On the DoF of the Multiple-Antenna Time Correlated Interference Channel with Delayed CSIT

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***Abstract*—We consider the time-correlated multiple-antenna interference channel where the transmitters have (i) delayed channel state information (CSI) obtained from a feedback channel as well as (ii) imperfect current CSIT, obtained e.g., from prediction on the basis of these past channel samples. We derive the degrees of freedom region for the two-user MISO interference channel under such conditions. In doing so we propose an optimal scheme relying on a form of space-time alignment combined with interference quantization. Extensions to some MIMO cases are also considered[1].**

I. Introduction

Although the determination of capacity region of the interference channel (IC) has been a long standing open problem, several interesting recent results shed light on the problem from various perspectives. When specializing to the large SNR regime, it is known that the characterization of the full capacity region can be conveniently replaced with the determination of the so-called degrees of freedom (DoF) region. Progress on that particular front was reported in [1] with the derivation of the DoF region for the two-user MIMO interference channel with M_1, M_2 transmit antennas and N_1, N_2 receive antennas, where the sum DoF $\min\{M_1 + M_2, N_1 + N_2, \max(M_1, N_2), \max(M_2, N_1)\}$ is shown to be optimal. Most of these advances suggest achievable schemes which require the full knowledge of channel state information (CSI) at both the transmitter and receiver sides. In fact, the cruciality of CSI at the transmitter side in particular is demonstrated to shrink dramatically when zero CSIT is available.

More recently, the impact of feedback delays providing the transmitter with outdated CSI over MIMO channels was considered in [2] for the broadcast channel (BC) and later extended to the IC [3], [4]. The key contribution in [2] was to establish the usefulness of even completely outdated channel state information in designing precoders achieving significantly better DoF than what is obtained without any CSIT. Considering the worst case scenarios, including those where the feedback delay extends beyond the coherence period of the time varying fading channels, the authors in [2] propose a space-time interference alignment-inspired strategy achieving an optimal sum DoF of 4/3 for the two-user MISO BC, in a setting when the no CSIT case yields no more than 1 DoF. The essential ingredient for the proposed scheme in [2] lies in the use of multi-slot protocol initiating with the transmission of unprecoded information symbols to the user terminals, followed by the analog forwarding of the interference overheard in the first time slot. Most recently, this strategy was generalized under similar principle to the interference channel setting [3], [4], again establishing DoF strictly beyond the ones obtained without CSIT in scenarios where the delayed CSIT bears no correlation with the current channel realization.

Albeit inspiring and fascinating in nature, such results nonetheless rely on the somewhat over-pessimistic assumption that no estimate for the *current* channel realization is available to the transmitter. Owing to the finite Doppler spread behavior of fading channels, it is however the case in many real life situations that the past channel realizations can provide information about the current one. Therefore a scenario where the transmitter is endowed with delayed CSI in addition to some (albeit imperfect) estimate of the current channel is practical relevance. This form of combined delayed and imperfect current CSIT was recently introduced in [5] for the multiple-antenna broadcast channel whereby a novel transmission scheme is proposed which extends beyond the MAT algorithm in allowing the exploitation of precoders designed based on the current CSIT estimate. The full characterization of the optimal DoF for the hybrid CSIT was reported in [6] and independently in [7]. The key idea behind the schemes in [5], [6] lies in the modification of the MAT protocol where i) the initial time slot involves transmission of *precoded* symbols, followed by the forwarding of *residual* interference overheard in the first time slot, and ii) the taking advantage of the reduced power for the residual interference (compared with full power interference in MAT) based on a suitable quantization method and digital transmission.

In this paper, we extend the results in [5], [6] and consider the two-user time-correlated multiple-antenna interference channel. A similar hybrid CSIT scenario is considered whereby each transmitter has access to delayed channel samples for the links it is connected to, as well as possessing an imperfect estimate of the current channel. The current CSIT estimate could be obtained from, e.g., a linear prediction applied to past samples [8], although the prediction aspects are not specified in this

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paper. Instead, the quality level for the current CSIT estimate is simply modeled in terms of an exponent of the transmit power level, allowing DoF characterization for various ranges of current CSIT quality. Thus our model bridges between previously reported CSIT scenarios such as the pure delayed CSIT of [2]–[4] and the pure instantaneous CSIT scenarios [1]. We assume each receiver has access to its own perfect instantaneous CSI and the perfect delayed CSI of other receivers (as in e.g. [2]–[4]), in addition to the imperfect current CSI.

In what follows we obtain the following key results:

- We establish an outer bound on the DoF region for the two-user temporally-correlated MISO interference channel with perfect delayed and imperfect current CSIT, as a function of the current CSIT quality exponent. This result is initially derived for the two-antenna transmitters and then generalized to a certain MIMO cases.
- We propose two schemes which achieve the key vertices of the outer bound with perfect delayed and imperfect current CSIT. The schemes build on the principles of time-slotted protocol, starting with the ZF precoded transmission of information symbols from the two interfering transmitters simultaneously and followed by forwarding of the quantized residual interferences in a digital fashion.

