

A Markov Random Field description of fuzzy color segmentation

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Abstract—Image segmentation is a fundamental task in many computer vision applications. In this paper, we describe a new unsupervised color image segmentation algorithm, which exploits the color characteristics of the image. The introduced system is based on a color quantization of the image in the Lab color space using the popular eleven culture colors in order to avoid the well known problem of oversegmentation. To partially overcome the problem of highlight and shadows in the image, which is one of the main aspect affecting the performance of color segmentation systems, the proposed approach uses a fuzzy classifier trained on an ad-hoc designed dataset. A Markov Random Field description of the full algorithm is moreover provided which helps to remove resilient errors trough the use of an iterative strategy. The experimental results show the good performance of the proposed approach which is comparable to state of the art systems even if based only on the color information of the image.

Keywords—color segmentation, fuzzy clustering, Markov Random Field, Iterated Conditional Modes.

I. INTRODUCTION

Image segmentation is the process of partitioning an image into disjoint and homogeneous regions, called *segments*, and is one of the classical problems in computer vision. It is widely accepted that a good segmentation should group image pixels into regions whose statistical characteristics are homogeneous and whose boundaries are simple and spatially accurate.

The desirable characteristics that a good image segmentation should exhibit have been clearly stated in [1] with reference to gray-level images. “Regions of an image segmentation should be uniform and homogeneous with respect to some characteristics such as gray tone or texture. Region interiors should be simple and without many small holes. Adjacent regions of a segmentation should have significantly different values with respect to the characteristic on which they are uniform. Boundaries of each segment should be simple, not ragged, and must be spatially accurate.”

In the scientific literature, many statistical models and methods have been introduced. Generally, one can distinguish among *unsupervised*, *semi-supervised* and *fully supervised* methods¹. Unsupervised approaches provide segmentation results without any prior knowledge about the image and do not require any user-interaction. One of the main directions of current research in this field is to define the segmentation

process as finding the labeling of an image that minimizes a specific energy term. Despite the success of such energy minimization methods, still simple appearance based methods like Mean Shift [3] are considered as state of the art in the field of color segmentation. Semi-supervised methods require a user to highlight some regions as a prior, mostly by drawing some kind of seeds into the image. These methods achieve impressively accurate results, but have the disadvantage that results heavily depend on the selection of the seeds. The correct placement of the seeds by the user needs some training and expertise, and therefore mostly cumbersome postprocessing is required to correct the results. Finally, fully supervised methods require labeled training data for the expected type of image, mostly for the purpose of detecting specific object categories in images. Of course, the need for accurately labeled training data limits the scope of these methods.

In most of the existing color image segmentation algorithms, for all the categories explained before, the definition of a region is based on the color characteristics of the image within a chosen color space. This process could be further augmented by joining information about the objects in the scene, such as shape or superficial properties. In any case, the assumption to use color similarity makes it difficult for any algorithms to separate the objects with highlights, shadow, shadings or texture which cause inhomogeneity of colors of the object's surface. Moreover, segmentation algorithms based on color are always affected by the oversegmentation problem which is partially reduced by using specific post processing functions.

In this paper, we introduce a new unsupervised image segmentation algorithm exploiting the color characteristics of the image in the Lab color space. The novelty of the proposed work is the use of culture colors as a basis for the segmentation of the image in order to avoid undesirable oversegmentation effect. The proposed approach, moreover, tries to partially overcome the problem of highlight and shadows in the image, typically affecting the performance of segmentation systems, by using a fuzzy classifier trained on an ad-hoc designed dataset. A Markov Random Field description of the full algorithm is also provided which helps to remove resilient errors through the use of an iterative algorithm.

¹for a survey on color segmentation algorithms please refer to [2].

II. PROPOSED FRAMEWORK

As explained in the introduction, the goal of the proposed work is to introduce a new approach to color segmentation of natural images exploiting the property of Lab color space to be robust in change in illumination conditions.

The proposed approach is described in Fig. 1.

