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Resource allocation for multiuser uplink systems

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Abstract

In this thesis we study the subject of resource allocation for uplink communication systems.

When users have target rate constraints and interference cancelation is used at the base station we provide the optimal decoding order and power allocation in order to minimize the power consumption. In addition conditions are derived under which the allocation can be done in a distributed way, with only some knowledge of the statistics of the system.

We then proceed to consider multiple-input multiple-output (MIMO) systems, and obtain the optimal precoding matrices such that each user maximizes its own ergodic transmission rate from the sole knowledge of the overall channel statistics. The benefits of using a coordination signal and successive decoding are analyzed.

Next, a scenario in which mobile terminals can be simultaneously connected to several base stations, using non-overlapping frequency bands is investigated. The optimal power allocation in terms of sum rate is derived for different receiver types and an iterative algorithm proposed to achieve the optimal allocation.

Finally, we consider decentralized medium-access control in which many pairwise interactions, where users compete for a medium access opportunity, occur between randomly selected users that belong to a large population. The choice of power level is done by each user, and both team and noncooperative scenarios are analyzed.

Résumé

Dans cette thèse nous étudierons l'allocation de puissance optimale pour des systèmes de communication multi utilisateur en lien ascendant.

L'ordre de décodage et l'allocation de puissance optimaux pour minimiser la consommation totale de puissance sont déterminés lorsque les utilisateurs ont des contraintes de débit et que la suppression d'interférence est utilisée dans la station de base. De plus, nous chercherons à déterminer dans quelles conditions il est possible de faire une allocation distribuée en ne se basant que sur les connaissances statistiques du système.

Par la suite nous considérerons les systèmes a entrées multiples sorties multiples, afin d'obtenir les matrices de précodage optimales pour que chaque utilisateur maximise son taux de transmission ergodique avec la seule connaissance des statistiques des canaux. Les bénéfices de l'utilisation d'un signal de coordination et de décodages successifs sont analysés.

Ensuite, nous étudierons un scénario dans lequel les terminaux mobiles ont la possibilité de se connecter simultanément à plusieurs stations de base en utilisant des bandes de fréquence non superposées. L'allocation de puissance optimale est dérivée pour différents types de récepteurs et un algorithme itératif est proposé pour obtenir l'allocation optimale.

Finalement, nous considérerons les contrôles d'accès au canal décentralisé entre utilisateurs choisis aléatoirement parmi une population nombreuse, avec de nombreuses interactions entre paires d'utilisateurs où les utilisateurs sont en concurrence pour une opportunité d'accès.

Le choix du niveau de puissance est fait par chaque utilisateur, et nous analyserons à la fois les scénarios d'équipe et non coopératifs.

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List of Abbreviations

AWGN Additive White Gaussian Noise
BS Base Station
CDIT Channel Distribution Information at the Transmitter
CDMA Code Division Multiple Access
CSI Channel State Information
CSIR Channel State Information at the Receiver
CSIT Channel State Information at the Transmitter
GSM Global System for Mobile communications
i.i.d. independent and identically distributed
MAC Multiple Access Channel
MF Matched Filter
MIMO Multiple-Input Multiple-Output
MMSE Minimum Mean Square Error
MS Mobile Station
NE Nash Equilibrium
OFDM Orthogonal Frequency Division Multiplexing
OFDMA Orthogonal Frequency Division Multiple Access
pdf probability density function
PoA Price of Anarchy
Qos Quality of service
SIC Successive Interference Cancellation
SIMO Single Input Multiple Output
SINR Signal to Interference plus Noise Ratio
SIR Signal to Interference Ratio
SISO Single Input Single Output

TD Time Division

TPS Throughput Per Slot

UMTS Universal Mobile Telecommunication System

WCDMA Wideband Code Division Multiple Access

List of Symbols

x	a scalar variable
$\mathbf{x}$	a vector variable
$\mathbf{X}$	a matrix variable
$\mathcal{X}$	a set
$\mathcal{N}$	The Set of natural numbers
$\mathcal{C}$	The Set of complex numbers
$\|x\|$	The absolute value of an scalar
$\|\|\mathbf{x}\|\|$	Euclidean norm of the vector **x**
$\|\mathcal{X}\|$	The cardinality of a set
$\mathbf{X^{-1}}$	The inverse of matrix **X**
$\mathbf{0}$	All zero matrix
$\mathbf{I}$	Identity matrix
$(.)^T$	Transpose operator
$(.)^H$	Hermitian transpose operator
$Tr(.)$	Trace operator
$\|\mathbf{X}\|$	Determinant of the matrix **X**
$\mathcal{E}(.)$	Expectation operator
$\log_2(.)$	The base 2 logarithm
$\ln(.)$	The natural logarithm
$I(X;Y)$	Mutual information between random variables X and Y
$mod(x/y)$	Remainder in dividing two integer numbers x and y
$\lfloor x \rfloor$	Largest integer smaller than x
$o(.)$	The little-o notation, i.e. f=o(g) means that $\frac{f}{g} \to 0$
$[.]^+$	$\max(.,0)$ operator

CHAPTER 1

Introduction

During the last years, the appearance of new wireless services such as streaming and real time multimedia applications, video calls, Internet browsing or file transfer has led to an increasing demand of higher data rates, while at the same time requiring different quality of service constraints depending on the particular application.

These increased rates pose an important challenge, since the higher required spectral efficiency calls for more aggressive frequency reuse schemes, resulting in turn in a higher level of interference affecting all the communication links. One way to address this challenge is through proper allocation of the wireless resources. In particular, power allocation has been used in both the uplink and downlink of different communication systems to tackle interference management. This is specially important for the uplink due to the limited battery budget available at the mobile stations, making energy conservation important for its lifetime, and power control helps minimize the total energy requirements.

Early power control research, intended for voice-centric wireless networks, focused either on balancing the signal to interference ratios (SIR) , where the objective is maximizing the minimum SIR level, or achieving a target SIR enabling a successful communication from the point of view of outage probability. A similar approach is adopted here in chapter 2, although in our case the target signal to interference plus noise ratio (SINR) are determined by the varying data rates or quality of service (Qos) constraints required for the different applications, in contrast to the constant SINR required in a voice network.

The remainder of this work, i.e. the contributions shown in chapters 3,4 and 5, concentrates however on power allocation in the context of data wireless networks, where in general throughput optimization becomes a more relevant figure of merit, due to the possibility of varying transmit rates, adapting them to the channel state conditions by using adaptive modulation and coding schemes. The advent of services requiring best effort traffic, gives an additional degree of flexibility when compared with traditional voice communications, allowing the allocation of more resources to users with better channel conditions in order to benefit from the so-called multiuser diversity and increase the total capacity of the system.

The focus of this dissertation is on resource allocation for uplink systems. In particular, an emphasis is given to the possibility of determining this allocation in a distributed way, obtaining conditions under which this is possible, or at least with a reduced amount of channel state information at the transmitter in order to reduce the feedback costs associated with it.

A common set of assumptions will be considered throughout most of the thesis, except where otherwise stated:

- We will consider mainly a cellular 4G system, although some of the results can also be applied in the adhoc network context.

- We will consider physical layer resource allocation, mainly power allocation (or precoding matrices when multiantenna transmitters are considered), but scheduling and other upper layer resource allocation issues will not be dealt with here.

- A single cell system is considered, so only intracell interference is present (intercell interference is implicitly taken into account into the noise). Only in chapter 4 a system with several base stations will be analyzed, but no intercell interference will be present either, as the different base stations are assumed to communicate over non overlapping frequency bands.

- The main figure of merit used will be the capacity or sum rate in its information theoretic sense. As a result, no assumptions are made regarding the particular modulation scheme. In addition, this means assuming ideal link adaptation, for which adaptive modulation and coding schemes must be used. This is a reasonable assumption since practical coding schemes performing close to Shannon limits exist.

- Multiple antennas or receive dimensions are assumed at the base station in order to deal with the multiuser interference.

Another common thread in the thesis is the use of two set of tools, Random Matrix Theory and Game Theory, which have created a great deal of interest in the last few years regarding their application to the wireless communications field.

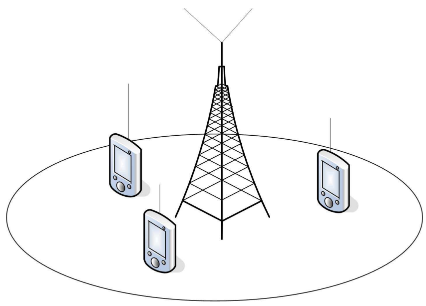
Random Matrix Theory allows to exploit the averaging properties of large systems (in the number of users, receive dimensions) and thus characterize the performance of a system in the asymptotic regime as a function of a few key parameters.

Game Theory is a convenient tool for the analysis of systems in which users have to make strategic choices independently (in our case selecting the transmission power level), and their success will depend also on other users' choices. Two different scenarios will be considered:

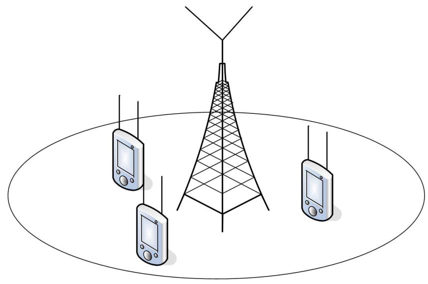
- The team problem, in which all users share the common objective of maximizing a global criterion.

- The non cooperative game, in which each user maximizes its own performance measure and where the solution concept is the Nash equilibrium. We will study both pure strategies, which determine the player's choice for any situation he could face, and mixed strategies, composed of a collection of pure strategies, each chosen with a given probability.

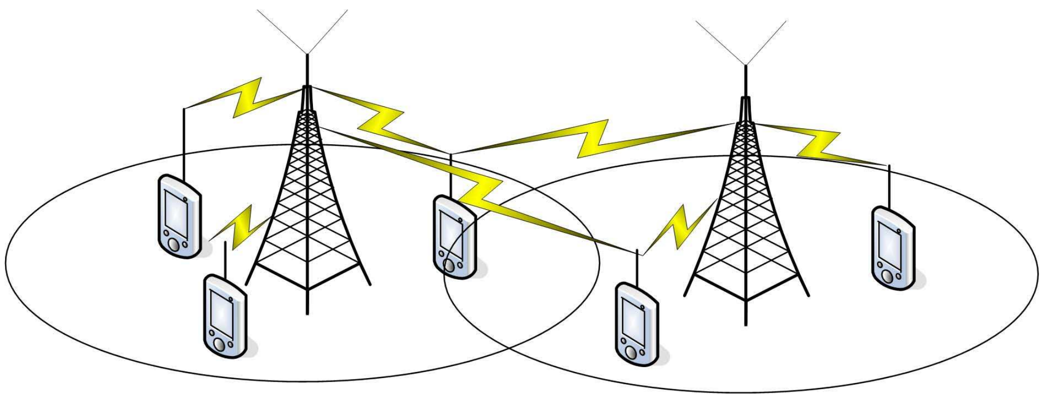
An outline and the contributions of each chapter are given below:



(a) Chapter 2: MAC channel with single antenna MS.



(b) Chapter 3: MIMO - MAC channel (multiple antenna MS).



(c) Chapter 4: Single antenna users communicating simultaneously to several BS.

Figure 1.1: Scenarios considered in the different chapters

Chapter 2 - Optimal decoding order and power allocation under target rate constraints

In this chapter, we consider the scenario illustrated in figure 1.1(a): several single-antenna users, with target rate constraints, are communicating with a common base station equipped with multiple receive dimensions (either multiple antennas or chips in the case of a code division multiple access (CDMA) system) and a multiuser receiver (matched filter successive interference cancellation (MF-SIC) and minimum mean square error SIC (MMSE-SIC) are considered) to deal with the interuser interference. Using results from Random Matrix Theory, the optimal decoding order at the base station and power allocation across users are obtained to minimize the total power consumption while satisfying the users' rate constraints. In addition, conditions under which the power allocation can be carried in a distributed way are discussed. The work in this chapter has been published in:

- A. Suárez, R. de Lacerda Neto, M. Debbah, and N. Linh-Trung "Power allocation under quality of service constraints for uplink multi-user MIMO systems" REV'06, *IEEE 10th Biennial Vietnam Conference on Radio and Electronics*, November 6-7, 2006, Hanoi, Vietnam

- A. Suarez, M. Debbah, L. Cottatellucci and E.Altman "Optimal decoding order under target rate constraints" *8th IEEE Workshop on Signal Processing Advances for Wireless Communications (SPAWC)*, June 17-20, 2007, Helsinki, Finland

Chapter 3 - MIMO multiple access channels: Distributed power allocation In this chapter, we consider multiuser MIMO multiple access channels (MAC) , in which mobile stations (MS) are also equipped with multiple antennas, as illustrated in figure 1.1(b). The problem is analyzed under a game theoretic perspective, deriving the optimal precoding matrices when users maximize their own ergodic rate under statistical channel state information (CSI) . Two scenarios are considered, in the first no coordination is available and thus single user decoding is performed at the base station (BS) , whereas in the second, there exists a random coordination signal which can be heard by all the MSs and the BS and is used to determine the decoding order of the different users when using an interference cancellation receiver. The work in this chapter has been published in part in:

- S, Lasaulce, A. Suárez, M. Debbah, and L. Cottatellucci, "Power allocation game for fading MIMO multiple access", in the *ACM Proceedings of the International Conference on Game Theory in Communications Networks (GAMECOMM)*, October 23-25, 2007, Nantes, France

Chapter 4 - Throughput Optimization in Heterogeneous Networks: Cross-System Diversity

In this chapter we introduce and study the problem where several users can be connected simultaneously to a set of BS (each of them using non overlapping frequency bands), as depicted in figure 1.1(c). The optimal power allocation is obtained, maximizing the ergodic sum-rate. Three different type of receivers are considered: optimum receiver, matched filter and MMSE. For the first, exact expressions are derived, whereas for the last two a concave approximation is analyzed and conditions for its validity studied. This work has been published in:

- S, Lasaulce, A. Suárez, R. de Lacerda Neto and M. Debbah, “Cross-system resources allocation based on random matrix theory” *Valuetools 2007, 2nd International Conference on Performance Evaluation Methodologies and Tools*, October 23-25, 2007, Nantes, France,
- S, Lasaulce, A. Suárez, R. de Lacerda Neto and M. Debbah, “Using cross-system diversity in heterogeneous networks : throughput optimization”, *Performance Evaluation, Elsevier*, Vol.65, N°11-12, November 2008 , pp 907-921

Chapter 5 - Team and Noncooperative Solutions to Access Control

In this chapter we consider a decentralized medium access control problem under both team and noncooperative game perspectives. It is shown that optimal pure policies do not exist in the team framework, but both an optimal solution as well as equilibria exist within the class of mixed policies. We establish structural properties as well as explicit characterization of these: We show that the optimal policy requires only three priority levels, whereas the noncooperative game possesses a unique symmetric equilibrium point that uses at most two priority levels. This work has been published in:

- E. Altman, I. Menache, A. Suarez “Team and noncooperative solutions to access control with priorities” *Infocom 2009, 28th IEEE Conference on Computer Communications*, April 19-25, 2009, Rio de Janeiro, Brazil

and submitted for publication in:

- A. Suarez, E. Altman, I. Menache “Team and noncooperative solutions to access control with priorities” *submitted to IEEE Transactions on Networking*

CHAPTER 2

Optimal decoding order and power allocation under target rate constraints

In this chapter we consider an uplink scenario in which each user has a target rate constraint that must be satisfied and the objective is to minimize the total power consumption. The BS is equipped with an interference cancellation receiver in order to deal with the multiuser interference. Successive interference cancellation is a simple scheme, which successively subtracts the decoded signal from the composite received signal, resulting in reduced interference for subsequent users. Here we will assume no decoding errors are made, and thus interference form previously decoded users is fully removed.

In its full generality, the target rate problem can be tackled through proper power allocation (when the rate regions are achievable) [Boche 2004, Jorswieck 2004b, Jorswieck 2003]. However, the power allocation scheme depends on the channel realizations of all users, the requested rates and the type of receiver structure [Müller 2000, Caire 2004, Meshkati 2005a]. In addition, when interference cancellation receivers are considered, it is also influenced by the decoding order used at the receiver. In fact, in order to meet the rate constraints, it is immediate to see that a given decoding order uniquely determines the associated power allocation. Thus, an added problem is the complexity increase of the power allocation algorithm with the number of users, since in general all the possible decoding orders have to be considered, making it a NP-hard problem.

In this chapter, we derive the optimal (which minimizes the total power) decoding order for MMSE-SIC and matched filter receivers for a given set of requested rates, considering rather general channel signatures. Similar work had been done previously in [Li 2004], but only for the MMSE-SIC receivers and the particular case of i.i.d. signatures.

Interestingly, it is also shown that the power allocation (in the case of independent and identically distributed (i.i.d) signatures) can be determined in a decentralized manner (each user can determine his decoding order and power allocation based only on the knowledge of the discrete set of possible rates, whereas in general, the base station computes the algorithm and allocates the powers) for a high number of users in the network. This result can be applied to reduce the downlink signaling of multi-user systems.

2.1 System model

A system composed of a base station, with N dimensions and K users, is considered. We are interested in the uplink scenario. Each user k is supposed to send a signal at a requested rate R_k. The input output relationship of the system is then given by:

$$\mathbf{y} = \mathbf{H}\mathbf{P}^{\frac{1}{2}}\mathbf{s} + \mathbf{n}, \tag{2.1}$$

where $\mathbf{y}$, $\mathbf{s}$, $\mathbf{n}$, $\mathbf{H}$ and $\mathbf{P}^{\frac{1}{2}}$ are respectively the received signal, transmitted signal, additive white Gaussian noise (AWGN) of variance σ^2, the mixing matrix, and diagonal matrix of transmitted powers. In the following, these terms are written out as: $\mathbf{y} = [y_1, y_2, \ldots, y_N]^T$, $\mathbf{s} = [s_1, s_2, \ldots, s_K]^T$, $\mathbf{n} = [n_1, n_2, \ldots, n_N]^T$,

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \ldots & h_{1K} \\ h_{21} & h_{22} & \ldots & h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1} & \ldots & \ldots & h_{NK} \end{bmatrix},$$

and

$$\mathbf{P}^{\frac{1}{2}} = \begin{bmatrix} p_1^{\frac{1}{2}} & 0 & 0 & \ldots & 0 \\ 0 & p_2^{\frac{1}{2}} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & p_K^{\frac{1}{2}} \end{bmatrix}.$$

The h_{ik} are independent zero mean gaussian variables with variances $|g_{ik}|^2$. In particular, the mixing matrix can be written as

$$\mathbf{H} = \mathbf{G} \odot \mathbf{W}$$

where $\mathbf{W}$ and $\mathbf{G}$ are respectively an N x K i.i.d. zero mean Gaussian matrix and the pattern mask specific to a given technology $\mathbf{G} = [g_{ik}]_{i=1\ldots N, k=1\ldots K}$. $\odot$ is the Hadamard product, defined as $(A \odot B)_{ij} = A_{ij}B_{ij}$.

The model is broad enough to incorporate several technologies: for instance, MIMO and flat fading CDMA systems.

In the following, column $\mathbf{h}_k$ corresponding to user k will be called a signature irrespective of the technology.

2.2 MMSE-SIC receiver

The MMSE receiver has several attributes that make it appealing for use. It is known to generate a soft decision output that maximizes the output SINR [Madhow 1994].

As far as the MMSE SINR is concerned and considering Eq.(2.1), the output of the MMSE detector, denoted by $\hat{\mathbf{s}} = [\hat{s}_1, \ldots, \hat{s}_K]^T$, is given by

$$\begin{aligned} \hat{\mathbf{s}} &= \mathbf{P}^{\frac{1}{2}}\mathbf{H}^H \left(\mathbf{H}\mathbf{P}\mathbf{H}^H + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{y} \\ &= \mathbf{P}^{\frac{1}{2}}\mathbf{H}^H\mathbf{A}^{-1}\mathbf{y}, \end{aligned}$$

with $\mathbf{A} = \mathbf{H}\mathbf{P}\mathbf{H}^H + \sigma^2\mathbf{I}_N$. Each component $\hat{s}_k$ of $\hat{\mathbf{s}}$ is corrupted by the effect of both thermal noise and “multi-user interference” due to the contributions of the other symbols

$\{s_l\}_{l\neq k}$. Let us now derive the expression of the SINR at one of the K outputs of the MMSE detector. Let $\mathbf{h}_k$ be the column of $\mathbf{H}$ associated to element s_k, and $\mathbf{U}$ the $N\times(K-1)$ matrix that remains after extracting $\mathbf{h}_k$ from $\mathbf{H}$. The component $\hat{s}_k$ after MMSE equalization has the following form:

$$\hat{s}_k = \eta_{\mathbf{h}_k} s_k + \tau_k,$$

where

$$\eta_{\mathbf{h}_k} = p_k^{\frac{1}{2}}\mathbf{h}_k^H\mathbf{A}^{-1}p_k^{\frac{1}{2}}\mathbf{h}_k, \tag{2.2}$$

$$\tau_k = p_k^{\frac{1}{2}}\mathbf{h}_k^H\mathbf{A}^{-1}\mathbf{H}P^{\frac{1}{2}}[s_1,\ldots,s_{k-1},0,s_{k+1},\ldots,s_K]^T \tag{2.3}$$

$$+\,p_k^{\frac{1}{2}}\mathbf{h}_k^H\mathbf{A}^{-1}\mathbf{n}. \tag{2.4}$$

The SINR_k at the output k of the MMSE detector can be shown to be expressed as (see e.g. [Tse 1999]):

$$\mathrm{SINR}_k = p_k\mathbf{h}_k^H\left(\mathbf{U}\mathbf{P}_k\mathbf{U}^H+\sigma^2\mathbf{I}_N\right)^{-1}\mathbf{h}_k,$$

where $\mathbf{P}_k$ is the power matrix, from which the k-th column and row have been removed.

The MMSE receiver has the advantage of a very low complexity implementation. This feature (due in part to its linearity) has triggered the search for other MMSE based receivers such as the MMSE Successive Interference Cancellation (MMSE-SIC) [Cioffi 1995b, Cioffi 1995a], which is at the heart of very famous schemes such as BLAST [Golden 1999].