II. System Model and Main Results

We consider a two-user MISO interference channel (IC), where two transmitters each equipped with 2 antennas[2] wish to send two private messages to their respective receivers each with a single antenna. The discrete time baseband signal model is given by

$$y(t) = \boldsymbol{h}_{11}^{\mathsf{H}}(t)\boldsymbol{x}_1(t) + \boldsymbol{h}_{12}^{\mathsf{H}}(t)\boldsymbol{x}_2(t) + e(t) \tag{1a}$$

$$z(t) = \boldsymbol{h}_{21}^{\mathsf{H}}(t)\boldsymbol{x}_1(t) + \boldsymbol{h}_{22}^{\mathsf{H}}(t)\boldsymbol{x}_2(t) + b(t), \tag{1b}$$

for any time instant t, where $\boldsymbol{h}_{ji}(t) \in \mathbb{C}^{2\times 1}$ is the independent and identically distributed (i.i.d.) channel vector from Tx-i to Rx-j with zero mean and covariance matrix $\mathbf{I}_2$; $e(t), b(t) \sim \mathcal{N}_{\mathbb{C}}(0,1)$ are normalized additive white Gaussian noise at the respective receivers; the coded input signal $\boldsymbol{x}_i(t)$ is subject to the power constraint $\mathbb{E}(\|\boldsymbol{x}_i(t)\|^2) \le P$, $\forall\, t$. Denote $\boldsymbol{H}(t) \triangleq \{\boldsymbol{h}_{11}(t), \boldsymbol{h}_{12}(t), \boldsymbol{h}_{21}(t), \boldsymbol{h}_{22}(t)\}$.

At each time instant t, Tx-i knows perfectly the delayed CSI $\{\boldsymbol{H}(k)\}_{k=1}^{t-1}$, and somehow predict/estimate imperfectly the current CSI $\hat{\boldsymbol{H}}(t)$, which can be modeled by

$$\boldsymbol{h}_{ji}(t) = \hat{\boldsymbol{h}}_{ji}(t) + \tilde{\boldsymbol{h}}_{ji}(t) \tag{2}$$

where the estimate $\hat{\boldsymbol{h}}_{ji}(t)$ and estimation error $\tilde{\boldsymbol{h}}_{ji}(t)$ are independent and assumed to be zero-mean and with variance $(1-\sigma^2)\mathbf{I}_2$, $\sigma^2\mathbf{I}_2$, respectively ($0 \le \sigma^2 \le 1$).

We assume that the estimation error σ^2 can be parameterized as an exponential function of the power P, so that we hope to characterize the DoF of the MISO IC with respect to this exponent. To this end, we introduce a parameter $\alpha \ge 0$, such that

$$\alpha \triangleq -\lim_{P\to\infty} \frac{\log \sigma^2}{\log P}. \tag{3}$$

This α indicates the quality of current CSIT at high SNR. While $\alpha = 0$ reflects the case with no current CSIT, $\alpha \to \infty$ corresponds to that with perfect instantaneous CSIT. As a matter of fact, when $\alpha \ge 1$, the quality of the imperfect current CSIT is sufficient to avoid the DoF loss, and ZF precoding with this imperfect CSIT is able to achieve the maximum DoF [8]. Therefore, we focus on the case $\alpha \in [0,1]$ hereafter. The connections between the above model and the linear prediction over existing time-correlated channel models with prescribed user mobility are highlighted in [5]. According to the definition of the estimated current CSIT, we have $\mathbb{E}(|\boldsymbol{h}_{ji}^{\mathsf{H}}(t)\hat{\boldsymbol{h}}_{ji}^{\perp}(t)|^2) = \sigma^2 \sim P^{-\alpha}$.

The main result of this paper is stated below:

Theorem 1. *In the two-user MISO IC with perfect delayed and imperfect current CSIT, the optimal DoF region can be characterized by*

$$d_1 \le 1, \quad d_2 \le 1 \tag{4a}$$

$$2d_1 + d_2 \le 2 + \alpha \tag{4b}$$

$$d_1 + 2d_2 \le 2 + \alpha. \tag{4c}$$

Proof: The achievability is provided in Section III and the outer bound is given in [10]. ■

For illustration, the DoF region for the two-user MISO IC is provided in Fig. 1. The DoF regions with pure perfect delayed CSIT, and perfect instantaneous CSIT are also plotted for comparison. It shows that the DoF region with perfect delayed and imperfect current CSIT is strictly larger than that with pure delayed CSIT and approaches the region with perfect CSIT as the quality of current CSIT increases.