The main steps of the proposed algorithm are the *Fuzzy K-Nearest Neighbors* (KNN) classifier which is applied to the *Lab* version of the original image using an ad-hoc training set, as described in Sec. II-B, and the *Iterated Conditional Mode* (ICM) which maximizes local conditional probabilities leading to a suboptimal solution, as explained in Sec. II-C. The complete algorithm can be described as a Markov Random Field (MRF) which assign to each pixel of the image a label corresponding to a segment in the segmentation process. The single steps are explained in details in the following subsections.

A. Markov Random Field

In this section we provide a short description of MRF theory in image processign applications, and, in particular, in segmentation problems.

Markov random field theory is a branch of probability theory for analyzing the spatial or contextual dependencies of physical phenomena [4]. The foundations of the theory of Markov random fields may be found in statistical physics of magnetic materials (Ising models, spin glasses, etc..) but Markov random fields are often used in image processing applications, because this approach defines a model for describing the coherence among neighboring pixels.

A random field $F = \{F_1, F_2, \dots, F_m\}$ is a family of random variables defined on a set \mathcal{S} , in which each random variable F_i takes a value f_i in \mathcal{L} .

F is said to be a Markov random field (MRF) on \mathcal{S} with respect to a neighborhood system N if and only if the two following conditions are satisfied:

$$P(f) > 0, \quad \forall f \in \mathcal{L}^m \quad (\text{positivity}) \quad (1)$$

$$P(f_i | f_{\mathcal{S}-\{i\}}) = P(f_i | f_{N_i}), \quad \forall i \in \mathcal{S} \quad (\text{Markov property}) \quad (2)$$

where $f = \{f_1, \dots, f_m\}$ is a configuration of F (corresponding to a realization of the field), $P(f)$ is the joint probability $P(F_1 = f_1, \dots, F_m = f_m)$ of the joint event $F = f$, and

$$f_{N_i} = \{f_{i'} | i' \in N_i\} \quad (3)$$

denotes the set of values at the sites neighboring i , i.e. the neighborhood N centered at position i . The positivity is due to technical reasons, since it is a necessary condition if we want that the Hammersley-Clifford theorem (see below) holds.

To exploit MRFs characteristics in a practical way we need to refer to the Hammersley-Clifford theorem for which the

probability distribution of a MRF has the form of a Gibbs distribution, i.e.:

$$P(f) = Z^{-1} \times e^{-\frac{1}{T}U(f)} \quad (4)$$

where Z is a normalizing constant called the partition function, T is a constant called the temperature and $U(f)$ is the energy function. The energy function

$$U(f) = \sum_{c \in \mathcal{C}} V_c(f) \quad (5)$$

is a sum of cliques potentials $V_c(f)$ over all possible cliques \mathcal{C} , where a clique c is defined as a subset of neighboring sites in \mathcal{S} . Thus the value of $V_c(f)$ depends on the local configuration on the clique c . The practical value of the theorem is that it provides a simple way of specifying the joint probability. $P(f)$ measures the probability of the occurrence of a particular configuration: the more probable configurations are those with lower energies.

In our case, we can model the segmentation of an image with a random field F defined on the set \mathcal{S} of the image pixels. The value assumed by each random variable represents the label corresponding to the segment to which the pixel belongs to. The advantage brought by MRF theory is that by letting the segment of a generic point (x, y) of the image depending on the segments of its neighboring points (let us indicate the neighborhood system of (x, y) with the notation $N(x, y)$), we can automatically impose that the resulting segmentation takes into account (with some constraints depending on the potential function we chose) the semantic correlation of the pixels in the image.

In the next sections we will see how we choose the neighborhood system N and the potential function V and how these information can be used to generate a segmented image.

B. Fuzzy clustering

The first step of the whole process is the Fuzzy KNN classifier. The idea of using a fuzzy classifier based on the Lab colorspace derives from a previous work we did aimed at understanding the ability of different colorspace in identifying colors in different illumination conditions.