The algorithm relies on a sequential detection of the received block [Wolniansky 1998]. At the first step of the method, an MMSE equalization of matrix $\mathbf{T}_{N,K} = \mathbf{H}$ is performed by a multiplication of $\mathbf{y}$ by matrix

$$\mathbf{F}_1 = \mathbf{P}^{\frac{1}{2}}\mathbf{T}_{N,K}^H(\mathbf{T}_{N,K}\mathbf{P}\mathbf{T}_{N,K}^H+\sigma^2\mathbf{I})^{-1}.$$

Suppose that the algorithm starts by decoding symbol s_K. The estimated symbol goes through a turbo-decoder chain in order to improve the reliability of the detection process. Assuming a perfect decision (this is possible if the information s_K has been encoded at a rate of $\log_2(1+\mathrm{SINR}_K)$), the resulting estimated symbol $\hat{s}_K$ is subtracted from the vector of received samples in the following manner:

$$\mathbf{r}_2 = \mathbf{r}_1 - p_K^{\frac{1}{2}}\hat{s}_K\mathbf{t}_K,$$

where $\mathbf{t}_i$ represents the i^{th} column of $\mathbf{T}_{N,K}$ and vector $\mathbf{r}_1 = \mathbf{y}$. This introduces one degree of freedom for the next cancelling vector choice which enables to reduce the noise plus interference influence and yields an increase in the decision process reliability.

The second step can be virtually represented by a completely new system of $K-1$ symbols $(s_1,\ldots,s_{K-1})$ transmitted with powers $(p_1,\ldots,p_{K-1})$ by an $N\times(K-1)$ matrix $\mathbf{T}_{N,K-1}$ on the same flat frequency fading channel. Equalizing with matrix

$$\mathbf{F}_2 = \mathbf{P}_{K-1}^{\frac{1}{2}}\mathbf{T}_{N,K-1}^H(\mathbf{T}_{N,K-1}\mathbf{P}_{K-1}\mathbf{T}_{N,K-1}^H+\sigma^2\mathbf{I}_N)^{-1},$$

one can retrieve symbol s_{K-1} which has been encoded at a rate of $\log_2(1+\text{SINR}_{K-1})$. The same process described at the beginning can be re-iterated. The advantage of such a scheme is that

$$\text{SINR}^{\text{SIC}}_{(K-1)} \geq \text{SINR}^{\text{MMSE}}_{(K-1)},$$

which means that one is able to convey more information on the second symbol (since the SINR increases) than with MMSE equalization.

This analysis can be extended to iteration i obtaining the corresponding SINR as

$$\text{SINR}_i = \gamma_i = p_i \mathbf{h}_i^H \left(\sum_{l=i+1}^{K} p_l \mathbf{h}_l \mathbf{h}_l^H + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{h}_i,$$

and the power

$$p_i = \frac{\gamma}{\mathbf{h}_i^H \left(\sum_{l=i+1}^{K} p_l \mathbf{h}_l \mathbf{h}_l^H + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{h}_i}.$$

2.2.1 Asymptotic SINR

The output SINR depends in an intricated manner on the different signature realizations. Interestingly, when the dimensions of the system increase at the same rate (i.e $N, K \to \infty$, $\frac{K}{N} = \alpha$), it can be shown [Girko 2001] and [Tulino 2005a] that the SINR, γ_k, at the output of the MMSE-SIC receiver is given by:

$$\gamma_k = \frac{p_k}{N} \sum_{i=1}^{N} \frac{|\, g_{ik} \,|^2}{\sigma^2 + \frac{1}{N} \sum_{l=k+1}^{K} \frac{p_l |g_{il}|^2}{1+\gamma_l}}.$$

Hence, the SINR does not depend on the channel realization.

2.2.2 Decoding order analysis

In this part, a flat fading scenario is considered, i.e. $g_{ik} = g_k$, where the SINR can be rewritten as

$$\gamma_k = p_k \frac{|\, g_k \,|^2}{\sigma^2 + \frac{1}{N} \sum_{l=k+1}^{K} \frac{p_l |g_l|^2}{1+\gamma_l}}. \tag{2.5}$$

Result: For the case of flat fading channels, the optimal decoding ordering depends on the user requested SINR, weighted by the individual path losses, and follows the ordering $\frac{1+\gamma_1}{|g_1|^2} < \frac{1+\gamma_2}{|g_2|^2} < \cdots < \frac{1+\gamma_K}{|g_K|^2}$. Moreover, the power allocated to each user has an explicit form given by:

$$p_k = \frac{\gamma_k}{|g_k|^2} \sigma^2 \prod_{i=k+1}^{K} [1 + \frac{1}{N} \frac{\gamma_i}{1+\gamma_i}].$$

Proof:

Define $\beta_k = \frac{p_k|g_k|^2}{\gamma_k}$. Thus

$\beta_{k-1} = \beta_k + \frac{1}{N}\frac{p_k|g_k|^2}{1+\gamma_k} = \beta_k\left(1 + \frac{1}{N}\gamma_k 1 + \gamma_k\right)$

So it can be easily seen that the exchange of the decoding order of 2 users would not affect the remaining ones. Let us now consider two possible orderings $[\gamma_{k-1}, \gamma_k]$ (user $k-1$ is here decoded before user k) and $[\gamma_k, \gamma_{k-1}]$ with respective power allocations $[p_{k-1}, p_k]$ and $[p_k^*, p_{k-1}^*]$. Then:

$$p_k = \frac{\beta_k\gamma_k}{|g_k|^2}$$

$$p_{k-1} = \frac{\beta_k\gamma_{k-1}}{|g_{k-1}|^2}\left(1 + \frac{1}{N}\frac{\gamma_k}{1+\gamma_k}\right)$$

$$p_{k-1}^* = \frac{\beta_k\gamma_{k-1}}{|g_{k-1}|^2}$$

$$p_k^* = \frac{\beta_k\gamma_k}{|g_k|^2}\left(1 + \frac{1}{N}\frac{\gamma_{k-1}}{1+\gamma_{k-1}}\right)$$

$$(p_k + p_{k-1}) - (p_k^* + p_{k-1}^*) = \frac{\beta_k}{N}\gamma_k\gamma_{k-1}\left(\frac{1}{|g_{k-1}|^2(1+\gamma_k)} - \frac{1}{|g_k|^2(1+\gamma_{k-1})}\right)$$

Then in order to minimize the total power consumption with the ordering $[\gamma_k, \gamma_{k-1}]$ we must have

$$|g_k|^2(1+\gamma_{k-1}) < |g_{k-1}|^2(1+\gamma_k)$$

$$\frac{1+\gamma_{k-1}}{|g_{k-1}|^2} < \frac{1+\gamma_k}{|g_k|^2}$$

2.3 Matched filter SIC receiver

The matched filter for user k is given by $\mathbf{u}_k^H = \mathbf{h}_k^H = (\mathbf{g}_k \odot \mathbf{w}_k)^H$. The signal at the output of the matched filter is given by

$$\mathbf{u}_k^H\mathbf{y} = \mathbf{p_k^{\frac{1}{2}}}|\mathbf{g_k} \odot \mathbf{w_k}|^{\mathbf{2}}\mathbf{s_k} + \sum_{\mathbf{i}\neq\mathbf{k}}\mathbf{u^H}\mathbf{p_i^{\frac{1}{2}}}(\mathbf{g_i} \odot \mathbf{w_i})\mathbf{s_i} + \mathbf{u^H n}$$

and the SINR can be expressed as

$$\gamma_k = \frac{p_k(\sum_{i=1}^N |w_{ik}|^2 g_{ik}^2)^2}{\sigma^2(\sum_{i=1}^N |w_{ik}|^2 g_{ik}^2) + \sum_{l=k+1}^K p_l|\sum_{i=1}^N w_{ik}^* w_{il} g_{ik} g_{il}|^2}.$$

2.3.1 Asymptotic SINR

In the case of a large number of users and dimensions increasing at the same rate (i.e $N, K \to \infty$ but the ratio $\frac{K}{N} = \alpha$, also known as the load of the system), and taking into account that the w_{ij} are independent and $E\{|w_{ij}|^2\} = 1/N$ and $E\{|w_{ij}|^4\} = \frac{1}{N^\alpha}$ with $\alpha > 1$, the SINR γ_k can be shown to be equal to

$$\gamma_k = \frac{p_k (\sum_{i=1}^N g_{ik}^2)^2}{N\sigma^2 \sum_{i=1}^N g_{ik}^2 + \sum_{l=k+1}^K (p_l \sum_{i=1}^N g_{ik}^2 g_{il}^2)} \tag{2.6}$$

2.3.2 Decoding order analysis

In this section a separable model will be considered for the channel energy profiles, i.e. $g_{ik} = a_i b_k$, which encompasses MIMO and frequency selective CDMA systems, among others. Hence equation (2.6) can be rewritten as

$$\gamma_k = \frac{p_k b_k^2 (\sum_{i=1}^N a_i^2)^2}{N\sigma^2 \sum_{i=1}^N a_i^2 + \sum_{l=k+1}^K (p_l b_l^2 \sum_{i=1}^N a_i^4)}$$

In the following, define $E_k = \frac{1}{N}\sum_{i=1}^N |g_{ik}|^2$ as the average energy of user k, then the following result holds:

Result: For the matched filter SIC receiver, the optimal decoding order is given in order of decreasing channel energies, i.e. $E_1 > E_2 > \cdots > E_K$ (where the index denotes the decoding order of the user), and the power allocation to satisfy the requested rates is given by

$$p_k = \frac{\gamma_k}{E_k} \prod_{l=k+1}^K (1 + \gamma_l \frac{\sum_{i=1}^N |g_{il}|^4}{E_l^2}) \tag{2.7}$$

Proof: The proof follows the same steps as the one in 2.2.

Let $A_2 = \frac{1}{N}\sum_{i=1}^N a_i^2$, and $A_4 = \frac{1}{N}\sum_{i=1}^N a_i^4$. Then the SINR is given by:

$$\gamma_k = \frac{p_k b_k^2 A_2^2}{\sigma^2 A_2 + \frac{1}{N} A_4 \sum_{l=k+1}^K p_l b_l^2}$$

Define $\beta_k = \frac{p_k b_k^2 A_2^2}{\gamma_k}$. Hence

$$\beta_k = \sigma^2 A_2 + \frac{1}{N} A_4 \sum_{l=k+1}^K p_l b_l^2$$

$$\beta_{k-1} = \beta_k (1 + \frac{1}{N}\gamma_k \frac{A_4}{A_2^2})$$

So it can be easily seen that the exchange of the decoding order of 2 users would not affect the remaining ones. Let us now consider two possible orderings $[\gamma_{k-1}, \gamma_k]$ (user $k-1$ is here decoded before user k) and $[\gamma_k, \gamma_{k-1}]$ with respective power allocations $[p_{k-1}, p_k]$ and $[p_k^*, p_{k-1}^*]$. Then:

$$p_k = \frac{\beta_k \gamma_k}{b_k^2 A_2^2}$$

$$p_{k-1} = \frac{\beta_k \gamma_{k-1}}{b_{k-1}^2 A_2^2}(1 + \frac{1}{N}\gamma_k \frac{A_4}{A_2^2})$$

$$p_{k-1}^* = \frac{\beta_k \gamma_{k-1}}{b_{k-1}^2 A_2^2}$$

$$p_k^* = \frac{\beta_k \gamma_k}{b_k^2 A_2^2}(1 + \frac{1}{N}\gamma_{k-1} \frac{A_4}{A_2^2})$$

So that

$$p_k + p_{k-1} - (p_k^* + p_{k-1}^*) = \frac{C}{b_{k-1}^2} - \frac{C}{b_k^2} \qquad \text{with} \quad C = \frac{1}{N}\frac{\gamma_k \gamma_{k-1} A_4}{A_2^4}$$

and the ordering to minimize the requested power depends only on the channel energies (since for user k, the energy is given by $E_k = b_k^2 \sum_{i=1}^N a_i^2$) and therefore the decoding order should be done in terms of decreasing energies ($b_{k-1}^2 > b_k^2$). The result follows therefore directly.

2.4 Distributed allocation

In many cases, the central entity can not feedback to the users the different powers in order to satisfy the requested rates. Moreover, the downlink overhead signaling may dramatically impact the useful rate as the number of users in the system increases. In these cases, a decentralized approach may be used where each user determines solely his power. Previous attempts for the analysis of decentralized schemes rely mainly on game theoretic approaches. In this section, we will show how asymptotic analysis can be used in this setting.

We consider a system in which users have a discrete set of M different available rates to choose from $R_1, ..., R_M$, as is the case in universal mobile telecommunication system (UMTS) or other wireless local area network standards. The number of users in each class rate is denoted by $K_1, ..., K_M$. The users are supposed to know the average fraction of users with a certain rate i.e $K_i^* = pr(R = R_i)K$ as well as the total number of users K in the system. The values $pr(R = R_i)$ are usually provided by previous measurements on the user's system behavior. In the case of a high number of users,

$$K_i \approx K_i^* = pr(R = R_i)K$$

.

2.4.1 Groupwise detection

In the case of i.i.d signatures, a user in rate class K_m can estimate his SINR and his decoding order since in this case, equation (2.5) boils down to:

$$\gamma_k \approx p_k \frac{1}{\sigma^2 + \frac{1}{N}\sum_{l=m}^{M} K_m^* \frac{p_l}{1+\gamma_l}} \tag{2.8}$$

The receiver in this case needs to implement a Groupwise SIC. Indeed, it is not possible for the user to determine precisely in which order he will be decoded among all the users with the same rate requirements, since this decision can be taken arbitrarily by the base station. Users in the same class can be decoded either in an MMSE filter or MMSE SIC fashion. In the latter case, users will have a better SINR then the targeted one which will reduce the probability of error. Moreover, as previously, the power allocation has an explicit form which depends only on the probabilities of the users to be in a certain class:

$$p^k = \gamma_k \sigma^2 + \frac{1}{N}\sum_{l=m}^{M} K_m^* \frac{p_l}{1+\gamma_l}. \tag{2.9}$$

For the MMSE-SIC, the groups of users should be decoded in order of increasing requested rates, by a derivation following the lines of the one in section 2.3.2.

Note that the same does not hold for the matched filter as the decoding order depends on the channel strength (which is the same in the i.i.d case) and not the target SINR's.

2.5 Simulations

In this section, some numerical results are presented to illustrate the theoretical claims. All simulation have been performed for an SNR (SNR= $\frac{1}{\sigma^2}$) of 10dB.

Figure 2.1 presents the requested and achieved rates for the SIC matched filter with optimal power allocation and decoding order. The users share a common power profile along the different dimensions and are supposed to be affected by random path losses. As one can see, the asymptotic results match for a reasonable system with $N = 256$ receive dimensions and $K = 100$ users.

In figure 2.2, the required power for a set of requested rates is plotted for the MMSE and matched filter SIC for different loads: $\alpha = 0.2$ and $\alpha = 0.6$. An important gain is achieved with the MMSE-SIC filter, especially as the load increases.

In figures 2.3 and 2.4, the achieved rates for the distributed power allocation scheme with a MMSE-SIC receiver are shown for $N = 64$ and $K = 30$ as well as $N = 256$ and $K = 100$. In the system, four available rates are considered (which are randomly requested by the users with equal probability). For each user, the requested and obtained rates for a certain channel realization are plotted. It can be seen that the results obtained are quite good already for a system with $N = 64$ and $K = 30$ when the users know only the probabilities of the requested rates.

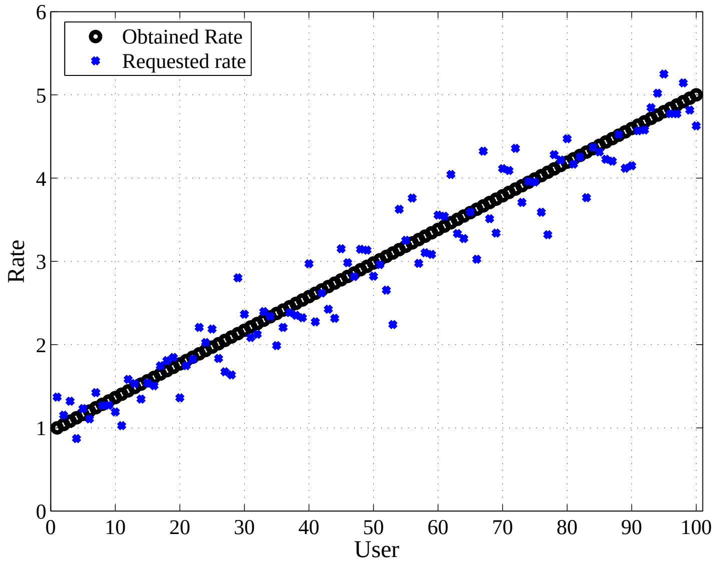


Figure 2.1: Matched filter with $N = 256$ receive dimensions and $K = 100$ users at 10dB.

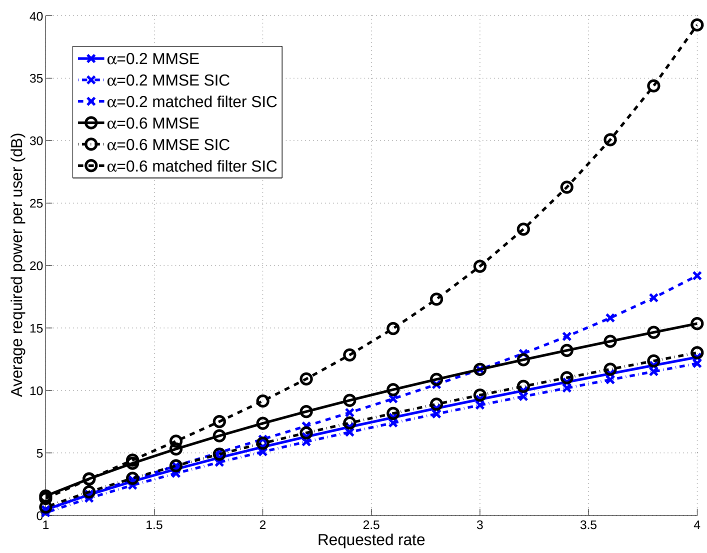


Figure 2.2: Total power required for MMSE, matched filter SIC and MMSE-SIC with $N = 128$ receive dimensions and respective loads $\alpha = 0.2, \quad \alpha = 0.6$ at 10dB.

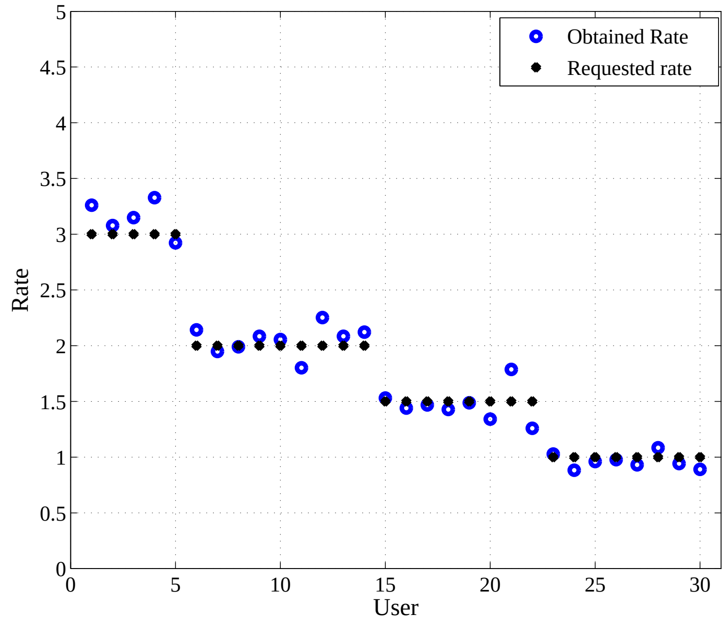


Figure 2.3: Distributed power allocation for MMSE-SIC with $N = 64$ receive dimensions and $K = 30$ users at 10dB.

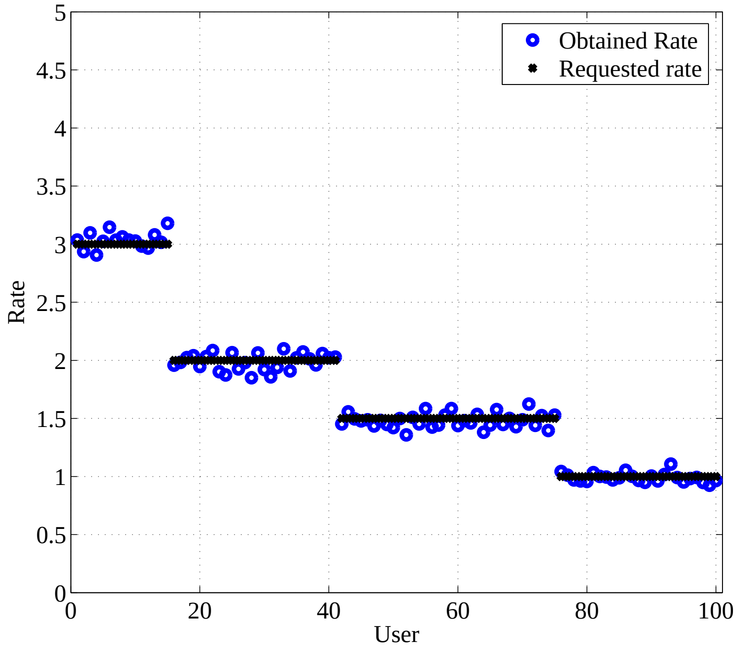


Figure 2.4: Distributed power allocation for MMSE-SIC with $N = 256$ receive dimensions and $K = 100$ users at 10dB.

2.6 Conclusions

In this chapter, the optimal decoding order and power allocation has been derived for SIC receivers, both MMSE and matched filter, considering different, realistic channel models. Remarkably, for the former, the optimal detection order depends on the requested SINR, eventually weighted, while in the latter, only on the channel energies. We have shown that the use of asymptotic tools from random matrix theory provide a neat framework for the analysis of SIC systems. It has also been shown that, under certain conditions, the power allocation can be determined in a decentralized manner (by each user individually) when considering a high number of users in the network.