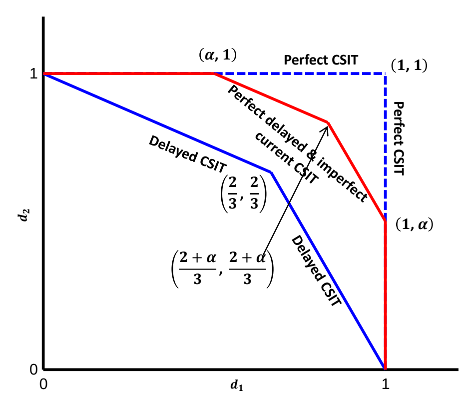

Fig. 1: DoF region for the two-user MISO IC (when $\alpha = 0.5$).

III. Achievability

In the following two schemes, we demonstrate the vertices $(1,\alpha)$ and $\left(\frac{2+\alpha}{3}, \frac{2+\alpha}{3}\right)$ are all achievable[3].

[2]The generalization is considered in Section IV.

[3]Note that we simply use $\hat{\boldsymbol{h}}_{ji}(t)$ for the range space while $\hat{\boldsymbol{h}}_{ji}^{\perp}(t)$ for the null space of $\hat{\boldsymbol{h}}_{ji}(t)$. The precoder design to improve the achievable rate is out of scope of this paper.

A. Achievability of $(1, \alpha)$

This vertex can be achieved by using only imperfect current CSIT within one single slot. The transmission with superposition coding can be given by

$$\boldsymbol{x}_1 = \begin{bmatrix} u_c \\ 0 \end{bmatrix} + \hat{\boldsymbol{h}}_{21}^{\perp} u_p, \quad \boldsymbol{x}_2 = \hat{\boldsymbol{h}}_{12}^{\perp} v_p \tag{5}$$

where u_c is a common message and decodable by both receivers but only desirable by Rx-1, and $u_p,\ v_p$ are private messages which can only be seen and decoded by their respective Rxs. These messages are assumed to satisfy the power constraints $\mathbb{E}|u_c|^2 \le P$ and rate $R_{u_c} \le (1-\alpha)\log P$, whereas $\mathbb{E}|u_p|^2 = \mathbb{E}|v_p|^2 \le P^{\alpha}$ and rate $R_{u_p} = R_{v_p} \le \alpha \log P$. At the receiver side, we have (the noise terms are omitted hereafter for conciseness)

$$y = \underbrace{h_{11,1}^* u_c}_{P} + \underbrace{\boldsymbol{h}_{11}^{\mathsf{H}} \hat{\boldsymbol{h}}_{21}^{\perp} u_p}_{P^{\alpha}} + \underbrace{\boldsymbol{h}_{12}^{\mathsf{H}} \hat{\boldsymbol{h}}_{12}^{\perp} v_p}_{P^{0}} \tag{6a}$$

$$z = \underbrace{h_{21,1}^* u_c}_{P} + \underbrace{\boldsymbol{h}_{22}^{\mathsf{H}} \hat{\boldsymbol{h}}_{12}^{\perp} v_p}_{P^{\alpha}} + \underbrace{\boldsymbol{h}_{21}^{\mathsf{H}} \hat{\boldsymbol{h}}_{21}^{\perp} u_p}_{P^{0}}, \tag{6b}$$

Both the common (cf. u_c) and private (cf. $u_p,\ v_p$) messages can be subsequently decoded by successive decoding, which takes three steps: (i) retrieve u_c first by treating others as noise, yielding total $1-\alpha$ DoF for Rx-1; (ii) reconstruct the whole terms with u_c, and consequently subtract them from received signals; (iii) recover the private messages u_p (resp. v_p) from the residual signal, yielding α DoF for each Rx from the private message. To sum up, the DoF pair $(1, \alpha)$ is achieved.

B. Achievability of $\left(\frac{2+\alpha}{3}, \frac{2+\alpha}{3}\right)$

To achieve this vertex, both perfect delayed and imperfect current CSIT are utilized. By using pure delayed CSIT, it was demonstrated the vertex $\left(\frac{2}{3}, \frac{2}{3}\right)$ is achievable by a three-slotted protocol [2]–[4], referred to as *MAT alignment*. The MAT alignment exploits the advantage of interference alignment in both the space and time domain. We first briefly review its application in the interference channel.

1) MAT Alignment in the Interference Channel: The MAT alignment in the interference channel is an extension from the broadcast channel, taking into account the distributive and uncooperative nature of the transmitters. The three-slotted protocol is described as follows:

Slot-1: Each Tx sends two independent symbols to its intended receiver without precoding, i.e.,

$$\boldsymbol{x}_1(1) = \boldsymbol{u} \quad \boldsymbol{x}_2(1) = \boldsymbol{v} \tag{7a}$$

and the received signals at both receivers are

$$y(1) = \boldsymbol{h}_{11}^{\mathsf{H}}(1)\boldsymbol{u} + \underbrace{\boldsymbol{h}_{12}^{\mathsf{H}}(1)\boldsymbol{v}}_{\eta_1} \tag{8a}$$

$$z(1) = \underbrace{\boldsymbol{h}_{21}^{\mathsf{H}}(1)\boldsymbol{u}}_{\eta_2} + \boldsymbol{h}_{22}^{\mathsf{H}}(1)\boldsymbol{v}, \tag{8b}$$

where η_1 and η_2 are interference terms overheard at the Tx-1 and Tx-2, respectively.