In [5] we proposed a new approach to provide a comparison of the most widely used color spaces in order to understand which one is the more suitable to identify colors in video. While the goal of the paper was completely different from color segmentation, we think that the analysis carried out is significant in understanding the problem of color identification in different illumination conditions, which is one of the main aspect affecting the performance of color segmentation algorithms.

We employed a statistical approach by conducting extensive data mining on video clips collected under various lighting conditions. The goal was to learn how individual colors can drift in different illumination conditions and with different color spaces. In order to provide a very general approach, we need to collect pixels describing different colors. Thus

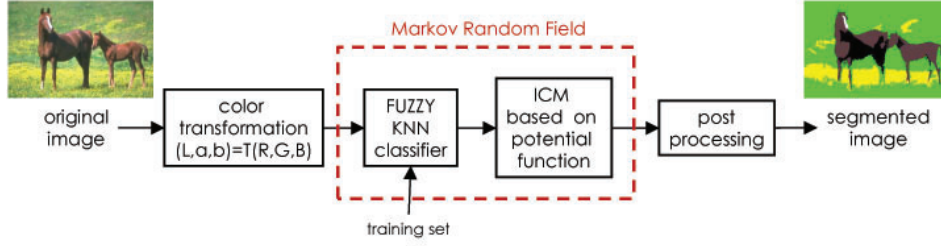


Fig. 1. Overall scheme of the proposed framework

we quantized the entire color space into eleven bins: black, white, red, yellow, green, blue, brown, purple, pink, orange, and grey. These colors are usually referred to as culture colors, which have been used in literature of different cultures in the past years to refer to colors [6]. The second step consisted in collecting pixels based on the above described color quantization, under various lighting conditions, from different cameras and from several distances from the cameras. Our idea to obtain a so diversified dataset of colors was to collect from the web video clips of eleven teams with the color of the uniforms corresponding to the eleven culture colors, and to obtain the sample colors from them. This procedure allowed us to obtain a great number of samples in real illumination conditions and, with very high probability, taken from different cameras.

We collected about 1200 samples for the training and about 1350 samples for the testing and we analyzed five of the most popular and widely used color spaces (RGB, normalized RGB, HSV, Lab, YUV) using a fuzzy clustering algorithm.

Fuzzy clustering is particular suited to color quantization since color boundaries are not well defined. In the proposed system we adopt the **fuzzy k-nearest neighbors algorithm** (KNN) introduced by Keller & al. [7], which works as follows. Let us assume that a training set of m samples vectors Z_1, Z_2, \dots, Z_m is available. Let X be a new vector considered as the input to be classified. For a fixed value of k , the first step consists in identifying, among these sample vectors, the k nearest neighbors Y_1, Y_2, \dots, Y_k of the input X . Then the membership vectors of the selected labeled samples Y are combined to find the membership vector of the input X , where the membership vector describes the probabilities of the membership to the possible C classes. Let $u_i(X)$ be the membership of the input X to the i^{th} class (with $i \leq C$), and w_{ij} the membership of its j^{th} neighbor Y_j to the same class ($w_{ij} = u_i(Y_j)$), then (with $m > 1$):

$$u_i(X) = \frac{\sum_{j=1}^k w_{ij} \left(\frac{1}{\|X - Y_j\|} \right)^{\frac{2}{(m-1)}}}{\sum_{j=1}^k \left(\frac{1}{\|X - Y_j\|} \right)^{\frac{2}{(m-1)}}} \quad (6)$$

In the above formula, the inverse distance is used to weight the membership degrees of the samples by assigning a higher

weight to closest vector.

The obtained results showed that the Lab color space is the one that provided the best correct classification rate (94.1%²) among the analyzed ones, thus it seems to be the more suitable in identify colors in different illumination conditions.

Based on the above considerations, we decided to use the discussed classifier to the problem of color segmentation. We apply the Fuzzy KNN algorithm to each pixel of the Lab version of the image using the same training test we used in the [5]. The classifier assign to each pixel a label corresponding to one of the classes given by the eleven culture colors. The obtained image is the segmented image where disjoint blobs (i.e. blobs of different colors or non connected blobs of the same colors) describe different segments. Even if fuzzy classifiers have been already adopted in image segmentation, the proposed approach based on culture colors has the advantage of avoiding the effect of oversegmentation and in the same time, thanks to the design of the training set, of reducing the errors due to shadows in the image.

