user is enough to estimate the DL channels at each MU. In addition each BS can send only one useful data stream, then the total number of DoF is $d_{tot} = min\{k, m\}$. The optimization function (6.17) then becomes (in the rest of the section we neglect the dependence on the SNR since it is not an optimization variable):

$$J(m, k) = [T - k] \min\{k, m\}$$

Two cases are then possible: $m \leq k$ or $m \geq k$.
$\mathbf{m \leq k}$
The cost function becomes:

$$J(m, k) = [T - k]m$$

this function is not convex (the Hessian is indefinite) then to find the optimum in the optimization domain we optimize the cost function over different borders of $[1, N] \times [1, K]$

- Border $\mathbf{m^* = 1}$
  Since the cost function $J(m, k)$ is linear in k with coefficient -1 then the optimum is in the left extremum of the domain, in this case $k^* = m^* = 1$.
- Border $\mathbf{m^* = N}$
  Similarly to the case above $k^* = \min\{N, K\}$.
- Border $\mathbf{k^* = 1}$
  Since the cost function $J(m, k)$ is linear in m with coefficient $(T - k^*) > 0$ then the optimum lies on the right extremum of the domain, in this case $m^* = k^* = 1$.
- Border $\mathbf{k^* = K}$
  Similarly to the case above $m^* = \min\{N, K\}$.

$\mathbf{m \geq k}$
The cost function can be written as:

$$J(m, k) = [T - k]k$$

that is independent of m then we can choose it minimum possible value: $m^* = k^*$. Now optimizing w.r.t. k we get

$$m^* = k^* = \min\{N, K, \frac{T}{2}\}. \tag{6.21}$$

From this result we can see that the optimum number of active antennas and users is equal and varies as $\frac{T}{2}$. This is an intuitive result because in SIMO IFC the number of DoF is equal to $min\{k,m\}$ then having an excess of antennas, compared to number of users, does not give any advantage in terms of DoF. Similarly, in the case of more user than antennas, there are users that cannot be served without interference, so an excess of users is only decremental in term of DoF.

The MISO IFC is characterized by a cost function that is different from the one of the SIMO case. In this system to set up a transmission CSIT is required at each BS, then also feedback from MUs is necessary. The time overhead, due to CSI acquisition is then composed of DL training and feedback, we can neglect the dedicated training phase. The cost function becomes:

$$J(m,k) = [T - (2k-1)m + 1] \min\{m,k\}$$

This cost function should be treated similarly to the SIMO case since it has an indefinite Hessian. Also in this case having the same number of Tx antennas and active users is the optimal solution. Equating the derivative of the cost function, assuming $m = k$, w.r.t. m we obtain the optimal solution of the form:

$$m^* = k^* = \min\{N, K, \frac{1}{6}(1 + \sqrt{6T+7})\}. \tag{6.22}$$

Also in this case the result is intuitive since similar considerations to the SIMO case can be stated about the number active users and Tx antennas. The different solution obtained in the MISO case is due to the different overhead necessary to set up a communication compared to the SIMO case, that is simpler so require less time to achieve the required CSIR to establish a communication.

Now we study the case of a MIMO IFC of the form $(m,m,d)^k$, that as we said before is the case where the relationship (6.18) between users, antennas and DoF is exact. We can study two possible cases, first we study the setting in which the feedback processing is done in a centralized way and then we focus on the distributed solution, proposed in section 6.3.3.

Centralized Square MIMO

In the centralized case the total length of the time overhead is:

$$T_{ovrhd} = T_T^{DL} + T_{FB} + T_{DL} = 2\frac{k^2+k-1}{k+1}m$$

where $T_{FB} = \frac{N_c}{K\min\{m,n\}}$. We can write (6.17) as follows for the case under investigation:

$$J(m,k) = \left(T - 2\frac{k^2+k-1}{k+1}m\right)\frac{2k}{k+1}m \tag{6.23}$$

this function is non convex hence we need to find the optimum over the borders of the optimization domain. In order to have a feeling on how the problem behaves as function of the coherence time what we propose is to study the cost function in the point (N, K). We want to understand when the full MIMO IFC is no longer optimal and hence some dimensions are shrunk if the coherence time T is shorten than a certain threshold. In particular we study the derivative of the cost function $J(m, k)$ w.r.t m and k separately evaluated at the point (N, K) and we find that:

$$\frac{\partial J(m,k)}{\partial m} = T - 4\frac{k^2+k-1}{k+1}m \bigg|_{\substack{m=N \\ k=K}} = T - 4\frac{K^2+K-1}{K+1}N \tag{6.24}$$

$$\begin{aligned} \tfrac{\partial J(m,k)}{\partial k} &= \tfrac{2m}{(k+1)^2}T - 4\tfrac{k^3+3k^2+3k-1}{(k+1)^3}m^2 \bigg|_{\substack{m=N \\ k=K}} \\ &= \tfrac{2N}{(K+1)^2}T - 4\tfrac{K^3+3K^2+3K-1}{(K+1)^3}N^2 \end{aligned} \tag{6.25}$$

now we study the sign of those derivatives as function of the coherence time T. In particular we study when, for example, $\frac{\partial J(m,k)}{\partial k}\Big|_{\substack{m=N \\ k=K}} \leq 0$. This means that the cost function evaluated at the point (N, K) is decreasing with k then if we use a $k_1 < K$ we have that $J(N, K) < J(N, k_1)$ so to maximize $J(m, k)$ we need to reduce the number of active users. Similar reasoning is valid for m. Solving this problem we find that

$$\begin{aligned} \tfrac{\partial J(m,k)}{\partial m} \leq 0 \quad \text{if} \quad T \leq T_m = 4N\tfrac{K^2+K-1}{K+1} \\ \tfrac{\partial J(m,k)}{\partial k} \leq 0 \quad \text{if} \quad T \leq T_k = 2N\tfrac{K^3+3K^2+3K-1}{(K+1)} \end{aligned} \tag{6.26}$$

It is easy to verify that $T_k > T_m \quad \forall K, N$, then the first quantity that decreases if the coherence time decreases is the number of users k. If we evaluate T_m, T_k for any arbitrary m, k we find that $T_k > T_m, \quad \forall k, m$. In addition the sign of the derivative of $J(m, k)$ w.r.t. m calculated in $(N, k = 1)$ is always positive for $T > 2N$, when it starts to becomes negative. We can conclude that the antennas and users distribution as function of coherence time behaves as follows:

1. $\mathbf{T > T_k}$
   Full (N, K) MIMO IFC is optimal, then $d_{tot} = \frac{2KN}{K+1}$

2. $\mathbf{T_k \leq T \leq 6N}$
   The number of user is not full, then (N, k^*) MIMO IFC is optimal where $k^* : k^3 + 3k^2 + (3 - \frac{T}{2N})k - (1 + \frac{T}{2N}) = 0$, the total DoF are $d_{tot} = \frac{2k^*(T)N}{k^*(T)+1}$

3. $\mathbf{6N \leq T \leq 2N}$
   SU-MIMO with N antennas is optimal so $d_{tot} = N$

4. $\mathbf{T \leq 2N}$
   Now also the number of antennas starts to shrink as $m^* = \min\{N, \frac{T}{2}\}$ that determines $d_{tot} = m^*$

What is remarkable in this analysis is the simple intuition that if the coherence time is too short SU-MIMO transmission is the best approach is indeed optimal. In addition, if we fix the number of transmit and receive antennas to be the same, as the coherence time start to reduce, it is optimal to reduce the number of users first up to the point SU-MIMO. The DoF behavior as function of the coherence time is schematically sketched in Fig. 6.4. If the coherence time is long enough then full DoF is achieved in the MIMO IFC, then reducing the coherence time implies that the number of user is reduced. This determines also a reduction of achievable DoF $d_{tot} = \frac{2k^*(T)N}{k^*(T)+1}$ until SU-MIMO is achieved. Finally if we further decrease T then also the number of active antennas starts to decrease, then the achieved DoF is given by the SU-MIMO formula as $d_{tot} = \min\{N, \frac{T}{2}\}$

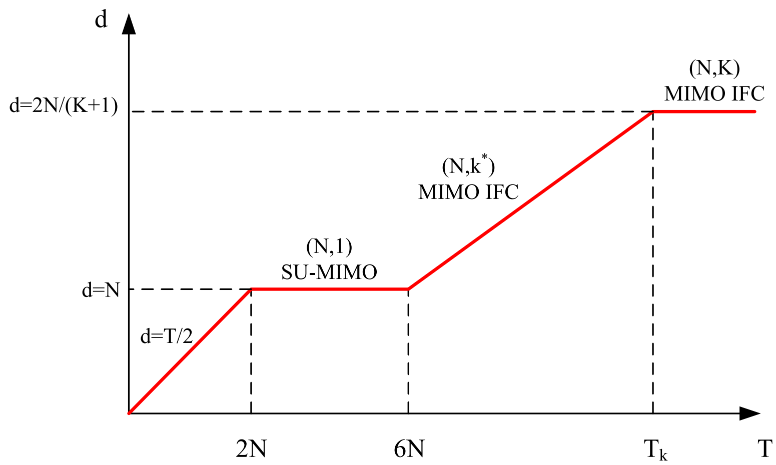

Figure 6.4: Behavior of the optimized DoF distribution for square symmetric MIMO IFC

Distributed Square MIMO

In the distributed case the total length of the time overhead is:

$$T_{ovrhd} = T_T^{DL} + T_{FB} + T_{DL} = \frac{k^2+1}{k+1} km$$

where $T_{FB} = \frac{N_c}{\min\{m,n\}}$. The cost function that we need to optimize becomes:

$$J(m,k) = \left(T - \frac{k^2+1}{k+1}km\right)\frac{2k}{k+1}m \tag{6.27}$$

As for the centralized case this cost function is non convex. To study the behavior of the antennas and users distribution as function of the coherence time we proceed using the same approach of the centralized case. From the partial derivatives of the cost function $J(m,k)$ w.r.t. to m and k separately, evaluated in the point (N,K), we get:

$$\frac{\partial J(m,k)}{\partial m} = \frac{2k}{k+1}T - 4\frac{k^2+1}{(k+1)^2}k^2 m\bigg|_{\substack{m=N\\k=K}} = \frac{2K}{K+1}T - 4\frac{K^2+1}{(K+1)^2}K^2 N \tag{6.28}$$

$$\begin{aligned}\frac{\partial J(m,k)}{\partial k} &= \frac{2m}{(k+1)^2}T - 4\frac{k^3+4k^2+1}{(k+1)^3}km^2\bigg|_{\substack{m=N\\k=K}} \\ &= \frac{2N}{(K+1)^2}T - 4\frac{K^3+4K^2+1}{(K+1)^3}KN^2.\end{aligned} \tag{6.29}$$

Now studying the sign of those derivative we obtain:

$$\begin{aligned}\frac{\partial J(m,k)}{\partial m} &\leq 0 \quad \text{if} \quad T \leq T_m = 2NK\frac{K^2+1}{K+1} \\ \frac{\partial J(m,k)}{\partial k} &\leq 0 \quad \text{if} \quad T \leq T_k = 2NK\frac{K^3+2K^2+1}{K+1}\end{aligned} \tag{6.30}$$

As for the centralized case $T_k > T_m \quad \forall K, N$, then the first quantity that decreases if the coherence time decreases is the number of users k. We can finally summarize the behavior of m and k as follows:

1. $\mathbf{T > T_k}$
   Full (N,K) MIMO IFC is optimal, then $d_{tot} = \frac{2KN}{K+1}$

2. $\mathbf{T_k \leq T \leq 4N}$
   The number of user is not full, then (N,k^*) MIMO IFC is optimal where $k^* : k^4 + 2k^3 + (1 - \frac{T}{2N})k - \frac{T}{2N} = 0$, the total DoF are $d_{tot} = \frac{2k^*(T)N}{k^*(T)+1}$

3. $\mathbf{4N \leq T \leq 2N}$
   SU-MIMO with N antennas is optimal so $d_{tot} = N$

4. $\mathbf{T \leq 2N}$
   Now also the number of antenna starts to shrink as $m^* = \min\{N, \frac{T}{2}\}$ that determines $d_{tot} = m^*$

Up to this point we have focused on a MIMO IFC constraining the system to have a symmetric square MIMO structure. If we want to relax this constraint the optimization problem becomes even more complex since the number of receiving antenna now becomes an optimization variable. In this situation the relation between users, antennas ans streams becomes:

$$d \leq \frac{m+n}{K+1}.$$

As explained before this equation is not always exact for non square systems then for some points in the optimization domain the cost function, constructed using this relation, is not exact. The cost function (6.17) then becomes, for example in the centralized case:

$$J(n,m,k) = (T - T_{ovrhd})\,\frac{k}{k+1}(m+n) \tag{6.31}$$

where

$$T_{ovrhd} = km + \frac{kn[(k-1)m-n]_+}{k\min\{m,n\}} + \frac{k}{k+1}(m+n) \tag{6.32}$$

From the definition of the time overhead above we can see that for a fixed k three different regimes can be observed in m and n.

$\mathbf{n > (k-1)m}$

In this case no alignment can be done so the entire processing should be done at the receiver. This can be seen from the following reasoning. Consider the $k = 2$ MIMO IFC. According to [25] the total number of DoF in the network are $d_{tot} = \min\{n, 2m\}$. If n streams are sent then with n antennas per user all the streams can be received with simple ZF receivers. On the other hand if $2m$ streams are transmitted this means that $2m < n$ so also in this case simple ZF receiver can be used to retrieve the transmitted streams. Consider now a general $k > 2$ user IFC. In the case where $n > (k-1)m$ the space spanned by the columns of the interference channel matrix at user i, $\mathbf{H}_{I_i} = [\mathbf{H}_{i,1}, \dots, \mathbf{H}_{i,i-1}, \mathbf{H}_{i,i+1}, \dots, \mathbf{H}_{i,k}]$, does not have any overlap so no alignment can be done at the transmitter [39]. Then all the processing should be done at the Rx side. This means that only CSIR necessary. Since each user has n antennas a maximum of $d = n/k$ streams per user can be sent, this implies that the minimum number of Tx antennas per user is $m = \frac{n}{k}$. To acquire the necessary CSIR a total DL training period of length $T_{DL} = kd = n$ is necessary so the optimization function (6.31) becomes:

$$J(n) = (T-n)n$$

from the equation above we can see that the optimum number of Rx, and consequently Tx, antennas, is

$$\begin{cases} n^* = \min\{N, \frac{T}{2}\} \\ m^* = \frac{n^*}{k} \end{cases}$$

$\mathbf{n \leq m}$
The time overhead becomes:

$$\begin{aligned} T_{ovrhd} = \quad & km + (k-1)m - n + \tfrac{k}{k+1}(m+n) = \\ & \tfrac{2k^2+2k-1}{k+1}m - \tfrac{n}{k+1} \end{aligned} \tag{6.33}$$

then cost function (6.31) becomes:

$$J(n,m,k) = \left(T - \frac{2k^2+2k-1}{k+1}m + \frac{n}{k+1}\right)\frac{k}{k+1}(m+n)$$

$\mathbf{m \leq n < (k-1)m}$
In this case T_{ovrhd} becomes:

$$\begin{aligned} T_{ovrhd} = \quad & km + \tfrac{[(k-1)m-n]n}{m} + \tfrac{k}{k+1}(m+n) = \\ & \tfrac{k(k+2)}{k+1}m + \tfrac{k^2+k-1}{k+1}n - \tfrac{n^2}{m} \end{aligned} \tag{6.34}$$

then the cost function is:

$$J(n,m,k) = \left(T - \frac{k(k+2)}{k+1}m - \frac{k^2+k-1}{k+1}n + \frac{n^2}{m}\right)\frac{k}{k+1}(m+n)$$

Solving the problem outlined above is not easy, what could be done is to develop a similar analysis proposed for the square symmetric MIMO case, where we studied the sign of the different partial derivatives. For example in the case $m \leq n < (k-1)m$ we find that the coherence times where the three derivatives, calculated at the point (N, N, K) become negative, are:

$$\begin{aligned} T_k &\leq 2N\tfrac{K^3+3K^2+3K-1}{K+1} \\ T_m &\leq 4N\tfrac{K(K+2)}{K+1} \\ T_n &\leq 4N\tfrac{K^2-2}{K+1} \end{aligned} \tag{6.35}$$

From the values found above we have that $T_k > T_m > T_n, \quad \forall K > 1$. This implies that if the coherence time decreases below T_k then the derivative of $J(n,m,k)$ w.r.t. k, evaluated in (N,N,K) becomes negative and hence full MIMO IFC, of the form (N,N,K), is no longer optimal and hence we should reduce the number of active users.

6.8 Conclusions

Optimal joint transmit and receive filter design, in MIMO IFC, assumes that each device has full channel knowledge of the entire network. This condition can not be always satisfied in real time-varying channels. To overcome this difficulty we analyzed a transmission protocol for the necessary CSI acquisitions at each BS and MU. The entire process is based on training and analog FB transmission. We also introduce the approach of output feedback where each MU feeds back directly the received samples during the DL training phase. In FDD communications this technique allows us to shrink the time overhead reducing partially the silent periods. Finally we optimized the sum rate of the MIMO IFC under investigation by focusing in particular on the resulting degrees of freedom. We showed that the optimal number of streams should vary as function of the channel coherence time. In addition if the coherence time is too short we showed that in some condition SU-MIMO transmission is optimal.

Part II

Cognitive Radio Channel

Chapter 7

Beamforming for the Underlay Cognitive MISO Interference Channel

7.1 Introduction and state of the art

Cognitive Radio (CR) is a set of techniques permitting an efficient utilization of the spectrum. This new communication paradigm allows spectrum reuse between legacy (primary) networks (PN) and secondary (possibly opportunistic) networks (SN) as long as the latter do not hamper the formers' communications in an overly adverse fashion. Different transmission paradigms have been introduced to describe the communication in a cognitive environment according to the level of cooperation between primary and secondary users [58]. In this chapter we focus on the underlay deployment. According to its definition an underlay CR communication is said to be in place when a given concerted level of interference from secondary Tx,usually called *Interference temperature constraint* [55], may be tolerated at the primary user (PU), refer to [58] for more on CR terminology.

Underlay CR using multiple antennas has recently come under intense focus since in such systems spatial dimensions can be exploited to shape interference towards primary users. One of the first attempt to study how it is possible to exploit the spatial dimension at the secondary users in a cognitive radio setting is reported in [64]. In this work the trade off between maximizing the secondary user's rate and controlling the interference caused at the primary receivers is studied for different CR settings. The authors exploit multi-antenna at the secondary transmitter to design the optimal transmit filter that effectively balances between spatial multiplexing for the secondary transmission and interference avoidance at the primary receivers. In [156] the cognitive beamformer design problem is studied with the objective of maximizing the secondary link throughput and satisfy a set of SINR constraints, at the secondary users, as well as limiting the interference caused to the primary users. In the proposed algorithm a user selection scheduling is also introduced. [157] addressed the problem of beamformer design to limit the probability of interference leakage at the primary receivers. The approach in [158] finds the beamformer weights, in presence of CSI errors, such that a set of SINR target are met limiting the interference power at the primary receivers to be below a fixed threshold. The problem of downlink beamforming and power allocation techniques at the cognitive base station that also ensure efficient control of the interference caused at primary receivers while maintaining a minimum required SINR for the secondary users is studied in [65, 159, 67]. There the secondary network is always represented as a BC channel that wants to communicate in presence of a set of primary receivers.

Much of the work in underlay CR systems has been done in the context of secondary broadcast (BC) networks coexisting with primary users. This is essentially due to relatively good understanding of BC beamforming and power allocation problem acquired in recent years [75] [76] based on the principle of UL-DL duality. Using this duality, the BF designed in the virtual (dual) uplink mode can be used in the actual downlink problem to achieve the same SINR values by choosing appropriate downlink power allocations. The design of secondary Tx beamformers under primary interference constraints has, for example, been studied in [160, 161, 162] with the objective of SINR balancing in the SN. In [160, 161] the role of UL-DL duality principle remains instrumental in the solution of this problem. In [159, 67] the BF at secondary base station is found introducing a new duality principle that allows to develop a simple iterative algorithm for the power minimization problem where a set of SINR constraints at the secondary Rxs and maximum interference powers level at the primary receiver are imposed. In [67] a robust version of the solution proposed in [159] is introduced. [162] addressed the problem of robust beamforming design with uncertainties in the channel, bounded by an Euclidean ball, the problem is modeled as a semidefinite program and is solved using a new technique without relaxing the rank constraints. In [163] the goal was to design optimal beamformers and rate allocation for the secondary users in a distributed fashion in order to maximize the smallest weighted rate among secondary users. This optimization problem includes a weighted sum-power constraint on the secondary users as well as the interference margin constraints imposed by the primary receivers. In [164] the authors studied the problem of joint transmit-receive filter design in MIMO cognitive broadcast channel with the objective of minimizing the total transmit power while targeting a fixed set of quality of service (QoS) requirements at the secondary multi-antenna receivers. At the same time the interference caused at each primary receiver, also equipped with multiple antennas, should be kept under a fixed threshold. When perfect CSIT is available, at the cognitive BS, they propose an iterative algorithm that solves the problem exactly. When the reduced cooperation with the primary users is such that perfect knowledge of the primary link is not available a robust algorithm is introduced to limit the interference to the primary with high probability. Both solutions rely on the conversion of the original problem into a Second Order Cone Programming (SOCP). In [68] the objective was to optimize the secondary network sum rate under the interference constraints at PUs. In this chapter the secondary network is model as an MISO interference channel that tries to optimize its own rate in presence of a primary receiver, the solution proposed is based on an iterative dual subgradient algorithm.

7.2 Contributions

In this chapter we focus on a secondary network (SN) that is no longer a BC but a MISO IFC. There is one fundamental difference between linear BF design and power allocation problems in BC and IFC, namely there are individual power constraints in the latter as opposed to a total power constraint in the former. Nevertheless, we argue that minimizing total Tx power in the IFC still makes sense from green wireless point of view and thus still makes a valid optimization problem. Here we focus on beamformer design of a secondary network, which is represented by a MISO interference channel (IFC), in presence of primary interference constraints. The main result in this work is that we use recent results on duality for IFC [12] and CR [159], to solve the optimization problem. With this analysis the primary users can be seen as set of receivers in the DL communication while as virtual primary Tx in the UL phase, thus as supplementary interference link that the secondary BSs should take into account while designing the Rx filter in the UL communication. We propose an iterative algorithm that efficiently solves the power minimization problem, at the secondary network, while a set of interference constraints are imposed on primary receivers.

7.3 MISO Cognitive Interference Channel

Fig. 7.1 depicts the cognitive radio scenario that we study in this work. The secondary system is represented as a K-user MISO IFC with K transmitter-receiver

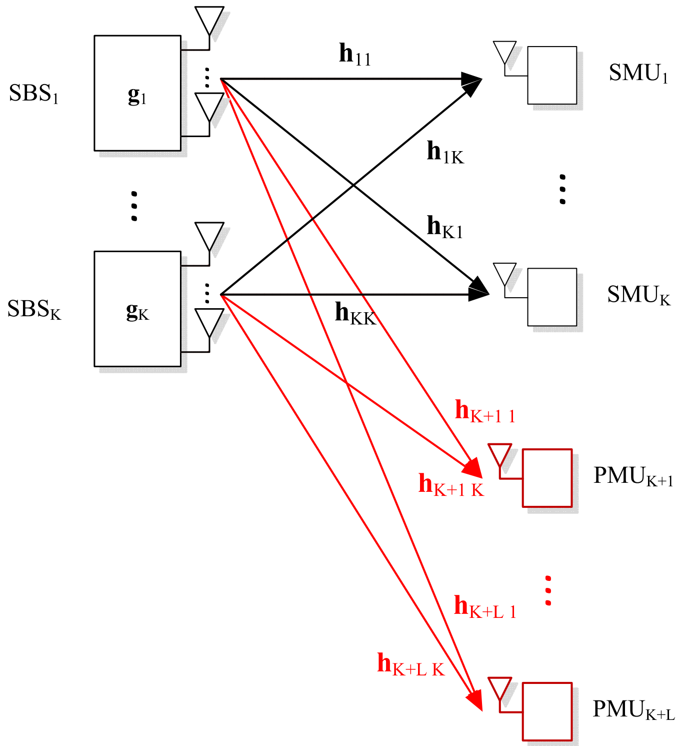

Figure 7.1: Cognitive Radio DL system

pairs. The k-th secondary Base Station (SBS) is equipped with M_k transmit antennas and corresponding secondary mobile user (SMU) is a single antenna node. The k-th transmitter generates interference at all $l \neq k$ receivers. Assuming the communication channel to be frequency-flat, the received signal y_k at the k-th receiver, can be represented as

$$y_k = \mathbf{h}_{kk}\mathbf{x}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{h}_{kl}\mathbf{x}_l + n_k \tag{7.1}$$

where $\mathbf{h}_{kl} \in \mathbb{C}^{1 \times M_l}$ represents the channel vector between the l-th transmitter and k-th receiver, $\mathbf{x}_k$ is the $\mathbb{C}^{M_k \times 1}$ transmit signal vector of the k-th transmitter and n_k represents (temporally white) AWGN with zero mean and variance σ_k^2. Each entry of the channel matrix is a complex random variable drawn from a continuous distribution.

We denote by $\mathbf{g}_k$, the $\mathbb{C}^{M_k \times 1}$ precoding matrix of the k-th transmitter. Thus $\mathbf{x}_k = \mathbf{g}_k s_k$, where s_k represents the independent symbol for the k-th user pair. We assume s_k to have a temporally white Gaussian distribution with zero mean and unit variance.