Slot-2 & Slot-3: At the end of Slot-1, the delayed CSI $\{\boldsymbol{h}_{21}(1)\}$ is available at the Tx-1, while $\{\boldsymbol{h}_{12}(1)\}$ is accessible at the Tx-2. Together with the transmitted symbols, the overheard interference terms are reconstructible at both Txs. By retransmitting the overheard interference terms $\eta_2 = \boldsymbol{h}_{21}^{\mathsf{H}}(1)\boldsymbol{u}$ at the Tx-1 and $\eta_1 = \boldsymbol{h}_{12}^{\mathsf{H}}(2)\boldsymbol{v}$ at the Tx-2 with time division, i.e.,

$$\boldsymbol{x}_1(2) = \begin{bmatrix} \eta_2 \\ 0 \end{bmatrix}, \quad \boldsymbol{x}_2(2) = \boldsymbol{0} \tag{9a}$$

$$\boldsymbol{x}_1(3) = \boldsymbol{0}, \quad \boldsymbol{x}_2(3) = \begin{bmatrix} \eta_1 \\ 0 \end{bmatrix}. \tag{9b}$$

We cancel the interference terms η_1 and η_2 at the Rx-1 and Rx-2, and importantly provide another linear combination of $\boldsymbol{u}$ (from η_2) and $\boldsymbol{v}$ (from η_1) to the Rx-1 and Rx-2, respectively. In the end, both receivers are able to recover their own symbols with high probability. The key idea behind is interference alignment in both space and time domain. At each receiver, the mutual interference aligns in one dimension, while the desired signal spans in a two-dimensional space. This enables each receiver to retrieve the desired signal from a three-dimensional space.

2) Integrating the Imperfect Current CSIT: As described above, MAT alignment takes into account the completely outdated CSIT, regardless of the correlation between current and previous channel states. In fact, however, the current CSI is predictable from the past states if the underlying channel exhibits some temporal correlation [5], [6]. In the following, a new scheme is proposed to demonstrate $\left(\frac{2+\alpha}{3}, \frac{2+\alpha}{3}\right)$ is achievable by integrating the estimated current CSIT with the delayed one. The achievable scheme also consists of 3 time slots.

Slot-1: The transmitted signals from both Txs are

$$\boldsymbol{x}_1(1) = [\hat{\boldsymbol{h}}_{21}(1)\ \hat{\boldsymbol{h}}_{21}^{\perp}(1)]\boldsymbol{u}(1) \tag{10a}$$

$$\boldsymbol{x}_2(1) = [\hat{\boldsymbol{h}}_{12}(1)\ \hat{\boldsymbol{h}}_{12}^{\perp}(1)]\boldsymbol{v}(1) \tag{10b}$$

where $\boldsymbol{u}(1), \boldsymbol{v}(1) \in \mathbb{C}^{2\times 1}$ are intended to Rx-1 and RX-2, respectively, satisfying $\mathbb{E}(\|\boldsymbol{u}(1)\|^2) = \mathbb{E}(\|\boldsymbol{v}(1)\|^2) \le P$. At both receivers, we have

$$y(1) = \boldsymbol{h}_{11}^{\mathsf{H}}(1)\boldsymbol{x}_1(1) + \underbrace{\boldsymbol{h}_{12}^{\mathsf{H}}(1)\boldsymbol{x}_2(1)}_{\eta_1}$$

$$z(1) = \underbrace{\boldsymbol{h}_{21}^{\mathsf{H}}(1)\boldsymbol{x}_1(1)}_{\eta_2} + \boldsymbol{h}_{22}^{\mathsf{H}}(1)\boldsymbol{x}_2(1),$$

where η_i $(i = 1, 2)$ is interference overheard at the Rx-i. According to $\mathbb{E}(|\boldsymbol{h}_{ji}^{\mathsf{H}}(1)\hat{\boldsymbol{h}}_{ji}^{\perp}(1)|^2) \sim P^{-\alpha}$, we make $\mathbb{E}(|\eta_1|^2) = \mathbb{E}(|\eta_2|^2) \sim P^{1-\alpha}$ by allocating $\mathbb{E}(|u_1(1)|^2) = \mathbb{E}(|v_1(1)|^2) = P^{1-\alpha}$ whereas $\mathbb{E}(|u_2(1)|^2) = \mathbb{E}(|v_2(1)|^2) = P - P^{1-\alpha} \sim P$. Accordingly, the effective transmission rate in Slot-1 is $(2-\alpha)\log P$ for each Tx-Rx pair.