Recalling MRF theory, the segmented image $f = \{f_1, \dots, f_{P \times Q}\}$, where $f_i \in \mathcal{L} = \{1, 2, \dots, 11\}$ and $P \times Q$ is the size of the image, is a possible realization of the field F .

The advantage of using a soft classifier (like the fuzzy classifier) with respect to the hard classifier is that to each pixel i is also associated the membership vector $\mathbf{u}(i) = \{u_1(i), u_2(i), \dots, u_{11}(i)\}$ describing the probabilities of the membership of the pixel to the 11 possible classes. This information will be exploited in the description of the ICM as explained in the next section.

C. Iterated Conditional Mode

As we said in Sec. II-A, an MRF is uniquely determined once the Gibbs distribution and the neighborhood system are defined. In the approach proposed here, for each pixel (for sake of clarity let us define the position of a pixel with the classical notation (x, y)) only four neighbors of first order and the corresponding four pair-site cliques are considered, as described by Fig.2.

The potential function we used is expressed by

²the other detection rates are: RGB= 91.3%, normalizde RGB= 89.9%, HSV= 86.8%, YUV= 91.4%

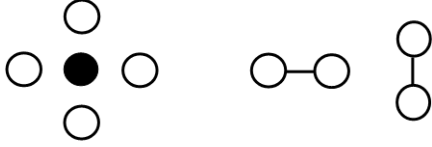


Fig. 2. Structure of a first order neighborhood system and corresponding pair-sites cliques.

$$\mathbf{V}_{((x,y),(\tilde{x},\tilde{y}))} = \frac{1}{\mathbf{u}(x,y) + \mathbf{u}(\tilde{x},\tilde{y})} \quad (7)$$

where $\mathbf{u}(x,y)$ is the membership vector associated to the pixel (x,y) and (\tilde{x},\tilde{y}) is a point belonging to $N(x,y)$, that is the 4-neighborhood system of (x,y) .

Since it is difficult to maximize the joint probability of a MRF as expressed by eq.4, it is common to use a deterministic algorithm called iterated conditional modes (ICM) which maximizes local conditional probabilities sequentially. The initial state of the ICM is the output of the fuzzy classifier. The ICM algorithm uses the greedy strategy in the iterative local maximization to visit all the points of the segmented image and update their values by minimizing the potential function in order to maximize the joint probability. When the ICM algorithm starts each pixel (x,y) of the image is randomly visited and its value is updated by trying to minimize the potential function in eq.7. Specifically, a local minimum is sought by letting

$$f_{\text{opt}}(x,y) = \arg \min_{f \in \mathcal{L}} \sum_{(\tilde{x},\tilde{y}) \in N(x,y)} V_{f((x,y),(\tilde{x},\tilde{y}))} \quad (8)$$

where $V_{f((x,y),(\tilde{x},\tilde{y}))}$ is the f^{th} component of the vector $\mathbf{V}_{((x,y),(\tilde{x},\tilde{y}))}$. The above equation assign to each pixel a value of the segment based on the probability of the membership of the neighboring pixels to the eleven segments, that is the selected value is the one that occurs with a higher probability in the neighborhood system. The above equation results in a local minimization of the Gibbs potential. Once each pixel is visited and the corresponding value updated, a new iteration starts. The algorithm ends when no new modification is introduced for a whole iteration, which is usually the case after 12-13 iterations.

The last step of the system is a post-processing function applied to connect close regions and remove isolated or small blobs.