CHAPTER 3

MIMO multiple access channels: Distributed power allocation

In this chapter we consider distributed power allocation algorithms for the MIMO multiple access channel. The main difference with respect to the previous one, is the presence of multiple antennas also on the MS, and thus the multiple receive dimensions at the BS will also be multiple antennas. In this context we want to investigate the optimum power allocation at the mobile stations when the signaling protocol overhead is absent or very reduced.

From an information theoretic point of view, the optimal centralized power and rate policies for the fast fading single input single output (SISO) MAC have been determined by [Gallager 1994][Shamai 1997] when channel state information at the receiver is assumed (CSIR) and by [Tse 1998] when CSI is assumed at the receiver and transmitters (CSIR and CSIT) , which leads to the MAC ergodic capacity region. Recently, the authors of [Soysal 2007][Soysal 2009] addressed the fast fading MIMO MAC with transmit antenna correlation and covariance feedback at the transmitters and determined the optimum power allocation policy in terms of ergodic sum capacity. We consider the same framework as the latter, fast fading MIMO MAC with CSIR and CDIT (channel distribution information at the transmitters) , but we also assume correlation at the receiver and much more importantly we do not assume the power allocation policies to be centralized. In our context each user wants to selfishly maximize its own utility instead of a global utility function such as the sum-capacity.

A convenient tool to address decentralized problems turns out to be game theory (see e.g. [Fudenberg 1991][Altman 2006a])). The authors of [Lai 2008] used a game theoretic approach to characterize the ergodic information rates of fast fading SISO and single input multiple output (SIMO) multiple access channels when perfect CSIR is assumed and each user knows his channel and those of the other users. Although reference [Lai 2008] is probably the closest work to ours we also note that other authors have worked on multiple access or interference channels from a game theoretic perspective. For example, in [Arslan 2007] the authors have chosen the individual mutual information as a utility function and assumed CSIR and CSIT for studying static MIMO interference channels. In [Scutari 2008] the authors have also considered the individual mutual information for studying static frequency-selective interference channels. Some authors have used different utility functions, such as those maximizing energy-efficiency (see e.g. [Meshkati 2005b][Meshkati 2006])), in order to study the existence and uniqueness of a Nash equilibrium (NE) in MACs.

Our work can be considered as a partial extension of [Lai 2008] in the sense that we address MIMO channels instead of SISO and SIMO channels but it differs from it at least in four important points. First, each user is only informed with the statistics of the different channels and not with their instantaneous knowledge. The CDIT assumption is generally considered to be more realistic in fast fading environments and in the particular case of decentralized systems it involves much less feedback signals from the base station, compared to the CSIT assumption. Second, the transmit and receive antennas can be correlated (this feature cannot be considered when assuming perfect CSI since each transmitter exploits the realization of the channel itself). Third, we exploit the theory of random matrices. Considering (moderately) large systems in terms of numbers of antennas has at least two advantages: the underlying averaging effect makes predictable certain quantities of interest, which allows each player to partially/totally predict the strategy of others, and more importantly it simplifies the derivation of distributed power allocation algorithms and the analysis of their properties. Concerning this point, random matrix theory will be used with the same approach as the authors of [Tulino 2005b], who studied the impact of antenna correlation on fading MIMO single-user channels. As a fourth point, we focus more on the sum-rate as a system performance criterion and present coordination schemes (there exists a random coordination signal which can be heard by all the MSs and the BS and is used to determine the decoding order of the different users when using an interference cancellation receiver), complementary to (and sometimes simpler to implement than) those developed in the Stackelberg formulation of [Lai 2008].

3.1 System model

We consider the uplink of a single cell with K active users. Each mobile station is equipped with n_t antennas whereas the base station has n_r antennas (thus we assume the same number of transmitting antennas for all the users). In our analysis the flat fading channel matrices of the different links vary from symbol vector (or space-time codeword) to symbol vector. We assume that the receiver knows all the channel matrices whereas each transmitter has only access to the statistics of the different channels. The equivalent baseband signal received by the base station can be written as

$$\mathbf{y} = \sum_{k=1}^{K} \boldsymbol{H}_k \mathbf{x}_k + \mathbf{n} \tag{3.1}$$

where $\mathbf{x_k}$ is the n_t-dimensional column vector of symbols transmitted by user k , $\mathbf{H}_k \in \mathbb{C}^{n_r \times n_t}$ is the channel matrix (stationary and ergodic process) of user k and $\mathbf{n}$ is a n_r-dimensional complex white Gaussian noise distributed as $\mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_r)$. Each channel input is subject to a power constraint $\mathrm{Tr}\left[\mathbb{E}(\mathbf{x}_k \mathbf{x}_k^H)\right] \triangleq \mathrm{Tr}(\mathbf{Q}_k) \leq n_t \overline{P}_k$. In order to take into account the antenna correlation effects at the transmitters and receiver we will assume the different channel matrices to be structured according to the Kronecker propagation model [Shiu 2000]:

$$\forall k \in \{1, ..., K\},\ \mathbf{H}_k = \mathbf{R}^{\frac{1}{2}} \boldsymbol{\Theta}_k \mathbf{T}_k^{\frac{1}{2}} \tag{3.2}$$

where $\mathbf{R}$ is the receive antenna correlation matrix, $\mathbf{T}_k$ is the transmit antenna correlation matrix for user k and $\boldsymbol{\Theta}_k$ is an $n_r \times n_t$ matrix whose entries are zero-mean independent and identically distributed complex Gaussian random variables with variance $\frac{1}{n_t}$. At last, note that for simplicity we will always assume $K = 2$ but all the results presented extend to $K-$user MACs, $K \geq 3$. In this respect, in some places K will be used instead of $K = 2$ and some numerical results will be provided for arbitrary K.

3.2 Scenarios considered

3.2.1 No coordination, single user decoding

We assume that the BS uses single user decoding (e.g. because the BS is neutral in the game or for limiting the receiver complexity). Each user treats the signal of the others as additive (colored) noise and wants to selfishly maximize its own transmission rate. The information rate achieved by user k equals the mutual information between $\mathbf{x}_k$ and $\mathbf{y}$ conditioned on the overall channel matrix $\mathbf{H} = [\mathbf{H}_1 \mathbf{H}_2 ... \mathbf{H}_K]$. As conditioning the mutual information by a random variable involves taking expectation over this random variable we have:

$$I(\mathbf{x}_k; \mathbf{y}|\mathbf{H}) = \mathbb{E}\left[\log_2 \left|\sum_{\ell=1}^{K} \mathbf{H}_\ell \mathbf{Q}_\ell \mathbf{H}_\ell^H + \sigma^2 \mathbf{I}\right|\right] - \mathbb{E}\left[\log_2 \left|\sum_{\ell \neq k} \mathbf{H}_\ell \mathbf{Q}_\ell \mathbf{H}_\ell^H + \sigma^2 \mathbf{I}\right|\right] \tag{3.3}$$

We see that the second term of the mutual information does not depend on $\mathbf{Q}_k$ and we can therefore omit it for the individual utility function of user $k \in \{1, ..., K\}$, which is chosen to be

$$u_k^{(SU)}(\mathbf{Q}_k, \mathbf{Q}_{-k}) = \mathbb{E}\left[\log_2 \left|\mathbf{I} + \rho \sum_{\ell=1}^{K} \mathbf{H}_\ell \mathbf{Q}_\ell \mathbf{H}_\ell^H\right|\right], \tag{3.4}$$

where $\mathbf{Q}_{-k} = (\mathbf{Q}_1, ..., \mathbf{Q}_{k-1}, \mathbf{Q}_{k+1}, ..., \mathbf{Q}_K)$ and $\rho = \frac{1}{\sigma^2}$. Clearly, the users have the same utility function but each user has to maximize it with respect to his *own* transmit covariance matrix. We see that, with the proposed choice of utility functions in the scenario where no coordination is possible and single-user decoding is assumed at the BS, the three concepts of the non-cooperative game, team problem and global optimization problem coincide. We effectively want to optimize the ergodic sum-rate of the MIMO MAC:

$$C_{sum} = \max_{\mathbf{Q}_1, ..., \mathbf{Q}_K} \mathbb{E}\left[\log_2 \frac{\left|\sum_{\ell=1}^{K} \mathbf{H}_\ell \mathbf{Q}_\ell \mathbf{H}_\ell^H + \sigma^2 \mathbf{I}\right|}{|\sigma^2 \mathbf{I}|}\right] \tag{3.5}$$

under the classical trace constraints. What characterizes our problem is that we only want to optimize the sum-rate over $\mathbf{Q}_k$ instead of $(\mathbf{Q}_1, ..., \mathbf{Q}_K)$. In the particular scenario under consideration the concavity of the ergodic sum-rate w.r.t. $(\mathbf{Q}_1, ..., \mathbf{Q}_K)$ is well known. We further note that the subset of non-negative Hermitian matrices verifying the trace constraints is convex. Therefore there exists a global maximum for the sum-rate. Now, since the players maximize the same function, we can draw the two following conclusions: (a) the global optimum is clearly a NE. This establishes the existence of a NE; (b) the strict concavity of the maximum sum-rate is equivalent to the diagonally strict concavity condition of [Rosen 1965], which implies that the NE is unique (see Theorem 2 of [Rosen 1965]).

The main technical issue is to determine the user strategies at the equilibrium point i.e. $(\mathbf{Q}_1^*, ..., \mathbf{Q}_K^*)$, which is done in Sec. 3.3.

3.2.2 Coordination, successive interference cancellation decoding

For the scenario considered now we assume the existence of a coordination signal denoted by S (see [Altman 2006b] where the authors apply a related idea for ALOHA protocol-based MACs to obtain a correlated equilibrium [Aumann 1974]). It could be obtained in practice, for example, by sampling a broadcast signal (e.g. an FM signal). The realizations of this signal, which are assumed to be equiprobable, are in the finite alphabet $\mathcal{S} = \{1, ..., K!\}$. For the case $K = 2$ it is therefore simply binary $S \in \{1, 2\}$. This signal is known both by the BS and the MSs. Here we assume that the decoding order does not depend on the realizations of $\mathbf{H}$, which are known to the BS but not to the MSs. Thus the coordination signal sent to the users does not provide them with any additional information on the channel conditions. For this reason we call this scheme "open loop coordination". In this framework, we allow the users to apply two different strategies: $\mathbf{Q}_1^{(1)}, \mathbf{Q}_1^{(2)}$ for user 1 and $\mathbf{Q}_2^{(1)}, \mathbf{Q}_2^{(2)}$ for user 2 where the notations $(.)^{(1)}$ and $(.)^{(2)}$ correspond to the realizations of the coordination signal. When $S = 1$, user 1 is privileged since it is decoded after user 2, and conversely for $S = 2$. Thus the achieved transmission rates are given by

$$\left\{\begin{array}{rcl} R_1^{(1)}(\mathbf{Q}_1^{(1)}, \mathbf{Q}_2^{(1)}) & = & \frac{1}{2}\mathbb{E}\left[\log_2\left|\mathbf{I} + \rho\mathbf{H}_1\mathbf{Q}_1^{(1)}\mathbf{H}_1^H\right|\right] \\ R_2^{(1)}(\mathbf{Q}_2^{(1)}, \mathbf{Q}_1^{(1)}) & = & \frac{1}{2}\mathbb{E}\left[\log_2\left|\mathbf{I} + \rho\mathbf{H}_1\mathbf{Q}_1^{(1)}\mathbf{H}_1^H + \rho\mathbf{H}_2\mathbf{Q}_2^{(1)}\mathbf{H}_2^H\right|\right] \\ & - & \frac{1}{2}\mathbb{E}\left[\log_2\left|\mathbf{I} + \rho\mathbf{H}_1\mathbf{Q}_1^{(1)}\mathbf{H}_1^H\right|\right] \end{array}\right. \tag{3.6}$$

when $S = 1$ and by

$$\left\{\begin{array}{rcl} R_2^{(2)}(\mathbf{Q}_2^{(2)}, \mathbf{Q}_1^{(2)}) & = & \frac{1}{2}\mathbb{E}\left[\log_2\left|\mathbf{I} + \rho\mathbf{H}_2\mathbf{Q}_2^{(2)}\mathbf{H}_2^H\right|\right] \\ R_1^{(2)}(\mathbf{Q}_1^{(2)}, \mathbf{Q}_2^{(2)}) & = & \frac{1}{2}\mathbb{E}\left[\log_2\left|\mathbf{I} + \rho\mathbf{H}_1\mathbf{Q}_1^{(2)}\mathbf{H}_1^H + \rho\mathbf{H}_2\mathbf{Q}_2^{(2)}\mathbf{H}_2^H\right|\right] \\ & - & \frac{1}{2}\mathbb{E}\left[\log_2\left|\mathbf{I} + \rho\mathbf{H}_2\mathbf{Q}_2^{(2)}\mathbf{H}_2^H\right|\right], \end{array}\right. \tag{3.7}$$

when $S = 2$. Therefore, when $S = 1$, user 1 sees a single-user MIMO system. The optimum input covariance matrix is obtained by choosing the eigenvectors of $\mathbf{Q}_1^{(1)}$ to be the eigenvectors of $\mathbf{T}_1$ and water-filling over its eigenvalues [Jafar 2004][Jorswieck 2004a]. User 1 has no interest in deviating from this strategy. User 2 knows it and its best strategy is to maximize the sum-rate w.r.t. $\mathbf{Q}_2$ given that $\mathbf{Q}_1 = \mathbf{Q}_1^{(1)}$. For this purpose he will choose its eigenvectors to be equal to those of $\mathbf{T}_2$ and water-fill over its eigenvalues. The same reasoning applies to the case $S = 2$. Thus, this clearly establishes the existence of a unique equilibrium. The users are thus following the coordination signal to adapt their strategies and have no interest in ignoring it. The described strategies can be checked to maximize the following utility functions:

$$\left\{\begin{array}{rcl} v_1^{(OL)}(\mathbf{Q}_1^{(1)}, \mathbf{Q}_1^{(2)}, \mathbf{Q}_2^{(1)}, \mathbf{Q}_2^{(2)}) & = & \frac{1}{2}R_1^{(1)}(\mathbf{Q}_1^{(1)}, \mathbf{Q}_2^{(1)}) + \frac{1}{2}R_1^{(2)}(\mathbf{Q}_1^{(2)}, \mathbf{Q}_2^{(2)}) \\ v_2^{(OL)}(\mathbf{Q}_1^{(1)}, \mathbf{Q}_1^{(2)}, \mathbf{Q}_2^{(1)}, \mathbf{Q}_2^{(2)}) & = & \frac{1}{2}R_2^{(1)}(\mathbf{Q}_2^{(1)}, \mathbf{Q}_1^{(1)}) + \frac{1}{2}R_2^{(2)}(\mathbf{Q}_2^{(2)}, \mathbf{Q}_1^{(2)}). \end{array}\right. \tag{3.8}$$

Here we casted the considered scenario into an open loop coordination-based equilibrium, but the game can also be seen as a hierarchical decision making problem. The present problem is a hierarchical decision making problem since, for a given realization of S, the last decoded user can be seen as a leader and the other as a follower. In our case however, the leader does not care about the follower since the actions of the follower have no impact on the leader. We note that the concept of the leader and followers are also present in the Stackelberg formulation. Indeed, by introducing a utility function for the BS, the Stackelberg formulation of [Lai 2008] could be applied here but in this case the BS should be involved and send a certain amount of control signal, which is not always negligible, especially when K increases. If the BS can use the information on the channels in order to choose the decoding order, then the signal S sent to the MSs provides them with some information on the channel conditions. This allows the users to have some information on the channel conditions and therefore we can refer to this scheme as closed-loop coordination. We can then replace $\mathbf{H}_k$ by $\mathbf{H}_k^s$ which has the interpretation of the channel condition of user k given that it receives the signal s. If the decision on the decoding order is such that the statistical assumptions on $\mathbf{H}_k^s$ are those we had on $\mathbf{H}_k$ (for example eq. (3.1) still holds with a possible dependence of the parameters with s) then we can still use eq. (3.6) and (3.7) for the utilities except that $\mathbf{H}$ will now also depend on the coordination signal. The equilibrium policies thus derived in the open loop case extend easily to the closed-loop situation.

3.3 Optimal precoding matrix

As in [Jafar 2004][Jorswieck 2004a][Soysal 2009] we distinguish two steps in the determination of the optimum covariance matrices: the optimum eigenvectors are determined in Sec. 3.3.1 by exploiting [Soysal 2009][Jorswieck 2004a] while the optimum eigenvalues are determined in Sec. 3.3.2 by approximating the utility functions under the large system assumption.

3.3.1 Optimal eigenvectors

In [Soysal 2009] the authors have determined the optimum structure for the transmit covariances matrices that maximizes the channel sum-rate. The proof of [Soysal 2009] can be reused and extended to the case where $\mathbf{R}$ is arbitrary in order to assert that there is no loss of optimality for $u_k^{(SU)}$ and $v_k^{(OL)}$ by restricting the search for the optimum covariance matrix by imposing the structure $\mathbf{Q}_k = \mathbf{U}_k \mathbf{P}_k \mathbf{U}_k^H$ where $\mathbf{T}_k = \mathbf{U}_k \mathbf{D}_k \mathbf{U}_k^H$ is the spectral decomposition of the transmit correlation matrix defined in (3.2) and the diagonal matrix $\mathbf{P}_k = \mathrm{Diag}(P_k(1), ..., P_k(n_t))$ represents the powers of user k allocated to the different eigenvectors. This is what states the following theorem, which is proved in Appendix A.1.

Theorem 3.3.1 (Optimum eigenvectors) For all $k \in \{1, 2\}$, let $\mathcal{Q}_k$ be the set of $n_t \times n_t$ Hermitian matrices such that $\mathrm{Tr}(\boldsymbol{Q}_k) \leq n_t \overline{P}_k$ i.e. $\mathcal{Q}_k = \left\{\boldsymbol{Q}_k \in \mathbb{C}^{n_t \times n_t} : \boldsymbol{Q}_k = \boldsymbol{Q}_k^H, \mathrm{Tr}(\boldsymbol{Q}_k) \leq n_t \overline{P}_k\right\}$. Additionally, let $\mathcal{S}_k$ be the subset of $\mathcal{Q}_k$

such that $\boldsymbol{Q}_k = \boldsymbol{U}_k \boldsymbol{P}_k \boldsymbol{U}_k^H$ where $\mathbf{U}_k$ represents the eigenvectors of $\mathbf{T}_k$. Then, for any $\mathbf{Q}_{-k} \in \mathcal{Q}_{-k}$:

$$\begin{cases} \displaystyle\max_{\mathbf{Q}_k \in \mathcal{Q}_k} u_k^{(SU)}(\mathbf{Q}_k, \mathbf{Q}_{-k}) & = \displaystyle\max_{\mathbf{Q}_k \in \mathcal{S}_k} u_k^{(SU)}(\mathbf{Q}_k, \mathbf{Q}_{-k}) \\ \displaystyle\max_{(\mathbf{Q}_k^{(1)}, \mathbf{Q}_k^{(2)}) \in \mathcal{Q}_k^2} v_k^{(OL)}(\mathbf{Q}_k^{(1)}, \mathbf{Q}_k^{(2)}, \mathbf{Q}_{-k}^{(1)}, \mathbf{Q}_{-k}^{(2)}) & = \displaystyle\max_{(\mathbf{Q}_k^{(1)}, \mathbf{Q}_k^{(2)}) \in \mathcal{S}_k^2} v_k^{(OL)}(\mathbf{Q}_k^{(1)}, \mathbf{Q}_k^{(2)}, \mathbf{Q}_{-k}^{(1)}, \mathbf{Q}_{-k}^{(2)}). \end{cases} \tag{3.9}$$

The best strategy for each user is always to choose an eigenvector basis which matches his own transmit correlation matrix and therefore does not depend on the channels of the other users. This reduces the power allocation game to the choice of the transmit powers only.

3.3.2 Optimal eigenvalues

We have shown that for the two decoding schemes considered and for each user, there is no loss of optimality by choosing the eigenvectors of $\mathbf{Q}_k$ to be equal to those of $\mathbf{T}_k = \mathbf{U}_k \mathbf{D}_k \mathbf{U}_k^H$. As a consequence, one can exploit the asymptotic results of [Tulino 2004][Tulino 2005b] derived for fading MIMO single-user channels with transmit and receive antenna correlation. This will lead us to simple approximations of the utility functions, which will make easier the optimization of the eigenvalues of the user transmit covariance matrices.

From now on, we assume the asymptotic regime in terms of the number of antennas, which is defined by: (a) $n_t \to \infty$; (b) $n_r \to \infty$; (c) $\displaystyle\lim_{n_t \to \infty, n_r \to \infty} \frac{n_t}{n_r} = c$ where $0 < c < \infty$. For each user $k \in \{1, ..., K\}$, we also suppose that $d_k(1), ..., d_k(n_t)$, which are the elements of the diagonal matrix $\mathbf{D}_k$ defined in Sec. 3.3.1, have an empirical distribution that converges to a p.d.f. $f_k(t)$ i.e. $\displaystyle\frac{1}{n_t} \sum_{i=1}^{n_t} \delta(t - d_k(i)) \to f_k(t)$.