At the end of Slot-1, Tx-1 can reconstruct $\eta_2 = \boldsymbol{h}_{21}^{\mathsf{H}}(1)\boldsymbol{x}_1(1)$ while Tx-2 can reconstruct $\eta_1 = \boldsymbol{h}_{12}^{\mathsf{H}}(1)\boldsymbol{x}_2(1)$. Instead of forwarding the interferences in an analog fashion, we first quantize the interference term η_i into $\hat{\eta}_i$ with $(1-\alpha)\log P$

bits each, modeled as,

$$\eta_i = \hat{\eta}_i + \Delta_i \tag{12}$$

where Δ_i is the quantization error with distortion $\mathbb{E}(|\Delta_i|^2) \sim \sigma_{\eta_i}^2 D$ and independent of $\hat{\eta}_i$. We let the normalized distortion D decay as $P^{-(1-\alpha)}$ (in turn $\mathbb{E}(|\Delta_i|^2) \sim P^0$) so that each receiver can decode it successfully and the quantization error is drowned in the noise. Then we encode the index of $\hat{\eta}_i$ to codeword c_i using a Gaussian channel codebook, and forward c_i to both receivers in the ensuing two slots.

Slot-2 & Slot-3: These two slots are similar. In Slot-2 (resp. Slot-3), apart from the forwarding of the codeword c_2 (resp. c_1), the Tx-1 (resp. Tx-2) sends a new extra symbol with superposition coding, while only a new symbol at the Tx-2 (resp. Tx-1). The transmitted signals in two slots are

$$\boldsymbol{x}_1(2) = \begin{bmatrix} c_2 \\ 0 \end{bmatrix} + \hat{\boldsymbol{h}}_{21}^{\perp}(2)u(2), \quad \boldsymbol{x}_2(2) = \hat{\boldsymbol{h}}_{12}^{\perp}(2)v(2) \tag{13a}$$

$$\boldsymbol{x}_1(3) = \hat{\boldsymbol{h}}_{21}^{\perp}(3)u(3), \quad \boldsymbol{x}_2(3) = \begin{bmatrix} c_1 \\ 0 \end{bmatrix} + \hat{\boldsymbol{h}}_{12}^{\perp}(3)v(3) \tag{13b}$$

where $c_i,\ (i = 1, 2)$ is treated as common messages with rate $R_{c_i} \leq (1-\alpha)\log P$ and power constraint $\mathbb{E}|c_i|^2 \leq P$, and $u(2), v(2), u(3), v(3)$ are private messages with each symbol of rate $\alpha \log P$, satisfying $\mathbb{E}|u(t)|^2 = \mathbb{E}|v(t)|^2 \leq P^{\alpha}$, $t = 2, 3$.

At the receivers, the received signals can be similarly written as in (6). By successive decoding, in Slot-2 for example, both the common (cf. c_2) and private [cf. $u(2)$ and $v(2)$] messages are recoverable, yielding extra α DoF for each Rx from the private message. The same strategy applies to Slot-3.

In the end, c_2 and c_1 can be all recovered (and in turn η_1 and η_2 with negligible quantization error) at both receivers, serving to cancel the overheard interference as well as to provide additional linearly independent equations for $\boldsymbol{v}(1)$ and $\boldsymbol{u}(1)$, respectively. Thus, an equivalent MIMO can be formulated to find the symbols $\boldsymbol{u}(1)$:

$$\begin{bmatrix} y(1) - \hat{\eta}_1 \\ \hat{\eta}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_{11}^{\mathsf{H}}(1) \\ \boldsymbol{h}_{21}^{\mathsf{H}}(1) \end{bmatrix} \boldsymbol{x}_1(1) + \begin{bmatrix} e(1) + \Delta_1 \\ -\Delta_2 \end{bmatrix} \tag{14}$$

for the Rx-1, and it is similar for the Rx-2. Hence, each Rx obtains $2 - \alpha + 2\alpha = 2 + \alpha$ in total over 3 slots.

Note that the vertices $(1, 0)$ and $(0, 1)$ are achievable by letting one pair communicate while keeping the other one silent. In conclusion, all vertices of the DoF region for two-user MISO IC are achievable, and in turn the entire region can be achieved by time sharing.