An example of the proposed approach on natural images is shown in Fig.3. The algorithm, let us call it MFseg (Markov Field based Segmentation), is applied on two images from the Berkeley database. Each row contains the original image (on the left), the first version of the segmented image which is the output of the fuzzy KNN classifier, the output image of the

ICM algorithm and the final segmented image (on the right) after some post processing operations. We can observe how the last two images are a refinement of the first segmented image. The MRF description of the proposed color segmentation algorithm exploits, through the use of the ICM algorithm, the probability of the membership of the pixels to the eleven classes by allowing a meaningful refinement of the output of the classifier.

III. EXPERIMENTAL RESULTS

In this section, we demonstrate the segmentation results of the proposed approach on natural images in the Berkeley segmentation database [10]. Berkeley provides 300 images and corresponding ground truth data obtained from human subjects (at least 4 human segmentations per image).

The performance evaluation is based on four quantitative measures widely used in the field of image segmentation: (1) The **Probabilistic Rand Index** (PRI) [8] counts the fraction of pairs of pixels whose labellings are consistent between the computed segmentation and the ground truth averaging across multiple ground truth segmentations. (2) The **Variation of Information** (VoI) metric [9] defines the distance between two segmentations as the average conditional entropy of one segmentation given the other, and thus measures the amount of randomness in one segmentation which cannot be explained by the other. (3) The **Global Consistency Error** (GCE) [10] measures the extent to which one segmentation can be viewed as a refinement of the other. Segmentations which are related are considered to be consistent, since they could represent the same image segmented at different scales. (4) The **Boundary Displacement Error** (BDE) [11] measures the average displacement error of boundary pixels between two segmented images. It defines the error of one boundary pixel as the distance between the pixel and the closest pixel in the other boundary image.

We compare the scores calculated for our algorithm to eleven state-of-the-art color segmentation algorithms (in chronological order): the Mean Shift method (Mshift) [3], the standard normalized cut algorithm (Ncuts) [13], the JSEG algorithm (Jseg) [16], the pixel affinity based method (Affin) [15], the spectral clustering method (Spect-Clust) [17], the graph based segmentation (Graph-Based) [18], the multi-scale normalized cut approach (Mscuts) [20], the seeded graph cuts method (Seed)[19], the MSER-based segmentation method (ROI-Seg) [21], the normalized partitioning tree (NormTree) [22] and the saliency driven method (TotVar) [12]. Results for the cited algorithms were taken from [12]. Table 1 summarizes the scores for all algorithms and the proposed approach (MFseg). The best two results for each measure are always highlighted in bold. As we can seen looking at the table, the proposed algorithm shows competitive results compared to state of the art systems. We rank in the top two for two different measures (PRI and BDE) and we achieve the best Boundary Displacement Error score of all.