3.3.2.1 No coordination, single user decoding

Under the assumptions made above, the capacity per receive antenna $\frac{C_{sum}}{n_r}$ can be shown to converge almost surely towards a limit, which can be obtained by applying Theorem 3.7 of [Tulino 2004]. It can be verified that:

$$\begin{aligned} \frac{C_{sum}}{n_r} &\to \frac{1}{n_r} \sum_{\ell=1}^{K} \sum_{i=1}^{n_t} \log_2 \left[1 + K\rho P_\ell(i) d_\ell(i) \alpha\right] \\ &+ \frac{1}{n_r} \sum_{j=1}^{n_r} \log_2 \left[1 + K\rho d^{(R)}(j) \beta\right] - \frac{n_t K^2}{n_r} \rho \alpha \beta \log_2 e \end{aligned} \tag{3.10}$$

where the coefficients $d^{(R)}(j)$ correspond to the spectral decomposition of the receive correlation matrix $\mathbf{R} = \mathbf{U}_R \mathbf{D}_R \mathbf{U}_R^H$ with $\mathbf{D}_R = \mathrm{Diag}(d^{(R)}(1), ..., d^{(R)}(n_r))$ and the pair (α, β) is the unique solution [Silverstein 1995b][Girko 2001] of the following system of equations:

$$\left\{ \begin{array}{rcl} \alpha & = & \displaystyle\frac{1}{Kn_t}\sum_{j=1}^{n_r}\frac{d^{(R)}(j)}{1+K\rho d^{(R)}(j)\beta} \\ \beta & = & \displaystyle\frac{1}{Kn_t}\sum_{\ell=1}^{K}\sum_{i=1}^{n_t}\frac{P_\ell(i)d_\ell(i)}{1+K\rho P_\ell(i)d_\ell(i)\alpha}. \end{array} \right. \tag{3.11}$$

In practice, for finite n_t, n_r the utility function $u_k^{(SU)}$ is therefore approximated by $\tilde{u}_k$ defined as $\tilde{u}_k = n_r \times \lim_{n_t\to\infty, n_r\to\infty} \frac{C_{sum}}{n_r}$. This defines an *approximate* game. For each user k, we want to determine the optimal way, in the sense of his approximated utility function $\tilde{u}_k$, to share its available power between the transmit antennas. To solve this constrained optimization problem we introduce the Lagrange multiplier λ_k and define the function

$$\mathcal{L}_{\lambda_k}(P_k(i)) \triangleq \tilde{u}_k - \lambda_k \times \left(\sum_{j=1}^{n_t} P_k(j) - n_t \overline{P}_k \right) \tag{3.12}$$

and search for the solution(s) $P_k^*(i)$ such that $\frac{\partial \mathcal{L}_{\lambda_k}}{\partial P_k(i)} = 0$. The solution of the corresponding optimization problem is stated through the following theorem.

Theorem 3.3.2 (Optimum eigenvalues for single-user decoding) Assume that the pair (α, β) is the solution of the system of equations (3.11). Then the spatial power allocation maximizing the constrained approximated utility function (3.12) is given by the following water-filling solution:

$$P_k^*(i) = \left[\frac{1}{n_r \ln 2\lambda_k} - \frac{1}{K\rho d_k(i)\alpha} \right]^+ \tag{3.13}$$

where we used the notation $[x]^+ = \max(x, 0)$.

The proof of this theorem is provided in Appendix A.2.

In the water-filling procedure the Lagrangian multiplier λ_k, for user k, is calculated in order to meet the power constraint $\sum_{i=1}^{n_t} P_k^*(i) = n_t \overline{P}_k$. Note that the power allocation for a given user k is based on the knowledge of the statistics of his channel but also others through β. We are now in position to describe the proposed iterative power allocation algorithm:

1. Initialize α with a value in the interval $[\alpha_{min}, \alpha_{max}]$ with

$$\alpha_{min} = \frac{1}{Kn_t}\sum_{j=1}^{n_r}\frac{d^{(R)}(j)}{1+K\rho d^{(R)}(j)}$$

$$\alpha_{max} = \frac{1}{Kn_t}\sum_{j=1}^{n_r} d^{(R)}(j)$$

.

2. Apply water-filling over the $d_k(i)$ by using equation (3.13) in order to find $P_k(i)$ for all $i \in \{1, ..., n_t\}$ and $k \in \{1, ..., K\}$.

3. By using the powers obtained at the previous step, update the value of α by searching for the solution of the system of equations (3.11).

4. If α has not converged (fix an arbitrary accuracy level on α) go to step 1. Otherwise, apply for the last time step 2 and stop the iterative procedure.

A similar algorithm has been used by [Dumont 2006] in order to derive the capacity of single-user Rician MIMO channels with antenna correlation. Based on their results one is ensured that the approximated utility function $\tilde{u}_k$ is a strictly concave function of the transmit power vectors $\{\mathbf{P}_1, ..., \mathbf{P}_K\}$, with $\forall k \in \{1, ..., K\}$, $\mathbf{P}_k = (P_k(1), ..., P_k(n_t))$, and if the iterative power allocation algorithm converges, it converges towards the global maximum (this result was only guaranteed in the *exact* game described in Sec. 3.3.1.

Now we provide a modified version of the iterative power allocation algorithm described above. In this modified version we exploit the idea of asymptotic water-filling, originally introduced by [Chuah 2002]. The asymptotic water-filling used in this version allows us to restrict the knowledge of the transmitters to the p.d.f. $f_k(t), k \in \{1, ..., K\}$ instead of the knowledge of the values of $d_k(1), ..., d_k(n_t)$. The drawback is that in order for the empirical distribution of the eigenvalues $d_k(1), ..., d_k(n_t)$ to be well approximated by the p.d.f. $f_k(t)$, n_t and n_r need to be relatively high. Indeed, the first version of the power allocation algorithm only relies on the approximation of the mutual information, which is accurate for small values of n_t, n_r as it will be seen in the simulations.

For the sake of clarity we assume here that $\mathbf{R} = \mathbf{I}$. By assuming a known law f_k for the diagonal terms $d_k(i)$, so that

$$\frac{1}{n_t} \sum_{i=1}^{n_t} \frac{1}{d_k(i)} \rightarrow \int \frac{f_k(t)}{t}, \tag{3.14}$$

we can see that the water level $\mu_k = n_r \ln 2\lambda_k$ can be expressed analytically and only depends on the distribution of $d_k(i)$ according to the following relation, which is obtained from (3.13) and the power constraints:

$$\overline{P}_k = \int_0^{+\infty} \left[\frac{1}{\mu_k} - \frac{1}{K\rho t\alpha}\right]^+ f_k(t)dt = \int_{\frac{\mu_k}{K\rho\alpha}}^{+\infty} \left(\frac{1}{\mu_k} - \frac{1}{K\rho t\alpha}\right) f_k(t)dt. \tag{3.15}$$

Therefore μ_k can be obtained through the following fixed-point equation:

$$\mu_k = \frac{\displaystyle\int_{\frac{\mu_k}{K\rho\alpha}}^{+\infty} f_k(t)dt}{\displaystyle\overline{P}_k + \frac{1}{K\rho\alpha}\int_{\frac{\mu_k}{K\rho\alpha}}^{+\infty} \frac{f_k(t)}{t}dt}. \tag{3.16}$$

In addition, a transmit correlation profile has to be chosen and derive the corresponding probability density function (pdf) . $f_k(t)$. For instance the authors of [Skupch 2005] have calculated it for an exponential correlation profile: $\forall(i,j) \in \{1,...,n_t\}^2,\ T_k(i,j) = r_k^{|i-j|}$ where r_k is the correlation coefficient characterizing the correlation matrix $\mathbf{T}_k$ (this model is assumed in the simulations in Sec. 3.4). It was shown that $f_k(t) = \dfrac{1}{\pi t\sqrt{-t^2+2a_k t-1}}$ if $\dfrac{1-r_k}{1+r_k} < t < \dfrac{1+r_k}{1-r_k}$ and 0 otherwise, with $a_k \triangleq \frac{1+r_k^2}{1-r_k^2}$.

3.3.2.2 Coordination, successive interference cancellation decoding

In the case where the BS applies successive decoding in the order indicated by the coordination signal the equilibrium and the iterative algorithm analyses can be conducted by using the same reasoning as used previously. In this section we will only provide the expressions of the optimum transmit powers. Assume that $S = 1$. Then the achievable transmission rates for the two users are:

$$\left\{\begin{array}{rcl} u_1^{(1)}(\mathbf{Q}_1^{(1)},\mathbf{Q}_2^{(1)}) &=& \frac{1}{2}\underbrace{\mathbb{E}\left[\log_2\left|\mathbf{I}+\rho\mathbf{H}_1\mathbf{Q}_1^{(1)}\mathbf{H}_1^H\right|\right]}_{\tau_1} \\ u_2^{(1)}(\mathbf{Q}_2^{(1)},\mathbf{Q}_1^{(1)}) &=& \frac{1}{2}\underbrace{\mathbb{E}\left[\log_2\left|\mathbf{I}+\rho\mathbf{H}_1\mathbf{Q}_1^{(1)}\mathbf{H}_1^H+\rho\mathbf{H}_2\mathbf{Q}_2^{(1)}\mathbf{H}_2^H\right|\right]}_{\tau_s} \\ && -\ \frac{1}{2}\underbrace{\mathbb{E}\left[\log_2\left|\mathbf{I}+\rho\mathbf{H}_1\mathbf{Q}_1^{(1)}\mathbf{H}_1^H\right|\right]}_{\tau_1}. \end{array}\right. \tag{3.17}$$

By exploiting the results of Sec. 3.3.1, Theorem 3.7 of [Tulino 2004] and choosing in this theorem K to be equal to the number of terms of the type $\mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^H$ present in the argument of the operator $\mathbb{E}[\log|.|]$ to be approximated, it can be checked that

$$\frac{\tau_1}{n_r} \rightarrow \frac{1}{n_r}\sum_{i=1}^{n_t}\log_2\left[1+\rho P_1^{(1)}(i)d_1(i)\alpha_1\right]+\frac{1}{n_r}\sum_{j=1}^{n_r}\log_2\left[1+\rho d^{(R)}(j)\beta_1\right]-\frac{n_t}{n_r}\rho\alpha_1\beta_1\log_2 e \tag{3.18}$$

where

$$\left\{\begin{array}{rcl} \alpha_1 &=& \dfrac{1}{n_t}\displaystyle\sum_{j=1}^{n_r}\dfrac{d^{(R)}(j)}{1+\rho d^{(R)}(j)\beta_1} \\ \beta_1 &=& \dfrac{1}{n_t}\displaystyle\sum_{i=1}^{n_t}\dfrac{P_1^{(1)}(i)d_1(i)}{1+\rho P_1^{(1)}(i)d_1(i)\alpha_1}. \end{array}\right. \tag{3.19}$$

The proof of Theorem 3.3.2 can be re-used here. Then, optimizing the approximated rate $\tilde{\tau}_1 = n_r \times \lim\limits_{n_t\rightarrow\infty, n_r\rightarrow\infty}\dfrac{\tau_1}{n_r}$ w.r.t. $P_1^{(1)}(i)$ leads to the following water-filling equation

$$P_1^{(1),*}(i) = \left[\frac{1}{n_r\ln 2\lambda_1}-\frac{1}{\rho d_1(i)\alpha}\right]^+. \tag{3.20}$$

We also know that user 2 will maximize the term $\tilde{\tau}_s = n_r \times \lim_{n_t \to \infty, n_r \to \infty} \frac{\tau_s}{n_r}$ by choosing his input covariance matrix to be structured as $\mathbf{Q}_2 = \mathbf{U}_2 \mathbf{P}_2 \mathbf{U}_2^H$ with

$$\begin{aligned} \frac{\tau_s}{n_r} \quad \to \quad & \frac{1}{n_r} \sum_{\ell=1}^{2} \sum_{i=1}^{n_t} \log_2 \left[1 + 2\rho P_\ell^{(1)}(i) d_\ell(i) \alpha_2 \right] \\ + \quad & \frac{1}{n_r} \sum_{j=1}^{n_r} \log_2 \left[1 + 2\rho d^{(R)}(j) \beta_2 \right] - \frac{4 n_t}{n_r} \rho \alpha_2 \beta_2 \log_2 e \end{aligned} \tag{3.21}$$

where

$$\left\{ \begin{array}{lcl} \alpha_2 & = & \frac{1}{2n_t} \sum_{j=1}^{n_r} \frac{d^{(R)}(j)}{1 + 2\rho d^{(R)}(j) \beta_2} \\ \beta_2 & = & \frac{1}{2n_t} \sum_{\ell=1}^{2} \sum_{i=1}^{n_t} \frac{P_\ell^{(1)}(i) d_\ell(i)}{1 + 2\rho P_\ell^{(1)}(i) d_\ell(i) \alpha_2}. \end{array} \right. \tag{3.22}$$

Eventually the powers for user 2 can be determined by

$$P_2^{(1),*}(i) = \left[\frac{1}{n_r \ln 2 \lambda_2} - \frac{1}{2\rho d_2(i) \alpha_2} \right]^+ . \tag{3.23}$$

3.4 Numerical results

First we show that in order to make the large system approximation accurate the numbers of antennas do not need to be very high. This is especially true when the metric of interest is the ergodic mutual information since one benefits from a double averaging effect, one from the randomness of the matrices into play and the other one from the expectation operator. Fig. 3.1 shows that the relative error is less than $4\ \%$ even for a 2×2 MIMO system.

Fig. 3.2 compares the simplest decentralized power allocation scheme, which is the uniform scheme, with the optimized power allocation scheme when no coordination and single-user decoding are assumed. Since single-user decoding is used at the BS, the system performance is inteference-limited, which clearly appears in the high SNR regime. We note a significant performance gap between the uniform and optimized schemes, which remarkably increases at high SNR.

Figs. 3.3 and 3.4 represent the sum-rate versus K for different power allocation schemes. Here we assumed $\overline{P}_1 = ... = \overline{P}_K$. We see that coordinating the system with an equiprobable random signal allows us to be quite close to the (centralized) MIMO MAC sum-capacity, which shows the interest in the proposed scheme in typical simulation scenarios.

3.5 Conclusions

Our goal was to design power allocation algorithms in fast fading MIMO channels with correlation while minimizing the amount of control signal from the BS. To this end we

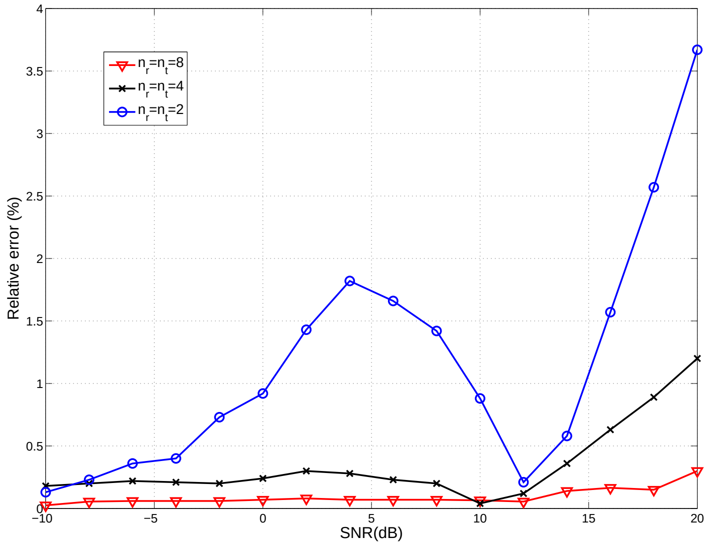


Figure 3.1: Relative error [%] on the mutual information as a function of SNR for different sizes of MIMO systems: $2\times 2, 4\times 4, 8\times 8$ with $K=1$, $r_1=0.5$, $\mathbf{R}=\mathbf{I}$.

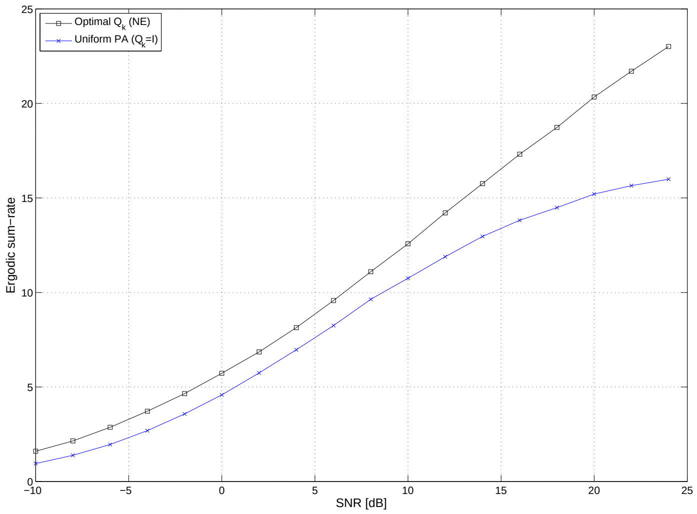


Figure 3.2: Ergodic sum-rate as a function of SNR for the optimized power allocation and uniform power allocation when $K=2, n_t=n_r=4, r_1=0.2, r_2=0.8$ in the scenario –no coordination + single-user decoding–.

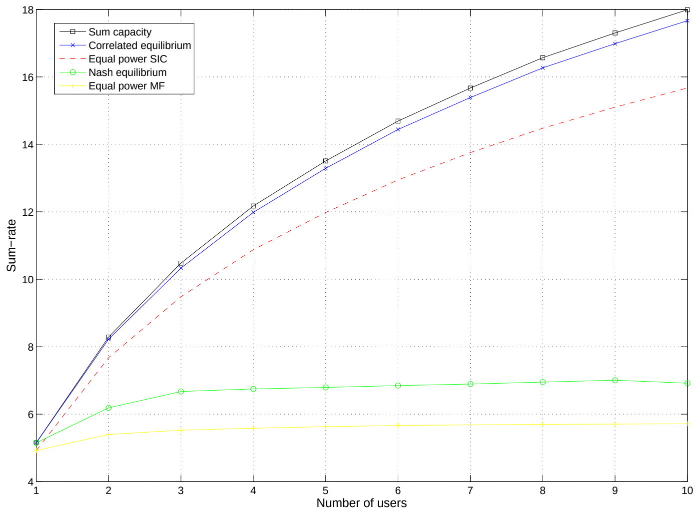


Figure 3.3: Sum-rate as a function of the number or users for different power allocation schemes: 1. Team game + SIC + optimal power allocation (sum-capacity); 2. Open loop coordination + SIC + optimal power allocation; 3. Open loop coordination + SIC + uniform power allocation; 4. No coordination + Single user decoding + optimal power allocation. Setup: $n_t = n_r = 4, r_k = 0.4, r_R = 0.2, \rho = 3$ dB.

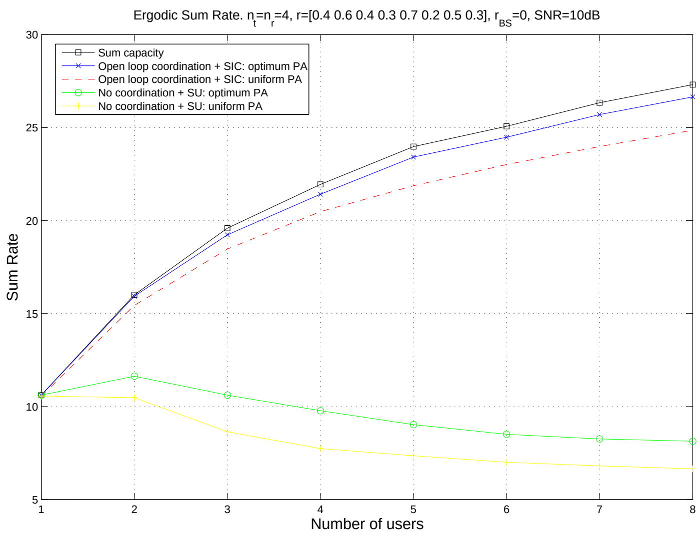


Figure 3.4: Sum-rate as a function of the number or users for different power allocation schemes: 1. Team game + SIC + optimal power allocation (sum-capacity); 2. Open loop coordination + SIC + optimal power allocation; 3. Open loop coordination + SIC + uniform power allocation; 4. No coordination + Single user decoding + optimal power allocation. Setup: $n_t = n_r = 4, \mathbf{r} = (0.4, 0.6, 0.4, 0.3, 0.7, 0.2, 0.5, 0.3), r_R = 0, \rho = 10$ dB.

only assumed CSIR and CDIT. A game theoretic setting was used to analyze both scenarios, determining the existence of equilibria, which effectively allows the mobiles to choose their power allocation policies in order to selfishly optimize their ergodic transmission rates. In addition, an iterative algorithm has been proposed to this end which is guaranteed to converge to the optimum if it does converge (extensive simulations indicate its convergence).

CHAPTER 4

Throughput Optimization in Heterogeneous Networks: Cross-System Diversity

The scenario considered in this chapter can be seen as an extension of that of chapter 2 where now the terminals have several BS to which they can communicate simultaneously, on non-overlapping frequency bands. In addition the objective function considered now is the maximization of the total network throughput.

As the number of wireless systems has increased over the last two decades, the idea of system convergence has been introduced (see e.g. [Molony 1998, Vrdoljak 2000]), in order to enable mobile terminals to operate with different standards. This convergence idea was one of the driving forces behind the design of reconfigurable terminals, also known as software defined radio, flexible radio [Mitola 1999] or cognitive radio [Fette 2006]. Mobile phones currently available on the market are usually multi-mode, which means that they can work with different standards. In addition, there are many situations where a terminal can have access to several signals in non-overlapping frequency bands: a Global System for Mobile communications (GSM) mobile station is able to listen to several GSM base stations; an UMTS MS can listen to Wideband Code Division Multiple Access (WCDMA) base stations, but also possibly time division CDMA (TD-CDMA) base stations. In all these examples, the terminal operates with only one standard at a time, depending on the user location and/or the type of service requested by the user.

Our contribution presented in this chapter is based on an information-theoretic approach, but it still provides elements to understand the aforementioned situations, and give some ideas of what could be done to optimize the overall uplink network throughput, by using all the systems simultaneously [Lee 1999], instead of sequentially (hard handover or best base station selection) as it is the case in existing systems or contributions [Feng 2007, Wang 1999]. This will provide an additional form of diversity at the terminals, which could be named cross-system diversity.

More specifically, we consider several mobile users and base stations, each of the latter using a different frequency band. We assume that the base stations are connected through perfect communication links. For instance, in UMTS networks, base stations are connected through a radio network controller and very reliable wired connection (e.g. optic fiber), which is not far from a perfect communication link. Users have wireless links towards the different base stations, and we want to derive the optimal power and rate allocations, given a fixed power constraint for each user. The uplink power allocation scheme is optimized in order to maximize the sum-rate (over the users and systems) of the overall network.