IV. Extension to MIMO Case

Here, we extend the aforementioned MISO case to a class of MIMO settings with antenna configuration (M, M, N, N), where M antennas at each transmitter and N antennas at each receiver, satisfying $M \geq 2N$. This includes a generalized MISO setting with more than 2 antennas at each transmitter. The discrete time baseband signal model is given by

$$\boldsymbol{y}(t) = \boldsymbol{H}_{11}(t)\boldsymbol{x}_1(t) + \boldsymbol{H}_{12}(t)\boldsymbol{x}_2(t) + \boldsymbol{e}(t), \tag{15a}$$

$$\boldsymbol{z}(t) = \boldsymbol{H}_{21}(t)\boldsymbol{x}_1(t) + \boldsymbol{H}_{22}(t)\boldsymbol{x}_2(t) + \boldsymbol{b}(t), \tag{15b}$$

for any time instant t, where $\boldsymbol{H}_{ji}(t) \in \mathbb{C}^{N\times M}$ is the channel matrix from Tx-i to Rx-j; the coded input signal $\boldsymbol{x}_i(t) \in \mathbb{C}^{M\times 1}$ is subject to the power constraint $\mathbb{E}(\|\boldsymbol{x}_i(t)\|^2) \leq P$, $\forall t$.

In analogy to the MISO case, we have the following result:

Corollary 1. *In the two-user (M, M, N, N) MIMO interference channel ($M \geq 2N$) with perfect delayed and imperfect current CSIT, the optimal DoF region can be characterized by*

$$d_1 \leq N, \quad d_2 \leq N, \tag{16a}$$

$$d_1 + 2d_2 \leq N(2 + \alpha), \tag{16b}$$

$$2d_1 + d_2 \leq N(2 + \alpha). \tag{16c}$$

Proof: The achievability is provided in the following and the outer bound is given in [10], [11]. ■

Remark: The DoF region is irrelevant to the number of transmit antennas as long as $M \geq 2N$. For the $M \times 1$ MISO case ($M \geq 2$), the DoF region is identical to that when $M = 2$.

Due to the fact that additional transmit antennas do not increase DoF as long as $M \geq 2N$ in both perfect instantaneous and delayed CSIT setting [1], [3], [4], we consider $M = 2N$ case here for illustration. In this case, we demonstrate two vertices $(N, N\alpha)$ and $\left(\frac{N(2+\alpha)}{3}, \frac{N(2+\alpha)}{3}\right)$ and their counterparts (with exchanged roles of both receivers) are all achievable.

Before proceeding further, we define

$$\boldsymbol{Q}_{ij}(t) \in \mathbb{C}^{2N\times N} \subseteq \mathcal{R}\{\hat{\boldsymbol{H}}_{ij}(t)\}, \tag{17}$$

$$\boldsymbol{Q}_{ij}^{\perp}(t) \in \mathbb{C}^{2N\times N} \subseteq \mathcal{N}\{\hat{\boldsymbol{H}}_{ij}(t)\}, \tag{18}$$

where $\mathbb{E}\|\boldsymbol{H}_{ji}(t)\boldsymbol{Q}_{ji}^{\perp}(t)\|_{\mathsf{F}}^2 \sim P^{-\alpha}$.

A. Achievability of $(N, N\alpha)$

This vertex can be achieved within one single slot. The transmission can be given by

$$\boldsymbol{x}_1 = \boldsymbol{Q}_{11}\boldsymbol{u}_c + \boldsymbol{Q}_{21}^{\perp}\boldsymbol{u}_p, \quad \boldsymbol{x}_2 = \boldsymbol{Q}_{12}^{\perp}\boldsymbol{v}_p, \tag{19}$$

where $\boldsymbol{u}_c \in \mathbb{C}^{N\times 1}$, $\boldsymbol{u}_p \in \mathbb{C}^{N\times 1}$ and $\boldsymbol{v}_p \in \mathbb{C}^{N\times 1}$. The transmitted symbols are assumed to satisfy the constraints of power $\mathbb{E}\|\boldsymbol{u}_c\|^2 \leq P$ and rate $R_{\boldsymbol{u}_c} \leq N(1-\alpha)\log P$, whereas $\mathbb{E}\|\boldsymbol{u}_p\|^2 = \mathbb{E}\|\boldsymbol{v}_p\|^2 \leq P^{\alpha}$ and rate $R_{\boldsymbol{u}_p} = R_{\boldsymbol{v}_p} \leq N\alpha \log P$. Although the symbol $\boldsymbol{u}_c$ is decodable by both receivers and hence referred to as a common message, it is only desirable by Rx-1. On the other hand, we refer to $\boldsymbol{u}_p$, $\boldsymbol{v}_p$ as the private messages which can only be seen and decoded by their corresponding receivers. At the receiver side, we have

$$\boldsymbol{y} = \underbrace{\boldsymbol{H}_{11}\boldsymbol{Q}_{11}\boldsymbol{u}_c}_{P} + \underbrace{\boldsymbol{H}_{11}\boldsymbol{Q}_{21}^{\perp}\boldsymbol{u}_p}_{P^{\alpha}} + \underbrace{\boldsymbol{H}_{12}\boldsymbol{Q}_{12}^{\perp}\boldsymbol{v}_p}_{P^0}, \tag{20a}$$

$$\boldsymbol{z} = \underbrace{\boldsymbol{H}_{21}\boldsymbol{Q}_{11}\boldsymbol{u}_c}_{P} + \underbrace{\boldsymbol{H}_{21}\boldsymbol{Q}_{21}^{\perp}\boldsymbol{u}_p}_{P^0} + \underbrace{\boldsymbol{H}_{22}\boldsymbol{Q}_{12}^{\perp}\boldsymbol{v}_p}_{P^{\alpha}}. \tag{20b}$$