Fig.4 shows some results of the proposed algorithm on

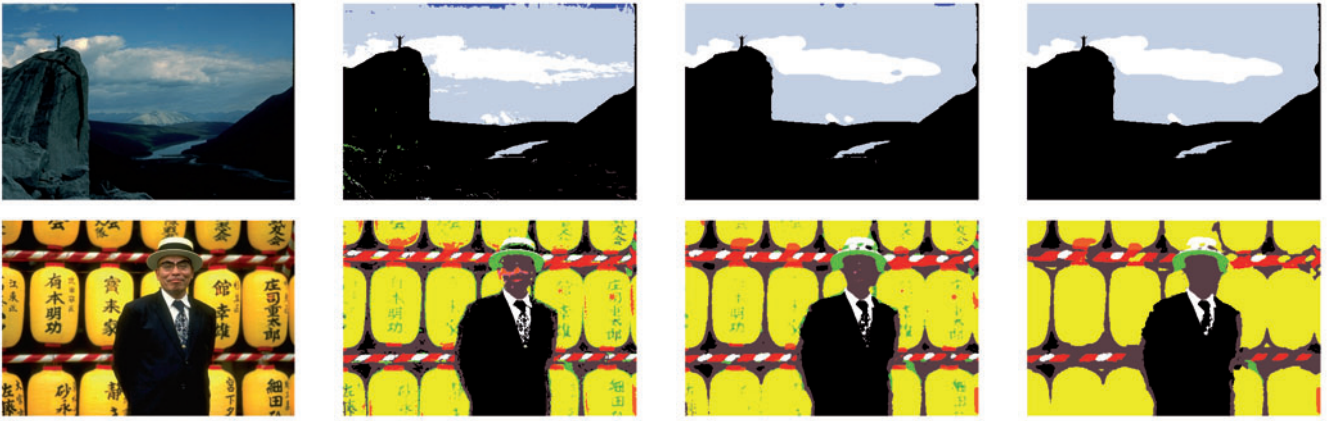


Fig. 3. Example of the proposed approach on two images from the Berkeley database. In order from left to right of each row: the original image, the segmented image output of the classifier, the result of the ICM algorithm, the final segmented image after post processing operations.



Fig. 4. Examples of original and segmented images from the Berkeley database.

images from the Berkeley database. These images are representative of the average result of the proposed approach on natural images. We would like to point out that, thanks to the fuzzy classifier, each pixel is assigned a label describing a culture color. This process allows to have a meaningful description of the segmented image in which each segment is described by a color as perceived by the Human Visual System³. This characteristic of the proposed system make it suitable to be integrated in several applications of image and video processing (an example could be to track or search a person based on clothes colors across the field of view of multiple camera in a video surveillance system).

To fairly assess an image-segmentation algorithm, we also need to investigate examples for which the algorithm has failed to produce good results. In Fig.5 we show two of such examples from the used database.

³for the sake of clarity, please note that for the classical description of the segmented images it is enough to describe blobs of different colors and non connected blobs of the same colors by different segments

By looking at the images in Fig.5, we can derive some considerations on the disadvantages of the proposed approach. The MFseg system is based uniquely on the analysis of the color of the image which is one of the most important features of the image but it can be not enough to discriminate objects in a scene. This is the case of the first image on the left: the system returns an unique segment for the hills in the background while the users usually associate them to different segments. This problem, that is a consequence of the choice to use a quantization based on only eleven colors, can be easily overcome by adding in the proposed approach an edge detection system to categorize different objects of the same color in different segments. The image on the right shows another problem affecting most of the state of the art systems: the presence of texture areas in the image. Texture areas are usually classified by segmentation algorithms as several segments (due to the presence of different range of colors, shapes, highlights, shadows) while users usually associate them to a unique segment. Several approaches are present in



Fig. 5. Examples of original and bad segmented images from the Berkeley database.

	PRI	VoI	GCE	BDE
Mshift	0.7958	1.9725	0.1888	14.41
Ncuts	0.7242	2.9061	0.2232	17.15
Jseg	0.7756	2.3217	0.1989	14.40
Affin		X	0.2140	X
SpectClust	0.7357	2.6336	0.2469	15.40
GraphBased	0.7139	3.3949	0.1746	16.67
Mscuts	0.7559	2.4701	0.1925	15.10
Seed	X	X	0.2090	X
ROI-Seg	0.7599	2.0072	0.1846	22.45
NormTree	0.7521	2.4954	0.2373	16.30
TotVar	0.7758	1.8165	0.1768	16.24
MFseg	0.7797	2.508	0.2010	14.3726

Table 1. Comparison of different methods on Berkeley image database using Probabilistic Rand Index (PRI), Variation of Information (VoI), Global Consistency Error (GCE) and Boundary Displacement Error. (BDE).

literature trying to overcome this problem even if a robust solution is still missing. It is thus necessary to include a texture analysis as well as an edge detection algorithm in the proposed approach in order to improve the performance of the system and increase the scores shown in table 1.

IV. CONCLUSIONS

In this paper we have introduced a new unsupervised color image segmentation algorithm. The main steps of the proposed algorithm are the Fuzzy KNN classifier and the ICM which maximizes local conditional probabilities leading to a suboptimal solution. The complete algorithm is described as a MRF. The experimental results show the good performance of the proposed approach which is comparable to state of the art systems even if only the color characteristic of the image is used for the segmentation process. For this reason, we are confident that the integration of an edge detection algorithm together with a texture analysis can improve the performance of the proposed system.

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