The secondary network coexists with a set of L primary mobile users (PMU) that are assumed to be single antenna nodes. To simplify the notation we denote the m-th PMU as $K+m,\ \forall m=1,\dots,L$. The downlink channel between the k-th SBS and the m-th PMU is represented by the vector $\mathbf{h}_{K+mk} \in \mathbb{C}^{1\times M_k}$.

In the following we use the results on UL-DL duality for the interference channel discussed in chapter 2, for this reason we introduce also the UL SIMO IFC system model. In the SIMO UL IFC the k-th BS applies a receiver filter $\overline{\mathbf{f}}_k$ to suppress interference and retrieve its desired symbol. The output of such a receive filter is then given by

$$\overline{r}_k = \overline{\mathbf{f}}_k^H \overline{\mathbf{h}}_{kk} \overline{s}_k + \sum_{\substack{l=1\\ l\neq k}}^{K} \overline{\mathbf{f}}_k^H \overline{\mathbf{h}}_{kl} \overline{s}_l + \overline{\mathbf{f}}_k^H \overline{\mathbf{n}}_k$$

7.4 Beamformer Optimization

In the CR scenario the BF design of the opportunistic users must take into account the presence of the primary nodes. In the underlay paradigm [58], the secondary users are allowed to transmit if the interference caused to the primary users is below a fixed threshold. The interference caused at the primary receiver can be handled in different ways. In this work we assume that the secondary BS are equipped with multiple antennas and hence can use the spatial dimension to satisfy the interference constraints.

7.4.1 CR Beamformer Design Under Per User Power Constraint

In the CR setting that we study the SBSs want to optimize their transmit BFs such that the total transmitted power is minimized while a set of quality of service (QoS) constraints, here expressed in term of target SINRs, are imposed at each SMU. At the same time the secondary network should be designed in such a way that the total interference power at each primary receiver is below a certain fixed threshold. From a mathematical point of view the optimization problem that we need to solve can be represented as follow:

$$\begin{aligned} &\min_{\{\mathbf{g}_k\}} \sum_{k=1}^{K} \mathbf{g}_k^H \mathbf{g}_k \\ &\mathbf{g}_k^H \mathbf{g}_k \leq P_k;\ k=1,\ldots,K \\ \text{s.t.}\quad & SINR_k^{DL} = \frac{\mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\sum_{l \neq k} \mathbf{g}_l^H \mathbf{h}_{kl}^H \mathbf{h}_{kl} \mathbf{g}_l + \sigma_k^2} \geq \gamma_k;\ k=1,\ldots,K \\ I_m^{DL} &= \sum_{k=1}^{K} \mathbf{g}_k^H \mathbf{h}_{K+mk}^H \mathbf{h}_{K+mk} \mathbf{g}_k \leq \frac{1}{\gamma_{K+m}};\ m=1,\ldots,L. \end{aligned} \tag{7.2}$$

The last inequality, in the optimization problem, represents the interference power constraint at each primary receiver and γ_{K+m} is a measure related to the interference level I_m^{DL} at mth PU.

The additional interference constraint in the cognitive optimization problem can now be related to virtual SISO primary Tx/Rx pairs in the cognitive IFC. This nevertheless does not change the structure of the problem if one considers further fictitious primary transmitting powers [159], p_m, $m = K+1,\ldots,K+L$ to the corresponding PU while causing zero interference to the K links (receivers) of the secondary network. Introducing this modification the interference constraint in (7.2) can be rewritten as:

$$\frac{1}{I_m^{DL}} = \frac{p_{K+m}}{\sum_{k=1}^{K} \mathbf{g}_k^H \mathbf{h}_{K+mk}^H \mathbf{h}_{K+mk} \mathbf{g}_k} \geq \gamma_{K+m};\ m=1,\ldots,L \tag{7.3}$$

with this equivalent expression the primary interference constraint can be interpreted as a fictitious additional SINR constraint at the primary receiver, where γ_{K+m} is the target SINR at PMU_{K+m}. The optimization problem for the BF design can be solved with standard optimization tools like Lagrange multipliers, then the Lagrangian of the optimization problem reported above is:

$$\begin{aligned} \mathcal{L}(\lambda_i,\mu_i,\mathbf{g}_i) &= \sum_{k=1}^{K} \mathbf{g}_k^H \mathbf{g}_k + \sum_{i=1}^{K} \mu_i \left[\mathbf{g}_i^H \mathbf{g}_i - P_i\right] \\ &+ \sum_{i=1}^{K} \lambda_i \left[-\frac{1}{\gamma_i} \mathbf{g}_i^H \mathbf{h}_{ii}^H \mathbf{h}_{ii} \mathbf{g}_i + \sum_{l \neq i} \mathbf{g}_l^H \mathbf{h}_{il}^H \mathbf{h}_{il} \mathbf{g}_l + \sigma_i^2\right] \\ &+ \sum_{m=1}^{L} \lambda_{K+m} \left[-\frac{p_{K+m}}{\gamma_{K+m}} + \sum_{k=1}^{K} \mathbf{g}_k^H \mathbf{h}_{K+mk}^H \mathbf{h}_{K+mk} \mathbf{g}_k + \sigma_{K+m}^2\right] \end{aligned} \tag{7.4}$$

where λ_k represents the Lagrange multiplier of the k-th SINR constraint and μ_k is the Lagrange multiplier associated to the Tx power constraint at user k.

From the Lagrangian reported above we can write the Lagrange dual problem of the original optimization problem (7.2) as:

$$\begin{aligned} \max_{\{\lambda_k\},\{\mu_k\},\{p_{K+m}\},} & \sum_{k=1}^{K} \lambda_k \sigma_k^2 - \sum_{m=1}^{L} \frac{\lambda_{K+m} p_{K+m}}{\gamma_{K+m}} - \sum_{k=1}^{K} \mu_k P_k \\ \text{s.t.} \quad & -\tfrac{\lambda_k}{\gamma_k} \mathbf{h}_{kk}^H \mathbf{h}_{kk} + \sum_{l \neq k}^{K+L} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I} \succeq 0;\ k = 1, \ldots, K \\ & \lambda_k \geq 0;\ k = 1, \ldots, K+L \\ & \mu_k \geq 0;\ k = 1, \ldots, K \end{aligned} \tag{7.5}$$

where $\eta_k = \mu_k + 1$.
The Lagrange dual of the DL beamforming problem (7.2) can be rewritten as an equivalent UL optimization problem for the Rx filter:

$$\bar{\mathbf{f}}_k = (\sum_{l \neq k}^{K+L} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I})^{-1} \mathbf{h}_{kk}^H \tag{7.6}$$

then (7.5) can be rewritten [90]:

$$\begin{aligned} \max_{\{\lambda_k\},\{\mu_k\},\{p_{K+m}\},} & \sum_{k=1}^{K} \lambda_k \sigma_k^2 - \sum_{m=1}^{L} \frac{\lambda_{K+m} p_{K+m}}{\gamma_{K+m}} - \sum_{k=1}^{K} \mu_k P_k \\ \text{s.t.} \quad & SINR_k^{UL} = \frac{\lambda_k \bar{\mathbf{f}}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \bar{\mathbf{f}}_k}{\bar{\mathbf{f}}_k^H \left[\sum_{l \neq k}^{K+L} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I} \right] \bar{\mathbf{f}}_k} \leq \gamma_k;\ k = 1, \ldots, K \\ & \lambda_k \geq 0;\ k = 1, \ldots, K+L \\ & \mu_k \geq 0;\ k = 1, \ldots, K \end{aligned} \tag{7.7}$$

in the equivalent UL optimization problem the Lagrange multipliers λ_l can be interpreted as dual UL Tx powers and μ_k represents a dual noise variance. Both quantities should now be optimized.

The original optimization problem (7.2) is expressed in the form of quadratically constrained quadratically programming that is in general non convex problems but it is possible to show that it can be expressed as a second order cone program that, at the contrary, is a convex problem [91].
The cost function and the constraints are quadratic in the optimization variables $\mathbf{g}_k$, then introducing a phase shift such that $\mathbf{h}_k \mathbf{g}_k \in \mathbb{R}$ does not influence the optimal

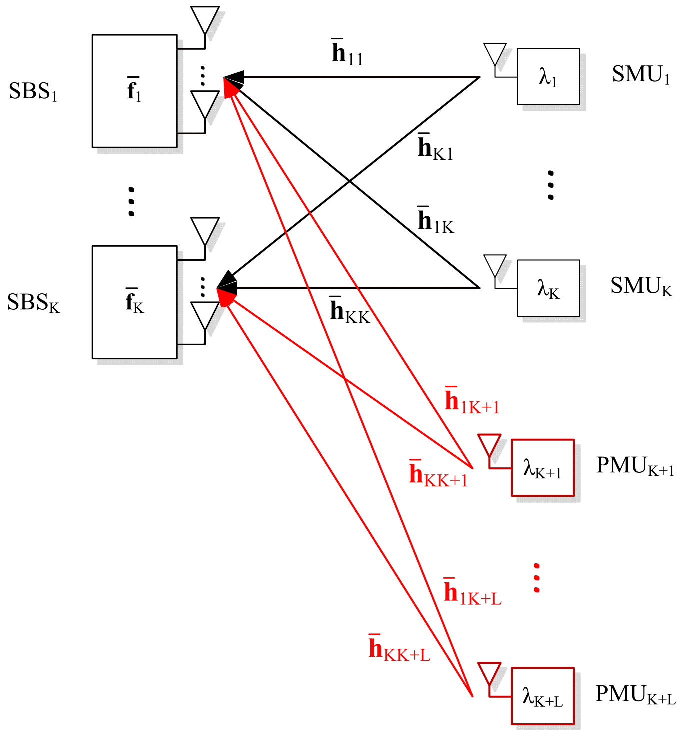

Figure 7.2: Cognitive Radio UL system

solution. Then the optimization problem can be rewritten as:

$$\begin{aligned} &\min_{\{\mathbf{g}_k\}} \sum_{k=1}^{K} \mathbf{g}_k^H \mathbf{g}_k \\ &\mathbf{g}_k^H \mathbf{g}_k \leq P_k;\ k = 1, \dots, K \\ \text{s.t. } &(1 + \tfrac{1}{\gamma_k})|\mathbf{h}_{kk}\mathbf{g}_k|^2 \geq \textstyle\sum_{l=1}^{k} |\mathbf{h}_{kl}\mathbf{g}_l|^2 + \sigma_k^2;\ k = 1, \dots, K \\ &\textstyle\sum_{k=1}^{K} |\mathbf{h}_{K+mk}\mathbf{g}_k|^2 \leq \frac{1}{\gamma_{K+m}};\ m = 1, \dots, L. \end{aligned} \tag{7.8}$$

Introducing the following quantities:

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{h}_{k1} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \ddots & & \\ & & & \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{h}_{kK} \end{bmatrix};\ \mathbf{H}_{K+m} = \begin{bmatrix} \mathbf{h}_{K+m1} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \ddots & & \\ & & & \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{h}_{K+mK} \end{bmatrix}$$

both of dimensions $K \times \sum_k M_k$, $\mathbf{g}^T = [\mathbf{g}_1^T, \dots, \mathbf{g}_K^T]^T$ we can further rewrite (7.8) as:

$$
\begin{aligned}
&\min_{\{\mathbf{g}_k\}} \sum_{k=1}^{K} \mathbf{g}_k^H \mathbf{g}_k \\
&\sqrt{P_k} \geq \|\mathbf{g}_k\|_2;\ k = 1, \ldots, K \\
\text{s.t.}\ &\sqrt{(1+\tfrac{1}{\gamma_k})}\mathbf{h}_{kk}\mathbf{g}_k \geq \left\| \begin{bmatrix} \mathbf{H}_k \mathbf{g} \\ \sigma_k \end{bmatrix} \right\|_2;\ k = 1, \ldots, K \\
&\sqrt{\tfrac{1}{\gamma_{K+m}}} \geq \|\mathbf{H}_{K+m}\mathbf{g}\|_2;\ m = 1, \ldots, L.
\end{aligned}
\tag{7.9}
$$

where $\|.\|_2$ denotes the Euclidean vector norm. The problem above is a quadratic problem with second order cone programming constraints that is a convex problem. We can also state that the duality gap between the original problem (7.2) and the dual problem (7.5) is zero because the optimization problem in convex hence strong duality holds. This implies that the optimal solution of the dual problem is also optimal for the original one.

From the modified expression of the UL optimization problem (7.7) we can see that introducing the per user power constraint brings to introduce a fictitious noise variance in the dual problem that should also be determined. At the optimum the SINR constraints in the UL and the DL problems must be satisfied with equality [159, 164], using this relationship it is possible to derive the DL BF from the UL receiver filter. Because a scaling factor in the receiver filter at the BS does not affect the SINR it is possible to show that the optimal DL BFs are given by:

$$\mathbf{g}_k = \sqrt{p_k}\, \bar{\mathbf{f}}_k \tag{7.10}$$

where p_k is such that the SINRs in the DL are satisfied with equality so:

$$\boldsymbol{p} = (\overline{\mathbf{D}}^{-1} - \overline{\boldsymbol{\Phi}})^{-1}\boldsymbol{\sigma} \tag{7.11}$$

where matrices $\overline{\mathbf{D}}$ and $\overline{\boldsymbol{\Phi}}$ are defined as:

$$[\overline{\boldsymbol{\Phi}}]_{ij} = \begin{cases} \bar{\mathbf{f}}_j^H \mathbf{h}_{ij}^H \mathbf{h}_{ij} \bar{\mathbf{f}}_j, & j \neq i \\ 0, & j = i \end{cases} \tag{7.12}$$

$$\overline{\mathbf{D}} = \mathrm{diag}\{\frac{\gamma_1}{\bar{\mathbf{f}}_1^H \mathbf{h}_{11}^H \mathbf{h}_{11} \bar{\mathbf{f}}_1}, \ldots, \frac{\gamma_K}{\bar{\mathbf{f}}_K^H \mathbf{h}_{KK}^H \mathbf{h}_{KK} \bar{\mathbf{f}}_K}\}. \tag{7.13}$$

In a similar way we can find the UL transmit powers such that the SINR constraints in the UL are satisfied with equality:

$$SINR_k^{UL} = \frac{\lambda_k \bar{\mathbf{f}}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \bar{\mathbf{f}}_k}{\bar{\mathbf{f}}_k^H \left[\sum_{l \neq k}^{K+L} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I} \right] \bar{\mathbf{f}}_k} = \gamma_k;\ k = 1, \ldots, K$$

The slack variable p_{K+m} has been introduced in the optimization procedure such that the primary user interference constraints are always satisfied with equality, then we have:

$$p_{K+m} = (\sum_{k=1}^{K} \mathbf{g}_k^H \mathbf{h}_{K+mk}^H \mathbf{h}_{K+mk} \mathbf{g}_k)\gamma_{K+m}$$

using p_{K+m} the corresponding Lagrange multiplier λ_{K+m} can also be found. The last variable that needs to be optimized is the Lagrange multiplier associated to the per-user power constraint μ_k. To find the optimal value what we propose to use a subgradient method [165] of the form:

$$\mu_k^{(i)} = \mu_k^{(i-1)} + t^{(i)}(\mathbf{g}_k^{(i)H}\mathbf{g}_k^{(i)} - P_k)$$

where $t^{(i)}$ represent the step size at iteration i. The proposed algorithm is summarized in **Algorithm 6**.

In the iterative algorithm the Lagrange multiplier λ_{K+m}, associated to the interference power constraints, are calculated as $\lambda_{K+m}^{(i)} = \lambda_{K+m}^{(i-1)} p_{K+m}^{(i)}$. The rational behind this formula is the following: if the interference generated at PMU_{K+m} is below the threshold $\frac{1}{\gamma_{K+m}}$, then $p_{K+m}^{(i)}$ is less then one, this means that $\lambda_{K+m}^{(i)}$ at the next iteration, should be reduced. If this is the case it contributes less in the calculation of the BF vectors in (7.6), then the BF will try to spend less effort to suppress the interference that it causes to PMU_{K+m}. On the other hand, when the interference is above the threshold, $p_{K+m}^{(i)}$ is greater than one, then $\lambda_{K+m}^{(i)}$ is increased, compare to the previous iteration. In this case the BFs at the secondary network will spend more effort to suppress interference to that particular PMU.

7.5 Simulation results

In this section we report the numerical performance of the proposed iterative algorithm for the optimization of the BF filter at the CR network in presence of a set of primary interference constraints. The performance of **Algorithm 6** is compared with the one of a second algorithm that solves the Lagrange dual problem (7.5) using the interior-point-method where the linear matrix inequality (LMI) constraints, that represents the dual UL SINR is handled using a logarithmic barrier [92]. Since strong duality holds solving the dual problem provides also the optimal solution for the the original problem (7.2). In Fig. 7.3 is plotted the Normalized Root Mean Square Error (NRMSE) of the proposed algorithm,

$$NRMSE = \frac{\sqrt{\frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{K}(\|\mathbf{g}_k^{(i)}(n)\|_2 - \|\mathbf{g}_k^{*}(n)\|_2)^2}}{\sqrt{\frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{K}(\|\mathbf{g}_k^{*}(n)\|_2)^2}}$$

Algorithm 6 Beamformer Design in CR Setting

Initialize: $i = 0$, $\lambda_k^{(0)} = 1, \forall k = 1, \ldots, K + L$, $\mu_k^{(0)} = 1, \forall k = 1, \ldots, K$

repeat

$i = i + 1$

For $k = 1, \ldots, K$ find the UL receiver filter as

$$\overline{\mathbf{f}}_k^{(i)} = (\sum_{l \neq k}^{K+L} \lambda_l^{(i-1)} \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k^{(i-1)} \mathbf{I})^{-1} \mathbf{h}_{kk}^H$$

Update $\lambda_k^{(i)}$ as

$$\lambda_k^{(i)} = \frac{\overline{\mathbf{f}}_k^{(i)H} (\sum_{l \neq k}^{K+L} \lambda_l^{(i-1)} \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k^{(i-1)} \mathbf{I}) \overline{\mathbf{f}}_k^{(i)}}{\frac{1}{\gamma_k} \overline{\mathbf{f}}_k^{(i)H} \mathbf{h}_{kk}^H \mathbf{h}_{kk} \overline{\mathbf{f}}_k^{(i)}}$$

Determine the optimal DL BF $\mathbf{g}_k^{(i)}$ using (7.11)

For $m = 1, \ldots, L$ update the quantity

$$p_{K+m}^{(i)} = (\sum_{k=1}^{K} \mathbf{g}_k^{(i)H} \mathbf{h}_{K+mk}^H \mathbf{h}_{K+mk} \mathbf{g}_k^{(i)}) \gamma_{K+m}$$

and find the UL power $\lambda_{K+m}^{(i)} = \lambda_{K+m}^{(i-1)} p_{K+m}^{(i)}$

Update $\mu^{(i)}$ using the subgradient method with step size $t^{(i)}$

$$\mu_k^{(i)} = \mu_k^{(i-1)} + t^{(i-1)} (\mathbf{g}_k^H \mathbf{g}_k - P_k) \tag{7.14}$$

until convergence

where $\|\mathbf{g}_k^{(i)}(n)\|_2$ represents the Euclidean norm of the DL BF determined using the iterative algorithm at iteration (i) for the n-th Monte Carlo run and $\mathbf{g}_k^*(n)$ is the DL BF obtained using the interior point method. The CR scenario that is simulated is represented by a secondary IFC of $K = 5$ users with $M = 9$ Tx antennas each and $L = 5$ PU. The target SINR are $\gamma_k = 6$ for all SUs and the interference constraints are $\gamma_{K+m} = 1, \ \forall m$, and the noise variance is equal to $-10dB$. As we

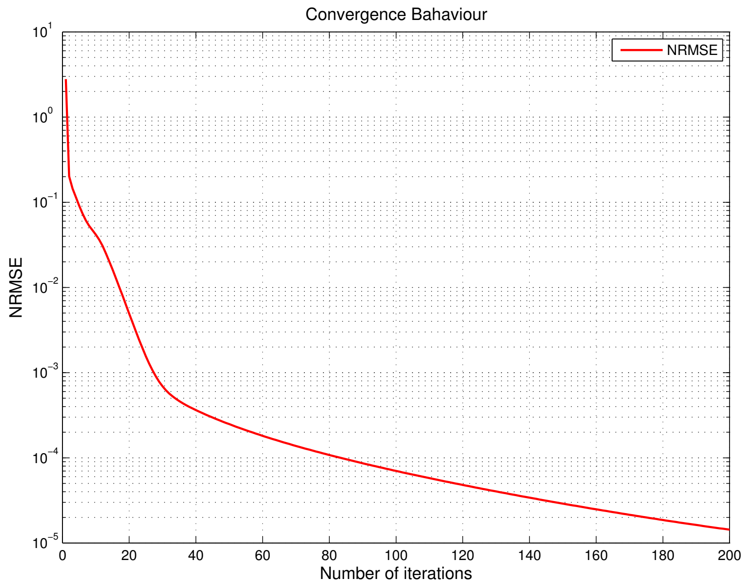

Figure 7.3: NRMSE for $K = 5, L = 5, M = 9$

can see the algorithm manifests good convergence behavior.

7.6 Conclusions

In this chapter we studied the problem of beamforming design in MISO cognitive IFC with objective the minimization of the total transmitted power. Our optimization problem included also a set QoS constraints at each secondary receiver, in addition the total interference generated at each primary receiver should not exceed a fixed threshold. We solved the problem using new results on UL/DL duality for CR channel. The primary users can be seen as a set virtual primary Txs in the UL communication, thus supplementary interference links that should be considered in the secondary Rx design in the UL communication. We proposed an iterative algorithm that efficiently solves the BF design problem.

Chapter 8

Spatial Interweave TDD Cognitive Radio Systems

8.1 Introduction and state of the art

In the previous chapter we focused on the Underlay CR paradigm while here we study a more stringent cognitive radio setting called Interweave (IW). In this situation a CR system exploits the unused communication resources, called *white spaces*, of the primary system in an opportunistic fashion. In this communication paradigm, secondary transmission can take place only if it does not cause any interference to the primary users. We can read the interweave paradigm as a more constrained underlay problem where the level of interference is zero. The unused primary resources can be time, frequency or, as recently introduced, space. Since we focus on a secondary system that relies on the spatial dimensions to opportunistically setup a communication, without deteriorating the primary system's transmission, we refer to this setting as *Spatial Interweave* cognitive radio.

In [166] a MISO CR setting is considered where the cognitive transmitter designs its BF in order to maximize the secondary received power with the constraint of causing zero interference to the primary receivers.

A MIMO broadcast (BC) cognitive radio scenario is studied in [167], they propose a Grassmannian beamforming technique for a limited feedback-based CR network based on the null space of the primary user channels. Cooperation between primary and secondary system is assumed.

In [70] the authors considered an heterogeneous system where the primary and secondary communications can coexist only if the latter keeps under control the interference generated at the primary receiver. They propose an iterative algorithm to design the secondary system transmission parameters to maximize the secondary users rate while imposing a maximum interference constraint to the primary receiver or the more stringent constraint of zero interference to the primary system. The proposed algorithm is completely decentralized and is based on iterative water filling (IWF). The author provide also an analytical description of the performances using the game theory framework. In this work the authors do not investigate how the cognitive users can acquire all the necessary information on channels and primary communication. In [73] a more practical setting is studied. They consider a system model where a primary and a secondary pair of users want to coexist and the secondary communication should take place without causing any interference to the primary receivers exploiting the spatial dimensions. Interestingly in this work any a-priory knowledge is assumed at the secondary network but the necessary information is acquired during a learning phase that exploit reciprocity of the primary TDD communication strategy. During this phase also partial knowledge of the primary signal subspace is acquired. They underline that the proposed scheme is better, in terms of degrees of freedom (DoF), than the previously proposed scheme in [64] because partial knowledge of the Rx subspace at the primary receivers increases the number of streams that can be Tx from the CTx. The authors call this opportunistic way of transmission of the secondary system as *opportunistic spatial sharing*.

The authors of [72] studied the same setting of [73] but with the objective of making their work more practical. A transmission scheme of three phases is introduced where the primary-to-secondary channel is acquired, then the channel between secondary users is estimated and finally the transmission takes place. In the proposed analysis the secondary channel estimation errors are taken into account in the secondary BF design and the interference caused at the secondary receiver, due to primary communication, is reduced introducing a receive filter at the secondary receiver.

In [168] the Pareto boundary of a secondary users' rate region is characterized. There the opportunistic system is described as a MISO interference channel with a set of additional constraints of causing no interference to the primary receivers. In addition a greedy secondary user selection algorithm is introduced to maximize the achievable sum rate.