By successive decoding, the two receivers decode common message $\boldsymbol{u}_c$ by treating other terms as noise. Then, subtracting the terms with $\boldsymbol{u}_c$, two receivers can decode their own private messages successfully, yielding total N and $N\alpha$ DoF for Rx-1 and Rx-2, respectively.

B. Achievability of $\left(\frac{N(2+\alpha)}{3}, \frac{N(2+\alpha)}{3}\right)$

To achieve such a vertex, three time slots are sufficient.

Slot-1: The transmitted signals from both transmitters are given by

$$\boldsymbol{x}_1(1) = \begin{bmatrix}\boldsymbol{Q}_{21}(1) & \boldsymbol{Q}_{21}^{\perp}(1)\end{bmatrix} \boldsymbol{u}(1), \tag{21a}$$

$$\boldsymbol{x}_2(1) = \begin{bmatrix}\boldsymbol{Q}_{12}(1) & \boldsymbol{Q}_{12}^{\perp}(1)\end{bmatrix} \boldsymbol{v}(1), \tag{21b}$$

where $\boldsymbol{u}(1) \triangleq \begin{bmatrix}\boldsymbol{u}_1^{\mathsf{T}}(1) & \boldsymbol{u}_2^{\mathsf{T}}(1)\end{bmatrix}^{\mathsf{T}}$, $\boldsymbol{v}(1) \triangleq \begin{bmatrix}\boldsymbol{v}_1^{\mathsf{T}}(1) & \boldsymbol{v}_2^{\mathsf{T}}(1)\end{bmatrix}^{\mathsf{T}}$ with $\boldsymbol{u}_i(1), \boldsymbol{v}_i(1) \in \mathbb{C}^{N\times 1}$. At both receivers, we have

$$\boldsymbol{y}(1) = \boldsymbol{H}_{11}(1)\boldsymbol{x}_1(1) + \underbrace{\boldsymbol{H}_{12}(1)\boldsymbol{x}_2(1)}_{\boldsymbol{\eta}_1 \in \mathbb{C}^{N\times 1}}, \tag{22a}$$

$$\boldsymbol{z}(1) = \underbrace{\boldsymbol{H}_{21}(1)\boldsymbol{x}_1(1)}_{\boldsymbol{\eta}_2 \in \mathbb{C}^{N\times 1}} + \boldsymbol{H}_{22}(1)\boldsymbol{x}_2(1)). \tag{22b}$$

By balancing the allocated power among those vectors, i.e., $\mathbb{E}\|\boldsymbol{u}_1(1)\|^2 = \mathbb{E}\|\boldsymbol{v}_1(1)\|^2 \sim P^{1-\alpha}$ and $\mathbb{E}\|\boldsymbol{u}_2(1)\|^2 = \mathbb{E}\|\boldsymbol{v}_2(1)\|^2 = P - NP^{1-\alpha} \sim P$, we approximate the total power of interference vectors as $\mathbb{E}\|\boldsymbol{\eta}_1\|^2 \sim P^{1-\alpha}$ and $\mathbb{E}\|\boldsymbol{\eta}_2\|^2 \sim P^{1-\alpha}$. A set of source codebooks $\{\mathcal{X}_{1i}, \mathcal{X}_{2i}, i = 1, \cdots, N\}$ with size $(1-\alpha)\log P$ bits each are generated to represent the quantized elements of the interference vectors $\boldsymbol{\eta}_2$ and $\boldsymbol{\eta}_1$ at the Tx-1 and Tx-2, respectively. The codewords representing the elements of $\boldsymbol{\eta}_2$ and $\boldsymbol{\eta}_1$ are chosen uniformly from $\{\mathcal{X}_{1i}\}$ and $\{\mathcal{X}_{2i}\}$ and concatenated as $\hat{\boldsymbol{\eta}}_2$ and $\hat{\boldsymbol{\eta}}_1$, respectively. Then, the indices of $\hat{\boldsymbol{\eta}}_i$ are encoded to $\boldsymbol{c}_i$ with rate $N(1-\alpha)\log P$ and power constraint $\mathbb{E}\|\boldsymbol{c}_i\|^2 \leq P$ using a Gaussian channel codebook, and then forwarded as common messages to both receivers in the following two slots.