There exist many works on how to optimally allocate the transmit power to the different sub-channels. To our knowledge, [Kim 2005] is the closest work to the one presented here. The authors address the problem of jointly allocating power and subcarriers in the context of orthogonal frequency division multiple access (OFDMA) systems. Our work differs from theirs on several points: we consider a more general channel model (fading channels instead of Gaussian channels), a very different context (heterogeneous networks), all the sub-channels are (possibly) used whereas in [Kim 2005] , only a subset of them is used by each transmitter and also the optimization problem of [Kim 2005] is not convex, in contrast with the power allocation problem for the optimum receiver investigated in this chapter. In addition, our main goal is to optimize a global performance criterion under local power constraints. Finally, our information theoretic approach exploits asymptotic random matrix theory [Girko 2001, Silverstein 1995b], in order to provide tractable expressions for the optimization problems under investigation. Hence, we will assume the dimensions of the systems as well as the number of users large enough, in order to benefit from the self-averaging properties of the matrices under consideration. In particular, an interesting feature of these self-averaging properties shows that only the parameters of interest to the problem (system load, signal to noise ratio, ...) are kept, whereas all irrelevant parameters disappear [Hachem , Moustakas 2003, Tulino 2004, Tulino 2005b]. This provides a neat analysis framework for multi-dimensional problems. Moreover, although the results are proved in the asymptotic regime, it turns out (due to fast convergence properties) that they are accurate even for rather small systems (see e.g. [Biglieri 2002, Dumont 2005, Dumont 2006] or results from previous chapters)).

We solve the optimal power allocation problem for three kinds of receivers: the optimum receiver, minimum mean square error and matched filters. Simulations validate our approach and illustrate the performance gain obtained by using several technologies simultaneously instead of one at a time.

4.1 System model

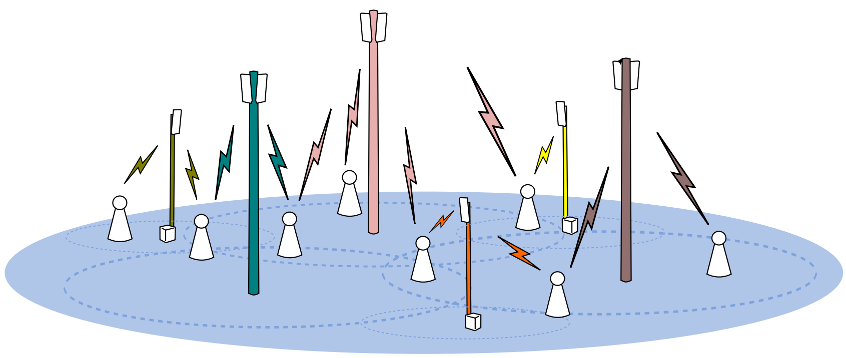


Figure 4.1: Cross-System scenario

The global system under investigation is represented in Fig. 4.1. It consists of K mobile terminals and S base stations using non-overlapping frequency bands (in Fig. 4.1, $S = 6$). Each mobile terminal has one single antenna, while the base station can possibly have multiple antennas depending on the radio technology. The number of dimensions associated with base station $s \in \{1, ..., S\}$ is denoted by N_s. For example, if a CDMA system is used, N_s represents the spreading factor; on the other hand, if the base station is equipped with multiple antennas, N_s represents the number of receive antennas. Assuming time selective but frequency non-selective channels, the equivalent baseband signals received by the base stations can be written as

$$\begin{cases} \mathbf{y}_1 &= \sqrt{\rho_1} \displaystyle\sum_{\ell=1}^{K} \mathbf{h}_{\ell,1} x_{\ell,1} + \mathbf{n}_1 \\ \mathbf{y}_2 &= \sqrt{\rho_2} \displaystyle\sum_{\ell=1}^{K} \mathbf{h}_{\ell,2} x_{\ell,2} + \mathbf{n}_2 \\ & \vdots \\ \mathbf{y}_S &= \sqrt{\rho_S} \displaystyle\sum_{\ell=1}^{K} \mathbf{h}_{\ell,S} x_{\ell,S} + \mathbf{n}_S \end{cases}, \tag{4.1}$$

where $\forall k \in \{1, ..., K\}, \forall s \in \{1, ..., S\}$, $x_{k,s}$ is the signal transmitted by user k to base station s, satisfying $\sum_{s=1}^{S} \mathbb{E}|x_{k,s}|^2 \leq 1$, $\mathbf{h_{k,s}}$ is the N_s-dimensional stationary and zero-mean ergodic complex Gaussian channel vector associated with user k for the system s, $\mathbf{n}_s$ is an N_s-dimensional complex white Gaussian noise distributed as $\mathcal{N}(\mathbf{0}, n_0 B_s \mathbf{I})$, where n_0 is the receive noise power spectral density, B_s the bandwidth of system s, ρ_s is the signal-to-noise ratio (SNR) in system s, defined as $\rho_s = \frac{P}{n_0 B_s}$, and P is the transmit power available at a given terminal. For simplicity and clarity, we henceforth implicitly assume that the mobile terminals have the same transmit power, which is a reasonable assumption (see *e.g.* [UMTS-World-Association] for more information). Otherwise, the case with distinct transmit powers could be easily taken into account. In our analysis the flat fading channel vectors of the different links can possibly vary from symbol vector (or space-time codeword) to symbol vector (or space-time codeword). We assume that the receivers (base stations) know their channel matrices (coherent communication assumption) and send the channel distribution information (CDI) through reliable links to a central controller. Knowing the channels of all users, the central controller implements the algorithm and indicates to each user how he has to share his transmit power between the different links. The transmitters therefore do not need any knowledge on the channels (neither channel state nor distribution information).

As we will consider the overall system sum-rate as the performance criterion, and assume a large system in terms of both the number of users and dimensions at the base stations ($N_1, ..., N_S$), it is convenient to rewrite the received signal in matrix form:

$$\left\{ \begin{array}{rcl} \mathbf{y}_1 & = & \sqrt{\rho_1}\mathbf{H}_1\mathbf{x}_1 + \mathbf{n}_1 \\ \mathbf{y}_2 & = & \sqrt{\rho_2}\mathbf{H}_2\mathbf{x}_2 + \mathbf{n}_2 \\ & \vdots & \\ \mathbf{y}_S & = & \sqrt{\rho_S}\mathbf{H}_S\mathbf{x}_S + \mathbf{n}_S \end{array} \right. , \tag{4.2}$$

where $\forall s \in \{1, ..., S\}$, $\mathbf{H}_s = [\mathbf{h}_{1,s} \dots \mathbf{h}_{K,s}]$ and $\mathbf{x}_s = (x_{1,s}, \dots, x_{K,s})^T$. We assume that the channel matrix of a given system can be factorized, in the sense of the Hadamard product, as a product of two matrices

$$\mathbf{H}_s = \mathbf{G}_s \odot \mathbf{W}_s, \tag{4.3}$$

where $\mathbf{W}_s$ is the matrix of the instantaneous channel gains which are assumed to be i.i.d zero-mean and unit variance, and $\mathbf{G}_s$ is the pattern mask specific to a given technology, containing the arbitrary variances of the elements of $\mathbf{H}_s$. This model is broad enough to incorporate several radio access technologies. Here are three typical examples:

- MIMO systems: N_s represents the number of antennas at the base station s and K the number of users (each equipped with a single antenna). The matrices $\mathbf{W}_s$ and $\mathbf{G}_s$ are respectively an i.i.d. zero mean Gaussian matrix and a $N_t \times K$ correlation matrix.

- Flat fading CDMA systems: N_s represents the spreading factor and K the number of users. For a block fading channel, $\mathbf{W}_s$ and $\mathbf{G}_s$ are respectively the code matrix, where each column represents the code of a given user, and the channel gains matrix, where the columns are identical (due to the fact that we consider flat fading models);

- Orthogonal Frequency Division Multiplexing (OFDM) systems: N_s represents the number of sub-carriers and K the number of users. Assuming for simplicity an OFDMA system where each user uses one subcarrier, $\mathbf{W}_s$ and $\mathbf{G}_s$ are respectively an i.i.d. zero mean Gaussian matrix and the truncated identity matrix (as the channel matrices are not necessarily square). Note that if $K < N_s$, some sub-carriers are not used.

4.2 Large Systems Scenario Analysis

In this section, we consider a much more realistic scenario for wireless communications. The different links between transmitters and receivers are now block fading and the numbers of users, systems and base station dimensions can be arbitrarily selected. Additionally, the base stations can have different bandwidths $B_1, ..., B_S$. The numbers of users and dimensions have to be large enough in order to make our asymptotic analysis sufficiently accurate. More precisely, we consider a scenario where $K \to +\infty$, $\forall s \in \{1, ..., S\}$, $N_s \to +\infty$ with $\lim\limits_{K\to\infty, N_s\to\infty} \dfrac{K}{N_s} = c_s$ and $0 < c_s < +\infty$. However, it is now well-known that many asymptotic results from random matrix theory under the large system assumption apply for relatively small systems [Biglieri 2002, Dumont 2005, Dumont 2006].

Under these assumptions our main objective is to derive the best power allocation scheme in the sense of the sum-rate of the global system for different types of receivers. One can notice that the selected performance criterion is global whereas the power constraints are local, which is a key difference with the conventional power sharing problem between different subchannels.

4.2.1 Optimum Receiver

When the optimum receiver is assumed at the base stations, maximizing the sum-rate leads to the Shannon sum-capacity of the global system. Considering the sum-rate point of the system, instead of an arbitrary operating point of the capacity region, has the advantage of simplifying the technical problem. In particular, considering the sum-rate as the performance criterion allows us to exploit some results obtained for single-user fading MIMO (*e.g.* [Tulino 2005b]). Note that the considered system consists of several MACs with multi-dimensional receivers and single-dimensional transmitters, under the assumption that CSIR but no CDIT is available. The sum-rate of each MAC is simply a special case of the general case analyzed by [Soysal 2007, Soysal 2009] for Rayleigh MIMO multiple access channels with input correlation with CSIR and CDIT. In our case where the dimension of the signal transmitted by a terminal is one, the CDIT assumption amounts for a user to knowing its transmit power. By considering the system of (orthogonal) equations (4.2) the network ergodic sum-capacity per user can be expressed as:

$$C = \max_{\mathbf{Q}_1,\dots,\mathbf{Q}_S} \mathbb{E}\left[\frac{1}{K}\left(\sum_{s=1}^{S} B_s \log_2\left|\mathbf{I}+\rho_s\mathbf{H}_s\mathbf{Q}_s\mathbf{H}_s^H\right|\right)\right] \tag{4.4}$$

where $\forall s \in \{1, ..., S\}$, $\mathbf{Q}_s = \mathbb{E}(\mathbf{x}_s\mathbf{x}_s^H)$. As long as the signals transmitted by the different users are independent, the matrices $\mathbf{Q}_s$ are diagonal: $\mathbf{Q}_s = \mathrm{Diag}\,(\alpha_{1,s}, \dots, \alpha_{K,s})$, where $\alpha_{k,s}$ denotes the fraction of its power user k employs in system s. As the mobile terminals have identical transmit power, we have $\forall k \in \{1, ..., K\}$, $\sum_{s=1}^{S} \alpha_{k,s} = 1$.

So far, we have not assumed anything about the numbers of users and base station dimensions. From now on, in order to simplify the optimization problem associated with equation (4.4) we will assume the asymptotic regime, as defined in the beginning of this section. Interestingly, in that case, an explicit equivalent for the network sum-rate can be obtained (from [Girko 2001]), whatever the pattern mask $\mathbf{G}_s$, as long as its continuous power profile, defined for $(\tau,\tau') \in [0,1]^2$ as $p_{N_s}(\tau,\tau') = g_s(i,j)$ with $\frac{i-1}{N_s} \leq \tau \leq \frac{i}{N_s}$ and $\frac{j-1}{N_sK} \leq \tau' \leq \frac{j}{N_sK}$, converges uniformly to a bounded and piecewise continuous function as $N_s \to \infty$ [Girko 2001], [Girko 1990, corollary 10.1.2]. However, if the pattern mask is not structured at all, the expression of the large system equivalent can be quite complicated and not always easy to exploit, whereas it is simpler for the class of separable channels (*e.g.* CDMA and MIMO channels). This is why we will mainly focus on this class of channels while having in mind that the proposed framework can be extended to other technologies. Note that the OFDM case needs a separate treatment since the power profile p_{N_s} does not converge uniformly. However, it is not difficult to see that one can obtain the same capacity expression as in the separable case [Hachem , Moustakas 2003, Tulino 2003, Tulino 2004, Tulino 2005b] with classical techniques. Therefore, for at least the three aforementioned types of technologies the constrained optimization under consideration can be simplified by finding a certain approximation $\tilde{C}$ of C, which can be obtained by exploiting the original results of [Girko 2001, Silverstein 1995a] which have been applied by [Hachem , Moustakas 2003, Tulino 2003, Tulino 2004, Tulino 2005b] to fading single-user vector channels. This is stated through the following proposition.

Proposition 4.2.1 (Equivalent of the network sum-rate) *An equivalent of (4.4) in the asymptotic regime, i.e. when* $K \to +\infty$, $\forall s \in \{1, \ldots, S\}$, $N_s \to +\infty$ *with* $\lim_{K\to\infty, N_s\to\infty} \frac{K}{N_s} = c_s$ *and* $0 < c_s < +\infty$, *is:*

$$\begin{aligned}\tilde{C} &= \max_{\alpha_1,\ldots,\alpha_K} \frac{1}{K}\left[\sum_{s=1}^{S}\sum_{\ell=1}^{K} B_s \log_2\left(1+\gamma_{\ell,s}\alpha_{\ell,s}r_s\right) + \frac{1}{K}\sum_{s=1}^{S}\sum_{j=1}^{N_s} B_s \log_2(1+\beta_{j,s}q_s)\right.\\ &\left. - \sum_{s=1}^{S} B_s v_s q_s r_s \log_2 e - \sum_{\ell=1}^{K}\lambda_\ell\left(\sum_{s=1}^{S}\alpha_{\ell,s}-1\right)\right] \qquad (4.5)\end{aligned}$$

where $\forall \ell \in \{1, \ldots, K\}$, λ_ℓ *is the Lagrange multiplier associated with the power constraint of user* ℓ, *guaranteeing that the sum of power fractions over the different systems equals one. The expression of* v_s *depends on the technology used by system* s: $v_s = K\rho_s$ *if* s *denotes the index of a MIMO system;* $v_s = \frac{K}{N_s}\rho_s$ *if* s *denotes the index of a CDMA system. In both cases the parameters* $\{(q_s, r_s)\}_{s\in\{1,\ldots,S\}}$ *are determined as the unique solution of the following system of equations:*

$$\left\{\begin{array}{lcl} r_s &=& \dfrac{1}{Kv_s}\displaystyle\sum_{j=1}^{N_s}\frac{\beta_{j,s}}{1+\beta_{j,s}q_s} \\ q_s &=& \dfrac{1}{Kv_s}\displaystyle\sum_{\ell=1}^{K}\frac{\gamma_{\ell,s}\alpha_{\ell,s}}{1+\gamma_{\ell,s}\alpha_{\ell,s}r_s}\end{array}\right. , \qquad (4.6)$$

$\mathbf{H}_s = \mathbf{R}_s^{\frac{1}{2}}\mathbf{\Theta}_s\mathbf{T}_s^{\frac{1}{2}}$, $\mathbf{\Theta}_s$ *is a matrix with i.i.d entries with unit-variance,* $\gamma_{\ell,s} = v_s d_{\ell,s}^{(T)}$, $d_{\ell,s}^{(T)}$ *is the* ℓ^{th} *eigenvalue of* $\mathbf{T}_s$, $\beta_{j,s} = v_s d_{j,s}^{(R)}$, $d_{j,s}^{(R)}$ *is the* j^{th} *eigenvalue of* $\mathbf{R}_s$. *For the OFDM case, equation (4.5) holds with* $r_s = \rho_s$, $q_s = 0$ *and* $\gamma_{\ell,s} = g_s^2(\ell,\ell)$.

The proof directly follows from [Tulino 2004, Tulino 2005b] since in our case the channels are also separable. In order to better understand and interpret the provided result and make this chapter self contained, we provide a special case drawn from [Tulino 2003]: a single MIMO system with SNR ρ, K inputs, N outputs and neither transmit nor receive correlation. The approximate capacity per receive antenna can be written in this case: $\tilde{C} = \frac{1}{N}\sum_{i=1}^{K}\log_2\left[1+\rho\alpha(i)r\right] + \log_2\left(\frac{N}{K}r\right) - \frac{N}{K}\left(\frac{N}{K}-r\right)\log_2 e$ where r is determined through the following fixed point equation

$$\left\{\begin{array}{lcl} r &=& \frac{N}{K}\frac{1}{1+\rho q} \\ q &=& \frac{1}{K}\sum_{i=1}^{K}\frac{\alpha(i)}{1+\rho\alpha(i)r}. \end{array}\right. \qquad (4.7)$$

Therefore we see that the large system approximation roughly allows to transform the exact capacity expression of the fast fading MIMO system into a sum of individual capacities similarly to a parallel set of Gaussian sub-channels. Now let us go back to the general case. In order to find the optimum power allocation scheme we need to derivate the argument of the maximum in equation (4.5), which we refer to as $\tilde{R}(\alpha_1, ..., \alpha_K)$. Obviously for all $s \in \{1, ..., S\}$, r_s and q_s are functions of the parameters to be optimized *i.e.* $\alpha_{1,s}, ..., \alpha_{K,s}$. It turns out that the partial derivative with respect to $\alpha_{k,s}$ is the same as it would be if r_s and q_s were assumed to be independent of this parameter, which is the purpose of the following lemma.

Lemma 4.2.2 (Property of the equivalent of the network sum-rate)
For all $(k, s) \in \{1, ..., K\} \times \{1, ..., S\}$, the derivative of the sum-rate approximation $\tilde{R}(\alpha_1, ..., \alpha_K)$ with respect to $\alpha_{k,s}$ is the same as that obtained when assuming r_s and q_s to be independent of $\alpha_{k,s}$.

This key property is proved in Appendix B.1. This property of the large dimension equivalent of the sum-rate is instrumental in the determination of the optimum power allocation policy because it considerably simplifies the optimization procedure and allows us to cope with the convergence issue of r_s and q_s towards strict constants as the numbers of users and dimensions grow. Based on this argument, the fact that $(\alpha_1, ..., \alpha_K) \mapsto \tilde{R}(\alpha_1, ..., \alpha_K)$ is a strictly concave function (its Hessian is strictly positive) and using the notation $B_s = b_s \times B$ (where $B = B_1 + ... + B_S$) in order to use dimensionless quantities, one can show that the optimum power fractions are given by the following proposition.

Proposition 4.2.3 (Power allocation for the optimum receiver) *In the asymptotic regime, the optimum power fraction of user k in system s is:*

$$\alpha^*_{k,s} = \left[\frac{b_s}{\sum_{t \in \mathcal{S}_k^+} b_t} \left(1 + \sum_{t \in \mathcal{S}_k^+} \frac{1}{\gamma_{k,t} r_t} \right) - \frac{1}{\gamma_{k,s} r_s} \right]^+, \tag{4.8}$$

where for each user k the set $\mathcal{S}_k^+$ represents the systems/sub-channels which receive a non-zero power; $|\mathcal{S}_k^+| \leq S$ by definition. User k will allocate power to system s if and only if the quantity $\frac{b_s}{\lambda_k \ln 2} - \frac{1}{\gamma_{k,s} r_s}$ is strictly positive.

We have a water-filling equation for the optimum power allocation scheme, which is due to the averaging effect induced by the large system assumption. Let us give one special case of equation (4.8): the case where the base stations have the same bandwidth (*e.g.* UMTS-FDD + UMTS-TDD base stations):

$$\alpha^*_{k,s} = \left[\frac{1}{|\mathcal{S}_k^+|} + \frac{1}{|\mathcal{S}_k^+|} \sum_{t \in \mathcal{S}_k^+} \frac{1}{\gamma_{k,t} r_t} - \frac{1}{\gamma_{k,s} r_s} \right]^+. \tag{4.9}$$

Here the optimum power fraction comprises a term corresponding to the uniform power allocation (*i.e.* the term $\frac{1}{|\mathcal{S}_k^+|}$) plus a term that characterizes the difference of quality between the system under consideration ($\frac{1}{\gamma_{k,s} r_s}$) and the average of all the systems ($\frac{1}{|\mathcal{S}_k^+|}\sum_{t\in\mathcal{S}_k^+}\frac{1}{\gamma_{k,t} r_t}$).

The capacity of the system under consideration is achieved if and only if all the water-filling equations (eq. 4.8) are verified simultaneously. This is obviously the case by construction of the derivation of the water-filling equations and the convexity of the optimization region. The main issue to be mentioned now is the way of implementing the proposed power allocation scheme. We propose an iterative algorithm to implement the optimal power allocation policy:

1. Initialization: assume a uniform power allocation scheme *i.e.* $\forall(k,s) \in \{1,...,K\} \times \{1,...,S\}, \alpha_{k,s} = \frac{1}{S}$.

2. Compute the corresponding value for r_s by using the fixed-point method: the first equation of system (4.6) can be written in the form: $r_s = f_s(r_s)$.

3. Iterate the procedure while the desired accuracy on the power fractions is not reached.

    - For users $k \in \{1,...,K\}$:
        – Update the power fractions by using the water-filling equation (4.8).
        – Update the value of r_s.

A similar algorithm has been recently used by [Dumont 2006, Dumont 2007] in order to derive the capacity of single-user Rician MIMO channels with antenna correlation. Based on the results of [Dumont 2006, Dumont 2007] one is ensured that the approximated ergodic mutual information is a strictly concave function of the transmit power fractions $\{\alpha_{\mathbf{1}},...,\alpha_{\mathbf{K}}\}$ and if the iterative power allocation algorithm converges, then it converges towards the global maximum. At each step of the iterative procedure, the total sum-rate of the system is therefore increasing and generally (all the simulations performed in [Dumont 2006, Dumont 2007] and here confirmed this point) converges to a limit. At the limit, all power fractions will verify the water-filling equations. As already mentioned, the system sum-capacity would be achieved by using a maximum likelihood receiver at all the base stations. More pragmatically we now turn our attention to sub-optimum receiver structures, which can be implemented more easily in real systems. One of the questions we want to answer is whether the optimal power allocation, in terms of the network sum-rate, for other types of receivers can also be expressed through a simple water-filling equation.