In the spatial interweave scenario the secondary transmitter can use *Interference Alignment* [28] to design its transmitted signal. The primary receiver sees the opportunistic transmission as interference but only in dimensions that it does not use for its communication. As a result there is no degradation of the performance of the primary system. This beamforming technique has been proposed in [169] where it is called *opportunistic interference alignment*. The authors assume perfect knowledge of all channels without investigating how to obtain this information. As shown in [170] acquisition of channel state information (CSI) is of crucial importance in a non cooperative system as the one considered here. The present work includes an inventory of tools needed to render coexistence of the two systems possible. In particular, the difficult problem of CSI acquisition is addressed. It is shown that the solution relies on Time-Division Duplex (TDD) mode of operation. TDD is desirable since, in theory, it allows the exploitation of uplink $\rightarrow$ downlink reciprocity of the underlying radio propagation channel. Using this transmission strategy the transceiver can obtain DL (UL) channel knowledge using an estimate of the UL (DL) channel. In this work, we prove that TDD is not just a *possible* option, but that it is crucial for spatial IW CR to work if unrealistic overheads and communications between the two systems are to be avoided. Unfortunately in practice, even in TDD, the channel reciprocity assumption only holds for one component of the overall channel, namely the propagation channel itself. More precisely, in order to exploit channel reciprocity one needs to compensate for the mismatch between the analog Tx/Rx circuitry at both ends: this process is called calibration. The calibration problem is generally addressed through two different approaches denoted as absolute and relative calibration [171]. The first one uses a third-party equipment, used as reference, in order to estimate and compensate the analog Tx/Rx circuitry impairments [172] offline. In the latter approach, UL and DL channel estimates obtained at each side of the communication link are exchanged at a low-rate from which calibration factors are deduced. New algorithm for relative channel calibration has been proposed in [173].

8.2 Contributions

In this chapter we study the joint optimization of the transmit-receive filter in a spatial interweave cognitive radio channel. The setting studied in this work is not novel, we describe the entire communication protocol required to acquire the necessary information at primary and secondary users in a spatial interweave cognitive radio setting. What really differentiates our work with previously proposed solutions, for example [73] and [72], is that no-one has studied, up to now, how to really get channel reciprocity in real TDD transmission using UL DL channel calibration. In addition we also underline how calibration influences transmit and receiver filter design at primary and secondary devices. In this paper we use relative calibration method to compensate for Tx/Rx electronics [171].

An important result that comes out of our analysis is that even though the opportunistic Tx needs to know the noise subspace at primary Rx, calibration between non cooperative Tx and Rx is not needed for secondary beamformer design. This is a crucially important result since otherwise in non-cooperative settings typical in CR, calibration would not be achieved and hence channel reciprocity could not be assumed. Finally we extend the results provided for the simple setting with one primary and one secondary pair to the case where the primary network is represented as a K-user interference channel. In this scenario we assume that the primary network designs the transmit and receive filters according to IA [28]. Then, thanks to IA duality, the secondary pair can blindly estimate the DL received subspace at all primary receivers from the transmitted signal subspace in the UL communication. Also for this case it is shown how calibration influences the beamformer design, at both primary and secondary network, and we prove that also for the case of multiple primary users calibration between non cooperative users is not required. This concept also applies to the different pairs of primary users. We evidence that also for IA design calibration is required in TDD communications but at the same time each user has to know only its own calibration filter. This information can be acquired doing calibration between users belonging to the same pair of primary users.

8.3 System Model

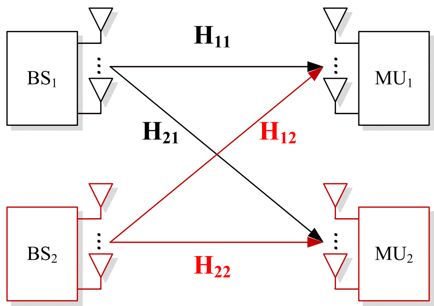

Figure 8.1: Downlink Channel

We focus on the MIMO interference channel where two point-to-point bidirectional links communicate using a TDD transmission scheme. Even if our work can be applied to a more general system, to simplify the notation we will refer to a primary link composed of a licensee Base Station (BS_1) that communicates with the respective Mobile User (MU_1) ignoring completely the presence of a secondary transmission in its vicinity. At the same time a cognitive Base Station (BS_2) tries to opportunistically communicate with a cognitive Mobile User (MU_2) without degrading the licensee's communication. The key assumption in this work is the lack of cooperation among the two systems, primary and secondary. We assume that all the information that the secondary system needs, such as synchronization and primary communication parameters, to design its communication strategy is acquired listening the over-the-air communication between primary BS and MU. Also the knowledge of the communication standard used in the legacy system gives useful information to the opportunistic user. These pieces of information can be acquired listening to the public control channels of the primary system. BS_1 and MU_1 are both equipped with N_1 antennas, BS_2 and MU_2 have both N_2 antennas. The results that we present in this paper can be easily generalized for the case of terminals with an arbitrary number of Tx and Rx antennas, M_i, N_j, $\quad i, j = \{1, 2\}$ respectively. We focus on the case where the opportunistic users have a number of antennas greater than or equal to the primary users $N_2 \geq N_1$. We denote with $\overline{(\cdot)}$ the quantities in the UL transmission, then matrices $\mathbf{H}_{ij}$ and $\overline{\mathbf{H}}_{ij} \in \mathbb{C}^{N_i \times N_j}$ are, respectively, the DL and UL channel matrices from transmitter j to receiver i, where $i, j \in \{1, 2\}$. The entries of these matrices are *i.i.d.* complex Gaussian random variables $\mathcal{N}(0, 1)$. We assume that all channels follow a block-fading model having a coherence time of T symbol intervals without variations. This corresponds to assuming that the channel remains constant for a sufficient number of TDD slots.

In a TDD transmission scheme, assuming perfect Tx/Rx calibration, the UL channel is the transpose of the relative downlink one [171] due to channel reciprocity.

$$\overline{\mathbf{H}}_{ij} = \mathbf{H}_{ji}^H \tag{8.1}$$

Thus an UL channel estimate can be used for designing the transmit beamformer in the DL communication. We assume that channel estimates are obtained through pilot symbols.

8.4 Transmission Techniques and Channel Estimation

In the Interweave cognitive scenario, licensee (primary) systems are not aware of the presence of secondary systems which should ideally cause no interference. The primary Tx is therefore assumed to be a Single User MIMO link (SU-MIMO). In this system the transmitter and receiver filters are designed in order to maximize the transmission rate. The capacity-achieving solution is based on a Beamforming matrix obtained from the singular value decomposition (SVD) of the channel matrix combined with Water-Filling power allocation [174]. Assuming low-rank Tx, the primary link can be decomposed into a signal and a complementary (noise) subspace,

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Delta}\mathbf{V}^H = [\mathbf{U}_s\mathbf{U}_n]\begin{bmatrix} \boldsymbol{\Delta}_s & \\ & \boldsymbol{\Delta}_n \end{bmatrix}\begin{bmatrix} \mathbf{V}_s^H \\ \mathbf{V}_n^H \end{bmatrix} \tag{8.2}$$

where subscripts s or n refer to signal subspace and noise subspace respectively. The matrices $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and $\boldsymbol{\Delta}$ is a diagonal matrix that contains the singular values of the channel matrix. In order to waterfill in UL and DL, both BS_1 and MU_1 must have complete knowledge of the primary channel and Rx noise variances. This information can be obtained partially through TDD reciprocity (pilots for channel estimation) and partially through (unavoidable) feedback.

In the interweave scenario unlicensed users must transmit without disturbing the licensed transmission. Because at low to medium signal-to-noise ratios (SNR) the primary transmitters are expected to exploit a limited number of channel modes, the opportunistic transmitter can beamform its signal in the noise subspace of the licensed communication. This has been labeled as interference alignment technique in [169]. To adapt its communication, the secondary Tx has to know the signal subspace of the primary Rx. As discussed in the following this subspace can be learnt by an opportunistic exploitation of the primary system signals.

All TDD frames in both UL and DL are composed of two time segments, one comprising possibly multiple data streams and the second pilots for channel estimation. In the primary link only the data part of the frame is beamformed but not the pilots. This implies that they span the entire channel space. On the other hand in the cognitive link, pilots are also beamformed, thus ensuring that they do not interfere with the primary transmission. We assume that the secondary TDD slots are aligned with the primary slots using classical spectrum sensing and synchronization techniques.

8.4.1 First TDD Slot

In this first slot all devices in the system should start to acquire the channel state information they need. In particular the licensed BS transmits without knowledge of the downlink channel and therefore cannot beamform. MU_1 can estimate the channel matrix $\mathbf{H}_{11}$ using pilots. The DL channel matrix has dimension $N_1 \times N_1$, then the minimum training length should satisfy :

$$T_T^P \geq N_1 \tag{8.3}$$

The primary BS sends orthogonal pilots with power P_T^P that can be represented as a matrix $\boldsymbol{\Psi}_P$ of dimension $N_1 \times T_T^P$. The total received $N_1 \times T_T^P$ matrix at the primary MU is:

$$\mathbf{Y}_1 = \sqrt{P_T^P}\mathbf{H}_{11}\boldsymbol{\Psi}_P + \mathbf{V} \tag{8.4}$$

where $\mathbf{V}$ represents the zero mean additive white Gaussian noise with variance σ_v^2. The DL Tx power can be related to the time duration of the corresponding Tx phase as

$$P_T^P = \frac{T_T^P}{N_1}\overline{P}_T^P. \tag{8.5}$$

where $\overline{P}_T^P$ represents the DL power constraint. Using an MMSE estimate on $\mathbf{Y}_1\mathbf{\Psi}_1$ each DL channel can be written as $\mathbf{H}_{11} = \widehat{\mathbf{H}}_{11} + \widetilde{\mathbf{H}}_{11}$ where:

$$\widehat{\mathbf{H}}_{11} \sim \mathcal{N}\left(0, \frac{P_T^P}{\sigma_v^2 + P_T^P}\mathbf{I}\right), \quad \widetilde{\mathbf{H}}_{11} \sim \mathcal{N}\left(0, \frac{\sigma_v^2}{\sigma_v^2 + P_T^P}\mathbf{I}\right) \tag{8.6}$$

we call $\sigma^2_{\widehat{\mathbf{H}}_{11}}$ and $\sigma^2_{\widetilde{\mathbf{H}}_{11}}$ the variance of the channel estimate and error respectively.

During this phase, cognitive users in particular MU_2, can use the pilot symbols of the primary communication to opportunistically estimate the cross channel $\mathbf{H}_{21}$. A similar analysis of (8.6) can be done for the channel $\mathbf{H}_{21} = \widehat{\mathbf{H}}_{21} + \widetilde{\mathbf{H}}_{21}$.

8.4.2 Second TDD Slot

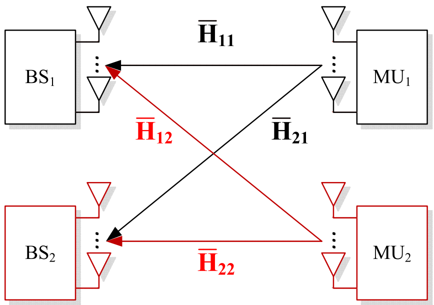

Figure 8.2: Uplink Channel

MU_1 now knows the downlink channel matrix and hence it can construct the beamforming subspace $\mathbf{T}_{MU_1} \in \mathbb{C}^{N_1 \times d_1}$ using reciprocity in equation (8.1). In the same UL frame BS_1 can estimate the UL channel, as done for the DL channel in the previous slot, exploring pilot symbols incorporated in each time segment. d_1 represents the number of transmitted streams, obtained using WF, and is equal to the signal subspace dimension. The received signal at BS_1 has the following structure.

$$\overline{\mathbf{y}}_1 = \overline{\mathbf{H}}_{11}\mathbf{T}_{MU_1}\overline{\mathbf{s}}_1 + \overline{\mathbf{n}}_1 \tag{8.7}$$

$\overline{\mathbf{y}}_1 \in \mathbb{C}^{N_1 \times 1}$ is the received signal vector, $\overline{\mathbf{s}}_1 \in \mathbb{C}^{d_1 \times 1}$ is the transmitted signal vector and $\overline{\mathbf{n}}_1 \in \mathbb{C}^{N_1 \times 1}$ is the spatially white Gaussian noise with zero mean and variance σ_1^2.

MU_1 proceeds with a SVD decomposition of the downlink dual channel, $\overline{\mathbf{H}}_{11} = \mathbf{H}_{11}^H = \mathbf{V}_1 \mathbf{\Delta}_{11} \mathbf{U}_1^H$, uses as Tx beamformer $\mathbf{T}_{MU_1} = \mathbf{U}_{1,s}$, taking the columns of $\mathbf{U}_1$ according to the WF solution. The BS_1 can design its Rx filter as $\mathbf{R}_{BS_1} = \mathbf{V}_{1,s}^H \in \mathbb{C}^{d_1 \times N_1}$ from the SVD of the UL channel. The signal at the output of the receiver filter at BS_1 is written as

$$\begin{aligned}\overline{\mathbf{r}}_1 &= \mathbf{R}_{BS_1} \overline{\mathbf{H}}_{11} \mathbf{T}_{MU_1} \overline{\mathbf{s}}_1 + \mathbf{R}_{BS_1} \overline{\mathbf{n}}_1 \\ &= \mathbf{V}_{1,s}^H \mathbf{H}_{11}^H \mathbf{U}_{1,s} \overline{\mathbf{s}}_1 + \mathbf{V}_{1,s}^H \overline{\mathbf{n}}_1 = \mathbf{\Delta}_{11,s} \overline{\mathbf{s}}_1 + \overline{\mathbf{n}}_1^{'}\end{aligned} \tag{8.8}$$

where $\mathbf{\Delta}_{11,s}$ is the diagonal matrix containing singular values of $\mathbf{H}_{11}^H$ corresponding to the signal subspace. Vector $\overline{\mathbf{n}}_1^{'}$ is the post-processed noise vector that, thanks to the unitary propriety of the Rx filter, preserves the distribution of the original noise vector and hence has variance σ_1^2.

At BS_2 the $N_2 \times 1$ Rx signal is given by

$$\overline{\mathbf{y}}_2 = \mathbf{H}_{12}^H \mathbf{T}_{MU_1} \overline{\mathbf{s}}_1 + \overline{\mathbf{n}}_2 = \mathbf{H}_{12}^H \mathbf{U}_{1,s} \overline{\mathbf{s}}_1 + \overline{\mathbf{n}}_2. \tag{8.9}$$

Assuming sufficient data samples at BS_2, we can obtain a consistent estimate of the primary Tx signal subspace from the autocorrelation matrix of the Rx signal $\mathbf{R}_{\overline{\mathbf{y}}_2\overline{\mathbf{y}}_2} = E\{\overline{\mathbf{y}}_2 \overline{\mathbf{y}}_2^T\}$. In practice we use the sample covariance matrix for the process of blind subspace estimation

$$\widehat{\mathbf{R}}_{\overline{\mathbf{y}}_2\overline{\mathbf{y}}_2} = \frac{1}{T_E} \sum_{t=1}^{T_E} \overline{\mathbf{y}}_2[t] \overline{\mathbf{y}}_2^H[t]. \tag{8.10}$$

From the eigenvalue decomposition of (8.10), $\widehat{\mathbf{R}}_{\overline{\mathbf{y}}_2\overline{\mathbf{y}}_2} = \widehat{\mathbf{U}} \widehat{\mathbf{\Lambda}} \widehat{\mathbf{U}}^H$, we can estimate the signal space dimension $\hat{d}_1$ using the information theoretic criteria described in [175]. Then we can determine the primary noise subspace estimate $\widehat{\mathbf{U}}_n$ from the last $N_2 - \hat{d}_1$ eigenvector of $\widehat{\mathbf{U}}$. This approach gives a consistent estimate for sufficient data samples T_E while for finite data samples the estimate is affected by an estimation error [176] that can be characterized as

$$\widetilde{\mathbf{U}}_n = \mathbf{U}_n - \widehat{\mathbf{U}}_n = \mathbf{S}_2^{H\dagger} \mathbf{N}^H \mathbf{U}_n \tag{8.11}$$

where $\mathbf{N} = [\overline{\mathbf{n}}_2[1], \ldots, \overline{\mathbf{n}}_2[T_E]]$ and

$$\mathbf{S}_2 = \mathbf{H}_{12}^H \mathbf{T}_{MU_1} \underbrace{[\overline{\mathbf{s}}_1[1], \ldots, \overline{\mathbf{s}}_1[T_E]]}_{\widetilde{\mathbf{S}}_2}. \tag{8.12}$$

$\mathbf{A}^\dagger$ denotes the pseudo-inverse of $\mathbf{A}$. In a similar way it is possible to determine the estimation error of the primary signal subspace $\mathbf{U}_s$ [177]. Due to channel reciprocity the Tx and Rx signal subspace in the primary link are the same, hence the knowledge of the primary Tx subspace is sufficient to determine the Rx signal subspace at MU_1.

Knowing $\mathbf{U}_s$, the BS_2 Tx beamformer $\mathbf{T}_{BS_2} \in \mathbb{C}^{N_2 \times d_2}$ can send at most d_2 streams while ensuring its signal lies in the noise subspace at the primary Rx. This implies that

$$\mathbf{R}_{MU_1}\mathbf{H}_{12}\mathbf{T}_{BS_2} = \mathbf{0} \Longrightarrow \mathbf{T}_{BS_2} \subseteq (\mathbf{R}_{MU_1}\mathbf{H}_{12})^{\perp}. \tag{8.13}$$

The equation above says that $\mathbf{T}_{BS_2}$ belongs to the subspace spanned by span$(\mathbf{R}_{MU_1}\mathbf{H}_{12})^{\perp}$, where $(\mathbf{R}_{MU_1}\mathbf{H}_{12})^{\perp}$ represents the orthogonal complement of the row space of the matrix $\mathbf{R}_{MU_1}\mathbf{H}_{12}$. In our case a possible choice for the secondary transmit filter is $\mathbf{T}_{BS_2} = \widehat{\mathbf{U}}_n$.

Including the receiver at MU_1 in the definition of $\mathbf{T}_{BS_2}$ has the advantage that in the low to medium SNR of the primary link, where the primary Tx sends only $d_1 < N_1$ of the total available signaling dimension N_1, the secondary Tx can (opportunistically) transmit at most $d_2 \leq N_2 - d_1$ streams. On the other hand in the high SNR region, when the primary link uses up its entire degrees of freedom (DoF) for spatial multiplexing, the secondary can always transmit $d_2 \leq N_2 - N_1$ streams.

8.4.3 Third TDD Slot

From this TDD time slot onwards starts the steady state of the system. This means that also the cognitive BS starts to transmit to MU_2. As for the reverse link, in the primary forward link BS_1 constructs its beamforming subspace using SVD of the channel matrix $\mathbf{H}_{11}$. Then the transmit beamformer is $\mathbf{T}_{BS_1} = \mathbf{V}_{1,s}$, and MU_1 uses as receiver $\mathbf{R}_{MU_1} = \mathbf{U}_{1,s}^H$. In this slot also the opportunistic BS starts to transmit its data symbols, hence the received signal at primary MU is

$$\mathbf{y}_1 = \mathbf{H}_{11}\mathbf{T}_{BS_1}\mathbf{s}_1 + \mathbf{H}_{12}\mathbf{T}_{BS_2}\mathbf{s}_2 + \mathbf{n}_1. \tag{8.14}$$

In order to extract the useful data MU_1 applies the Rx filter to the received signal: $\mathbf{r}_1 = \mathbf{R}_{MU_1}\mathbf{y}_1$. BS_2 beamformed signal lies in the noise subspace (8.13), hence MU_1 sees no interference. On the other hand MU_2 receives signals from both BS_1 and BS_2:

$$\mathbf{y}_2 = \mathbf{H}_{22}\mathbf{T}_{BS_2}\mathbf{s}_2 + \mathbf{H}_{21}\mathbf{T}_{BS_1}\mathbf{s}_1 + \mathbf{n}_2. \tag{8.15}$$

MU_2 needs to estimate the noise and signal subspaces of the primary communication to design its beamformer. This can be done using semi-blind estimation procedure. The definition semi-blind comes from the fact that part of the information is obtained using usual training and the remaining information comes from

blind subspace estimation.
Using the beamformed pilots incorporated into the secondary data frame, the secondary receiver can estimate the cascade of secondary direct channel and beamformer $\mathbf{H}_{22}\mathbf{T}_{BS_2}$ that has dimensions $N_2 \times d_2$. The training length should now satisfy:

$$T_T^S \geq d_2. \tag{8.16}$$

Once the secondary direct link has been estimated, MU_2 has to estimate the signal and noise subspaces of the primary DL transmission. To accomplish this task the cognitive device can reconstruct the transmitted signal from BS_2 during the secondary pilot transmission and then subtract it from the Rx signal vector:

$$\mathbf{y}_2^{'} = \mathbf{y}_2 - \widehat{\mathbf{H}_{22}\mathbf{T}_{BS_2}}\mathbf{s}_2 = \mathbf{H}_{21}\mathbf{T}_{BS_1}\mathbf{s}_1 + \mathbf{n}_2. \tag{8.17}$$

In (8.17) we assume the estimate $\widehat{\mathbf{H}_{22}\mathbf{T}_{BS_2}}$ is obtained without error to simplify the analysis.

Using the reconstructed signal $\mathbf{y}_2^{'}$ MU_2 determines the signal and noise subspaces, denoted as $\mathbf{V}_s$ and $\mathbf{V}_n$ respectively, of the primary downlink signal using second-order statistics (SOS). This estimation procedure can follow the same steps as the one proposed in section 8.4.2. The estimated noise and signal subspace will be also affected by similar error contribution of (8.11).
Finally MU_2 designs its beamformer subspace such that it creates zero interference at the primary BS:

$$\mathbf{T}_{MU_2} \subseteq (\mathbf{R}_{BS_1}\mathbf{H}_{21}^H)^\perp \tag{8.18}$$

a possible choice is $\mathbf{T}_{MU_2} = \mathbf{V}_n$.

8.4.4 Fourth TDD slot

In this slot all nodes have the required knowledge to transmit to corresponding receivers. The received signal of the primary UL transmission is

$$\overline{\mathbf{y}}_1 = \mathbf{H}_{11}^H\mathbf{T}_{MU_1}\overline{\mathbf{s}}_1 + \mathbf{H}_{21}^H\mathbf{T}_{MU_2}\overline{\mathbf{s}}_2 + \overline{\mathbf{n}}_1 \tag{8.19}$$

The Rx filter at BS_1 suppresses the opportunistic signal, transmitted from MU_2, thanks to the proper design of $\mathbf{T}_{MU_2}$ in (8.18). The received signal at BS_2 nevertheless contains interference due to MU_1.

$$\overline{\mathbf{y}}_2 = \mathbf{H}_{22}^H\mathbf{T}_{MU_2}\overline{\mathbf{s}}_2 + \mathbf{H}_{12}^H\mathbf{T}_{MU_1}\overline{\mathbf{s}}_1 + \overline{\mathbf{n}}_2. \tag{8.20}$$

To suppress this interference contribution standard linear MIMO receiver can be used.

8.5 Secondary Link Optimization

The secondary link beamformer subspace, designed to cause zero interference at the primary receivers, is invariant to a multiplication by a square $d_2 \times d_2$ matrix $\mathbf{Q}_{BS_2}$,$\mathbf{T}_{BS_2}\mathbf{Q}_{BS_2} \in \text{span}(\mathbf{T}_{BS_2})$ hence $\mathbf{R}_{MU_1}\mathbf{H}_{12}\mathbf{T}_{BS_2}\mathbf{Q}_{BS_2} = \mathbf{0}$. The remaining degrees of freedom in $\mathbf{Q}_{BS_2}$ can be used for the optimization of the secondary link communication.

The received signal at MU_2 is given in (8.15). To find the matrix $\mathbf{Q}_{BS_2}$ we need to solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{Q}_{BS_2}} \log & \left| \mathbf{I} + \mathbf{Q}_{BS_2}^H \underbrace{\mathbf{T}_{BS_2}^H \mathbf{H}_{2,2}^H \mathbf{R}_{int}^{-1} \mathbf{H}_{2,2} \mathbf{T}_{BS_2}}_{\mathbf{K}} \mathbf{Q}_{BS_2} \right| \\ \text{s.t.} \quad & \text{Tr}(\mathbf{T}_{BS_2}\mathbf{Q}_{BS_2}\mathbf{Q}_{BS_2}^H\mathbf{T}_{BS_2}^H) \le P_2 \end{aligned} \tag{8.21}$$

where P_2 represents the transmit power constraint at the secondary link and $\mathbf{R}_{int} = \mathbf{H}_{2,1}\mathbf{T}_{BS_1}\mathbf{S}_1\mathbf{T}_{BS_1}^H\mathbf{H}_{2,1}^H + \sigma_n^2\mathbf{I}$ is the interference plus noise covariance matrix with $\mathbf{S}_1 = \mathbb{E}\{\mathbf{s}_1\mathbf{s}_1^H\}$. We use the common notation $|\mathbf{A}| = \det(\mathbf{A})$. The Lagrangian of the optimization problem in (8.21) can be written as:

$$\mathcal{L} = \log\left|\mathbf{I} + \mathbf{Q}_{BS_2}^H\mathbf{K}\mathbf{Q}_{BS_2}\right| - \lambda[\text{Tr}(\mathbf{T}_{BS_2}\mathbf{Q}_{BS_2}\mathbf{Q}_{BS_2}^H\mathbf{T}_{BS_2}^H) - P_2]. \tag{8.22}$$

where λ represents the Lagrangian multiplier associated to the secondary user power constraint P_2. Introducing the eigenvalue decomposition $\mathbf{K} = \mathbf{U}_\mathbf{K}\boldsymbol{\Delta}_\mathbf{K}\mathbf{U}_\mathbf{K}^H$, the matrix $\mathbf{Q}_{BS_2}$ can be parametrized as $\mathbf{Q}_{BS_2} = \mathbf{U}_\mathbf{K}\mathbf{P}_2^{1/2}$, where $\mathbf{P}_2^{1/2}$ represents a diagonal matrix with the power allocation for the d_2 streams. Then (8.22) can be rewritten as:

$$\begin{aligned} \mathcal{L} &= \log|\mathbf{I} + \boldsymbol{\Delta}_\mathbf{K}\mathbf{P}_2| - \lambda(\text{Tr}(\underbrace{\mathbf{U}_\mathbf{K}^H\mathbf{T}_{BS_2}^H\mathbf{T}_{BS_2}\mathbf{U}_\mathbf{K}}_{\boldsymbol{D}}\mathbf{P}_2) - P_2) \\ &= \sum_{i=1}^{d_2} log(1 + \delta_i p_i) - \lambda(\sum_{i=1}^{d_2} \mathbf{D}_{ii}p_i - P_2) \end{aligned} \tag{8.23}$$

To determine the optimal power allocation $\mathbf{P}_2$ we need to derive the Lagrangian (8.23) w.r.t. p_i, the i-th diagonal element of the power allocation matrix. Equating the result to zero we have:

$$p_i = \left[\frac{1}{\lambda\mathbf{D}_{ii}} - \frac{1}{\delta_i}\right]_+ \tag{8.24}$$

where $[a]_+ = max\{a, 0\}$. The solution of this problem corresponds to the traditional water-filling in colored noise because in the Noise covariance matrix $\mathbf{R}_{int}$ we accounted also the interference due to the primary communication.