Slot-2 & Slot-3: These two slots are similar. The objective of the Slot-2 (resp. Slot-3) is to convey the coded common message $\boldsymbol{c}_2$ (resp. $\boldsymbol{c}_1$) carrying the information of $\hat{\boldsymbol{\eta}}_2$ (resp. $\hat{\boldsymbol{\eta}}_1$), together with a new symbol vector at the Tx-1 (resp. Tx-2), while only a new symbol vector is sent at the Tx-2 (resp. Tx-1). The transmitted signals in two slots are

$$\text{Slot-2:} \quad \begin{aligned}\boldsymbol{x}_1(2) &= \boldsymbol{Q}_{21}(2)\boldsymbol{c}_2 + \boldsymbol{Q}_{21}^{\perp}(2)\boldsymbol{u}(2)\\ \boldsymbol{x}_2(2) &= \boldsymbol{Q}_{12}^{\perp}(2)\boldsymbol{v}(2)\end{aligned} \tag{23a}$$

$$\text{Slot-3:} \quad \begin{aligned}\boldsymbol{x}_1(3) &= \boldsymbol{Q}_{21}^{\perp}(3)\boldsymbol{u}(3)\\ \boldsymbol{x}_2(3) &= \boldsymbol{Q}_{12}(3)\boldsymbol{c}_1 + \boldsymbol{Q}_{12}^{\perp}(3)\boldsymbol{v}(3)\end{aligned} \tag{23b}$$

where $\boldsymbol{u}(2), \boldsymbol{v}(2), \boldsymbol{u}(3), \boldsymbol{v}(3) \in \mathbb{C}^{N\times 1}$ with rate $N\alpha \log P$ each, satisfying $\mathbb{E}\|\boldsymbol{u}(2)\|^2 = \mathbb{E}\|\boldsymbol{u}(3)\|^2 \leq P^{\alpha}$ and $\mathbb{E}\|\boldsymbol{v}(2)\|^2 = \mathbb{E}\|\boldsymbol{v}(3)\|^2 \leq P^{\alpha}$.

In Slot-2, the received signals are give by (the received signals in Slot-3 can be similarly obtained)

$$\begin{aligned}\boldsymbol{y}(2) = \underbrace{\boldsymbol{H}_{11}(2)\boldsymbol{Q}_{21}(2)\boldsymbol{c}_2}_{P} &+ \underbrace{\boldsymbol{H}_{11}(2)\boldsymbol{Q}_{21}^{\perp}(2)\boldsymbol{u}(2)}_{P^{\alpha}}\\ &+ \underbrace{\boldsymbol{H}_{12}(2)\boldsymbol{Q}_{12}^{\perp}(2)\boldsymbol{v}(2)}_{P^{0}},\end{aligned} \tag{24a}$$

$$\begin{aligned}\boldsymbol{z}(2) = \underbrace{\boldsymbol{H}_{21}(2)\boldsymbol{Q}_{21}(2)\boldsymbol{c}_2}_{P} &+ \underbrace{\boldsymbol{H}_{21}(2)\boldsymbol{Q}_{21}^{\perp}(2)\boldsymbol{u}(2)}_{P^{0}}\\ &+ \underbrace{\boldsymbol{H}_{22}(2)\boldsymbol{Q}_{12}^{\perp}(2)\boldsymbol{v}(2)}_{P^{\alpha}}.\end{aligned} \tag{24b}$$

By treating other terms as noise, $N \times 1$ vector $\boldsymbol{c}_2$ is retrievable at both receivers with high probability provided N linearly independent equations. After that, $\boldsymbol{u}(2)$ and $\boldsymbol{v}(2)$ are also recoverable from N linear equations at the Rx-1 and Rx-2 by subtracting the terms with $\boldsymbol{c}_2$ from the received signals. The same strategy applies to Slot-3.

In the end, $N \times 1$ codeword vector $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ can be all recovered (and in turn $\hat{\boldsymbol{\eta}}_1$ and $\hat{\boldsymbol{\eta}}_2$) at both receivers, serving to cancel the overheard interference as well as to provide additional linearly independent equations for $\boldsymbol{v}(1)$ and $\boldsymbol{u}(1)$, respectively. With $2N$ linearly independent equations, the $2N \times 1$ vectors $\boldsymbol{u}(1)$ and $\boldsymbol{v}(1)$ are both recoverable with high probability at its respective receiver. Hence, the total $N(2+\alpha)$ DoF for each receiver is achieved within three time slots, and in turn the DoF pair is achievable by symmetry.

V. Conclusion

We characterize the DoF region of the two-user MISO and certain MIMO interference channels where the transmitter has access to both delayed CSI as well as an estimate of the current CSI. Our DoF region covers a family of CSIT settings, coinciding with previously reported results for extreme situations such as pure delayed CSIT and pure current CSIT. For intermediate regimes, the DoF achieving scheme relies on the forwarding to users of a suitably quantized version of prior interference obtained under imperfect linear ZF precoding at the two transmitters.

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