4.2.2 MMSE Receiver

The MMSE receiver is known to be the best linear multi-user receiver in terms of SINR. In our context, the MMSE receiver at base station $s \in \{1,...,S\}$ for user $k \in \{1,...,K\}$ can be written as:

$$\mathbf{w}_{k,s}^H = \mathbf{h}_{k,s}^H\left(\sum_{\ell=1}^{K}\alpha_{\ell,s}\mathbf{h}_\ell\mathbf{h}_\ell^H + \sigma^2\mathbf{I}\right)^{-1}, \tag{4.10}$$

and the SINR is given by:

$$\eta_{k,s}^{(mmse)} = \alpha_{k,s}\mathbf{h}_{k,s}^{H}\left(\sum_{\ell=1,\ell\neq k}^{K}\alpha_{\ell,s}\mathbf{h}_{\ell}\mathbf{h}_{\ell}^{H}+\sigma^{2}\mathbf{I}\right)^{-1}\mathbf{h}_{k,s}. \tag{4.11}$$

In order to express the sum-rate achieved by the overall system when the MMSE receiver is used at the base stations, one just needs to determine the SINR at the input of each MMSE receiver. It turns out that each of these SINRs converges to a limit and is especially easy to express in the large dimensions regime (see *e.g.* [Debbah 2002] or Chapter 2). Let $\tilde{\eta}_{\ell,s}^{(mmse)}$ be the asymptotic SINR for user ℓ in the output of the MMSE receiver at base station s. The achievable approximate ergodic sum-rate is then given by:

$$\tilde{R}_{sum}^{(mmse)} = \mathbb{E}\Big[\sum_{s=1}^{S}\sum_{\ell=1}^{K}\underbrace{\log_2\left(1+\tilde{\eta}_{\ell,s}^{(mmse)}\right)}_{\tilde{R}_{k,s}^{(mmse)}}\Big]. \tag{4.12}$$

The asymptotic SINR expression in the MMSE output can be shown to be (see *e.g.* Chapter 2):

$$\forall \ell\in\{1,...,K\},\ \tilde{\eta}_{\ell,s}^{(mmse)} = \frac{\alpha_{\ell,s}}{N_s}\sum_{i=1}^{N_s}\frac{g_s^2(i,\ell)}{\sigma^2+\frac{1}{N_s}\sum_{j\neq\ell}^{K}\frac{\alpha_{j,s}g_s^2(i,j)}{1+\tilde{\eta}_{j,s}^{(mmse)}}}. \tag{4.13}$$

To find the amount of power user k has to allocate to system s one needs to derivate the sum-rate (eq. (4.12)) w.r.t. $\alpha_{k,s}$. Unlike the asymptotic sum-rate achieved by the optimum receiver, the asymptotic sum-rate achieved by using the MMSE receiver is not always a concave function of $(\alpha_1,...,\alpha_K)$. In order to obtain an analytical solution (otherwise an exhaustive numerical optimization of the sum-rate can always be performed) and avoid using possibly computationally demanding numerical optimization techniques, we propose to approximate the asymptotic sum-rate by a concave function by introducing the two approximations (given below). This leads to the following proposition.

Proposition 4.2.4 (Optimum power allocation for the MMSE receiver) *Assume that*

1. $\tilde{\eta}_{k,s}^{(mmse)} = a_{k,s}^{(mmse)}\times\alpha_{k,s}$ *with* $\dfrac{\partial a_{k,s}}{\partial\alpha_{k,s}} = 0$;

2. $\left|\dfrac{\partial\tilde{R}_{k,s}^{(mmse)}}{\partial\alpha_{k,s}}\right| >> \left|\displaystyle\sum_{\ell\neq k}\dfrac{\partial\tilde{R}_{\ell,s}^{(mmse)}}{\partial\alpha_{k,s}}\right|$.

In the asymptotic regime, the optimum power fraction of user k in system s is:

$$\alpha_{k,s}^{(mmse)} = \left[\omega_k - \frac{1}{a_{k,s}^{(mmse)}}\right]^{+} \tag{4.14}$$

where $\omega_k \triangleq \frac{1}{\lambda_k\ln 2}$ is the water-level for user k and

$$a_{k,s}^{(mmse)} \triangleq \frac{1}{N_s}\sum_{i=1}^{N_s}\frac{g_s^2(i,k)}{\sigma^2+\frac{1}{N_s}\sum_{j\neq k}^{K}\frac{\alpha_{j,s}g_s^2(i,j)}{1+\tilde{\eta}_{j,s}^{(mmse)}}}. \tag{4.15}$$

Proof By setting the derivative of the constrained asymptotic sum-rate to zero, one directly obtains that:

$$
\begin{aligned}
\frac{\partial}{\partial \alpha_{k,s}} \left[\tilde{R}_{sum}^{(mmse)} - \sum_{\ell=1}^{K} \lambda_\ell \left(\sum_{s=1}^{S} \alpha_{\ell,s} - P \right) \right] &= 0 \\
\Leftrightarrow \qquad \frac{1}{\ln 2} \frac{\frac{\partial \tilde{\eta}_{k,s}^{(mmse)}}{\partial \alpha_{k,s}}}{1 + \tilde{\eta}_{k,s}^{(mmse)}} - \lambda_k &= 0.
\end{aligned} \tag{4.16}
$$

The validity of assumptions (1) and (2) is discussed below and will also be commented in the simulation part. The first assumption is actually exactly verified in the finite case and we would also like its large system equivalent to have this property. The second assumption is motivated by the fact that in a many user network the behavior of a single user should have almost no impact on the SINR of another user of this network. Mathematically, as the proof above shows, the motivations for assuming (1) and (2) is that the optimization problem becomes very similar to the one investigated for the optimum receiver. Therefore, like the optimum receiver, the *approximate* optimum power allocation policy is given by a simple water-filling equation.

4.2.2.1 Approximating the asymptotic system sum-rate by a concave function

For the user of interest (*i.e.* user k):

$$
\begin{aligned}
\frac{\partial \tilde{\eta}_{k,s}^{(mmse)}}{\partial \alpha_{k,s}} = & \frac{1}{N_s} \sum_{i=1}^{N_s} \Bigg\{ \frac{g_s^2(i,k)}{\sigma^2 + \frac{1}{N_s} \sum_{j \neq k}^{K} \frac{\alpha_{j,s} g_s^2(i,j)}{1+\tilde{\eta}_{j,s}^{(mmse)}}} \times \\
& \left[\sigma^2 + \frac{1}{N_s} \sum_{j \neq k}^{K} \frac{\alpha_{j,s} g_s^2(i,j)}{1 + \tilde{\eta}_{j,s}^{(mmse)}} \left(1 + \alpha_{k,s} \frac{\partial \tilde{\eta}_{j,s}^{(mmse)}}{\partial \alpha_{k,s}} \frac{1}{1 + \tilde{\eta}_{j,s}^{(mmse)}} \right) \right] \Bigg\}
\end{aligned} \tag{4.17}
$$

For all $\ell \neq k$,

$$
\begin{aligned}
\frac{\partial \tilde{\eta}_{\ell,s}^{(mmse)}}{\partial \alpha_{k,s}} = & -\frac{\alpha_{\ell,s}}{N_s} \sum_{i=1}^{N_s} \Bigg\{ g_s^2(i,\ell) \times \\
& \frac{\frac{1}{N_s} \frac{g_s^2(i,k)}{1+\tilde{\eta}_{k,s}^{(mmse)}} + \frac{1}{N_s} \sum_{j \neq \ell} \alpha_{j,s} g_s^2(i,j) \left(\frac{-\partial \tilde{\eta}_{j,s}^{(mmse)}}{\partial \alpha_{k,s}} \right) \frac{1}{(1+\tilde{\eta}_{j,s}^{(mmse)})^2}}{\left(\sigma^2 + \frac{1}{N_s} \sum_{j \neq \ell} \frac{\alpha_{j,s} g_s^2(i,j)}{1+\tilde{\eta}_{j,s}^{(mmse)}} \right)^2} \Bigg\}
\end{aligned} \tag{4.18}
$$

Let $|\tilde{\eta}'_M|$ and g_M be the maxima of $\left| \frac{\partial \tilde{\eta}_{\ell,s}^{(mmse)}}{\partial \alpha_{k,s}} \right|$ and $g_s(i,\ell)$ over all the triplets (i,ℓ,s). By definition $\left| \frac{\partial \tilde{\eta}_{\ell,s}^{(mmse)}}{\partial \alpha_{k,s}} \right| \leq |\tilde{\eta}'_M|$. In fact, under reasonable assumptions, one can tighten this bound, this is the purpose of what follows. The main point is to assume that the entries $g_s(i,j)$ take finite values and do not vanish. Note that for MIMO systems the entries of the mask matrix $g_s(i,j)$ are effectively bounded and they do not scale with N_s. However, for CDMA and OFDM systems this is not true since for both case they represent the realizations of the channel impulse. As a Rayleigh distribution is assumed for the channel gains, they are not bounded mathematically. However, many works applying random matrix theory (see *e.g.* [Debbah 2003]) assume that the channel has a compact support. In practice, for physical reasons, the channel gains do not strictly vanish and stay effectively in a finite interval and therefore the proposed assumption makes sense.

For all (k,s) in $\{1,...,K\} \times \{1,...,S\}$ one can easily check that

$$\left|\frac{\partial \tilde{\eta}^{(mmse)}_{\ell,s}}{\partial \alpha_{k,s}}\right| \leq \frac{\alpha_\ell}{N_s}\sum_{i=1}^{N_s}\frac{g_s^2(i,\ell)}{N_s}\sum_{j\neq\ell} g_s^2(i,j)\frac{\alpha_j}{\left(1+\tilde{\eta}^{(mmse)}_{j,s}\right)^2}\left|\frac{\partial \tilde{\eta}^{(mmse)}_{j,s}}{\partial \alpha_{k,s}}\right| \times \frac{1}{\sigma^4} \tag{4.19}$$

$$\leq \frac{1}{N_s^2}\frac{g_M^4}{\sigma^4}\sum_{i=1}^{N_s}\sum_{j\neq\ell}\left|\frac{\partial \tilde{\eta}^{(mmse)}_{j,s}}{\partial \alpha_{k,s}}\right| \tag{4.20}$$

$$\leq \left(\frac{g_M}{\sigma}\right)^4 \frac{K}{N}|\tilde{\eta}'_M|. \tag{4.21}$$

Therefore we see that a sufficient condition for the MMSE output SINR of user ℓ to be considered as independent of the power allocation of user $k \neq \ell$ is that the ratio $\frac{K}{N}$ has to be small. Under this sufficient but not necessary condition the approximate SINR $\tilde{\eta}_{k,s}$ can be considered to be proportional to $\alpha_{k,s}$ (Assumption (1)). For the second assumption to hold a sufficient but stronger condition is that the quantity $\frac{K^2}{N}$ is small. We therefore see that the validity of the proposed assumptions depends on the scenario under consideration.

4.2.3 Matched Filter

Now we go a step further in decreasing the receiver complexity. We assume a matched filter at all the base stations. The MF for user k at base station s simply consists in multiplying the received signal $\mathbf{y}_s$ by $\mathbf{h}^H_{k,s}$. The signal at the MF output is expressed as

$$\mathbf{h}^H_{k,s}\mathbf{y}_s = ||\mathbf{h}_{k,s}||^2 x_{k,s} + \sum_{\ell\neq k}\mathbf{h}^H_{k,s}\mathbf{h}_{\ell,s}x_{\ell,s} + \mathbf{h}^H_{k,s}\mathbf{z}_{k,s}, \tag{4.22}$$

and the corresponding SINR follows:

$$\eta^{(mf)}_{k,s} = \frac{||\mathbf{h}_{k,s}||^4\alpha_{k,s}}{\sigma^2||\mathbf{h}_{k,s}||^2 + \sum_{\ell\neq k}\alpha_{\ell,s}|\mathbf{h}^H_{k,s}\mathbf{h}_{\ell,s}|^2}. \tag{4.23}$$

In the asymptotic regime the SINR becomes (see sec. 2.3.1)

$$\tilde{\eta}^{(mf)}_{k,s} = \frac{\alpha_{k,s}\left(\sum_{i=1}^{N_s} g_s^2(i,k)\right)^2}{\sigma^2 N_s\sum_{i=1}^{N_s} g_s^2(i,k) + \sum_{\ell\neq k}\alpha_{\ell,s}\sum_{i=1}^{N_s} g_s^2(i,k)g_s^2(i,\ell)}. \tag{4.24}$$

The asymptotic system sum-rate achieved by using the MF at the reception is:

$$\tilde{R}^{(mf)}_{sum} = \mathbb{E}\left[\sum_{s=1}^{S}\sum_{\ell=1}^{K}\log_2\left(1+\tilde{\eta}^{(mf)}_{\ell,s}\right)\right]. \tag{4.25}$$

The optimum power allocation for the marched filter is then given by the following proposition.

Proposition 4.2.5 (Optimum power allocation for the MF) *Assume that* $\left|\frac{\partial \tilde{R}_{k,s}^{(mmse)}}{\partial \alpha_{k,s}}\right| >> \left|\sum_{\ell \neq k} \frac{\partial \tilde{R}_{\ell,s}^{(mmse)}}{\partial \alpha_{k,s}}\right|$. *In the asymptotic regime, the optimum power fraction of user k in system s is:*

$$\alpha_{k,s}^{(mf)} = \left[\omega_k - \frac{1}{a_{k,s}^{(mf)}}\right]^+, \tag{4.26}$$

where

$$a_{k,s}^{(mf)} = \frac{\left(\sum_{i=1}^{N_s} g_s^2(i,k)\right)^2}{\sigma^2 N_s \sum_{i=1}^{N_s} g_s^2(i,k) + \sum_{\ell \neq k} \alpha_{\ell,s} \sum_{i=1}^{N_s} g_s^2(i,k) g_s^2(i,\ell)}, \tag{4.27}$$

and $\omega_k \triangleq \frac{1}{\lambda_k \ln 2}$ *is the water-level for user k.*

Proof A quick look at the sum-rate expression shows that the situation is similar to that encountered with the MMSE receiver. The only difference is that one does not need to introduce assumption (1) since the SINR $\eta_{k,s}^{(mf)}$ is always proportional to $\alpha_{k,s}$, whatever the dimensions of the system. The stated result follows.

4.2.3.1 Approximating the asymptotic system sum-rate by a concave function

First, note that Assumption (1) is exactly verified both in the finite and large dimensions settings. So, here we focus on the validity of Assumption (2). In a given system s, we have

$$\frac{\partial \tilde{R}_s^{(mf)}}{\partial \alpha_{k,s}} \triangleq \frac{\partial}{\partial \alpha_{k,s}} \sum_{\ell=1}^{K} \log_2\left(1 + \tilde{\eta}_{\ell,s}^{(mf)}\right) = \frac{1}{\ln 2} \sum_{\ell=1}^{K} \frac{\partial \tilde{\eta}_{\ell,s}^{(mf)}}{\partial \alpha_{k,s}} \frac{1}{1 + \tilde{\eta}_{\ell,s}^{(mf)}} \tag{4.28}$$

with

$$\begin{cases} \frac{\partial \tilde{\eta}_{k,s}^{(mf)}}{\partial \alpha_{k,s}} = \frac{\tilde{\eta}_{k,s}^{(mf)}}{\alpha_{k,s}} \\ \frac{\partial \tilde{\eta}_{\ell,s}^{(mf)}}{\partial \alpha_{k,s}} = -\tilde{\eta}_{\ell,s}^{(mf)} \frac{\sum_{i=1}^{N_s} g_s^2(i,k) g_s^2(i,\ell)}{\sigma^2 N_s \sum_{i=1}^{N_s} g_s^2(i,\ell) + \sum_{j \neq \ell} \alpha_{j,s} \sum_{i=1}^{N_s} g_s^2(i,\ell) g_s^2(i,j)} & \text{for all } \ell \neq k. \end{cases} \tag{4.29}$$

Define $g_M^2 = \max_{(\ell,s,i)} g_s^2(i,\ell)$ and $g_m^2 = \min_{(\ell,s,i)} g_s^2(i,\ell)$ and upper bound the quantity of interest that is

$$\left|\sum_{\ell\neq k}\frac{\partial\tilde{R}_{\ell,s}}{\partial\alpha_{k,s}}\right| = \left|\frac{1}{\ln 2}\sum_{\ell=1}^{K}\frac{\partial\tilde{\eta}^{(mf)}_{\ell,s}}{\partial\alpha_{k,s}}\frac{1}{1+\tilde{\eta}^{(mf)}_{\ell,s}}\right| \tag{4.30}$$

$$\leq \frac{1}{\ln 2}\sum_{\ell\neq k}\left|\frac{\partial\tilde{\eta}^{(mf)}_{\ell,s}}{\partial\alpha_{k,s}}\right| \tag{4.31}$$

$$\leq \frac{1}{\ln 2}\sum_{\ell\neq k}\left|\tilde{\eta}^{(mf)}_{l,s}\right|\frac{\sum_{i=1}^{N_s} g_s^2(i,k)g_s^2(i,\ell)}{\sigma^2 N_s\sum_{i=1}^{N_s} g_s^2(i,\ell)} \tag{4.32}$$

$$\leq \frac{1}{\ln 2}\sum_{\ell\neq k}\left|\tilde{\eta}^{(mf)}_{l,s}\right|\frac{N_s g_M^4}{\sigma^2 N_s\sum_{i=1}^{N_s} g_s^2(i,\ell)}. \tag{4.33}$$

At this point we have to distinguish between MIMO systems on the one hand and CDMA and OFDM systems on the other hand. For MIMO systems we know that $\sum_{i=1}^{N_s} g_s^2(i,\ell) \geq N_s g_m^2$ where g_m is finite and different from zero. For CDMA and OFDM systems, as the channel realizations are into play, we exploit the central limit theorem, which allows us to write $\sum_{i=1}^{N_s} g_s^2(i,\ell) = N_s\left(\mu + o\left(\frac{1}{\sqrt{N_s}}\right)\right)$ where μ is the average energy of the channel gain (assumed to be normalized to one). In any case, the sum of interest can be bounded by $const. \times \frac{K}{N_s}$, which gives us a sufficient condition in order for Assumption 2 to hold for the matched filter.

4.3 Numerical results

In all the simulations the following channel model will be assumed. The entries of $\mathbf{W}_s$ will be chosen to be i.i.d. with zero-mean and variance 1. For the CDMA case, the entries of $\mathbf{G}$ will be generated according to a Rayleigh distribution with variance 1, with independent columns and all the elements in each of them equal, corresponding to flat fading, and for MIMO a matrix of ones (no correlation). First we assume the optimum receiver at the base stations. We want to evaluate the performance gain brought by exploiting the available cross-system diversity, in comparison with the standard power allocation scheme (hard handover). For this, let us assume the following typical simulation setup in a cellular system: 50 active users ($K = 50$) and 4 CDMA base stations ($S = 4$) with different spreading factors ($(N_1, N_2, N_3, N_4) = (4, 8, 16, 32)$). Fig. 4.2 shows that for medium and high SNRs the performance loss induced by using only one technology at a time can be very significant, greater than 4 dB typically, which means that the mobile transmit power could be divided by a factor greater than 2 w.r.t. to the conventional strategy. On the other hand, for low SNRs, the hard handover solution performs better than the uniform power allocation, which shows the potential interest in implementing the optimum power allocation, which provides the best performance whatever the SNR. Also, in contrast to single-user MIMO systems, it can be seen that the gap in performance between uniform and optimum power allocation schemes does not shrink as the SNR increases. This observation has also been made in other simulation scenarios. Figure 4.3 shows a scenario with the same parameters as the one just analyzed but now both CDMA and MIMO systems are considered, obtaining relatively similar results. In all the tested scenarios the convergence of the proposed iterative power allocation algorithm was obtained after at most 10 iterations; note that the algorithm is said to have converged if the optimum power fractions are determined with an accuracy of 10^{-4}.

Now we assume the simplest receiver at the base stations, namely the matched filter. There are two base stations and two users. The BS are equipped with multiple antennas: $N_1 = 2, N_2 = 4$. Fig. 4.4 shows the network sum-rate achieved by using the MF for four different power allocation schemes: the optimum power allocation obtained by an exhaustive numerical search, the approximate power allocation obtained by assuming the two hypotheses stated in Sec. 4.2.2 and 4.2.3, the uniform power allocation scheme and the hard handover. First, the figure shows that the corresponding approximation of the sum-rate is not very good but it still provides a performance gain over the other power allocation schemes. Second, this simulation confirms that the uniform power allocation becomes more and more suboptimal w.r.t. to the exact optimum power allocation as the SNR increases. Third, we clearly see that handover based power allocation suffers from a significant performance loss for medium and high SNRs. To sum up, we can say that, as a rule of thumb, the uniform power allocation can always be used and will provide significant gains with the advantage of being very simple to implement (no feedback mechanism required in particular).

The last figure, *i.e.* Fig. 4.5 sums up the network performance for the three receivers investigated in this chapter in the typical scenario $K = 20$, $S = 3$, $(N_1, N_2, N_3) = (4, 8, 32)$. It allows one to better evaluate the benefits from using the optimum receiver over the MMSE receiver and MF. A typical information that can be drawn from this figure is as follows: by simply using a MMSE receiver with uniform power allocation instead of the MF with hard handover (as used in current networks) a huge performance gain could be obtained by exploiting the available cross-system diversity. Of course, this comment holds for medium and high SNRs. If the network is also likely to operate in the low SNR regime, the optimum power allocation should be used or a SNR-based switching mechanism between the hard handover and uniform power allocation could be introduced.

4.4 Conclusion

In this chapter, a cross-system power allocation algorithm has been provided in the context of MIMO, CDMA and OFDM technologies in order to exploit the available cross-system diversity. Interestingly, in the asymptotic regime, a radio access technology can be characterized, from the information-theoretic point of view, by only a few parameters. Indeed, the solution for all the receivers turns out to be dependent only on a limited number of parameters: the dimensions of the system, number of users, channel gains, path loss, noise variance and correlation at the transmitter and the receiver.