8.5.1 Feedback Requirements and Differential Feedback

To find the solution of the optimization problem above, BS_2 should know the covariance matrix $\mathbf{K}$. Note that, even using TDD, there is no way for BS_2 to know the interference plus noise covariance matrix, $\mathbf{R}_{int}$ at MU_2. A feedback of $\mathbf{K}$ to BS_2 is therefore necessary. In order to reduce the rate penalty due to feedback, we propose to use differential feedback [178]. In this technique the Rx and Tx both generate a common random codebook of Hermitian matrices from which they choose the appropriate matrix. The receiver, according to the received signal, chooses the Hermitian matrix that is closest to the real covariance matrix. The information that is fed back is the index corresponding to the chosen matrix in the codebook. Using the index, and the corresponding random matrix, the transmitter finds the Tx filter through WF. This process continues until convergence or a certain number of iterations is reached, refer to [178] for more details.

The main advantage of the differential method is that the amount of feedback is not related to the matrix dimensions [178]. The number of bits required is $b = log_2(Q)$, where Q is the cardinality of the codebook. The disadvantage of this method is that it is sensitive to transmission error, in particular if the transmitter chooses the wrong matrix, due to feedback errors, the beamformer matrix is no longer optimal. Fortunately, it turns out that differential feedback is robust against transmission errors introducing a little modification in the feedback procedure. At every iteration, before finding the new covariance matrix in the random codebook to be fed back, the receiver should verify if the transmitter has used the right covariance matrix to design the beamformer. In particular it checks if the received covariance matrix is the same that it would have received if the transmitter would have used the covariance matrix corresponding to the correct fed back index. It compares the results and if they are different it tries to find out the covariance matrix that the transmitter has used for designing the BF. Then it uses this matrix to initialize the next feedback iteration step.

8.6 Rate loss due to blind subspace estimation

As we described in section 8.4.2 the blind estimation of the signal and noise subspace of the primary transmission could be affected by some estimation error (8.11). This implies that when the secondary transmitter sends data using a BF based on the estimated noise subspace some interference leaks in the signal subspace at the primary receiver creating interference. Naturally this interference determines some loss in terms of primary achievable rate.

The received signal at primary MU (8.14), after the Rx filter $\mathbf{R}_{MU_1}$, can be written as:

$$\mathbf{r}_1 = \mathbf{R}_{MU_1}\mathbf{H}_{11}\mathbf{T}_{BS_1}\mathbf{s}_1 + \mathbf{R}_{MU_1}\mathbf{H}_{12}\mathbf{T}_{BS_2}\mathbf{s}_2 + \mathbf{R}_{MU_1}\mathbf{n}_1$$
$$\mathbf{r}_1 = \mathbf{R}_{MU_1}\mathbf{H}_{11}\mathbf{T}_{BS_1}\mathbf{s}_1 + \mathbf{R}_{MU_1}\mathbf{H}_{12}\widetilde{\mathbf{U}}_n\mathbf{s}_2 + \mathbf{R}_{MU_1}\mathbf{n}_1.$$

Denoting with $\mathbf{R}_{\overline{\mathbb{I}}}$ the interference plus noise covariance matrix:

$$\mathbf{R}_{\overline{\mathbb{I}}} = \underbrace{\mathbf{R}_{MU_1}\mathbf{H}_{12}\widetilde{\mathbf{U}}_n\mathbf{Q}^2_{BS_2}\widetilde{\mathbf{U}}_n^H\mathbf{H}_{12}^H\mathbf{R}_{MU_1}^H}_{\mathbb{I}_1} + \underbrace{\sigma_1^2\mathbf{I}}_{\mathbf{R}_{n_1}}$$

the rate at primary MU can be written as:

$$\widetilde{\mathcal{R}}_{MU_1} = \log|\mathbf{I} + \underbrace{\mathbf{R}_{MU_1}\mathbf{H}_{11}\mathbf{T}_{BS_1}\mathbf{S}_1\mathbf{T}_{BS_1}^H\mathbf{H}_{11}^H\mathbf{R}_{MU_1}^H}_{\mathbf{D}_{MU_1}}\mathbf{R}_{\overline{\mathbb{I}}}^{-1}| \tag{8.25}$$

where $\mathbf{S}_1 = \mathbb{E}\{\mathbf{s}_1\mathbf{s}_1^H\}$. The average rate loss due to the estimation error can be determined simply:

$$\begin{aligned}\Delta\mathcal{R} &= \mathbb{E}\mathcal{R}_{MU_1} - \mathbb{E}\widetilde{\mathcal{R}}_{MU_1}\\ &= \mathbb{E}\log|\mathbf{I} + \mathbf{D}_{MU_1}\mathbf{R}_{n_1}^{-1}| - \mathbb{E}\log|\mathbf{I} + \mathbf{D}_{MU_1}\mathbf{R}_{\overline{\mathbb{I}}}^{-1}|\\ &= \mathbb{E}\log|\mathbf{I} + \mathbf{D}_{MU_1}\mathbf{R}_{n_1}^{-1}| - \mathbb{E}\log|\mathbf{I} + (\mathbf{D}_{MU_1} + \mathbb{I}_1)\mathbf{R}_{n_1}^{-1}|\\ &+\mathbb{E}\log|\mathbf{I} + \mathbb{I}_1\mathbf{R}_{n_1}^{-1}|\\ &\overset{(a)}{\leq} \mathbb{E}\log|\mathbf{I} + \mathbb{I}_1\mathbf{R}_{n_1}^{-1}| \overset{(b)}{\leq} \log|\mathbf{I} + \mathbb{E}\{\mathbb{I}_1\}\mathbf{R}_{n_1}^{-1}|\end{aligned} \tag{8.26}$$

where (a) is due to the fact that $|\mathbf{I} + (\mathbf{D}_{MU_1} + \mathbb{I}_1)\mathbf{R}_{n_1}^{-1}|$ dominates $|\mathbf{I} + \mathbf{D}_{MU_1}\mathbf{R}_{n_1}^{-1}|$, finally (b) comes from the Jensen inequality. To determine theoretically the value of the rate loss we need to compute the expectation $\mathbb{E}\{\mathbb{I}_1\}$. This can be done following the steps below.

$$\begin{aligned}\mathbb{E}\{\mathbb{I}_1\} &= \mathbb{E}\{\mathbf{R}_{MU_1}\mathbf{H}_{12}\widetilde{\mathbf{U}}_n\mathbf{Q}^2_{BS_2}\widetilde{\mathbf{U}}_n^H\mathbf{H}_{12}^H\mathbf{R}_{MU_1}^H\}\\ &= \mathbb{E}\{\mathbf{R}_{MU_1}\mathbf{H}_{12}\mathbf{S}_2^{H\dagger}\mathbf{N}^H\mathbf{U}_n\mathbf{Q}^2_{BS_2}\mathbf{U}_n^H\mathbf{N}\mathbf{S}_2^{\dagger}\mathbf{H}_{12}^H\mathbf{R}_{MU_1}^H\}\end{aligned} \tag{8.27}$$

because the noise samples in $\mathbf{N}$ are iid the expectation $\mathbb{E}\{\mathbf{N}^H\mathbf{U}_n\mathbf{Q}^2_{BS_2}\mathbf{U}_n^H\mathbf{N}\} = \sigma_2^2\mathrm{Tr}\{\mathbf{Q}^2_{BS_2}\} = \sigma_2^2P_2$. Then (8.27) becomes

$$\begin{aligned}\mathbb{E}\{\mathbb{I}_1\} = {}& \sigma_2^2P_2\mathbb{E}\left\{\mathbf{U}_{1,s}^H\mathbf{H}_{12}\big[\mathbf{H}_{12}^H\mathbf{U}_{1,s}\left(\mathbf{U}_{1,s}^H\mathbf{H}_{12}\mathbf{H}_{12}^H\mathbf{U}_{1,s}\right)^{-1}\times\right.\\ &\left(\widetilde{\mathbf{S}}_2\widetilde{\mathbf{S}}_2^H\right)^{-1}\widetilde{\mathbf{S}}_2\widetilde{\mathbf{S}}_2^H\left(\widetilde{\mathbf{S}}_2\widetilde{\mathbf{S}}_2^H\right)^{-1}\left(\mathbf{U}_{1,s}^H\mathbf{H}_{12}\mathbf{H}_{12}^H\mathbf{U}_{1,s}\right)^{-1}\mathbf{U}_{1,s}^H\mathbf{H}_{12}\big]\\ &\left.\mathbf{H}_{12}^H\mathbf{U}_{1,s}\right\}\\ = {}& \sigma_2^2P_2\mathbb{E}\left\{\left(\widetilde{\mathbf{S}}_2\widetilde{\mathbf{S}}_2^H\right)^{-1}\right\}.\end{aligned} \tag{8.28}$$

In the equation above we used the property of pseudo-inverse that if $\mathbf{S}_2$ can be parameterized as $\mathbf{S}_2 = \mathbf{CD}$ then $\mathbf{S}^\dagger = \mathbf{D}^H(\mathbf{DD}^H)^{-1}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H$. According to the definition of $\widetilde{\mathbf{S}}_2$ in (8.12), $\left(\widetilde{\mathbf{S}}_2\widetilde{\mathbf{S}}_2^H\right)^{-1}$ is distributed as a complex inverse Wishart matrix distributed as $\mathcal{W}_{d_1}^{\mathbb{C}^{-1}}\left(T_E, \mathbf{S}_1^{-1}\right)$ [179]. Then

$$\mathbb{E}\{\mathbb{I}_1\} = \frac{\sigma_2^2 P_2}{T_E - d_1}\mathbf{S}_1^{-1}$$

we can finally conclude that the upper bound of rate loss is :

$$\Delta\mathcal{R} \leq \log|\mathbf{I} + \frac{\sigma_2^2 P_2}{\sigma_1^2(T_E - d_1)}\mathbf{S}_1^{-1}| \tag{8.29}$$

Assuming that $\mathbf{S}_1$ is roughly proportional to the primary transmit power P_1, form (8.29) we can see that if both powers, P_1 and P_2, grow at the same rate, $\Delta\mathcal{R}$ stays constant. This means that at high SNR the estimation error in the noise subspace at secondary devices determines only a loss in term of SNR offset. On the other hand the multiplexing gain (or degrees of freedom (DoF)) achievable by the primary users remains constant. This will be more clear in the simulation results of Section 8.11.

The rate loss can be decreased thanks to enhanced channel estimation, using, for example, more symbols, increasing T_E, to perform the estimate at the receiver. The analysis done in this section refers to the DL transmission phase, a similar analysis can be carried out for the uplink phase.

8.7 Uplink Downlink Calibration

Up to this point we have considered UL and DL channel to be perfectly reciprocal. In practice this is true only after perfect calibration. In this section we describe the basic principle of calibration. Then we will introduce the new calibration algorithm for MIMO systems.

The overall UL and DL channels, Fig. 8.3, can be written as:

$$\mathbf{U}_{ii} = \mathbf{R}_B\mathbf{H}_{ii}^T\mathbf{T}_M \tag{8.30}$$

$$\mathbf{D}_{ii} = \mathbf{R}_M\mathbf{H}_{ii}\mathbf{T}_B \tag{8.31}$$

where the matrices $\mathbf{T}_B$, $\mathbf{R}_B$ and $\mathbf{T}_M$, $\mathbf{R}_M$ model the transmit and receive circuitry at the BS and MU respectively, with dimensions $N_i \times N_i$. It is possible to express the DL channel as function of the UL channel, and vice versa:

$$\mathbf{D}_{ii} = \underbrace{\mathbf{R}_M\mathbf{T}_M^{-T}}_{\mathbf{P}_{MU_i}}\mathbf{U}_{ii}^T\underbrace{\mathbf{R}_B^{-T}\mathbf{T}_B}_{\mathbf{P}_{BS_i}} \tag{8.32}$$

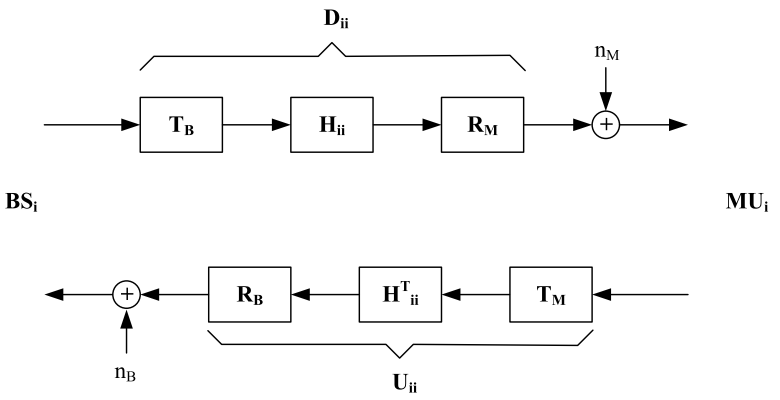

Figure 8.3: Reciprocity Model

The calibration matrices $\mathbf{P}_{MU_i}$ and $\mathbf{P}_{BS_i}$ only depend on the electronic components at respective sides. The objective of relative calibration is to find these matrices using estimates of the UL and DL channel obtained through classical training and channel feedback operation [171]. Complete calibration requires an UL to DL and another DL to UL training phase between users. Several techniques in MIMO CR exploit directly the reciprocity without a calibration process. Until now, it was really challenging to deal with the difficulty to find simultaneously the MIMO calibration matrices like mentioned in [171], where they first propose to simplify the problem, in subdividing the MIMO channel into $N_i \times N_j$ single channels calibrated separately. However, this solution fails compensate the antenna coupling effects since it assumes that the calibration matrices are diagonal [173]. Therefore, they propose an iterative method where each calibration matrix is found alternatively, but the convergence of this technique has not been proved yet. In the sequel, we will describe a technique to find simultaneously the MIMO calibration matrices.

In our study, according to the relative calibration principle the question is: ”How to calibrate the cross links in a CR system where communication between primary and secondary systems is not allowed?”. As we shall see in the following despite the stringent secondary beamformer requirement (the interference should lie in the crosslink Rx noise subspace) no calibration is required between crosslink Tx-Rx devices. This result is a key element to implement spatial interweave CR systems.

It must be noted that in our CR scenario, the calibration phase of secondary link will interfere a little with the primary link (and vice versa) but considering that the training phase for calibration is infrequent, the interference caused is negligible.

8.8 Beamforming Design with Channel Calibration

8.8.1 Primary Beamformer Design

In this section we will discuss how the calibration of Tx-Rx electronics can be incorporated in the beamformer design.

BS_1 performs an SVD decomposition of the UL channel $\mathbf{U}_{11} = \mathbf{ZDW}^H$ that it estimates directly using pilots transmitted by MU_1. The primary DL channel can be written as function of the UL channel SVD decomposition using the calibration filters as:

$$\mathbf{D}_{11} = \mathbf{P}_{MU_1}\mathbf{U}_{11}^T\mathbf{P}_{BS_1} = \mathbf{P}_{MU_1}\mathbf{W}^*\mathbf{DZ}^T\mathbf{P}_{BS_1} \tag{8.33}$$

in order to diagonalize the DL channel BS_1 designs its beamformer subspace as $\mathbf{T}_{BS_1} = \mathbf{P}_{BS_1}^{-1}\mathbf{Z}^*$, and hence the receiver filter at MU_1 is given by: $\mathbf{R}_{MU_1} = \mathbf{W}^T\mathbf{P}_{MU_1}^{-1}$.

During UL transmission it is possible to design the transmit and receive filters using the UL channel as reference. In doing so, calibration filters do not appear in the expression and thus the transmit matrix at MU_1 is $\mathbf{T}_{MU_1} = \mathbf{W}$ and the receive filter at BS_1 is: $\mathbf{R}_{BS_1} = \mathbf{Z}^H$.

8.8.2 Secondary Beamformer Design

The signal at secondary BS due to primary and secondary Tx is expressed as

$$\overline{\mathbf{y}}_2 = \mathbf{U}_{21}\mathbf{T}_{MU_1}\overline{\mathbf{s}}_1 + \mathbf{U}_{22}\mathbf{T}_{MU_2}\overline{\mathbf{s}}_2 + \overline{\mathbf{n}_2} \tag{8.34}$$

Knowing $\mathbf{U}_{22}\mathbf{T}_{MU_2}$ estimated through MU_2 beamformed pilots, BS_2 can determine the MU_1 Tx subspace $\mathbf{U}_{21}\mathbf{W}$ using second order statistics.

Now let us consider the signal at MU_1, after the Rx filter, which is given by

$$\mathbf{r}_1 = \underbrace{\mathbf{R}_{MU_1}\mathbf{D}_{11}\mathbf{T}_{BS_1}\mathbf{s}_1}_{\mathbf{r}_{1,s}} + \underbrace{\mathbf{R}_{MU_1}\mathbf{D}_{12}\mathbf{T}_{BS_2}\mathbf{s}_2}_{\mathbf{r}_{1,int}} + \mathbf{n}_1 \tag{8.35}$$

where $\mathbf{r}_{1,s}$ represents the useful signal part and $\mathbf{r}_{1,int}$ contains the interference term. The objective of the secondary user is to transmit without causing any interference to the primary system. So BS_2 must design its beamformer subspace such that $\mathbf{r}_{1,int} = 0$. Expressing the DL channel $\mathbf{D}_{12}$ as function of the UL channel and the calibration filters we can write

$$\mathbf{r}_{1,int} = \mathbf{R}_{MU_1}\mathbf{D}_{12}\mathbf{T}_{BS_2}\mathbf{s}_2 = \mathbf{W}^T\mathbf{U}_{21}^T\mathbf{P}_{BS_2}\mathbf{T}_{BS_2}\mathbf{s}_2 \tag{8.36}$$

because BS_2 knows the calibration filter $\mathbf{P}_{BS_2}$ it is possible to parameterize $\mathbf{T}_{BS_2} = \mathbf{P}_{BS_2}^{-1}\hat{\mathbf{T}}_{BS_2}$, so it is possible to design the beamformer subspace, in order to cause zero interference at MU_1 after its receiver filter, as

$$\hat{\mathbf{T}}_{BS_2} \subseteq (\mathbf{W}^T\mathbf{U}_{21}^T)^{\perp} \tag{8.37}$$

Similar treatment applies to the design of MU_2 beamformer which are not discussed here.

It is important to remark that the secondary transmitter can design the beamformer subspace using only its own calibration factor, obtained during the calibration phase only with its intended receiver. Then the UL channel and the receiver subspace at MU_1 are estimated using second order statistics of the received signal. Calibration with non cooperative users is not required.

8.9 Practical Considerations in Spatial IW CR

Despite a pragmatic approach taken in this work to spatial interweave CR design, we nevertheless make a strong assumption, namely the Tx/Rx subspace is the same in the primary system. In practical system, this condition may not be satisfied for several reasons, for example a different ratio of power constraint and noise variance between the BS_1 and MU_1 may lead to different number of streams in UL and DL. One subspace will be the subset of the other. A more drastic difference could be the presence at one end of colored noise instead of white noise or different colored noises at the two ends in which case whitened channels may lead to unrelated Tx/Rx subspaces. In such cases, secondary systems can resort to zero-forcing beamforming on the crosslink if enough degrees of freedom are available. This implies a smaller number of secondary Tx streams but the IW paradigm is still satisfied.

If the primary link is affected by colored noise due to secondary link leakage, one may observe that the CR is no longer strictly spatial interweave and fits the underlay paradigm [58]. When this happens, TDD is not enough to design Tx/Rx filters and feedback is also required between BS_1 and MU_1. Furthermore, estimation of interference plus noise covariance matrices is needed for channel whitening and primary beamformer design. In some way, the CR problem starts resembling a classical MIMO interference channel.

8.10 Extension to multiple Primary pairs

The system model described so far can be easily extended to the situation where a cognitive system wants to coexist with a set of K primary transmitter and receiver pairs, Fig. 8.4. This problem formulation depicts the scenario where a femto-cell is deployed at the cell edge of a macro cell, thus the femto communication suffers from the interference received from the surrounding macro-cells. The primary sys-

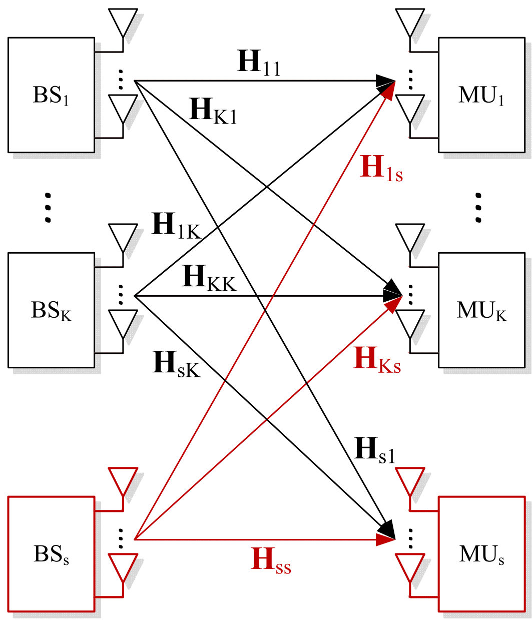

Figure 8.4: Setting with multiple primary pairs

tem can be interpreted as a K-user MIMO interference channel (IFC). To mitigate the interference that each macro user receives from the other macro transmissions we assume that an interference alignment transmission strategy is used at the level of macro communication. This strategy has been shown to maximize the degrees of freedom for the K-user MIMO IFC [28]. In this section we generalize the system model assuming that the k-th primary BS is equipped with M_k antennas while the corresponding primary MU has N_k antennas and they want to exchange d_k streams. We denote the number of antennas at secondary BS and MU as M_s, N_s respectively. To simplify the notation we define with $\mathbf{G}_l$ and $\mathbf{F}_l$ the transmit and receive filter matrices at primary pair number l and with $\mathbf{G}_s$ and $\mathbf{F}_s$ the same quantities at the secondary pair. As discussed in chapter 3 the transmit beamformers are designed such that the interference caused by all transmitters at each non-intended Rx lies in a common interference subspace. Then with a ZF receiver the interference can be completely suppressed. The interference alignment conditions can be simply described as:

$$\mathbf{F}_k \mathbf{H}_{kl} \mathbf{G}_l = \mathbf{0} \quad \forall l \neq k \tag{8.38}$$

$$\mathrm{rank}(\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k) = d_k \quad \forall k \in \{1, 2, \ldots, K\} \tag{8.39}$$

This last rank condition leads to the traditional single user MIMO constraint $d_k \leq \min(M_k, N_k)$ for d_k streams to be able to pass over the k-th link. Since we suppose to use a TDD communication protocol thanks to channel reciprocity IA duality still holds, then :

$$\overline{\mathbf{F}}_k\overline{\mathbf{H}}_{kl}\overline{\mathbf{G}}_l = \mathbf{0} \quad \forall l \neq k \tag{8.40}$$

$$\mathrm{rank}(\overline{\mathbf{F}}_k\overline{\mathbf{H}}_{kk}\overline{\mathbf{G}}_k) = d_k \quad \forall k \in \{1, 2, \ldots, K\} \tag{8.41}$$

where $\overline{\mathbf{F}}_l = \mathbf{G}_l^H$, $\overline{\mathbf{G}}_l = \mathbf{F}_l^H$ are the UL-DL relationship between transmit and receive filters. From this conditions we can see that, as in the previous case with one single primary pair, the transmit signal subspace corresponds to the receive signal subspace.
The signal at the output of the k-th primary MU can be written as:

$$\begin{aligned} \mathbf{r}_k = & \; \mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\mathbf{s}_k + \sum_{l \neq k} \mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l\mathbf{s}_l + \mathbf{F}_k\mathbf{H}_{ks}\mathbf{G}_s\mathbf{s}_s + \mathbf{F}_k\mathbf{n}_k \\ = & \; \mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\mathbf{s}_k + \mathbf{F}_k\mathbf{H}_{ks}\mathbf{G}_s\mathbf{s}_s + \mathbf{n}_k^{'} \end{aligned}$$

where we denoted with $\mathbf{H}_{ks}$ the channel matrix between the secondary BS and the k-th primary MU and with $\mathbf{n}_k^{'}$ the noise at the output of the receive filter.
The stated objective of our investigation is to design the transmit filter at the secondary network such that the interference generated at all primary receivers is zero: $\mathbf{F}_k\mathbf{H}_{ks}\mathbf{G}_s = \mathbf{0}, \quad \forall k$. The received signal at cognitive BS in the UL transmission phase can be written as:

$$\overline{\mathbf{y}}_s = \overline{\mathbf{H}}_{ss}\overline{\mathbf{G}}_s\overline{\mathbf{s}}_s + \sum_{l=1}^{K} \overline{\mathbf{H}}_{sk}\overline{\mathbf{G}}_k\overline{\mathbf{s}}_k + \overline{\mathbf{n}}_s \tag{8.42}$$

As shown in section 8.4.2 and 8.4.3 from the received signal at the secondary BS we can estimate the primary signal subspace generated at the secondary BS using semi-blind subspace estimation. Due to channel reciprocity and duality of IA the Tx and Rx signal subspace at each primary device are the same:

$$\overline{\mathbf{H}}_I = \left[\overline{\mathbf{H}}_{s1}\overline{\mathbf{G}}_1, \ldots, \overline{\mathbf{H}}_{sK}\overline{\mathbf{G}}_K\right] = \begin{bmatrix} \mathbf{F}_1\mathbf{H}_{1s} \\ \vdots \\ \mathbf{F}_K\mathbf{H}_{Ks} \end{bmatrix}^H = \mathbf{H}_I^H$$

$\mathbf{H}_I$ is the composite secondary to primary channel as seen at the joint outputs of the primary UEs. Then the signal subspace at all primary MUs in the DL communication, spanned by $\mathbf{H}_I$, can be estimated from the received signal at the secondary BS

(8.42), using semi-blind techniques. The knowledge of the primary Tx subspace is enough to design the Tx filter at the secondary BS such that the interference that the secondary communication generates at each primary receiver lies in the noise subspace. This implies:

$$\mathbf{H}_I \mathbf{G}_s = \mathbf{0} \Longrightarrow \mathbf{G}_s \subseteq \text{span}(\mathbf{H}_I^H)^\perp$$

The last relationship says that the BF chosen by the secondary BS should be in the orthogonal complement of the subspace spanned by the matrix $\mathbf{H}_I$ In order to have a possibility to design the Tx filter each cognitive device should be equipped with a number of antenna greater than the total number of streams transmitted in the primary network: $N_s, M_s > \sum_k^K d_k$.

8.10.1 Transmit and receive filter design with calibration filters

In this section we show how to design the IA filters for the primary network and the transmit and receive filter at the cognitive users when calibration filters need to be included in the filter design.
Initially we consider the IA design at the primary network.

Primary BSs and MUs, with the estimate of the UL channels, calculate the transmit and receive filters for the UL transmission using an iterative algorithm available in literature, for example [34] or [33]. Then the UL IA conditions are satisfied:

$$\overline{\mathbf{F}}_k \mathbf{U}_{kl} \overline{\mathbf{G}}_l = \mathbf{0}.$$

To apply the UL filters in the DL communication each terminal should pre-compensate for the UL-DL channel mismatch, as done in section 8.8:

$$\begin{aligned} \mathbf{G}_k &= \mathbf{P}_{BS_k}^{-1} \overline{\mathbf{F}}_k^T \\ \mathbf{F}_l &= \overline{\mathbf{G}}_l^T \mathbf{P}_{MU_l}^{-1} \end{aligned} \tag{8.43}$$

Applying the IA filter, found above, in the DL transmission we get:

$$\mathbf{F}_l \mathbf{D}_{lk} \mathbf{G}_k = \mathbf{F}_l \mathbf{P}_{MU_l} \mathbf{U}_{kl}^T \mathbf{P}_{BS_k} \mathbf{G}_k = \overline{\mathbf{G}}_l^T \mathbf{U}_{kl}^T \overline{\mathbf{F}}_k^T = (\overline{\mathbf{F}}_k \mathbf{U}_{kl} \overline{\mathbf{G}}_l)^T = \mathbf{0}$$

then the IA conditions also in the DL are satisfied. This concludes the filter design in the primary network. To understand how the secondary network can find the BF matrices for the concurrent transmission we fist study the received signal at the secondary BS in the UL transmission:

$$\overline{\mathbf{y}}_s = \mathbf{U}_{ss} \overline{\mathbf{G}}_s \overline{\mathbf{s}}_s + \sum_{l=1}^{K} \mathbf{U}_{sk} \overline{\mathbf{G}}_k \overline{\mathbf{s}}_k + \overline{\mathbf{n}}_s \tag{8.44}$$

from the received signal above, the subspace spanned by the matrix $\overline{\mathbf{H}}_I = \left[\mathbf{U}_{s1}\overline{\mathbf{G}}_1, \ldots, \mathbf{U}_{sK}\overline{\mathbf{G}}_K\right]$ can be estimated. The objective of the secondary BF design is to cause zero interference at all the primary receivers, then the interference contribution at, for example, primary MU_k can be written as:

$$\mathbf{r}_{k,int} = \mathbf{F}_k\mathbf{D}_{ks}\mathbf{G}_s\mathbf{s}_2 = \overline{\mathbf{G}}_k^T\mathbf{U}_{sk}^T\mathbf{P}_{BS_s}\mathbf{G}_s\mathbf{s}_2. \tag{8.45}$$

We parameterize the BF at the secondary BS as $\mathbf{G}_s = \mathbf{P}_{BS_s}^{-1}\widehat{\mathbf{G}}_s$ because BS_s knows the calibration filters obtained during the secondary calibration phase, then the secondary BF subspace can be designed such that $(\mathbf{U}_{sk}\overline{\mathbf{G}}_k)^T\widehat{\mathbf{G}}_s = \mathbf{0},\ \forall k$, then we choose:

$$\mathbf{G}_s \subseteq \text{span}(\mathbf{H}_I^T)^\perp$$

In this section we have shown that it is possible to extend the results obtained for spatial interweave in the simple scenario of one primary and secondary pair also to the case of multiple primary users when the transmit and receiver filter design of the primary system is IA. Here we have proved that also in this setting the calibration between primary and secondary users is not required for the calculation of the secondary transmitters. A sub-product of this analysis is that also to find an IA solution, in the pure MIMO IFC, using UL-DL duality each device only needs its own calibration coefficient, so calibration between pairs of primary users is also not required. This is another important remark because if we want to exploit channel reciprocity also for IA design we have shown that calibration is necessary and this must be done only between users of the same pair.

8.11 Numerical Results

Fig. 8.5 depicts the rate curves for the primary and secondary links. We compare the performances of a cognitive radio system where the licensed users have $N_1 = 4$ transmitting and receiving antennas. We report in the same figure the rate performances of a secondary system that have two possible antenna configurations: $N_2 = 4$ and $N_2 = 7$. The primary communication is not affected by the opportunistic transmission thanks to the proper beamformer design of the secondary devices. On the other hand the rate of the secondary is very dependent of the number of antennas. The plot shows that if the secondary users have the same number of antennas of the primary the transmission takes place only in the low SNR region because the opportunistic users can only communicate using unused modes of the primary communication. When licensed users use all the possible modes there is no room for secondary transmission and hence the rate curve goes to zero. Different is the situation of an opportunistic user that is equipped with

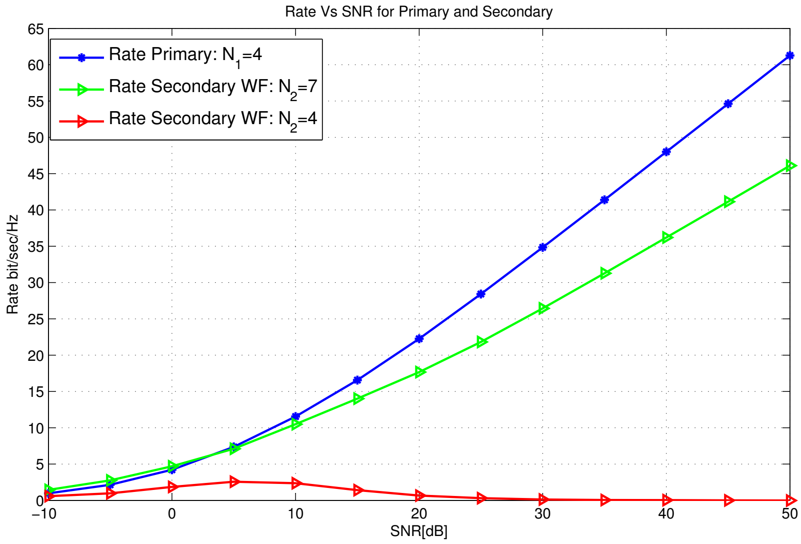

Figure 8.5: Rate Comparisons

more antennas than the licensed one. In this case the cognitive user can transmit in all SNR regimes. In particular at high SNR the secondary system is able to sustain a significant rate.

In Fig. 8.6 we report the rate curves of a cognitive system where the licensed users have $N_1 = 1$ antenna and the secondary transmitter and receiver have $N_2 = 9$ antennas. Here we want to study the effect of differential feedback on the secondary transmission. As we can see having imperfect CSIT deteriorates the performances. In particular we can see that increasing the number of iteration for the feedback acquisition corresponds to an increases of the CSIT quality and this reflects into an increase of performances. For both number of feedback iterations that we consider, $ITER_{FB} = \{5, 10\}$, there is a loss in term of multiplexing gain achieved by the secondary users. This is revealed from the loss in slope of the two red curves compared to the green one that represents the perfect CSIT case. Usually for digital feedback in order not to loose in multiplexing gain the number of bits used for feedback transmission should increase with the SNR. Here we see that to obtain the same effect the number of iterations should increase as the SNR increases. A different transmit strategy could be used, like analog feedback, that does not require iterations but only the transmission of un-quantized channel coefficients.

Finally we want to study the effect of estimation error in the blind subspace es-

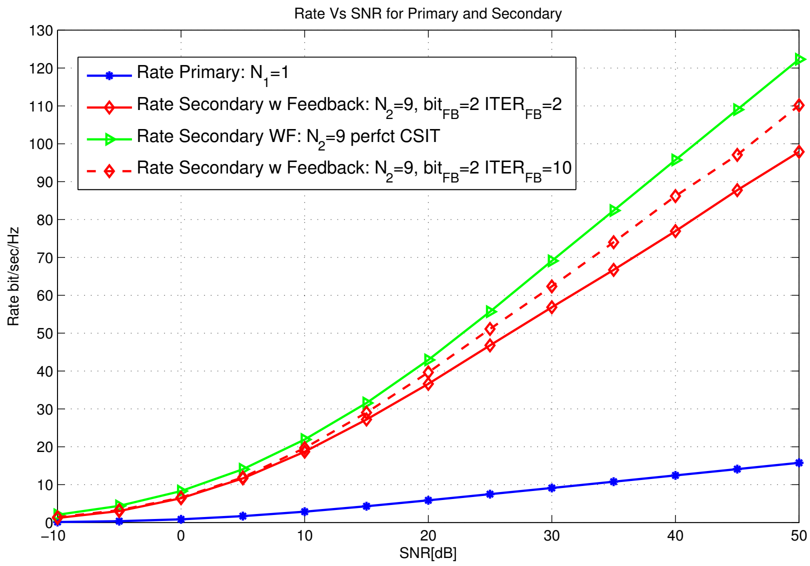

Figure 8.6: Rate comparisons with feedback

timation procedure at the secondary transmitters. In Fig. 8.7 we report the rate curves achieved by the primary user with and without error in the subspace estimation procedure at the secondary user. We can notice that if an error is present then the beamforming design at the secondary is not perfect and hence a residual interference is present in the primary signal. In figure 8.7 we also compare the rate at the primary user when different number of samples T_E is used in the subspace estimation procedure. As we were expecting the longest the estimation period is the better the estimate is, hence the rate loss decreases. We can also see that the rate loss due to signal subspace estimation affects only the SNR offset and not the multiplexing gain. This can be seen comparing the slopes of the three curves. In Fig. 8.8 we compare the theoretical rate loss, $\Delta\mathcal{R}_{theo}$ upper bound found in section 8.6, with the experimental one $\Delta\mathcal{R}_{exp}$. As we can see the upper bound that we found becomes tighter as the number of samples T_E used for the estimation procedure increases. The rate loss is not constant over the SNR because it also depends on the number of transmitted streams d_1, so it increases with the increase of d_1. When the primary transmitter uses all the available modes then $\Delta\mathcal{R}$ remains constant.

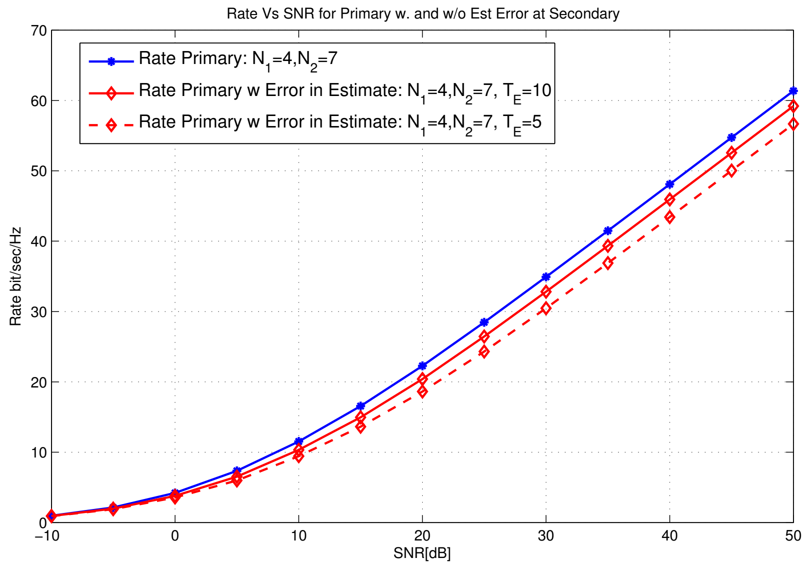

Figure 8.7: Rate comparisons with estimation error at the secondary transmitter

8.12 Conclusions

We addressed the beamformer design for a secondary communication system in a spatial interweave CR system. The practical problem of opportunistic CSI acquisition was addressed by exploiting primary signal statistics and reciprocity of the underlying TDD channel. Beamformer for secondary Tx is designed so that the secondary signal lies in the noise subspace of the primary signal. It must be noted that spatial interweaving of secondary's signal with the primary's relies on reciprocity of the TDD channel. Tx/Rx calibration is therefore mandatory.

The main contribution of this chapter is the discovery that despite the requirement for channel reciprocity between noncoperative users, calibration between crosslinks is not required. We also extended the results to a CR system where a single secondary pair coexists with multiple primary user organized as an interference channel.

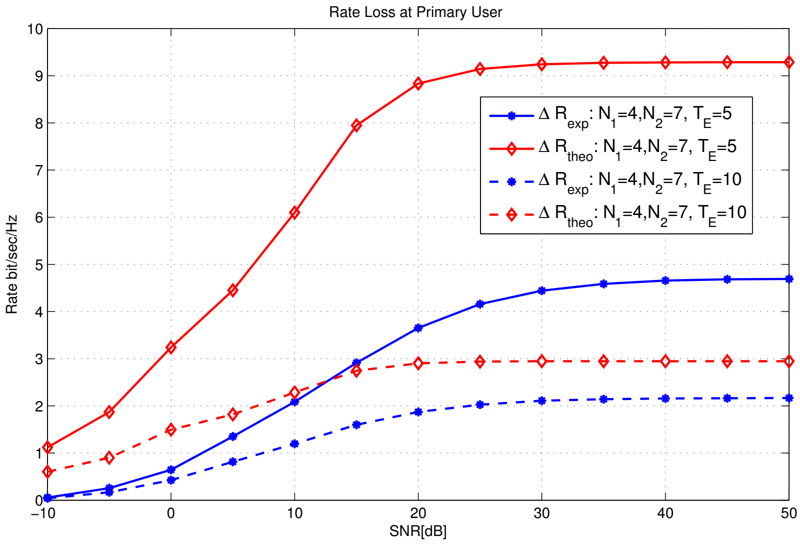

Figure 8.8: Rate loss comparisons

Chapter 9

Spatial Interweave Cognitive Radio Interference Channel with Multiple Primaries

9.1 Introduction and state of the art

In the previous chapter we have introduce the concept of *Spatial Interweave* where exploiting the unused spatial dimensions of the primary system we can setup an opportunistic communication. In this chapter we further exploit the spatial dimension in the interweave paradigm considering a secondary network modeled as a MIMO interference channel, with multiple transmit-receive pairs, and where the BF matrices are designed according to IA among secondary users while constraining the interference caused to the primary receiver to be in a subspace of reduced dimensions. A similar setting, but with only one pair of primary and secondary users, was considered in [169]. In [180] the authors extended the setting in [169] to multiple secondary pairs but constraining the number of primary users to only one pair of transmit-receive terminals, each transmitter is a single antenna device while all the receivers are equipped with multiple antennas. For this setting the authors introduced the concept of *Opportunistic Spatial Orthogonalization* (OSO) that allows the coexistence of multiple secondary pairs and primary users. The idea is based on a selection process, done at the primary receiver, that selects the secondary pair that interfere the least with the primary communication. This concept relies on the randomness and independence of the channel vectors to take advantage of the multiuser diversity. The authors of [181] also extend the setting in [169] to multiple secondary pairs with only one pair of primary transmit-receive terminals considering multi antenna terminals at both ends. Extending the results on IA feasibility in [37] they provide a condition for IA feasibility in the described CR setting. In addition an iterative algorithm, for secondary users's beamformer design is introduced. A similar setting was studied in [182] where the secondary network, model as an interference channel, coexists with a single primary users using an IA transmission strategy. Two settings are considered: SISO and MIMO. For the SISO case an IA solution based on symbol extension is introduced while for the MIMO case an iterative algorithm, based on gradient method, is used for the secondary users' beamformer design. In [183] a MIMO interference channel is considered as primary network where IA transmission strategy is used to exchange useful data. At the same time a set of secondary multi-antenna users desire to access to the primary network and this can be done only if the primary users' transmission is not modified. The authors present a condition for user admission and a set of beamforming design solutions for the secondary users with the objective of maximizing the secondary users' rate.

9.2 Contributions

In our work we consider an arbitrary number of secondary users pair that want to set up a communication in presence of L primary multi antenna receivers. The secondary network applies IA beamforming strategy constraining the interference subspace, generated at each primary receiver, to have a given dimension. With the cognitive constraint the correct number of primary users' streams can be retrieve at the primary receivers. We study the feasibility of an IA solution of the cognitive radio system under investigation based on the results presented in chapter 3. The solution obtained takes also into account the additional cognitive constraints that allow the secondary interweave communications. In addition we introduce an iterative algorithm that allows us to design the secondary users' transmit and receive filters. Introducing a fictitious zero forcing receive filter at each primary receiver we rewrite the rank constraints, on the interference subspace at the primary user, as an additional set of IA constraints. This allows us to extend the algorithm proposed in [33] to the proposed cognitive radio setting.

9.3 Signal Model

The cognitive radio setting that we consider in this chapter is depicted in Fig. 11.1. The system setting that we study can be used to model the coexistence of a set K of femto-cells with the presence of L macro-users.

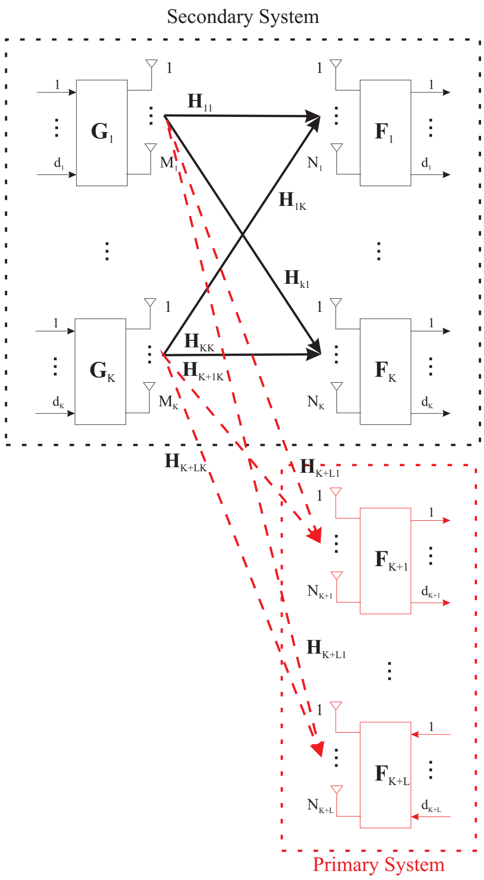

Figure 9.1: Cognitive Radio System

The secondary network is a K-link MIMO interference channel with K transmitter-receiver pairs. To differentiate the two transmitting and receiving devices we assume that each of the K pairs is composed of a secondary Base station (SBS) and a secondary Mobile user (SMU). This is only for notational purposes. The k-th SBS and its corresponding SMU are equipped with M_k and N_k antennas respectively. The k-th transmitter generates interference at all $l \neq k$ receivers. The received signal $\mathbf{y}_k$ at the k-th SMU, can be represented as

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{x}_k + \sum_{\substack{l=1 \\ l\neq k}}^{K} \mathbf{H}_{kl}\mathbf{x}_l + \mathbf{n}_k \tag{9.1}$$

where $\mathbf{H}_{kl} \in \mathbb{C}^{N_k \times M_l}$ represents the channel matrix between the l-th SBS and k-th SMU, $\mathbf{x}_k$ is the $\mathbb{C}^{M_k \times 1}$ transmit signal vector of the k-th SBS and the $\mathbb{C}^{N_k \times 1}$ vector $\mathbf{n}_k$ represents (temporally white) AWGN with zero mean and covariance matrix $\mathbf{R}_{n_k n_k}$. The channel is assumed to follow a block-fading model having a coherence time of T symbol intervals without channel variation. Each entry of the channel matrix is a complex random variable drawn from a continuous distribution. It is assumed that each transmitter has complete knowledge of all channel matrices.

We denote by $\mathbf{G}_k$, the $\mathbb{C}^{M_k \times d_k}$ precoding matrix of the k-th transmitter. Thus $\mathbf{x}_k = \mathbf{G}_k\mathbf{s}_k$, where $\mathbf{s}_k$ is a $d_k \times 1$ vector representing the d_k independent symbol streams for the k-th user pair. We assume $\mathbf{s}_k$ to have a spatio-temporally white Gaussian distribution with zero mean and unit variance, $\mathbf{s}_k \sim \mathcal{N}(0, \mathbf{I}_{d_k})$. The k-th receiver applies $\mathbf{F}_k \in \mathbb{C}^{d_k \times N_k}$ to suppress interference and retrieve its d_k desired streams. The output of such a receive filter is then given by

$$\mathbf{r}_k = \mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\mathbf{s}_k + \sum_{\substack{l=1 \\ l\neq k}}^{K} \mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l\mathbf{s}_l + \mathbf{F}_k\mathbf{n}_k$$

In this work we design the Tx and Rx filter matrix at the cognitive receiver according to interference alignment (IA). For more details on IA and the corresponding IA conditions please refer to chapter 3 and [28]. The secondary network wants to coexist with a set of L multi antenna primary mobile users (PMU). To simplify the notation we index the L PMUs from $K+1$ to $K+L$. With this notation the channel matrix between the SBS_k and the PMU_{K+l} is denoted $\mathbf{H}_{K+lk}$ and has dimensions $N_{K+l} \times M_k$, where N_{K+l} represents the number of antennas at PMU number l. The receiver filter applied at the PMU_{K+l} is denotes as $\mathbf{F}_{K+l}$, in this chapter we do not consider the optimization of the primary transmission so the receiver $\mathbf{F}_{K+l}$ is a general receiver. We only assume it involves a fixed number of transmitted stream d_{K+l}. In the following we consider the situation where the primary transmitter (PBS) is located far from the secondary system and hence no interference is caused to the secondary network from the primary communication. We constraint our attention to the scenario where each primary receiver has to suppress only the interference coming from the secondary network. Primary user receiver design is not considered here. In this chapter we do not make any assumption on the antenna configuration at the primary and secondary network but we should underline that two possible situations can occur. In particular if the number of antennas in the secondary network is grater that the number of antennas at the primary users then blind channel estimation is possible. On the other hand if the secondary network has less antennas than the primary users then the primary training signal should be explored for the channel estimation process.