As a consequence, for the optimum receiver a simple cross-layer algorithm, analogous to the water-filling algorithm, can be implemented at the central controller to schedule the powers of all the users in order to maximize the network capacity, and this can be done in

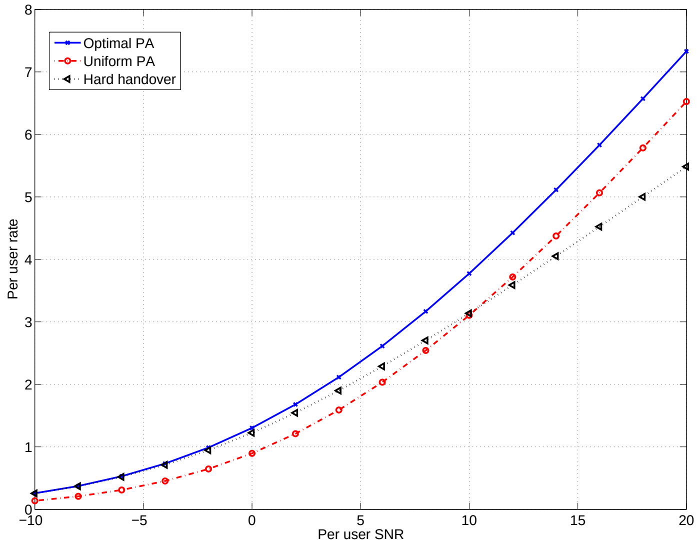


Figure 4.2: Optimal receiver. Performance gains brought by cross-system diversity (4 CDMA systems with $N_s = [32, 16, 8, 4]$ receive dimensions and $K = 50$ users).

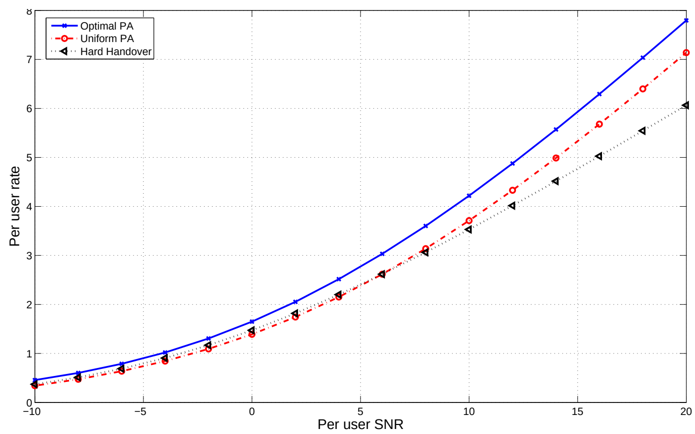


Figure 4.3: Optimal receiver. Performance gains brought by cross-system diversity (2 CDMA systems with $N_s = [32, 16]$ receive dimensions and 2 MIMO systems with $N_s = [8, 4]$ receive antennas. $K = 50$ users).

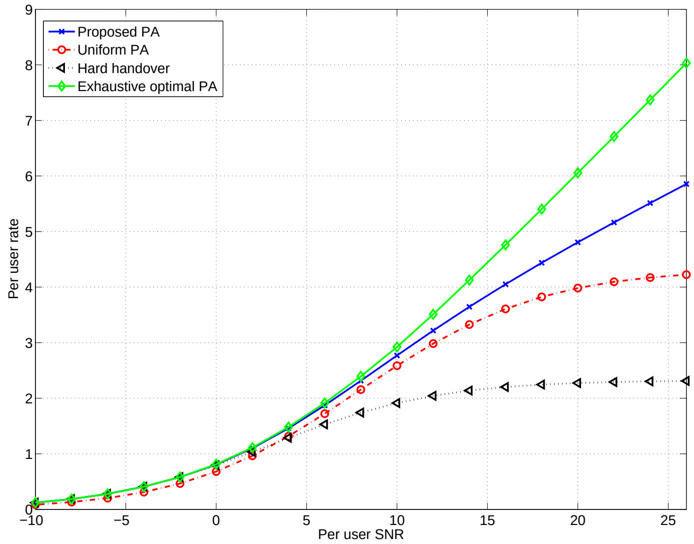


Figure 4.4: Matched filter performance for the optimum (calculated exhaustively), approximate optimum, uniform and hard handover power allocations. (2 CDMA systems with $N_s = [2, 4]$ receive dimensions and $K = 2$ users)

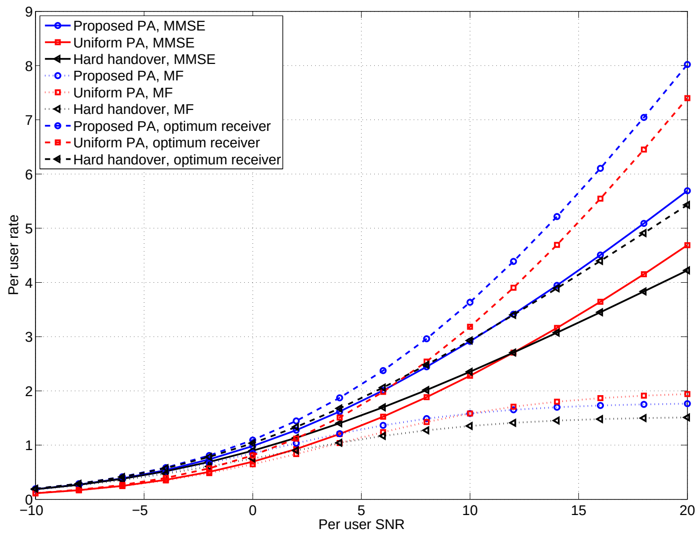


Figure 4.5: Average user rate vs. SNR for Optimum receiver, MMSE receiver and matched filter, comparing the obtained power allocation, with hard handover and uniform power allocation. (3 CDMA systems with $N_s = [32, 8, 4]$ receive dimensions and $K = 20$ users)

a simple, iterative way, which generally converges to the optimum.

For the MF and MMSE receivers a water-filling solution can still be obtained by introducing two additional assumptions, which simplify the optimization problem but at the price of a performance loss that has to be evaluated in the situations of interest. For the typical scenarios considered in this chapter, we saw that they were reasonable. The potential performance gain of cross-system diversity was shown to be important in several typical simulation setups. For instance, by simply using MMSE receivers at the base stations and uniform power allocation over the different systems, the mobile transmit power could be divided by a factor greater than 10 with respect to a standard network using the MF and hard handover power allocation scheme.

The proposed work could be extended by considering the outage probability in order to further analyze the benefits of cross-system diversity, which will allow one to complete our comparisons between the hard handover, uniform and optimum power allocation schemes. It would also be interesting to study a more heterogeneous network, for instance by introducing CDMA base stations with multiple antennas and exploiting the results derived by [Hanly 2001]. As mentioned here, more technologies can be considered since the condition on the patter mask matrices $\mathbf{G}_s$ are mild and the strong results of [Girko 2001] can be directly applied in the proposed framework.

To conclude this chapter, it should be mentioned the fact that our approach is information theoretical, and obviously, many issues would need to be addressed to implement the proposed power allocation schemes. The way of coordinating base stations using different technologies is just one example of this kind of issues.

CHAPTER 5

Team and Noncooperative Solutions to Access Control

In this chapter we consider decentralized medium-access control in which many pairwise interactions, where users compete for a medium access oportunity, occur between randomly selected users that belong to a large population. A given user has a fixed number of access attempts and a fixed budget for buying different power levels (in a more general setting, they could be considered priority levels, with only some minor differences that will be pointed out). We consider situations in which the choice of power level is done by each user, without knowing in advance the choices of other users. In each time-slot, the access is attributed to the user with the largest power level. The performance criterion is the expected number of successful access attempts that a user may obtain within a given budget.

We consider both the team framework, in which all users share the common objective of maximizing the above criterion averaged over the whole population, as well as the noncooperative framework, in which each user maximizes its own performance measure and where the solution concept is the Nash equilibrium. We restrict to a regime of weak interactions in which upon an access attempt, a user is either faced with no other simultaneous attempt or might face a single opponent that attempts to access the network at the same time. This framework is similar to the pairwise interaction paradigm in evolutionary game theory (see, e.g., [7]), and may correspond, for example, to sparse network topologies (such as ad-hoc networks). Due to the impliocit symmetry assumption on the channels made by the model, both multiple acces and interference channels can be considered.

Our analysis reveals that unlike many standard team problems, optimal pure policies do not exist in the team framework, but both an optimal solution, as well as equilibria exist within the class of mixed policies. Focusing on symmetric working points, we fully characterize both the team solution and the equilibrium point, which turn out to be unique. We show that the optimal policy requires only three priority (or power) levels, where the Nash equilibrium uses only two priority levels. This result is significant from an engineering perspective, as network architectures usually limit the number of power levels or priority classes out of practical concerns.

5.1 System model

5.1.1 General setting

We consider a large population of mobiles. Each has a battery with K energy units. Time is discrete. At each time unit a mobile has a transmission opportunity. If it has $k \leq K$ energy units left then it can transmit with any *integer* energy level $1 \leq l \leq k$. If $k = 0$ then it cannot transmit. Every N time units the battery is replaced with a new one with energy level K. Assume that there are pairwise interactions: when a mobile attempts transmission, the receiver is with probability $(1 - \delta)$ in the range of yet another mobile which is randomly selected from the whole population. At each transmission opportunity the interaction occurs with another randomly selected mobile. The time slots are common to all mobiles but when a mobile is at the ith stage in his battery lifetime, it interacts with a mobile that is at a random stage j, uniformly distributed[1] between 1 and N.

User Policy. Due to the above assumptions, a general transmission policy u may be characterized by the number of times each power level is used, since the specific times in which each level is applied are insignificant. Hence, a (pure) policy u will be described by a $K + 1$ vector $u = (n_0, n_1, \ldots, n_K)$, where n_i represents the number of slots during the lifetime of the battery in which a power of a_i is used for transmission (n_0 stands for the number of slots in which there is no transmission). The following constraints must obviously be met for every feasible user policy:

$$\sum_{i=0}^{K} n_i = N \tag{5.1}$$

$$\sum_{i=1}^{K} a_i n_i \leq K. \tag{5.2}$$

Let $x_i \equiv x_i(N) := n_i/N$ denote the fraction of time that power level a_i is employed for a given policy u. Throughout the chapter, we shall alternatively use the vector $x = (x_0, x_1, \ldots, n_K)$ to represent a policy.

Our model may allow for mixed policies as well. A mixed policy σ is a a randomized choice among a collection of pure policies $(u(1), \ldots, u(m))$, where policy $u(i)$ is chosen with probability q_i and $q_i >= 0, \quad \sum q_i = 1$. If policy $u(i)$ is selected then it is used throughout the battery life.

Reception Rule. At any given time, a transmission attempt with power level $i > 0$ is successful, if and only if (i) there is no simultaneous transmission, or (ii) the interfering transmission uses a power level j such that $\frac{i}{j} \geq \beta$, where β is a positive SIR threshold strictly greater than 1. In this case if a power level i_1 is used, and assuming symmetric policies, power levels $i_1 + 1, ..., \beta i_1 - 1$ offer no advantage with respect to it in terms of interference avoidance, whereas requiring extra power. As a result it is immediate than an optimal policy will only use powers 0 and $\beta^k, k \in \mathbb{N}$, thus returning to a problem where

[1] More generally, we may consider any distribution for the random stage of the opponent. In that case, the results in this chapter will remain the same, if we plausibly assume that users choose the number of times that each power level will be used, and then uniformly randomize over the possible permutation for a given choice.

whenever different power levels are used the higher one succeeds with probability one. If we redefine $n_i, i > 0$ as the number of times power level β^{i-1} and n_0 the number of times the user is idle, the constraints 5.1 and 5.2 become

$$\sum_{i=0}^{K} n_i = N \tag{5.3}$$

$$\sum_{i=1}^{K} \beta^{i-1} n_i \leq K. \tag{5.4}$$

5.1.1.1 Team problem

We denote by $g_{N,K}(\sigma)$ the expected number of successful transmissions per battery lifetime of a mobile when all mobiles use the same mixed policy σ for given parameters N and K. Accordingly, $g_{N,K}(\sigma)$ would be regarded as the utility of the mobile. The objective in the team problem is to set a unified policy which maximizes the utility $g_{N,K}(\sigma)$ over σ. The chosen σ can be regarded as a fixed access protocol that all mobiles must obey.

In order to be able to compare strategies for different parameters N, K we introduce the Throughput Per Slot (TPS) criterion which divides the former criterion by number of slots N, i.e., $TPS(\sigma) = \frac{g_{N,K}(\sigma)}{N}$. Obviously, maximizing $TPS(\sigma)$ is an equivalent problem to maximizing $g_{N,K}(\sigma)$.

When restricting ourselves to pure policies u, the team-objective becomes to maximize $g_{N,K}(u)$ over u, where

$$g_{N,K}(u) = \delta(N - n_0) + (1 - \delta)\frac{1}{N}\sum_{i=1}^{K}\sum_{j=0}^{i-1} n_i n_j. \tag{5.5}$$

Indeed, when there is no interference, all non-zero power levels lead to a successful transmission, whereas in the presence of interference, the probability that a transmission with power level i is successful is given by $\sum_{j=0}^{i-1} n_j/N$.

5.1.1.2 Noncooperative Game

In a noncooperative framework, users are self-optimizing and are free to determine their own policy in order to maximize their expected number of successful transmissions (or alternatively their expected TPS). A Nash equilibrium point is a collection of user strategies for which no user can obtain a higher number of expected successful transmissions by unilaterally modifying its transmission strategy. In the current work, we shall focus on *symmetric* Nash equilibria. A symmetric Nash equilibrium is a working point where all mobiles use the same strategy σ, and furthermore, for all other strategies $\tilde{\sigma}$,

$$g_{N,K}(\sigma) \geq g_{N,K}(\tilde{\sigma}, \sigma), \tag{5.6}$$

where $g_{N,K}(\tilde{\sigma}, \sigma)$ is the utility of a user who deviates to the policy $\tilde{\sigma}$, while the rest of the population uses σ.

For simplicity, we shall restrict our attention in the bulk of this work to the case where $N = K$. A feasible policy under this setting is to use a power level of one at all time slots. Obviously, such policy would result in zero TPS when $\delta = 0$, hence it is not an optimum nor an equilibrium for this value of δ. However, for the other extreme of $\delta = 1$, the same policy becomes an optimal solution as well as an equilibrium point.

5.1.2 Numeric Examples

We provide below some numeric examples and derive some interesting properties. For simplicity, we consider cases where $N = K$ and $\delta = 0$, which corresponds to the case where a user interacts with probability one with another user in each of its stages. In addition, we focus below on pure strategies. We use the format $(n_0, n_1, \dots)$ to describe a policy.

The case of $N = 3$ The feasible policies that use all energy are $(0,3,0,0)$, $(1,1,1,0)$, $(2,0,0,1)$. The expected number of packets transmitted successfully in a cycle of duration 3, if all use the same policy is $g_{3,3}(0,3,0,0) = 0$, $g_{3,3}(1,1,1,0) = 1$, $g_{3,3}(2,0,0,1) = 2/3$. The policy $(1,1,1,0)$ is seen to be the best pure strategy[2]. It is an equilibrium (in pure strategies) as well; a deviation to $(0,3,0,0)$ or to $(2,0,0,1)$ decreases the utility from 1 to 1/3. $(0,3,0,0)$ is not an equilibrium as a deviation of a player to $(1,1,1,0)$ or to $(2,0,0,1)$ increases its utility to 1/3. $(2,0,0,1)$ is not an equilibrium since a player deviating to $(0,3,0,0)$ increases its utility from 2/3 to 2.

The case of $N = 4$ The feasible policies that use all energy are $(0,4,0,0,0)$, $(1,2,1,0,0)$, $(2,0,2,0,0)$, $(2,1,0,1,0)$, $(3,0,0,0,1)$. The policies $(1,2,1,0,0)$ and $(2,1,0,1,0)$ are both optimal pure policies for the team problem, and obtain $g_{4,4}(1,2,1,0,0) = g_{4,4}(2,0,2,0,0) = 5/4$. None of the above policies is an equilibrium: any deviation from $(0,4,0,0,0)$ strictly increases the utility of the deviator. By deviating from $(1,2,1,0,0)$ to $(2,0,2,0,0)$ the utility of the deviator increases to 6/4. A deviation from $(2,0,2,0,0)$ or from $(2,1,0,1,0)$ to $(0,4,0,0,0)$ increases the utility to 2. Finally, deviating from $(3,0,0,0,1)$ to $(0,4,0,0,0)$ increases the utility to 3.

In the list below we provide the optimal pure policies for the team problem and the associated TPS up to $N = 10$.

- $N = 2$: $TPS = 0.25$
- $N = 3$: $TPS = 0.333$
- $N = 4$: $TPS = 0.313$
- $N = 5$: $(2,1,2,0,0,0), (2,2,0,1,0,0), TPS = 0.32$
- $N = 6$: $(2,2,2,0,..,0), (3,1,1,1,0,..,0), TPS = 0.333$

[2]It can be easily shown that there always exists an optimal policy that uses all the available energy. Indeed, given a policy that does not use all energy, we may always construct a policy that does use all energy and obtains the same TPS (by assigning the access energy to the highest used power level).

- $N = 7$: $(3, 2, 1, 1, 0, .., 0)$, $TPS = 0.347$
- $N = 8$: $(3, 3, 1, 1, 0, .., 0)$, $TPS = 0.344$
- $N = 9$: $(3, 3, 3, 0, .., 0)$, $(4, 2, 2, 1, 0, .., 0)$, $TPS = 0.346$
- $N = 10$: $(4, 3, 2, 1, 0, ..0)$, $TPS = 0.35$

We observe the following properties from our numerical study.

1. There need not be a (symmetric) equilibrium point in pure strategies.
2. A power greater than three is not used for the team problem.
3. The optimal TPS under pure strategies is not monotone in N.

The potential of using mixed policies is highlighted in the next example. Let $N = 5$, and consider the mixed policy of using with probability of $1/2$ each of the two policies $(2, 1, 2, 0, 0, 0), (2, 2, 0, 1, 0, 0)$. Note that the TPS in this case is equivalent to the one obtained for $N = 10$ and $(4, 3, 2, 1, 0, .., 0)$, which is also the optimal (pure) policy for $N = 10$. The latter policy thus obtains $TPS = 0.35$, which is a strictly higher value than the one obtained while restricting the mobiles to pure strategies.

In the next section we show that a TPS of 0.35 is a tight upper bound on *any* policy (pure or mixed). We further show that it can be obtained for any $N = K$ by the use of mixed policies. The in-existence of an equilibrium in pure policies motivates the study of mixed policies for the noncooperative framework as well, which is covered in Section 5.3.

5.2 The team problem

In this section we consider the team problem, in which a central authority assigns a unified policy to all users, who must obey it. The policy can be thus be viewed as a *protocol*. The natural objective is to find a protocol that maximizes the average number of successful transmissions (or the TPS) across users. In Section 5.2.1 we consider this optimization problem under pure policies, and obtain some structural properties of the best policy. In Section 5.2.2 we derive an upper bound on the TPS for any N. In Section 5.2.3 we show that the upper bound is always achievable when mixed policies are allowed. Implications of these results are discussed in Section 5.2.4.

5.2.1 Pure Strategies

In this subsection we restrict attention to the set of pure policies, and analyze the optimal policy among this set. From a practical-engineering viewpoint, the underlying complexity in implementing pure strategies can be lower compared to mixed policies, which require randomization between several pure policies.

We start our analysis with a lemma that provides an alternative expression for $g_{N,K}$, which will be central in our subsequent analysis of the problem.

Lemma 5.2.1 *Let $u = (n_0, n_1, \dots, n_K)$ be a unified transmission policy. Then*

$$g_{N,K}(u) = \delta(N - n_0) + (1 - \delta)\frac{1}{2N}\left(N^2 - \sum_{i=0}^{K} n_i^2\right). \tag{5.7}$$

Proof Note first that

$$N^2 = (n_0 + n_1 + \dots n_K)^2 = 2\sum_{i=1}^{K}\sum_{j=0}^{i-1} n_i n_j + \sum_{i=0}^{K} n_i^2$$

. Hence,

$$\sum_{i=1}^{K}\sum_{j=0}^{i-1} n_i n_j = \frac{N^2 - \sum_{i=0}^{K} n_i^2}{2}. \tag{5.8}$$

Substituting (5.8) into (5.5) gives (5.7). □

The following result is a direct consequence of Lemma 5.2.1.

Proposition 5.2.2 *There always exists an optimal unified policy which satisfies the following relation*

$$n_K \leq n_{K-1} \leq \dots \leq n_1. \tag{5.9}$$

Proof Let $u = (n_0, \dots, n_K)$ be an optimal unified policy. Assume that $n_i > n_j$ for some indexes i and j such that $i > j$. Consider now the modified policy $\tilde{u} = (n_0, \dots, \tilde{n}_N)$, where $\tilde{n}_k = n_k$, for every $k \neq i, j$, $\tilde{n}_i = n_j$, $\tilde{n}_j = n_i$. Then $\tilde{u}$ obviously obeys the constraints (5.1)–(5.2). Moreover, noting (5.7), $\tilde{u}$ achieves the same throughput as u, hence it is an optimal policy as well. □

The above monotonicity result suggests that there is no benefit in using higher power levels more frequently than lower power levels are used. Note that for the case of $\delta = 0$ it can be further shown that $n_K \leq n_{K-1} \leq \dots \leq n_1 \leq n_0$, i.e., the number of no-transmissions is higher than the number of transmission at any power level. However, this inequality need not hold for general δ.

In the remaining of this subsection, we consider the case of $N \geq K$, which may be relevant, for example, in ad-hoc or sensor wireless networks, in which energy is relatively limited. Our main result for that case suggests that a power level greater than 3 would not be used in *any* optimal unified policy (regardless of how large N and K are). Formally,

Theorem 5.2.3 *Assume that $N \geq K$. Let u be an optimal unified policy. Then $n_i = 0$ for $i > 3$.*

For the proof of the theorem we require four lemmas.

Lemma 5.2.4 *For* every *policy u*

$$n_0 \geq n_2 + 2n_3 + 3n_4. \tag{5.10}$$

Proof Combining (5.1) and (5.4) and recalling that $N \geq K$ we get that

$$n_0 + n_1 + \cdots + n_K \geq n_1 + \beta n_2 + \beta^2 n_3 + \cdots \geq n_1 + 2n_2 + 3n_3 + \ldots$$

.