9.4 Interference Alignment for Cognitive Radio System

In this section we specify the IA conditions, presented in chapter 3, for the cognitive radio system that we consider in this work. As explained before the focus of our work is to design a set of K IA beamformers and receiver filters such that the interference at each primary MU is constrained in the subspace of fixed dimensions. This means that on top of the usual IA conditions we need to impose the additional interweave constraints:

$$\mathbf{F}_k^H \mathbf{H}_{kl} \mathbf{G}_l = \mathbf{0} \quad \forall l \neq k \tag{9.2}$$

$$\mathrm{rank}(\mathbf{F}_k^H \mathbf{H}_{kk} \mathbf{G}_k) = d_k \quad \forall k \in \{1, 2, \ldots, K\} \tag{9.3}$$

$$\mathrm{rank}\left[\sum_{k=1}^{K} \mathbf{H}_{K+lk} \mathbf{G}_k\right] \leq N_{K+l} - d_{K+l} \quad \forall l = 1, \ldots, L \tag{9.4}$$

The rank requirements at the primary receiver described above can be interpreted in an alternative way. If we assume that each PMU applies a fictitious interference suppressing filter $\mathbf{F}_{K+l}$ such that it retrieves d_{K+l} interference free streams, condition (9.4) reads:

$$\mathbf{F}_{K+l}\left[\sum_{k=1}^{K} \mathbf{H}_{K+lk} \mathbf{G}_k\right] = \mathbf{0} \quad \forall l = 1, \ldots, L \tag{9.5}$$

This condition says that the *Interference Leakage* [33] at each PMU should be equal to zero. The receiver $\mathbf{F}_{K+l}$ is introduced only for the derivation of an iterative algorithm it is not the real receiver applied at the PMUs. With this modification we can interpret the entire network as an asymmetric IFC with K transmitters and $K + L$ receivers. Using the results proposed in [33] we can extend their algorithm to the CR setting that we consider here.
The objective of the algorithm is to find a set of BF and Rx filters such the the leakage interference at each receiver is minimized. If an interference alignment solution exists the residual interference will be completely suppressed.
The interference leakage at receiver k is defined as:

$$IL_k = \mathrm{Tr}\left[\ \mathbf{F}_k \mathbf{R}_{\overline{k}} \mathbf{F}_k^H\ \right] \ \forall k = 1, \ldots, K + L$$

where the interference covariance matrix at receiver k is defined as

$$\mathbf{R}_{\overline{k}} = \begin{cases} \sum_{l \neq k}^{K} \frac{P_l}{d_l} \mathbf{H}_{kl} \mathbf{G}_l \mathbf{G}_l^H \mathbf{H}_{kl}^H, & k = 1, \ldots, K \\ \sum_{l=1}^{K} \frac{P_l}{d_l} \mathbf{H}_{kl} \mathbf{G}_l \mathbf{G}_l^H \mathbf{H}_{kl}^H, & k = K+1, \ldots, K+L \end{cases}$$

P_l represents the Tx power for user l. The algorithm to determine the Tx and Rx filters is based on *Reciprocity of IA* solutions [33]. It iterates between the original and the reciprocal system. The reciprocal network can be the real dual system or a fictitious network used only in the BF design algorithm. In our case the reciprocal, dual network, is described by a dual channel $\overline{\mathbf{H}}_{kl} = \mathbf{H}_{lk}^H$, the reciprocal Tx and Rx filters are $\overline{\mathbf{F}}_k = \mathbf{G}_k^H$, $\overline{\mathbf{G}}_k = \mathbf{F}_k^H$. With those definitions the leakage interference in the reciprocal network is:

$$\overline{IL}_k = \text{Tr}\left[\ \overline{\mathbf{F}}_k \overline{\mathbf{R}}_{\overline{k}} \overline{\mathbf{F}}_k^H \ \right] \ \forall k = 1, \ldots, K \tag{9.6}$$

where the dual interference covariance matrix is defined as:

$$\overline{\mathbf{R}}_{\overline{k}} = \sum_{l \neq k}^{K+L} \frac{P_l}{d_l} \mathbf{H}_{kl} \overline{\mathbf{G}}_l \overline{\mathbf{G}}_l^H \mathbf{H}_{kl}^H \tag{9.7}$$

as we can see from the definitions above there is a difference between original and reciprocal network due to the non symmetric structure of our system. As described in [33] to find the Tx and Rx filters we need to minimize the leakage interference in the original and reciprocal system in particular for all $k = 1, \ldots, K+L$ we have to solve the following:

$$\min_{\mathbf{F}_k \mathbf{F}_k^H = \mathbf{I}} IL_k \quad \forall k = 1 \ldots, K+L \tag{9.8}$$

The optimal solution of this problem is given by the eigenvectors of $\mathbf{R}_{\overline{k}}$ corresponding to the d_k smallest eigenvalues. In a second step we solve the same problem but for the reciprocal UL system, determining the UL Rx filters at SBS_k for $k = 1, \ldots, K$. The solution is obtained solving, as done for the DL problem, the following UL problem:

$$\min_{\overline{\mathbf{F}}_k \overline{\mathbf{F}}_k^H = \mathbf{I}} \overline{IL}_k \quad \forall k = 1 \ldots, K \tag{9.9}$$

Algorithm 7 Iterative Algorithm for Cognitive IA

Fix the initial set of precoding matrices $\mathbf{G}_k,\ \forall \in k = \{1, 2 \ldots K\}$
repeat
  Find $\mathbf{F}_k\ \ k = 1 : K + L$ as the d_k eigenvector corresponding to the smallest eigenvalue of $\mathbf{R}_{\overline{k}}$
  Reverse the system and solve in the reciprocal system
until convergence

9.5 Interference Alignment Feasibility

To determine the existence of an IA solution for a given DoF allocation in our CR scenario we the translate the IA equations into a set of conditions that need to be satisfied to admit an IA solution.

$$\mathbf{F}_k \mathbf{H}_{kl} \mathbf{G}_l = \mathbf{0} \quad \forall l \neq k \tag{9.10}$$

$$\mathrm{rank}(\mathbf{F}_k \mathbf{H}_{kk} \mathbf{G}_k) = d_k \quad \forall k \in \{1, 2, \ldots, K\} \tag{9.11}$$

$$\mathrm{rank}\left[\sum_{k=1}^{K} \mathbf{H}_{K+lk} \mathbf{G}_k\right] \leq N_{K+l} - d_{K+l} \quad \forall l = 1, \ldots, L \tag{9.12}$$

The approach we adopt in this chapter is of formulating the given IA problem as finding a solution to a system of equations with limited number of variables dictated by the dimensions of the overall system. The interference aligning beamformer matrix $\mathbf{G}_k$ aligns the transmit signal of the k-th user to the interference subspace at all $l \neq k$ users while ensuring the rank of the equivalent channel matrix $\mathbf{F}_k \mathbf{H}_{kk} \mathbf{G}_k$ is d_k. The only requirement on the $(d_k \times d_k)$ matrix that mixes up the desired streams is that it be of full rank. The beamforming matrix is defined up to an arbitrary $(d_k \times d_k)$ square matrix. Thus, of the total number of $(M_k \times d_k)$ variables available for the design of $\mathbf{G}_k$ matrix reduces to $d_k(M_k - d_k)$.
Considering all the SBS the total number of variable available at the Tx side is:

$$\sum_{i=1}^{K} d_i(M_i - d_i) \tag{9.13}$$

The IA scheme essentially requires that all alignment is done at the Tx. Therefore every Tx imposes a set of constraints on the entire system whenever it transmits a stream to its Rx. An IA solution will be feasible only if the total number of variables available in the system is greater than or equal to the total number of constraints to be satisfied. Moreover, the variables should be distributed appropriately

at each of the Tx. Here we propose a method of counting the number of variables available for the design of beamformers and comparing them with the number of constraints imposed on the system.
The main idea behind our method is to convert the alignment requirements at each Rx into a rank condition of an associated interference matrix.
Because in our CR system we have a set of additional requirements for the alignment at the primary receiver we have to consider also the interference matrix that the secondary transmissions span at each PMU. For this reason we first study the problem of the alignment at the secondary network and then we consider the primary constraints.

At SMU k, the interference due to all other $(K-1)$ secondary transmitters is grouped into a $(N_k \times \sum_{l=1;l\neq k}^{K} d_l)$ matrix

$$\mathbf{H}_{IS}^{[k]}=[\mathbf{H}_{k1}\mathbf{G}_1, ...\mathbf{H}_{k(k-1)}\mathbf{G}_{(k-1)}, \mathbf{H}_{k(k+1)}\mathbf{G}_{(k+1)}, ...\mathbf{H}_{kk}\mathbf{G}_K],$$

that spans the interference subspace. The total signal-space dimension at SMU k is given by the total number of receive antennas N_k and d_k are to be reserved for the signal from the k-th PBS. This is achieved when the interference from all other transmitters lies in an independent subspace whose dimension can be at most $(N_k - d_k)$. Thus the dimension of the subspace spanned by the matrix $\mathbf{H}_{IS}^{[k]}$ must satisfy

$$\text{rank}(\mathbf{H}_{IS}^{[k]}) = r_{IS}^{[k]} \leq N_k - d_k \tag{9.14}$$

Imposing a rank $r_{IS}^{[k]}$ on $\mathbf{H}_{IS}^{[k]}$ implies imposing

$$(N_k - r_{IS}^{[k]})(\sum_{\substack{l=1\\ l\neq k}}^{K} d_l - r_{IS}^{[k]})$$

constraints at Rx k. In general the rank $r_{IS}^{[k]}$ should satisfy the following upper bound

$$r_{IS}^{[k]} \leq \min(d_{tot}, N_k) - d_k \tag{9.15}$$

where $d_{tot} = \sum_{k=1}^{K} d_k$.

At PMU $K+l$ the interference coming from the entire secondary network can be identified with an interference matrix of dimensions $(N_{K+l} \times d_{tot})$:

$$\mathbf{H}_{IP}^{[K+l]}=[\mathbf{H}_{K+l1}\mathbf{G}_1, \ldots, \mathbf{H}_{K+lK}\mathbf{G}_K].$$

To satisfy the CR constraint the interference matrix $\mathbf{H}_{IP}^{[K+l]}$ should span a subspace of dimensions

$$\text{rank}(\mathbf{H}_{IP}^{[K+l]}) = r_{IP}^{[k]} \leq N_{K+l} - d_{K+l}. \tag{9.16}$$

According to the rank requirement and the dimensions of the interference matrix $\mathbf{H}_{IP}^{[K+l]}$ satisfies the following upper bound:

$$r_{IP}^{[K+l]} \leq \min(d_{tot}, N_{K+l} - d_{K+l}) \tag{9.17}$$

Imposing a rank constraint (9.12) on the interference matrix at the PMU implies imposing

$$(N_{K+l} - r_{IP}^{[K+l]})(d_{tot} - r_{IP}^{[K+l]})$$

constraints. Once we know how to calculate the number of variable available to design the IA precoding matrices and the number of constraints that the IA solution imposes on the system under investigation we can write the final relation in (9.19).

To evaluate the existence on an IA solution it is not only important that the number of variable is enough to satisfy the constraints that the IA imposes on our system but we should study also how this variables are distributed among all the users. To consider this aspect we propose a recursive procedure based on studying IA feasibility on a subsystem built by successively adding one transmitter at a time [184]. At each step k of the recursion, (9.19) accumulates the total number of variables available for designing an IA solution in an associated sub-problem comprising of a k-link MIMO IFC in the LHS of (9.19), where $\underline{d}_k = \sum_{i=1}^{k} d_i$. In the considered subproblem only k transmitters are transmitting non-zero streams and aligning their streams into some interference subspace of all non-intended receivers. The RHS accumulates the total number of constraints at all receivers that arise due to these transmitters.

Consider a network where the secondary system is symmetric hence $M_k = N_k = N_S,\ d_k = d_S\ \forall k = 1, \ldots, K$ and a primary system with $N_{K+l} = N_P,\ d_{K+l} = d_P\ \forall l = 1, \ldots, L$. In this particular scenario we can specify a condition that the antenna distribution in the secondary network should attain to obtain the desired stream allocation satisfying, at the same time, the rank requirement at the PMU. Neglecting trivial cases as $N_P > K d_S$ and $N_S > K d_S$ we can specify condition (9.19) as follows:

$$N_S \geq \frac{K+1}{2} d_S + \frac{L d_P}{2K d_S}(K d_S - (N_P - d_P)) \tag{9.18}$$

From the condition above we can see that compared to the simple K-users MIMO IFC introducing a set of primary user interference constraint causes a reduction in terms of performances. In particular to obtain the same DoF of a traditional MIMO IFC additional $\frac{L d_P}{2K d_S}(K d_S - (N_P - d_P))$ antennas are required in order to handle the interference to the primary users. If equation (9.18) is derived for the case where the PMU does not have any noise subspace ($(N_P - d_P)$) our conditions for symmetric systems becomes similar to the equivalent condition given in [181].

$$
\begin{aligned}
\sum_{i=1}^{k} d_i(M_i - d_i) \geq & \sum_{i=1}^{k}(N_i - \underbrace{(\min(\underline{d}_k, N_i) - d_i)}_{r_{IS}^{[i]}})(\underline{d}_k - \min(\underline{d}_k, N_i)) \\
& + \sum_{i=k+1}^{K} (N_i - \underbrace{\min(\underline{d}_k, (N_i - d_i))}_{\overline{r}_{IS}^{[i]}})(\underline{d}_k - \min(\underline{d}_k, (N_i - d_i))) \\
& + \sum_{i=K+1}^{K+L} (N_i - \underbrace{\min(\underline{d}_k, (N_i - d_i))}_{r_{IP}^{[i]}})(\underline{d}_k - \min(\underline{d}_k, (N_i - d_i)))
\end{aligned} \tag{9.19}
$$

9.6 Simulation Results

In this section we present some simulation results for the cognitive radio scenario that we presented. In Fig. 9.2 we report the sum rate of the primary and secondary system. In particular there is a single primary receiver with $N_P = 2$ antennas. To calculate its rate we assume that it communicates with a primary transmitter according to a single user MIMO communication without receiving interference from the secondary communication. Thus the the primary Tx and Rx are built according to water filling like technique. In high SNR regime this will lead to a maximum of $d_P = 2$ transmitting streams. The secondary network is modeled as a $K = 2$ MIMO IFC where each secondary pairs wants to send $d_k = d_S = 1$ stream each. To satisfy interference alignment requirements and the interference rank constraints to the primary, according to (9.18), each Tx and Rx pair should be equipped with $M_k = N_k = N_S = 3$ transmitting and receiving antennas.
As we can the two curves are parallel in the high SNR regime. This means that the secondary network is able to achieve the same DoF of the primary network hence the total required number of streams has been sent. The rate curve of the secondary system is characterized by an higher SNR offset, this is due to the higher number of antennas of the cognitive devices compare to the primary users that determines an antenna gain.

9.7 Conclusions

We address the problem of BF design in the CR system where the secondary network is a K-user MIMO IFC. At the same time a set of L multi-antenna primary receivers are affected by the interference generated from the SBS transmitted sig-

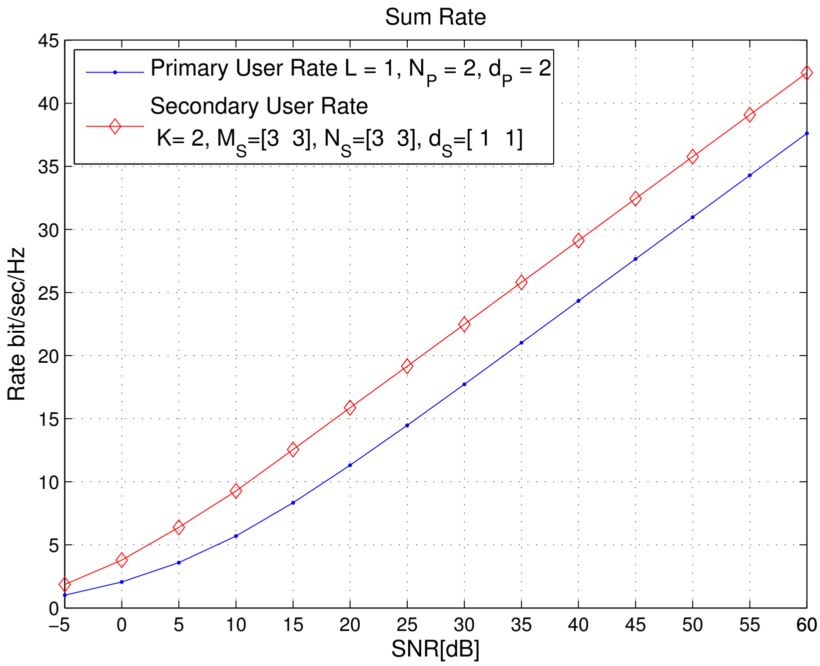

Figure 9.2: Sum rate performances

nals. The objective of our investigation is to design IA BF for the secondary network constraining the interference to the primary receiver to span a subspace of proper dimensions. To accomplish this objective we propose an iterative algorithm. In addition we present a set of IA feasibility conditions that if not satisfied immediately rule out the possibility of designing such cognitive IA beamformers.

Chapter 10

Conclusions

In this thesis we mainly tackled the transmit and receive filter design for different interfering systems: interference and cognitive radio channels.

In the first part we were mainly interested in studying the interference channel with particular focus on multi antenna systems. We first studied the beamforming problem in a MISO interference channel. Then we moved to the more complex problem of joint transmit and receive filter design in MIMO interference channel. Interference alignment and weighted sum rate approach were considered. Those approaches rely on full CSIT and CSIR that in more realistic scenarios is difficult to be achieved. We then focus on a robust approach to design optimal transmit beamformers when stochastic CSI is available at the transmit side. Finally we proposed two transmission protocols to achieve the required channel state information at both ends with the consequent optimization of the number of transmitted streams as function of the coherence time.

In the second part of the thesis the cognitive radio channel was studied. In particular we first studied the beamforming problem for a MISO underlay cognitive interference channel and then we introduced the concept of spatial interweave. There transmitters and receivers, at the secondary network, were jointly optimized to cause zero interference to the primary receivers. The proposed solutions rely on the spatial dimensions to control the interference generated to primary users.
In the following we give the conclusions and the corresponding future extensions of the main themes treated in this thesis.

- **Beamforming in the MISO Interference Channel**

We studied the problem of $\max\min$ SINR with minimum QoS constraints and per-user power constraints for a MISO IFC. We derived an iterative algorithm to solve the problem based on the equivalence between SINR balancing and the power minimization problem, that allows distributed implementation. The solution is based on the recent results on UL/DL duality for IFC. We showed that when the IFC is separable the optimum of the SINR balancing problem is achieved when all users transmit with full power. On the other hand if none of the users can apply a ZF beamformer then only one user transmits with full power. These pieces of information can be used to develop an algorithmic solution in simplified problems. The interest behind the weighted SINR balancing problem resides on the possibility to use its solutions to characterize the complete Pareto boundary of the SINR (Rate) region of a MISO IFC.

The natural extension of this work includes the analysis of the MIMO interference channel. There the main difficulty is the definition of QoS constraints in multi stream communication. The optimal approach should be to work with per-receiver rate constraints that can also include joint decoding of the received streams. This approach is indeed difficult to handle so a possible suboptimal approach would be to work in a per stream fashion assuming linear receive filters, that now should also be optimized. The solution of the $\max\min$ rate problem with per user rate constrains can, in any case, give some hints in the characterization of the rate region of the general MIMO interference channel.

- **Interference Alignment in the MIMO Interference Channel**
  We considered the problem of analytically evaluating the feasibility of an interference alignment solution for a given degrees of freedom allocation in a general K-link MIMO IFC. We introduced a systematic method to check the feasibility of an interference alignment solution for a given MIMO IFC. In addition we also showed that exploring the fact that IA feasibility is unchanged when the MIMO crosslink channel matrices have a reduced rank, we propose a new way to study the problem using numerical continuation method. Finally we observe that using real signal constellations, in place of complex constellations, transmission over a complex channel can be interpreted as transmission over a real channel of double the original dimensions. This doubling of dimensions provide additional flexibility in achieving the total DoF available in the network providing a finer granularity in the system design.

  In a recent paper [101] the authors showed that our conditions are only necessary and for some particular cases they are not conclusive. Our future perspectives on this subject include the definition of necessary and sufficient conditions. This should go in the direction to incorporate, in our test, also the interaction between interference subspaces generated at non intended receivers. This, for the moment, is not taken into account since our procedure is based on counting variables and constraints on a per-receiver basis.

- **Weighted Sum Rate maximization in the MIMO Interference Channel**
  We studied the weighted sum rate maximization for the MIMO IFC introducing an iterative algorithm to solve this optimization problem. The proposed solution converges to local optima due to the non-convexity of the MIMO IFC rate region. To reduce the probability to be trapped in one suboptimal point we introduced Deterministic Annealing. This approach allows to track the variation of the known solution of one version of the problem into the unknown solution of the desired version by a controlled variation of a parameter called temperature. In our problem the temperature is related to the inverse of the SNR. From our analysis we introduced a sum rate duality for the MIMO IFC, where the optimal BF solution resulted to be an MMSE Rx filter in the dual communication with proper UL Tx covariance matrix and dual noise variance. The dual of an interference channel is still an interference channel so duality does not simplify the solution of the problem. We can use duality principle to have better interpretation of the provided solutions. We also introduced a more robust version of the WSR algorithm when stochastic CSIT are available. The proposed approach seems to achieve the correct DoF distribution in high SNR, if the channel uncertainty scales inversely proportional with the SNR.

  The problem of WSR maximization is still not solved completely since how to get a global optimal solution has not been shown. The approach that we introduced, with deterministic annealing, can give some useful directions. It is of interest studying analytically the evolution of the cost function at different phase splits. This will help to understand how the global optima evolves as function of the SNR. In addition also some other different approach can be used in the design an algorithm for WSR maximization. A possible line of research include approaches in which power and Tx directions are optimized separately. In this way we can have a better handling on DoF allocation between different users that can probably reduce the occurrence of local optima.

- **CSI Acquisition in the MIMO Interference Channel**
  The problem of joint transmit and receive filter design, in MIMO IFC, re-

quires full CSI at both side of the communication link. To achieve this information we analyzed different transmission protocols for the necessary CSI acquisitions at each BS and MU based on training and analog feedback transmission. Our final objective was the optimization of the achievable net degrees of freedom in the network. In particular the time overhead, due to CSI acquisition, reduce substantially the time left for useful data transmission. We showed that the optimal number of streams should vary as function of the channel coherence time. In addition if the coherence time is too short we showed that, in some condition, SU-MIMO transmission is optimal.

In this chapter we only introduced the problem of DoF optimization as function of the coherence time. The results provided are given only in some particular cases in order to simplify the problem formulation. The analysis for more general antenna distribution should be developed. With these further results a more optimal CSI acquisition stage can be design in order to maximize the total achievable DoF. In addition the recent results on IA with delayed CSIT (DCSIT) can be included in the analysis to design a more optimal communication protocol that uses DCSIT for BF design until full, or perfect, CSI is achieved with training and feedback.

- **Underlay Cognitive MISO Interference Channel**
  We proposed an iterative algorithm to solve the problem of beamforming design in MISO cognitive IFC with objective the minimization of the total transmitted power. In order to meet the underlay requirements we imposed a set of interference constraints at each primary receiver. The solution proposed is based on new results on UL/DL duality for CR channel. The primary users can be seen as a set virtual primary Txs in the UL communication, thus supplementary interference links that should be considered in the secondary Rx design in the UL communication.

  The main problem that still need to be solved in this setting is the feasibility study of the problem. In particular given a set of QoS constraints at the secondary receiver, per-transmitter power constraints and the set of maximum interference level tolerated at the primary receivers we should be able to state if the given problem admits a solution or not. The other natural evolution of this problem is the extension to a MIMO setting. There the problem definition is more critical, as explained in the study of the beamforming problem for traditional MISO interference channel, due to multi stream communications.

- **Spatial Interweave TDD Cognitive Radio Systems**
  We studied the problem of opportunistic CSI acquisition and secondary beam-

former design in a spatial interweave setting posing particular attention to the practical implementation. We showed that the problem can be solved exploiting primary signal statistics and reciprocity in TDD communications. In order to take advantage of channel reciprocity, Tx/Rx calibration is therefore mandatory.

The main contribution of this chapter is the discovery that despite the requirement for channel reciprocity between non-cooperative users, calibration between crosslinks is not required. We also extended the results to a CR system where a single secondary pair coexists with multiple primary users organized as an interference channel. Also in this case the beamformer design relies on channel reciprocity. We also showed how IA design is influenced by channel calibration and how the calibration coefficients influence the Tx/Rx filter design. Then we focused on a different setting in which the secondary network is organized as a MIMO IFC. There the objective is to design IA cognitive beamformers constraining the interference subspace dimensions to the primary receivers. We developed an iterative algorithm that solves the problem and we also derived a set of feasibility conditions for IA design in CR networks.

French Summary

Les systèmes traditionnels de communication sans fil sont conçus de telle sorte que la zone de couverture est divisée en zones dites *cellules*. Dans chaque cellule une station de base (BS) assure la communication pour les utilisateurs qui se trouvent dans la cellule correspondante. Afin d'éviter ou de réduire les interférences générées par la communication dans les cellules voisines une configuration de réutilisation des fréquences a été introduit [2]. Cette approche pour traiter l'interférence empêche la réutilisation d'une ressource spectrale à l'intérieur d'un ensemble de cellules appelées *cluster*. La réduction d'interférence obtenu avec un facteur de réutilisation de fréquence se fait au prix d'une perte d'efficacité spectrale. Pour cette raison, dans la prochaine génération de cellulaires normes de communication sans fil, par exemple, Code Division Multiple Access (CDMA), un facteur de réutilisation de fréquence de un a été utilisé.
Facteur de réutilisation de fréquence l'un entrane, d'autre part, une réduction drastique de la capacité du réseau en raison de l'augmentation de l'interférence. Les performances des utilisateurs au bord de la cellule sont gravement touchés par ce configuration de réutilisation de fréquence agressif en raison de l'augmentation de l'interférence inter-cellules que l'expérience de ces utilisateurs. Pour traiter ces problèmes les systèmes actuels de communication comprennent différentes solutions de gestion des interférences. Même si les interférences venant de l'extérieur de la cellule de transmission peut être réduit en utilisant une planification minutieuse ou instaurant une coopération entre cellules voisines, ces techniques ne sont parfois pas suffisantes pour garantir un débit élevé à utilisateur au bord del la cellule . Pour cette raison, les principaux organismes de normalisation sont maintenant y compris les stratégies de coordination des interférences dans la prochaine génération de normes de communication cellulaire. Par exemple, dans les futures versions de la norme de communication cellulaire appelée Long Term Evolution Advanced (LTE-A) ces techniques sont regroupées dans ce qu'on appelle emph Coordinate multipoint transmission et réception [3]. Ces techniques sont fondées sur la coopération des stationes de base plus conscient d'interférences.

Les évolutions les plus récentes de cette particulière technique de communica-

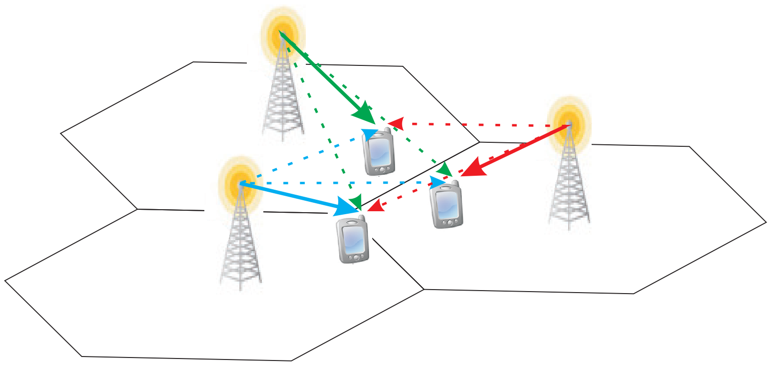

Figure 11.1: Cell-edge users problem representation

tion coopérative est ce qu'on appelle *Network* ou *Virtual* MIMO (Multiple-Input Multiple-Output), o le concept principal est d'introduire une collaboration plus étroite entre les stations de base voisines telles que chaque utilisateur est desservi par plusieurs stations de base. Ce scénario peut être considéré comme un système MIMO broadcast (BC) distribué. Pour l'introduction complète sur les résultats récents sur ce sujet s'il vous plat se référer à [4]. Pour parvenir à ce résultat tout le BS doit être connecté à un système centralisé de traitement/contrle parce que la pleine coopération au niveau de signal est nécessaire, en particulier toutes les stations de base doivent être conscients de tous les messages destinés à tous les utilisateurs du réseau. Ces techniques de coopération, dans leur mise en uvre dans LTE-A, se sont révélés apporter une amélioration significative de l'efficacité spectrale pour les utilisateurs au bord de la cellule, tandis que le gain résultant de la couverture cellulaire complète est presque négligeable [5]. Bien que très utile, au moins pour améliorer la performance de la cellule de pointe utilisateurs, ces techniques présenter quelques difficultés dans les systèmes réels. Réaliser le nécessaire collaboration et la coordination entre les différentes stations de base pose des problèmes différents dans des systèmes réels avec une capacité backhaul limitée et une latence finie.

11.1 Interference channel: Overview

Une autre façon de voir le problème les utilisateurs au bord de la cellule est de décrire mathématiquement le réglage comme un K-utilisateurs canal interférences . Dans ce système K paires de transmetteurs et de récepteurs transmettre dans la même ressource de fréquence. Chaque émetteur souhaite communiquer uniquement au récepteur correspondant, chaque communication génère des interférences avec les $K-1$ récepteurs non prévues. Ce modèle de système diffère de l'approche network MIMO parce que le niveau de coopération entre les émetteurs s'arrête à la connaissance du canal (CSI). Moins de signalisation est donc requis entre les stations de base. En particulier, selon la technique de transmission utilisée, différents degrés de connaissance du canal sont échangées entre les émetteurs.

Interférences de canal a été au centre d'intenses recherches au cours des dernières décennies, à partir du célèbre papier de Carleial [6]. D'un point de vue de théorie de l'information sa région de capacité, conçue comme tous les tuples de taux possibles qui peuvent être atteints simultanément par tous les utilisateurs, en général reste un problème ouvert et n'est pas bien comprise, même pour les cas simples. Dans [7] le résultat contre-intuitif que si l'interférence est assez forte (régime dit de fortes interférences) l'interférence ne limite pas les performances d'un canal avec deux utilisateurs. Cela montre que l'exploitation de l'interférence au lieu de le traiter comme du bruit est la stratégie optimale. Les autres résultats connus est que le traitement de l'interférence comme du bruit est optimal dans le régime d'interférence faible.

Dans [8] les auteurs montrent que même pour le système avec 2 utilisateurs, le cas le plus étudié, afin d'atteindre la capacité du système à l'intérieur d'un bit schémas de transmission très complexes sont nécessaires, qui doivent être adaptés au particulière régime d'interférence de le système. Pour parvenir à ce résultat l'auteur emploi un schéma de type Han-Kobayashi [9]. Ce système de codage est basée sur le partage de l'information transmise dans les deux utilisateurs d'une *message privé*, qui peut être décodé uniquement par le récepteur destiné, et une *message commun*, qui peut être décodé au niveau des deux récepteurs. L'innovation essentielle est ici moduler la puissance du message privé de telle sorte que le signal correspondant est reçu au niveau de bruit. De cette façon, l'interférence générée par le récepteur non prévu peut être négligée.

11.1.1 MISO Interference Channel

Avec l'introduction de plusieurs antennes sur le récepteur, les systèmes dits single-input-multiple-output (SIMO), il est possible d'augmenter la capacité obtenu [2], si le récepteur possède une connaissance canal approprié (CSIR). Ce résultat est attribuable au gain de puissance obtenu en combinant le signal reçu de toutes les antennes de réception. Un résultat similaire peut être obtenu si le transmetteur est équipé d'antennes multiples, système appelé multiple-input-single-output (MISO). Dans ce cas, si l'émetteur a des informations d'état de canal (CSIT), puis un gain de puissance est obtenue également pour les systèmes MISO. Ces simples résultats peuvent également être étendu à des systèmes plus complexes où un émetteur veut communiquer avec plusieurs récepteurs en même temps [10]. également la capacité d'un canal d'interférence a été étudiée lorsque l'émetteur ou le récepteur est équipé d'antennes multiples. Par exemple, dans [11] la capacité d'un canal interférences avec deux utilisateurs MISO/SIMO est étudié en fournissant la région de capacité pour une classe de MISO IFC dans le régime de fortes interférences. Une nouvelle limite extérieure est également prévu pour un général MISO IFC, mais la capacité d'un des canaux d'interférence plus générales, avec un nombre arbitraire d'utilisateurs, est encore un problème ouvert. Ensuite, des approches plus pratiques ont été ajoutés pour optimiser les performances du système en utilisant des émetteurs et des récepteurs linéaires. Dans [12, 13] les beamformer pour un K-user MISO IFC sont déterminés à réduire la puissance d'émission totale imposant un ensemble de qualité de service par utilisateur (QoS) à chaque récepteur. [14, 15] traiter le problème de maximiser minimum Signal to Interference plus Noise ratio (*max min SINR*) pour un MISO IFC.

Dans [16, 17, 18] solutions distribuées pour le problème de conception du BF sont étudiées, dont l'objectif principal est de réduire l'échange de signalisation entre des paires d'utilisateurs. Certaines de ces techniques utilisent des concepts de la théorie des jeux pour décrire les algorithmes proposés.

Une autre ligne de recherche se trouve dans [19, 20, 21, 22] dont l'objectif est la caractérisation de la région un taux de MISO IFC o un traitement linéaire est utilisée au cté émetteur. La région étudiée est définie comme l'ensemble des tuples de taux qui peuvent être obtenu simultanément par les paires de transmission. L'objectif principal de cette analyse est la définition de la frontière de Pareto de la région de capacité, définie comme l'ensemble des points o la performance d'un utilisateur ne peut pas être incrémenté sans réduire les performances des autres utilisateurs.

11.1.2 MIMO Interference Channel

Avec la découverte que l'utilisation de plusieurs antennes sur l'émetteur et le récepteur peut apporter une augmentation significative du débit du système [23], la communication multiple-input multiple-output-(MIMO) a été largement appliquée à tous les systèmes de communication, y compris l'interférence canal.

Interference Alignment

Comme nous l'avons déjà vu la difficulté de trouver la capacité d'un canal d'interférence est un problème difficile qui n'a pas été encore été entièrement résolue. Le problème devient encore plus complexe avec l'introduction de paires MIMO dans le réseau d'interférence. Pour simplifier le problème une approche différente a été introduite récemment. L'objectif devient alors le rapprochement la capacité à un haut rapport signal sur bruit (SNR). Dans ce régime, la courbe des sum rate peut être complètement décrite à l'aide du facteur Prelog, aussi appelées degrés de liberté (DoF):

$$C(\rho) = d\log(\rho) + o(\log(\rho))$$

o $C(\rho)$ représente la capacité de somme, ρ est le SNR et d est le facteur pre-log. Il peut être interprété comme le nombre de dimensions sans interférence disponibles dans le système. Il peut également être défini comme suit:

$$d = \lim_{\rho\to\infty} \frac{C(\rho)}{\log(\rho)}$$

Il a été introduit dans [24] pour un lien MIMO avec un seul utilisateur et il est devenu immédiatement instrumentale aussi pour des systèmes plus complexes. Pour un 2-user MIMO IFC le DoF réalisable a été étudiée dans [25], pour le canal des interférences avec d'autres utilisateurs l'utilisation de *Alignement interférences* (IA) devient instrumentale [26, 27]. Dans [28] les auteurs ont démontré la possibilité d'atteindre un facteur capacité de Prelog de $K/2$ dans un K-user canal interférence SISO, puis la moitié du DoF d'un réseau sans interférence peut être atteint. L'idée principale derrière l'alignement d'interférence est de traiter le signal d'émission (flux de données) à chaque émetteur, de manière à aligner tous les signaux non désirés à chaque récepteur dans un sous-espace de dimension convenable.

Le canal interférence MIMO est plus difficile à manipuler et quelques résultats récents sur DoF pour ce cas sont rapportés dans [29, 30]. Même si IA a la propriété prometteuse de maximiser le DoF, une expression analytique pour les filtres BF n'est pas connue en général. Dans [31, 32] une solution est proposée pour K-user MIMO IFC o chaque paire d'utilisateurs est équipé de $N = K - 1$ antennes. Pour trouver une solution IA pour des configurations plus générales algorithmes itératifs doit être utilisé [33, 34, 35, 36], o fonctions de cot différentes sont utilisées pour déterminer un ensemble de IA BF utilisant des solutions numériques. Ces algorithmes peuvent être également utilisé pour évaluer l'existence d'une solution IA à travers des simulations. L'existence d'une solution IA pour MIMO IFC a été étudié dans plusieurs documents [37, 38, 39] o différentes séries de conditions doivent être satisfaites par un K-user MIMO IFC pour admettre une solution IA.

Sum Rate Maximization

L'objectif de la transmission IA est de maximiser le DoF qui représente une bonne approximation de la courbe des taux à haut SNR. Le même concept ne peut pas être appliquée à des régimes de SNR moyenne et basse, pour cette raison IA manifeste mauvaises performances dans ces SNR régimes . C'est pourquoi des approches différentes ont été proposées pour la conception de transmetteur et de récepteurs dans un K-user MIMO IFC. Une approche possible est la maximisation du sum rate. Dans le travail séminal [40] les auteurs ont noté que la capacité du réseau en général n'est ni convexe ni concave fonction des matrices de covariance de transmission et donc son optimisation est un problème difficile. L'approche de théorie des jeux a été utilisé dans [41] pour étudier la modélisation de le problème MIMO IFC comme un jeu non coopératif. La solution proposée est prouvé d'atteindre un équilibre de Nash, mais ce point peut être très loin du point sum rate optimale. Le problème de maximisation du weighted sum rate (WSR) a été étudiée dans certains documents récents [42, 43, 44]. Dans [42] le seul flux MIMO IFC est étudiée, en proposant un algorithme itératif pour la maximisation de WSR. Une approche différente est utilisée dans [43] o le problème est résolu en utilisant second order cone program (SOCP). Enfin, dans [44] la maximisation du WSR est atteint, dans un canal interfering broadcast, élargissant les résultats proposés pour un BC dans [45]. La solution repose sur le lien entre la maximisation WSR et la minimisation de weighted sum mean squared error (WSMSE).

Channel State Information Acquisition

Pour déterminer une série de beamformer qui maximise le DoF à haut SNR, en utilisant IA, ou pour maximiser le débit total, en utilisant les approches décrites ci-dessus, diverses formes deinformations d'état de canal (CSI) sont obligatoires. Dans la plupart des cas, la CSI aux deux terminaux, l'émetteur et le rècepteur, est nécessaire pour réaliser la bonne conception conjointe des filtres d'émission et de rèception. Ceci est généralement acquises à l'aide d'une phase de training et une phase de rètroaction entre émetteurs et récepteurs. Le probléme de comme la rétroaction influence la conception IA beamformer a été étudiée dans [46, 47, 48]. Dans [46, 47], en utilisant une rètroaction du canal quantifié, il est montré que le gain de multiplexage complet peut être atteint que si le débit de rétroaction échelles suffisamment rapide avec le SNR. Les auteurs de [48] présenter la r̀action analogique pour l'acquisition de CSIT. Ils montrent que l'utilisation de rétroaction analogique, pour l'acquisition de la CSIT et la conception de IA beamformers , n'encourt aucune perte de gain de multiplexage si la puissance de rétroaction échelles avec le SNR.

Dans [49] un modèle staggered block fading channel est la seule hypothése nécessaire pour atteindre IA. Le gain résultant de multiplexage est toutefois beaucoup plus faible que dans le cas de la pleine CSI. Ces techniques sont maintenant connus par les termes *delayed CSIT* (DCSIT) ou *IA rétrospective*. Le probléme de l'étude de la DoF maximale rèalisable en utilisant DCSIT a récemment attirè beaucoup d'effort de recherche. [50, 51] a introduit un nouveau protocole de transmission qui maximise le DoF réalisable dans un canal BC. Dans [52] les auteurs étendent les résultats de [50] aux MISO IFC avec deux utilisateurs.

11.2 Cognitive Radio

Les organismes de réglementation du spectre, depuis leur fondation au début du 20e siècle, ont alloué des parties du spectre des fréquences aux différents services sans fil de façon fixe et statique. Cela a été fait avec l'objectif deéviter / réduire la possibilité de générer des interférences. Avec la croissance rapide des services sans fil la politique d'allocation des fréquences rigide, utilisée jusqu'à présent, il a été démontré être très inefficace en terme d'utilisation du spectre. En plus la quasi-totalité des bandes de fréquences ont déjà été attribué, Fig. 11.2. La rareté du spectre par conséquent a un effet significatif sur la communication sans fil fournisseurs de services puisque aujourd'hui les bandes de fréquences sont attribuées au plus offrant dans les ventes publiques, l'acquisition de fréquence représente l'un des cots les plus importants pour les opérateurs. Dans une récente campagne de mesure [53], menée par la Federal Communications Commission (FCC) aux Etats-Unis, a montré que l'utilisation du spectre est généralement concentrée sur certaines bandes de fréquences, alors qu'une quantité significative de les bandes licencees reste inutilisé ou sous-utilisé pour 90% du temps. Ce problème a inspiré le travail pionnier [54] o la notion de *Cognitive Radio* (CR) a été mis en place. Selon ce paradigme de communication, développée dans [55], un système radio cognititve est défini comme un ensemble de dispositifs intelligents qui sont conscients de l'environnement en adaptant leurs paramètres de communication avec l'objectif deune communication fiable et plus efficiente utilisation du spectre. Le scénario le plus courant est composé d'un ensemble d'utilisateurs secondaires, qui représentent les utilisateurs cognitives, qui veulent coexister avec un ensemble d'utilisateurs primaires, les porteurs du spectre existants. La caractéristique la plus importante des dispositifs cognitifs, comme son nom l'indique, est la capacité d'apprendre de l'environnement et de réagir correctement. Ce problème a donné naissance à une ligne intense de recherche dont l'objectif principal est d'étudier comment il est possible de comprendre si, dans une bande de fréquences déterminée une transmission a lieu ou non. Cela va sous le nom de détection du spectre, se référer à [56] et les référence à l'intérieur pour un examen approfondi des contributions majeures. Un de la première tentative de rendre les principes CR une réalité était la norme IEEE 802.22, qui avait pour objectif d'utiliser les espaces blancs de la télévision pour développer un système de communication sans fil pour les réseaux régionaux (WRANs). En 2009, une nouvelle proposition de la norme IEEE 802.11af, considérée comme modifiant à la fois les couches PHY et MAC

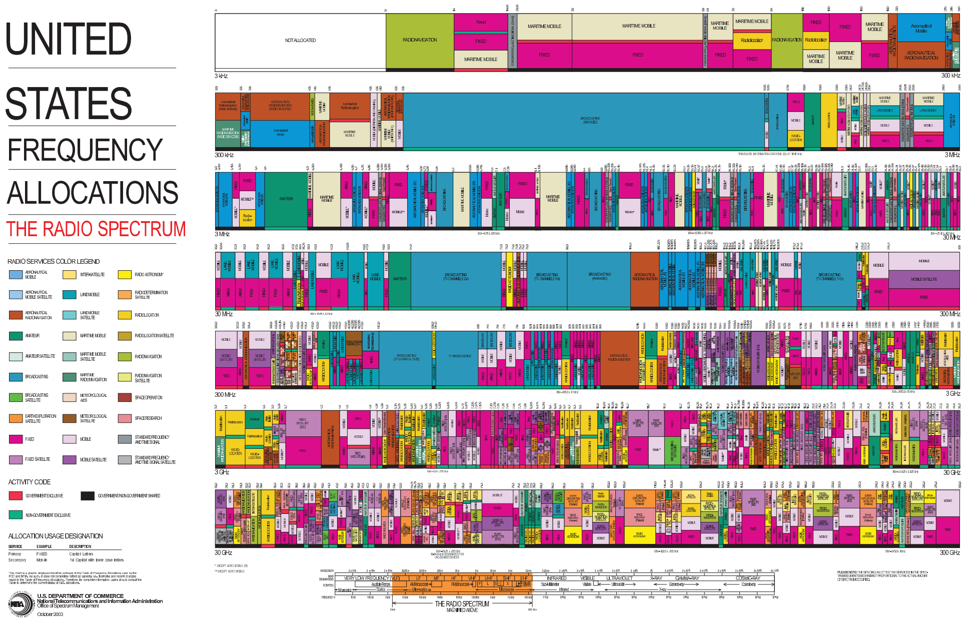

Figure 11.2: US frequency allocation chart, *www.ntia.doc.gov/osmhome/allochrt.pdf.*

du 802,11 à utiliser espace blanc de la télévision . Pour plus d'informations sur les normes CR référer à [57].

D'un point de vue théorique, les différents paradigmes de communication cognitifs ont été mis en place en fonction de la quantité d'informations échangées entre les utilisateurs primaires et secondaires et les contraintes imposées à la communication secondaire. Dans [58] les scénarios de communication suivants radio sont introduits: *Overlay*, Underlay et *Interweave*.

Overlay Paradigm

Overlay CR est une technique de coopèration dans lequel les signaux secondaires sont destinès à compenser toute dègradation qu'ils pourraient causer aux communications primaires, nècessitant un partage des connaissances des dictionnaires et des schèmas de modulation. Avec ce supplèment d'informations certaine forme de coopèration asymètrique peut être ètablie. Par exemple l'utilisateur opportuniste peut diviser sa part de rate consacrant sa part de sa puissance d'èmission à diffuser aussi le message dèsignè à l'opèrateur primaire. Avec les ressources restantes, il transmet son message pour une communication privèe avec le rècepteur secondaire. D'autres stratègies d'encodage [58] peut être utilisè pour règler une communication overlay comme du dirty paper coding (DPC) ou du rate splitting. Dans ce scènario la communication primaire n'est pas prèjudice ou pourrait même etre amèliorè à la suite d'un gain de relais. Ce règlage CR peut ègalement être lu comme une combinaison de canaux BC et d'interfèrence avec ensembles de messages dègradès [59]. Même si le Overlay CR est le plus ètudiè d'un point de vue thèorique, la capacitè d'un tel système n'est pas encore connu en gènèral. Il est connu que dans certains règimes particuliers. Dans [60] le règime de *faible interfèrence* est ètudiè, les auteurs ont montrè que, dans ce règime, o le lien entre le Tx cognitif et Rx primaire est faible, la capacitè du canal overlay est rèalisè en utilisant une combinaison de DPC et codage de superposition. Le mode cognitif exploite la connaissance du message primaire pour coder son message de telle sorte qu'il soit reçu à rècepteur cognitive libre l'interfèrence. En même temps, en utilisant un codage de superposition, il utilise une partie de sa puissance disponible pour transmettre ègalement le message primaire et la puissance rèsiduelle est utilisèe pour la transmission cognitive.

Dans le règime inverse, *forte interfèrence* vu à niveau des deux rècepteurs, [61] constatè que la capacitè du canal est atteint à l'aide de superposition de codage dans l'èmetteur cognitive.

Underlay Paradigm

Underlay CR permet la coexistence d'un réseau primaire (généralement sous licence) et un secondaire (cognitive), contraignant l'interférence causée par émetteurs secondaires sur les récepteurs primaires d'être sous un certain seuil, généralement appelé *Interference temperature constraint* [55]. Pour atteindre ces contraintes des interférences différentes techniques peuvent être utilisées variant de procédés de codage à l'utilisation de la dimension spatiale (multiantenna systems). Le problème de étudier la région de capacité, de différents systèmes, ce qui limite la puissance reçue à certains utilisateurs a été explorée dans [62], ces contraintes de modifier sensiblement la structure du problème. Dans [63] un réglage de la radio cognitive Underlay est étudiée dans des environnements avec du fading. Il est démontré qu'un gain de capacité importante peut être réalisée par l'utilisateur opportuniste dans les canaux touchés par fading sévère, car la probabilité que la liaison trans primaire-secondaire pour être en fade est non négligeable et donc le système secondaire peut atteindre un rate de plus en plus importante sans interférer de façon significative avec la communication primaire. Dans le paradigme Underlay contraignant les interférences au niveau des récepteurs primaires est l'objectif principal des émetteurs cognitives. Fournir aux utilisateurs cognitives des antennes multiples améliore la capacité de contrler les interférences générées au niveau des récepteurs primaires, pour cette raison, le problème de conception de beamforming en systèmes cognitifs a été l'objet d'intenses recherches ces dernières années. [64] étudie le problème de la maximisation du rate de l'utilisateur secondaire contrler l'interférence causée au niveau des récepteurs primaires. Une autre ligne de recherche se concentre sur la satisfaction d'un minimum de qualité de service nécessaire aux utilisateurs cognitives dans un scénario underlay [65, 66, 67].

Là la réseau secondaire est toujours modélisé comme un canal BC qui souhaite communiquer, en présence d'un ensemble de récepteurs primaires. Dans [68, 69] l'objectif était d'optimiser le débit total du réseau secondaire, modélisé comme un canal d'interférence, sous reçus les contraintes de puissance d'interférence à des utilisateurs primaires.

Interweave Paradigm

Enfin, Interweave (IW) CR exploite les ressources de communication inutilisés, appelé *white spaces*, du système primaire dans un mode opportuniste. Dans ce paradigme de communication, la transmission secondaire peut prendre placer que si elle ne cause pas d'interférence à l'utilisateur principal. Le ressources primaires inutilisée peut prendre du temps, de la fréquence ou, comme l'a récemment mis en place, l'espace.

Le problème de la radio cognitive a été étudiée également dans une perspective de théorie des jeux dans [70], les auteurs proposent un algorithme décentralisé, basé sur iterative water filling, afin d'optimiser les performances du système secondaire. Une description analytique en profondeur dans le cadre de la théorie des jeux est également disponible. Dans [71] un aperçu détaillé de la théorie des jeux et son application au problème de CR est fourni.

Dans ce paradigme de communication l'utilisation de plusieurs antennes est encore plus bénéfique que dans le Underlay. Un premier document á étudier la dimension spatiale dans les systèmes CR était [64]. Quelques tentatives de faire le CR pratique peut être trouvé dans [72, 73]. Les auteurs proposent un schéma de transmission o la communication primaire est exploitée dans le but d'apprendre l'environnement et de concevoir correctement les beamformers á les utilisateurs secondaires. Dans l'analyse proposée les erreurs d'estimation de canal secondaire sont prises en compte dans les conception du BF secondaire . L'interférence causée au récepteur secondaire, cause d'une communication primaire, est réduite introduisant un bon filtre de réception à récepteurs secondaires.

Dans [74] une nouvelle approche pour établir une transmission cognitive a été proposé pour des canaux sélectifs en fréquence. Les auteurs ont proposé d'appliquer un précodeur Vandermonde comme filtre d'émission à l'utilisateur cognitive, pour cette raison, il est appelé Vandermonde Frequency Division Multiplexing (VFDM). Le précodeur Vandermonde est construit en utilisant les L roots de les canal L-tap qui connecte les émetteur cognitive avec les récepteur primaire. Avec cet émetteur l'interférence au récepteur primaire est complètement zeroforced. Cette approche a l'avantage que la coopération ne est pas nécessaire entre le primaire et le secondaire pour configurer une communication Interweave.

11.3 Thesis Outline and Contributions

Cette thèse est divisée en deux parties principales. Première partie traite du canal d'interférence, o nous étudions d'abord le problème de beamforming design dans un canal d'interférence MISO introduisant des principes de dualité, qui peut être considéré comme une extension au IFC du les résultats obtenus pour le canal broadcast. Alors le problème de $\max\min$ SINR beamforming design est adressée. Dans les chapitres suivants, nous introduisons plus d'antennes aussi du cté du récepteur, nous étudions le problème de la conception conjointe des filtres de transmission-réception d'interférence dans le canal MIMO. Nous étudions l'alignement d'interférence, avec un accent particulier sur l'analyse de faisabilité et sur la maximisation du sum rate. Enfin, le problème de l'acquisition des informations d'état de canal, pour résoudre les problèmes précédemment introduites, est étudiée à l'aide de rétroaction analogique.

Partie II traite des scénarios de radio cognitive. Dans un premier temps, nous étudions le problème de beamforming design dans la conception underlay MISO cognitive IFC pour résoudre le problème de minimisation de puissance sous contraintes de puissance par l'utilisateur et de limiter le montant maximum des interférences générées aux utilisateurs primaires. Puis, dans les chapitres suivants, nous introduisons le concept de *Spatial Interweave*. Dans le chapitre 8, nous décrivons toutes les phases de transmission nécessaires pour concevoir de façon opportuniste le beamformers secondaire dans les communications TDD. Pour exploiter réciprocité du canal, en raison de la transmission TDD, nous considérons également le problème de calibration et la façon dont cette opération supplémentaire influe sur le problème de conception. Nous découvrons que le calibrage entre utilisateurs non coopératifs n'est pas nécessaire ce qui implique que le réglage CR spatial interweave est possible dans la pratique, sans aucune coopération entre les utilisateurs primaires et secondaires. Le réglage simple avec une paire primaire et secondaire on est étendu à de multiples paires secondaires et primaire dans le chapitre 9. Dans ce chapitre, le problème de conception IA est étudiée dans un contexte o le problème de faisabilité est également introduite et étudiée fournissant un ensemble de conditions de faisabilité.

Dans les paragraphes suivants, nous donnons un bref aperçu de la thèse décrivant le contenu des différents chapitres soulignant leurs contributions.

Chapter 2 - MISO Interference Channel

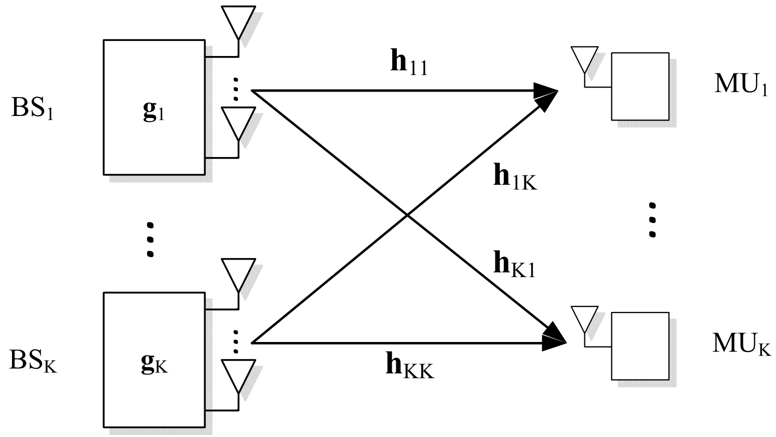

Figure 11.3: MISO Interference Channel

Dans ce chapitre, nous commençons à introduire certaines principes de dualité Uplink-Downlink(UL-DL), initialement introduit pour le canal BC en les adaptant à MISO IFC, Fig. 11.3. Puis la dualité UL-DL est utilisée pour la solution du problème d'équilibrage de weighted SINR (WSINR) pour MISO IFC avec des contraintes de puissance individuelles. Nous introduisons une nouvelle algorithme itératif qui permet de résoudre le problème d'équilibrage du WSINR lorsqu'une seule contrainte de puissance est active. Ensuite, nous proposons un algorithme itératif qui permet de résoudre le problème de façon décentralisée o rien ne peut être dit sur le nombre de contraintes de puissance active. L'algorithme permet de résoudre le problème en utilisant une séquence de problèmes de minimisation de puissance avec un ensemble approprié de contraintes de QoS. L'algorithme proposé peut être utilisé pour trouver tous les points de la frontière de Pareto de la région de capacité de MISO IFC, Fig. 11.4.

Les contributions à la recherche de ce chapitre ont été publiés dans

- F. Negro, M. Cardone, I. Ghauri, and D. T. M. Slock, "SINR balancing and beamforming for the MISO interference channel," *in Personal Indoor and Mobile Radio Communications (PIMRC), 2011 IEEE 22st International Symposium on, Sept. 2011.*

- F. Negro, I. Ghauri, and D. T. M. Slock, "On duality in the MISO interference channel," *in Signals, Systems and Computers (ASILOMAR), 2010 Conference Record of the Forty Fourth Asilomar Conference on, Nov. 2010, pp. 2104 -2108.*

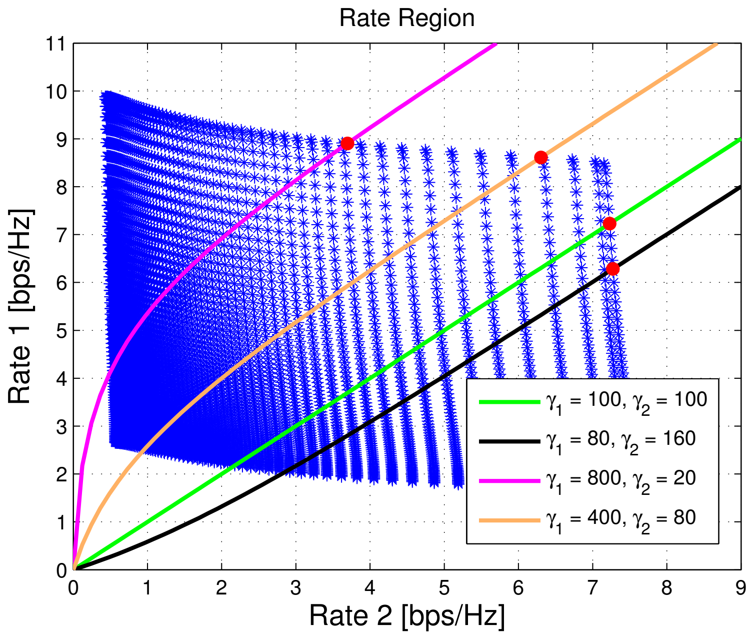

Figure 11.4: Rate region for a 2-user MISO IFC for $\sigma_k^2 = 30$ dB

Chapter 3 - Interference Alignment Feasibility for MIMO interference channel

L'objectif de ce chapitre est l'étude de faisabilité de solutions d'alignement d'interférence pour un canal MIMO IFC. Nous présentons d'abord le modèle du système général d'un K-user MIMO IFC, Fig. 11.5 qui sera également utilisé dans les chapitres suivants. Ensuite, nous proposons une méthode systématique pour vérifier la faisabilité de solutions IA pour une allocation DoF arbitraire. Nous validons l'approche proposée en utilisant des exemples numériques, en comparant le résultat de notre vérification de faisabilité avec la propriété de convergence d'un algorithme itératif pour déterminer des solutions IA. Nous discutons de la dualité alignement d'interférence et l'interprétation des IA comme une contrainte comprimé SVD.

Les résultats présentés dans ce chapitre sont également publiés dans les papiers suivants:

- F. Negro, S. Shenoy, D. T. M. Slock, and I. Ghauri, "Interference alignment limits for K-User frequency-flat MIMO interference channels," *in Proc. European Signal Proc. Conf. (Eusipco), Glasgow, Scotland, Aug. 2009.*

- F. Negro, S. P. Shenoy, I. Ghauri, and D. T. M. Slock, "Interference align-

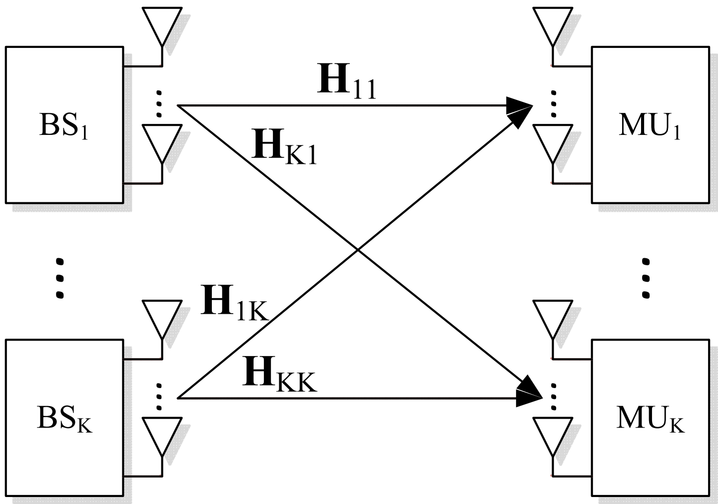

Figure 11.5: MIMO Interference channel

ment feasibility in constant coefficients MIMO interference channel," *in Proc. 11th IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC 2010), June 2010.*

- F. Negro, I. Ghauri, and D. T. M. Slock, "Deterministic annealing design and analysis of the noisy MIMO interference channel," *in Information Theory and Applications Workshop (ITA), 2011, feb. 2011, pp. 1 -10.*

Chapter 4 - Sum rate maximization for the noisy MIMO interference channel

Dans ce chapitre, nous introduisons la maximisation du WSR pour un canal interférences MIMO. Nous proposons une nouvelle algorithme itératif basé sur l'extension de la relation entre la maximisation du WSR et la minimisation de la somme pondérée erreur quadratique moyenne (WMSE). Ensuite, nous spécifions l'algorithme proposé lorsque le WSR est maximisée sous un approach per-stream. L'approche per-stream nous aide à mettre en place une dualité WSR pour le MIMO IFC o la transmission optimale est un filtre récepteur MMSE dans un dual UL communication avec une matrice de covariance d'émission approprié et la variance du bruit dual. Afin de réduire la possibilité de converger vers une solution optimale locale, nous introduisons une nouvelle approche basée sur Deterministic Annealing. Enfin, nous décrivons comment optimiser le WSR à haut SNR. Quelques résultats de simulation sont fournis pour valider l'algorithme proposé numérique Fig. 11.6.

Dans les documents suivants sont reportées la recherche décrite dans ce chapitre:

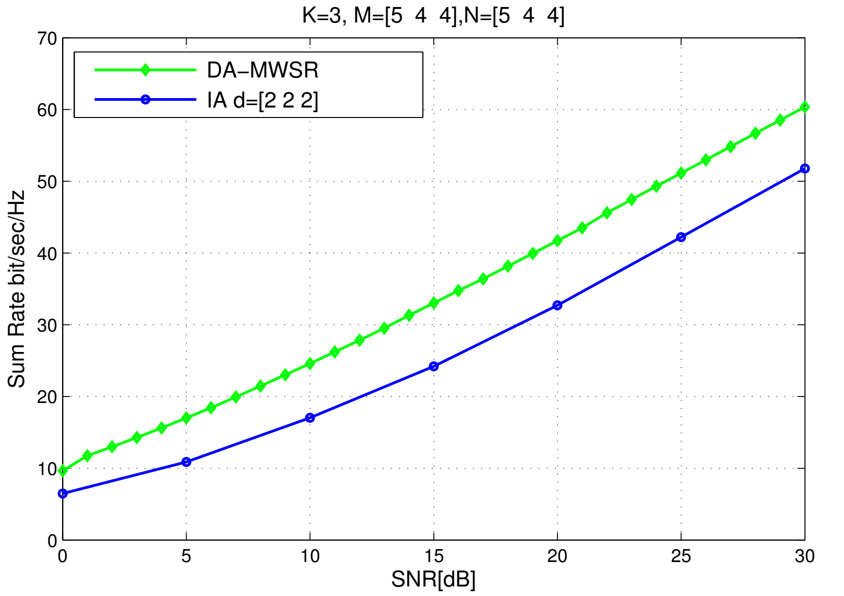

Figure 11.6: WSR for $K = 3$, $M_1 = N_1 = 5,\ M_i = N_i = 4,\ i = 2, 3$, $d_k = 2\ \forall k$

- F. Negro, S. Shenoy, I. Ghauri, and D. T. M. Slock, "On the MIMO interference channel," *in Information Theory and Applications Workshop (ITA), 2010, 31 2010-Feb. 5 2010, pp. 1 -9.*

- F. Negro, S. Shenoy, I. Ghauri, and D. T. M. Slock, "Weighted sum rate maximization in the MIMO interference channel," *in Personal Indoor and Mobile Radio Communications (PIMRC), 2010 IEEE 21st International Symposium on, Sept. 2010, pp. 684 -689.*

- F. Negro, I. Ghauri, and D. T. M. Slock, "Deterministic annealing design and analysis of the noisy MIMO interference channel," *in Information Theory and Applications Workshop (ITA), 2011, Feb. 2011, pp. 1 -10.*

- F. Negro, I. Ghauri, and D. T. M. Slock, "Optimizing the noisy MIMO interference channel at high SNR," *in Communication, Control, and Computing (Allerton), 2010 48th Annual Allerton Conference on, 29 2010-Oct. 1 2010, pp. 254 -261.*

Chapter 5 - Sum Rate Maximization with Partial CSIT via the Expected Weighted MSE

Dans cette partie de la thèse, nous nous concentrons sur la conception robuste du beamformers pour un canal MIMO IFC avec l'objectif de maximiser le sum rate. Nous supposons que chaque émetteur a des informations d'état de canal (CSI) stochastique, tandis que le récepteur a idéal CSI. La solution proposée pour la conception robuste de beamformers est basé sur la relation entre la WSR et WMSE mis en place pour le canal MIMO interférences dans le chapitre 4. Ici, les filtres optimaux de beamformer sont obtenus à partir de la minimisation de la somme des WMSE moyenne, puis un algorithme itératif est introduit pour résoudre le problème. Les performances de la solution proposée est finalement validée numériquement, Fig. 11.7. Comme nous nous attendions à maximiser la WSR surperforme la solution IA aussi pour le cas CSIT partiel. D'autre part l'utilisation des connaissances de canal partiel entrane une perte en terme de SNR offset, mais pas en terme de pente. Nous pouvons donc conclure que l'algorithme proposé permet d'obtenir, avec la CSIT partielle, le même DoF de IA avec CSIT parfait si la qualité CSI augmente avec le SNR. Les résultats décrits dans ce chapitre sont

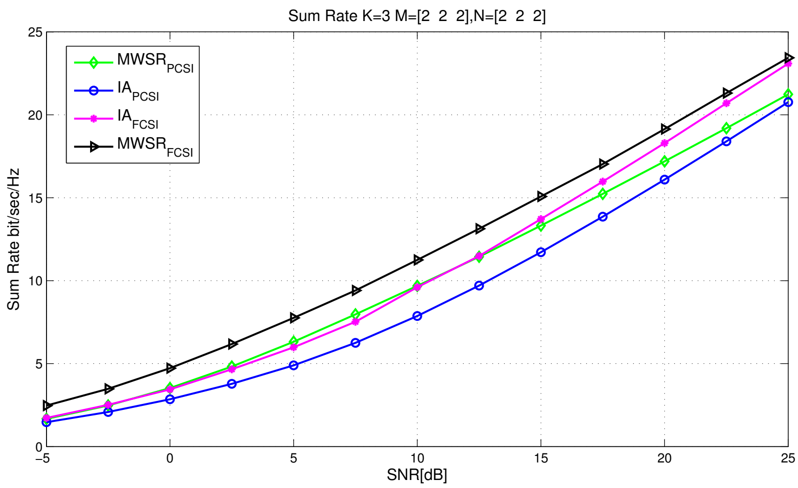

Figure 11.7: Sum Rate comparisons for $K = 3$ $M_k = N_k = 2$, $\forall k$

publiés dans:

- F. Negro, I. Ghauri, and D. T. M. Slock, "Sum Rate maximization in the Noisy MIMO Interfering Broadcast channel with partial CSIT via the expected weighted MSE," *in Wireless Communication Systems (ISWCS), 2012*

IEEE 4th International Symposium on 28-31 August 2012.

Chapter 6 - CSI acquisition in the MIMO interference channel via analog feedback

Toutes techniques conjoints de conception de filtres d'émission-réception présentés dans cette thèse, la maximisation WSR et IA, nécessitent une certaine forme de CSI aux deux terminaux. Dans ce chapitre, nous étudions le problème de l'acquisition de CSI à transmettre et recevoir cté l'introduction de deux protocoles de transmission qui sont basés sur training du canal st du retour analogique (FB). Nous étudions aussi le problème de l'optimisation du sum rate, en se focalisant en particulier sur les degrés de liberté (DOF), en fonction du temps de cohérence. Cette approche nous permet d'optimiser les paramètres du système, le nombre d'antennes d'émission et les flux transmises, compte tenu de la surcharge de l'acquisition de CSI. Dans les articles suivants sont rapportés les résultats présentés dans ce chapitre:

- F. Negro, U. Salim, I. Ghauri, and D. T. M. Slock, "The noisy MIMO interference channel with distributed CSI acquisition and filter computation," *in Signals, Systems and Computers (ASILOMAR), 2011 Conference Record of the Forty Fifth Asilomar Conference on, 2011.*
- F. Negro, D. T. M. Slock, I. Ghauri, "On the noisy MIMO interference channel with CSI through analog feedback," *in Communications Control and Signal Processing (ISCCSP), 2012 5th International Symposium on (ISCCSP), 2012 , pp. 1 - 6*

Chapter 7 - Beamforming for the Underlay Cognitive MISO Interference Channel

Ici, nous nous concentrons sur le problème de la conception d'un beamformer pour réseau CR modélisée comme une canal d'interférence MISO, Fig. 11.8. Comme nous supposons de travailler dans un cadre uderlay nous avons encore imposer un ensemble de contraintes de puissance d'interférence au niveau des récepteurs primaires. D'étendre les résultats sur UL-DL dualité a la radio cognitive nous concevons le beamformers sur les émetteurs secondaires afin de minimiser la puissance totale émise. Nous proposons un algorithme itératif qui permet de résoudre efficacement le problème de minimisation de puissance, sur le réseau secondaire, tout un ensemble de contraintes d'interférence sont imposées sur les récepteurs primaires. Les contributions à la recherche dans ce chapitre sont présentés également dans le document suivant:

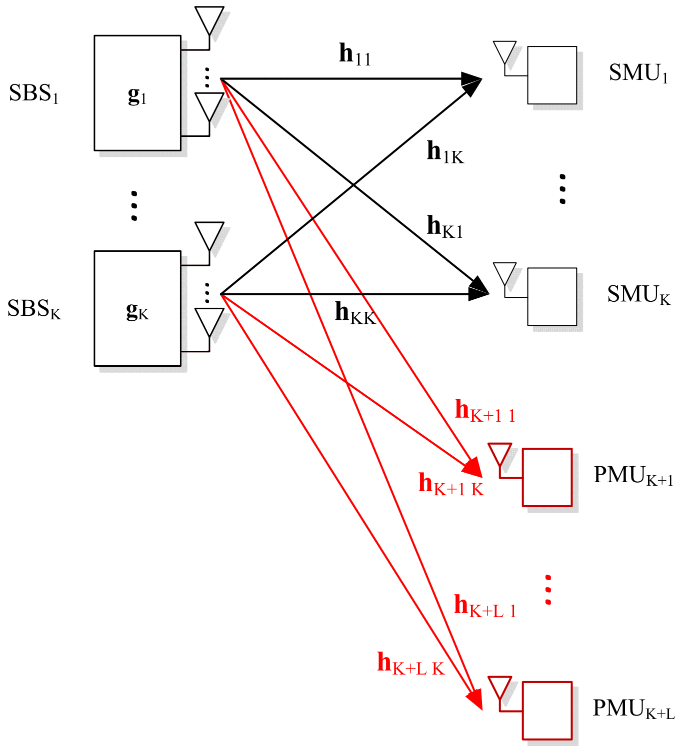

Figure 11.8: Cognitive Radio DL system

- F. Negro, I. Ghauri, and D. T. M. Slock, "Beamforming for the underlay cognitive MISO interference channel via UL-DL duality," *in Cognitive Radio Oriented Wireless Networks Communications (CROWNCOM), 2010 Proceedings of the Fifth International Conference on, june 2010, pp. 1 -5.*

Chapter 8 - Spatial Interweave TDD Cognitive Radio Systems

Dans ce chapitre, nous étudions l'optimisation conjointe des filtres d'émission-réception dans un canal spatial interweave cognitive radio, nous décrivons toutes les phases de communication nécessaires pour acquérir les informations nécessaires aux utilisateurs primaires et secondaire. Nous mettons l'accent en particulier sur la façon d'exploiter réellement réciprocité du canal de transmission en TDD à l'aide UL DL calibrage étudiant comment calibrage influe émission et la conception du filtre du récepteur sur les périphériques primaires et secondaires. Un résultat important qui ressort de notre analyse est que la calibration non coopératif entre Tx et Rx ne sont pas nécessaires pour la conception de beamformers secondaire. Nous

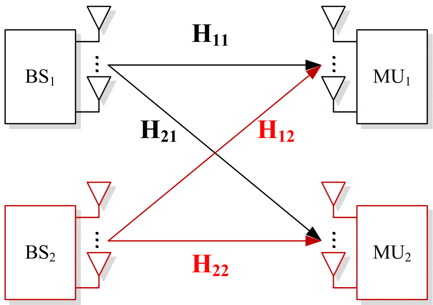

Figure 11.9: Downlink Channel

introduisons une extension des résultats au cas par plusieurs paires de émetteurs et récepteurs principal. Si la conception des beamformers de réseaux primaire selon ses IA, grce à la dualité IA, la paire secondaire peut estimer aveuglement le sous-espace reçu à tous les récepteurs primaires du sous-espace du signal transmis dans la communication UL. Problèmes calibrage sont également étudié dans ce contexte prouvant que la calibration entre les utilisateurs non coopératifs n'est pas nécessaire aussi dans le scénario approfondi. Quelques résultats de simulation sont présentés pour valider les solutions proposées Fig.11.10. Les résultats décrits dans

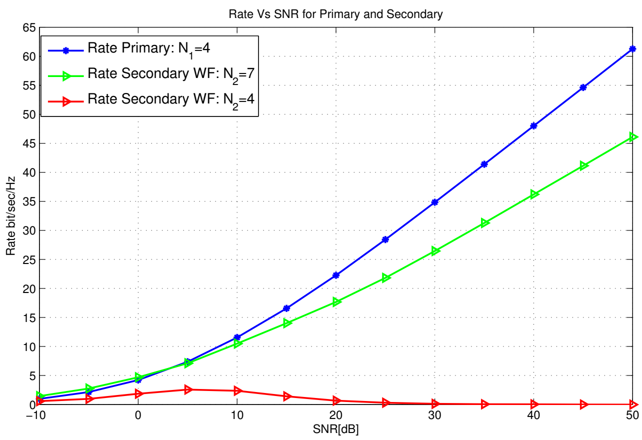

Figure 11.10: Rate Comparisons

ce chapitre sont partiellement publié dans:

- F. Negro, I. Ghauri, and D. T. M. Slock, "Transmission techniques and channel estimation for spatial interweave TDD cognitive radio systems," *in Pro-*

ceedings of the 43rd Asilomar conference on Signals, systems and computers, Asilomar'09, 2009, pp. 523-527.

Chapter 9 - Spatial Interweave Cognitive Radio Interference Channel with Multiple Primaries

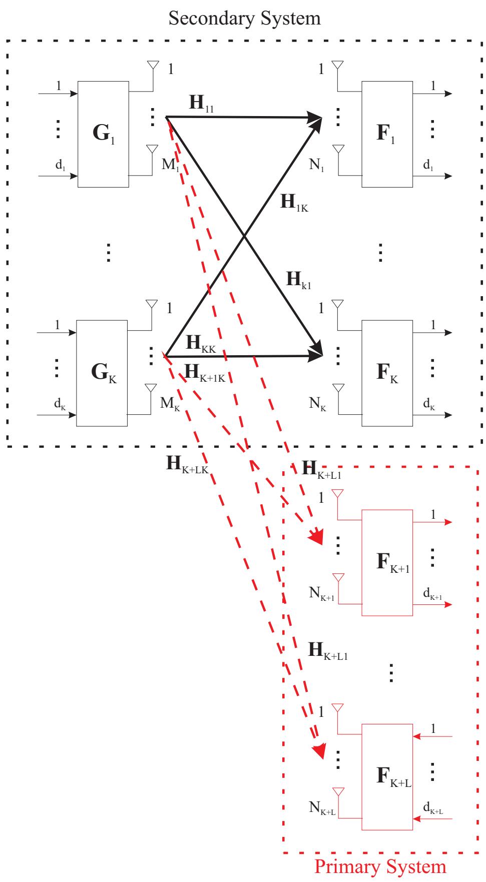

Figure 11.11: Cognitive Radio System

Dans cette partie du travail que nous considérons un réseau secondaire modélisé comme un K-user MIMO IFC qui veut communiquer en présence de L récepteurs primaires multi-antennes, Fig.11.11. Les beamformers aux utilisateurs secondaires sont conçus selon IA, à l'entrelacement des contraintes supplémentaires pour générer un sous-espace d'interférence, au niveau de chaque récepteur primaire, d'une dimension donnée. Nous étudions la faisabilité d'une solution IA dans le système de radio cognitive sous enquête sur la base des résultats présentés dans le chapitre 3. Ensuite, nous proposons un algorithme itératif qui trouve les émetteur et recevoir IA secondaire satisfaisant aux contraintes s'entrecroisent au niveau des récepteurs primaires. Les contributions de ce chapitre peuvent être trouvés dans le document suivant:

- F. Negro, I. Ghauri, and D. T. M. Slock, "Spatial interweave for a MIMO secondary interference channel with multiple primary users," *in 4th International Conference on Cognitive Radio and Advanced Spectrum Management, (CogART 2011), October 2011.*

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