Thus

$$\begin{aligned} n_0 &\geq n_2 + 2n_3 + 3n_4 + \cdots + (k-1)n_k + \ldots \\ &\geq n_2 + 2n_3 + 3n_4. \end{aligned}$$

□

Lemma 5.2.5 *Assume that $N \geq K$. Further assume that u is an optimal unified policy with $n_4 > 0$ then $n_1 > 0$.*

Proof Note first that $n_4 > 0$ implies that $n_0 \geq 3$ by (5.10). Assume by contradiction that $n_1 = 0$ and consider the modified policy $\tilde{n}_4 = n_4 - 1$, $\tilde{n}_1 = 2$, $\tilde{n}_0 = n_0 - 1$, and $\tilde{n}_k = n_k$ for $k \neq 1, 2, 4$. Note that this policy obeys the constraints (5.1)–(5.2). We next show that $2\left(g_{N,K}(\tilde{u}) - g_{N,K}(u)\right) > 0$ which contradicts the optimality of u. Using (5.7), $2\left(g_{N,K}(\tilde{u}) - g_{N,K}(u)\right) =$

$$2\delta + (1-\delta)(n_4^2 + n_1^2 + n_0^2 - (n_4 - 1)^2 - (n_1 + 2)^2 - (n_0 - 1)^2) =$$

$= 2\delta + (1-\delta)(2n_4 + 2n_0 - 6)$,

which is obviously strictly positive since $n_4 \geq 1$ and $n_0 \geq 3$. □

Lemma 5.2.6 *Assume that $N \geq K$. Let u be an optimal unified policy with $n_4 > 0$ then*

$$n_0 - n_1 \leq n_3 - n_4 + 2, \tag{5.11}$$

$$n_1 - n_2 \leq n_3 - n_4 + 2. \tag{5.12}$$

Proof To prove (5.11), consider the modified policy $\tilde{u}$ with $\tilde{n}_4 = n_4 - 1$, $\tilde{n}_3 = n_3 + 1$, $\tilde{n}_1 = n_1 + 1$, $\tilde{n}_0 = n_0 - 1$ (note that $n_0 > 0$ from (5.10) and the lemma's conditions, hence $\tilde{n}_0 \geq 0$), and $\tilde{n}_k = n_k$ for $k \neq 4, 3, 1, 0$. Note that $\tilde{u}$ is a valid policy, since it obeys (5.1) and (5.2) because u does (the energy investment of both policies is equal). Since u is an optimal policy we must have $2\left(g_{N,K}(\tilde{u}) - g_{N,K}(u)\right) \leq 0$. Using (5.7) this means that

$$2\delta + (1-\delta)\big[n_4^2 + n_3^2 + n_1^2 + n_0^2 - (n_4 - 1)^2$$
$$-(n_3 + 1)^2 - (n_1 + 1)^2 - (n_0 - 1)^2\big] \leq 0.$$

Noting that 2δ is non-negative and rearranging terms in the inequality above, this inequality holds if

$$2n_4 - 2n_3 - 2n_1 + 2n_0 - 4 \leq 0$$

which is easily seen to be equivalent to (5.11). The inequality (5.12) is proven similarly, yet instead of shifting an energy unit from n_0 to n_1, we shift an energy unit from n_1 to n_2 (note that such shift is possible by Lemma 5.2.5). □

Lemma 5.2.7 *Let u be an optimal unified policy for some N and K so that $N \geq K$. Then $n_4 = 0$*

Proof Assume by contradiction that $n_4 > 0$. Then

$$3n_4 + 2n_3 + n_2 - n_1 \leq n_0 - n_1 \leq n_3 - n_4 + 2, \tag{5.13}$$

where the first inequality follows from (5.10) and the second one from (5.11). Hence, $4n_4 + n_3 - 2 \leq n_1 - n_2 \leq n_3 - n_4 + 2$, where the first inequality follows from (5.13) and the second one from (5.12). The last set of inequalities suggests that $5n_4 \leq 4$ which contradicts the assumption that $n_4 > 0$. □

We are now ready to prove the theorem. Note first that $n_4 = 0$ for every optimal unified policy by Lemma 5.2.7. Assume by contradiction that there exists an optimal policy with $n_i > 0$ for some $i > 4$. Then as in the proof of Proposition 5.2.2, the policy $\tilde{u}$, with $\tilde{n}_k = n_k, k \neq i, 4$, $\tilde{n}_4 = n_i > 0$, $\tilde{n}_i = n_4 = 0$ is optimal as well. But this contradicts Lemma 5.2.7. □

5.2.2 Asymptotic Analysis

We henceforth restrict attention to the case $K = N$. In the remaining of this section, we use the vector $x = (x_0, x_1, \ldots, x_N)$ for representing a policy, where $x_i \equiv n_i/N$. With this representation, (5.7) can be written as

$$TPS(x) = \delta(1 - x_0) + \frac{1}{2}(1 - \delta)\left(1 - \sum_{i=0}^{K} x_i^2\right). \tag{5.14}$$

The battery lifetime constraint (5.1) is

$$\sum_{i=0}^{\infty} x_i = 1, \tag{5.15}$$

while the energy constraint (5.4) is

$$\sum_{i=1}^{\infty} \beta^{i-1} x_i \leq 1. \tag{5.16}$$

In addition there is an "integrity" constraint: the x_i's are restricted to multiples of N^{-1}.

We now consider the problem with N very large. x_i is then interpreted as the long-run fraction of time (or *frequency*) that a power of i units is used. The integrity constraint disappears, and we are left with an optimization problem, which is easily seen to be a strictly convex one.

Lemma 5.2.8 *The problem of maximizing $TPS(x)$ in (5.14) subject to (5.15)– (5.16) is a strictly convex optimization problem.*

Proof Since $TPS(x)$ is quadratic in x_i with a negative multiplicative term $-(1-\delta)$, and the constraints are affine, the optimization problem is (strictly) convex. Note that in the case of $\delta = 1$ the trivial unique solution of this problem is $x_1 = 1$. □

The optimal TPS in the asymptotic case is of course an upper bound to the maximal TPS that can be obtained for every N (with the integrity constraint present). We emphasize that the last statement is valid not only for pure strategies, but also for mixed strategies, as the solution for the case of $N \to \infty$ may be viewed as the frequency in which each power level should be used, regardless if the frequencies are obtained under pure or mixed policies. A complete characterization of the optimal policy for the asymptotic case is provided below.

Theorem 5.2.9 *Assume $N = K$ and let $N \to \infty$. The optimal frequencies x_i as a function of δ and the corresponding TPS are given by:*

- $0 \le \delta \le \frac{1}{3}$: $\begin{cases} x_0 = \frac{4-7\delta}{10(1-\delta)}; \qquad x_i = \frac{(3-2i)\delta+4-i}{10(1-\delta)}, i = 1,2,3; \\ TPS = \frac{7-2(\delta+\delta^2)}{20(1-\delta)}. \end{cases}$

- $\frac{1}{3} < \delta \le \frac{2}{3}$: $\begin{cases} x_0 = \frac{2-3\delta}{6(1-\delta)}; \qquad x_i = \frac{2+3\delta(1-i)}{6(1-\delta)}, i = 1,2, \\ TPS = \frac{1}{12}\frac{4-3\delta^2}{1-\delta} \end{cases}$

- $\delta > \frac{2}{3}$: $\begin{cases} x_1 = 1; \\ TPS = \delta. \end{cases}$

Proof Noting that

$$TPS(x) = \frac{1}{2}\left(1 - \sum_{i=0}^{K} x_i^2\right) + \delta\left(\frac{1}{2} - x_0 + \frac{1}{2}\sum_{i=0}^{K} x_i^2\right),$$

we introduce the Lagrangian

$$\begin{aligned} \mathcal{L}(\mathbf{x}) = \frac{1}{2}(\delta+1) + \frac{1}{2}(\delta-1)\sum_{i=0}^{K} x_i^2 - \delta x_0 \\ + \lambda\left(\sum_{i=0}^{K} x_i - 1\right) + \mu\left(\sum_{i=1}^{K} \beta^{i-1} x_i - 1\right), \end{aligned} \tag{5.17}$$

where λ is the Lagrange multiplier associated with the number of time slots, and μ with the power constraint. We ignore in (5.17) the positivity constraints for each x_i, assuming that x_i involved are all positive, yet directly consider this constraint in our analysis below.

We recall from Proposition 5.2.2 that the optimal solution satisfies $x_1 \ge x_2 \ge x_3$. Depending on δ, the largest i for which $x_i > 0$ is either 3, 2, or 1. This is a direct consequence of Theorem 5.2.3, which holds for every N (and also in the limit $N \to \infty$). We shall denote this largest i by i^*. Assume that $i^* > 1$ (the case $i^* = 1$ is treated separately below). In this case, the extremum of the Lagrangian corresponds to an interior point. Indeed, since for $1 \le i \le i^*$, we focus on optimal solutions that satisfy $x_i > 0$

and we are thus away from the boundary $x_i = 0$ for these indices; additionally $x_0 > 0$, since a power level larger than one is being used. The optimal solution is thus obtained by equating the gradient of the Lagrangian to zero, which leads to the following equations

$$\frac{\partial \mathcal{L}}{\partial x_0} = (\delta - 1)x_0 - \delta + \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_i} = (\delta - 1)x_i + \lambda + \mu\beta^{i-1} = 0 \quad \text{for} \quad i = 1, \dots, i^*$$

or equivalently

$$x_0 = \frac{\delta - \lambda}{\delta - 1}, \quad x_i = -\frac{\lambda + \mu\beta^{i-1}}{\delta - 1}. \tag{5.18}$$

We now consider the different alternatives for i^*. Assume $i^* = 3$. Substituting (5.18) in the constraint equations (5.15)–(5.16) (recall that the inequality (5.16) is active in the optimum, see Footnote 2) and taking into account that $x_i = 0$ for $i \geq 4$, we obtain that $\mu = \frac{3-\beta-\beta^2-4\delta}{3\beta^4-2\beta^3+\beta^2-2\beta+3}$ and $\lambda = \frac{\beta^2-\beta+\delta}{3\beta^2-5\beta+3}$. Substituting these quantities back in (5.18) yields $x_3 = \frac{(3\beta^2-\beta-1)\delta+\beta^3-3\beta^2+\beta}{(\delta-1)(3\beta^4-2\beta^3+\beta^3-2\beta+3)}$.

We will consider now the particular case where there is always an interferer, i.e. $\delta = 0$. Then the previous expressions become $\mu = \frac{3-\beta-\beta^2}{3\beta^4-2\beta^3+\beta^2-2\beta+3}$, $\lambda = \frac{\beta^2-\beta}{3\beta^2-5\beta+3}$, $x_3 = -\frac{\beta^3-3\beta^2+\beta}{(3\beta^4-2\beta^3+\beta^3-2\beta+3)}$.

Since the non-negativity constraints for the x_i have not been explicitly considered in the formulation of the problem, we have to deal now with them.

For $\frac{3-\sqrt{5}}{2} < \beta < \frac{3+\sqrt{5}}{2}$, $x_3 > 0$, and we have $x_2 = \frac{\beta^4-\beta^3-\beta^2+2\beta}{(3\beta^4-2\beta^3+\beta^3-2\beta+3)}$, $x_1 = \frac{\beta^4-\beta^2-2\beta+3}{(3\beta^4-2\beta^3+\beta^3-2\beta+3)}$ and $x_0 = \frac{\beta^2-\beta}{3\beta^2-5\beta+3}$.

For the rest of values of β, it is negative, and thus only power levels up to 2 will be used. Proceeding analogously we get then $\mu = \frac{2-\beta}{2(\beta^2-\beta+1)}$, $\lambda = \frac{(\beta-1)\beta}{2(\beta^2-\beta+1)}$, $x_2 = \frac{1}{2}\frac{\beta}{(\beta^2-\beta+1)}$ always greater than 0, $x_1 = \frac{1}{2}\frac{(\beta^2-2\beta+2)}{(\beta^2-\beta+1)}$ and $x_0 = \frac{(\beta-1)\beta}{2(\beta^2-\beta+1)}$.

The evolution of the optimal power allocation as a function of β is summarized in Fig. 5.1.

5.2.3 Optimal policy in mixed policies

As shown in Section 5.1.2, the use of mixed strategies may increase the TPS. The upper bound on performance obtained in Section 5.2.2, leads to the objective of achieving this bound via mixed strategies. We next establish that the upper-bound is indeed achievable for every N, and explicitly derive the mixed policy that leads to the corresponding optimal performance.

With some abuse of notations, we use the notation $u = (n_0, n_1, n_2, n_3)$ for a policy which uses a maximal power level of 3. Consider the following three pure policies:
$u(1) = (0, N, 0, 0)$,
$u(2) = (N - \lfloor N/2 \rfloor - \mathrm{mod}(N/2), \mathrm{mod}(N/2), \lfloor N/2 \rfloor, 0)$,
$u(3) = (N - \lfloor N/3 \rfloor - \mathrm{mod}(N/3), \mathrm{mod}(N/3), 0, \lfloor N/3 \rfloor)$
(where $\lfloor y \rfloor$ stands for the largest integer smaller than y, and $\mathrm{mod}(y/z)$ is the reminder in

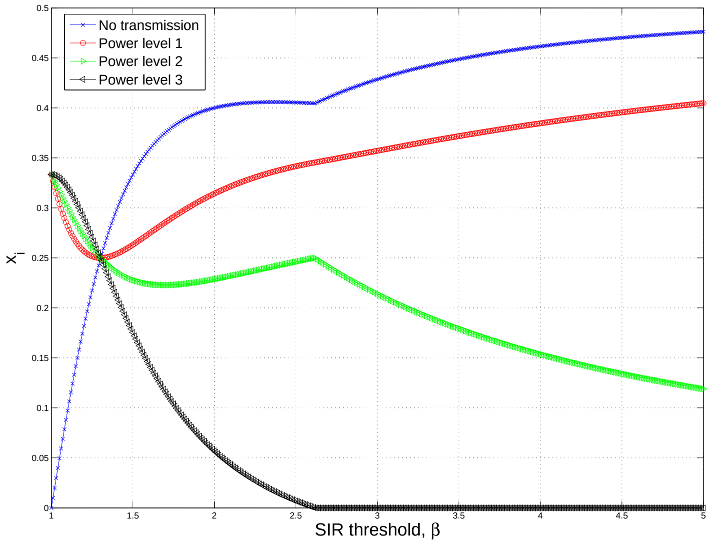


Figure 5.1: Optimal distribution of power levels as a function of the SIR threshold.

dividing two integer numbers y and z). We show below that any required frequency vector (x_0, x_1, x_2, x_3) can be obtained by a mixed policy that uses the above three pure policies.

Theorem 5.2.10 *Any required frequency vector (x_0, x_1, x_2, x_3) is attained by a mixed policy that uses the pure policies $u(1), u(2), u(3)$ with probabilities $p_3 = x_3 \frac{N}{\lfloor N/3 \rfloor}$, $p_2 = x_2 \frac{N}{\lfloor N/2 \rfloor}$, and $p_1 = 1 - p_2 - p_3$.*

Proof Note first that the battery lifetime constraint (5.15) is obeyed since $\sum_{i=0}^{3} n_i = N$ for each of the three pure policies. Observe next that a power level of 3 is used only in $u(3)$. Hence, the probability of transmitting at this power level is $\frac{p_3 \lfloor N/3 \rfloor}{N} = x_3$. Similarly, a power level of 2 is used only in $u(2)$. Hence, the probability of transmitting at this power level is $\frac{p_2 \lfloor N/2 \rfloor}{N} = x_2$. In order to obey the total energy constraint (5.16), it remains to be shown that $x_1 = 1 - 3x_3 - 2x_2$. To that end, we examine the probability for using a power level of 1 in each pure policy, and multiply it be the probability of using that policy. This gives

$$
\begin{aligned}
&(1 - p_2 - p_3) + \frac{\mathrm{mod}(N/2)}{N} p_2 + \frac{\mathrm{mod}(N/3)}{N} p_3 \\
&= 1 - p_2 \left(1 - \frac{\mathrm{mod}(N/2)}{N}\right) - p_3 \left(1 - \frac{\mathrm{mod}(N/3)}{N}\right) \\
&= 1 - x_2 \frac{N - \mathrm{mod}(N/2)}{\lfloor N/2 \rfloor} - x_3 \frac{N - \mathrm{mod}(N/3)}{\lfloor N/3 \rfloor} \\
&= 1 - x_2 \frac{N - \mathrm{mod}(N/2)}{\frac{1}{2}(N - \mathrm{mod}(N/2))} - x_3 \frac{N - \mathrm{mod}(N/3)}{\frac{1}{3}(N - \mathrm{mod}(N/3))},
\end{aligned}
$$

which means that $x_1 = 1 - 3x_3 - 2x_2$. □

The significance of Theorem 5.2.10 is that the upper bound TPS can be obtained for every N by implementing the optimal frequencies obtained in Theorem 5.2.9 via the mixed policy derived above.

5.2.4 Discussion

The combination of Theorems 5.2.9 and 5.2.10 leads to a globally optimal (mixed) policy that achieves the upper bound on performance and hence can be set as a unified protocol. It is important to emphasize that the number of pure policies that are used in the optimal mixed policy remains a constant (three), and does not grow with N. In addition, the complexity in computing the optimal mixed policy relates to calculating expressions such as $N/2$ and $N/3$, which do not become much more complex for a large N. Hence, the optimal policy is appealingly implementable.

At a higher perspective, we note that the approach used in Theorems 5.2.9–5.2.10 can be applied in more general contexts, besides throughput optimality. For example, assume that half of the population should be given some priority in terms of the obtained TPS, compared to the other half. The precise definition of the QoS differentiation between the two sub-populations can be casted as a (continuous) optimization problem. After solving the problem and obtaining the frequencies for each subset of the population, Theorem 5.2.10 can be invoked in order to implement the corresponding protocol.

5.3 The Noncooperative Game

This section is dedicated to the study of the noncooperative framework and the underlying Nash equilibria. Our main focus is on symmetric equilibria 5.6, which may be regarded as protocols, from which no user has an incentive to unilaterally deviate. In Section 5.3.1 we prove the uniqueness of the symmetric equilibrium point, and further provide a complete characterization thereof. Using the characterization, Section 5.3.2 compares the performance of the optimal policy obtained in Section to the unified equilibrium policy via the so-called price-of-anarchy (PoA) performance measure. We conclude this section by showing that asymmetric equilibria exist in general, yet leave their full analysis for future work. Throughout this section, we shall focus on the case of $N = K$, which enables us to provide a concrete comparison between optimal and equilibrium performance.

5.3.1 Symmetric Equilibria

We start our analysis by showing that in any symmetric equilibrium point (5.6), power levels equal or greater than three would never be used.

Theorem 5.3.1 *Let u be a unified equilibrium point. Then $x_i = 0$ for every $i \geq 3$.*

Proof The idea behind the proof is to establish first that a power level of three would not be used in any best response. The theorem's claim would then follow by induction on x_i. The proof proceeds in the following steps.

Step 1: *There is no best-response with* $x_3 > 0$: Consider a policy $u' = (x'_0, x'_1, ..., x'_K)$, for all players . Let $u = (x_0, ..., x_j)$ be a best response to u'. Note that the energy constraint (5.2) is met with equality for a best-response, hence $\sum_{i=1}^{j} ix_i = 1$. Assume by contradiction that $x_j > 0$. Introduce also the policy $\widehat{u} = (\widehat{x}_0, \widehat{x}_1, \widehat{x}_{j-1}, 0)$ where $\widehat{x}_{j-1} = x_{j-1} + x_j$, $\widehat{x}_0 = x_0 - x_j$, $\widehat{x}_1 = x_1 + x_j$ (note that $\widehat{u}$ obeys the energy constraint (5.2)). We show below that $\widehat{u}$ obtains a larger value compared to u contradicting the optimality of the latter.

$$
\begin{aligned}
TPS(u, u') &= \delta(1 - x_0) + (1 - \delta) \sum_{i=1}^{j} \sum_{l=0}^{i-1} x_i x'_l \\
&= \delta(1 - x_0) + (1 - \delta)(x_1 x'_0 + \sum_{i=2}^{j-2} \sum_{l=0}^{i-1} x_i x'_l + x_{j-1} \sum_{l=0}^{j-2} x'_l + x_j \sum_{l=0}^{j-1} x'_l) \\
&= \delta(1 - x_0) + (1 - \delta)(x_1 x'_0 + x_j x'_{j-1} + \sum_{i=2}^{j-2} \sum_{l=0}^{i-1} x_i x'_l + (x_{j-1} + x_j) \sum_{l=0}^{j-2} x'_l) \\
&< \delta(1 - \widehat{x}_0) + (1 - \delta)(\widehat{x}_1 x'_0 + + \sum_{i=2}^{j-2} \sum_{l=0}^{i-1} \widehat{x}_i x'_l + \widehat{x}_{j-1} \sum_{l=0}^{j-2} x'_l) \\
&= g(\widehat{u}, u')
\end{aligned}
$$

the inequality follows from $\widehat{x}_0 < x_0$ and also from $x'_0 > x'_{j-1}$.
Step 2: $x_3 = 0$ *in any best-response*. Consider now a general policy $u' = (x'_0, x'_1, x'_2, x'_3, \dots)$ employed by all users, and a best-response of a deviator $u = (x_0, x_1, x_2, x_3, \dots)$. The utility for the deviating user can be decomposed as:

$$
\begin{aligned}
g(u, u') = g(u_{(0-3)}, u'_{(0-3)}) + g(u_{(0-3)}, u'_{(\geq 4)}) \\
+ g(u_{(\geq 4)}, u'_{(0-3)}) + g(u_{(\geq 4)}, u'_{(\geq 4)}),
\end{aligned}
\tag{5.19}
$$

where for every $I \subset \mathbb{N}$, the notation $u_{(I)}$ stands for the sub-vector $\{x_i\}_{i \in I}$ (thus, for example , $g(u_{(\geq 4)}, u'_{(0-3)})$ is the number of successful transmissions obtained in interactions where all users use power levels $0 - 3$ and the deviator uses power levels greater or equal to 4). Obviously, $g(u_{(0-3)}, u'_{(\geq 4)}) = 0$. As before, assume by contradiction that $x_3 > 0$ and consider an alternative policy for the deviating user $\widehat{u} = (\widehat{x}_0, \widehat{x}_1, \widehat{x}_2, 0, \dots)$ where $\widehat{x}_2 = x_2 + x_3$, $\widehat{x}_0 = x_0 - x_3$, $\widehat{x}_1 = x_1 + x_3$, $\widehat{x}_i = x_i$ for $i \geq 4$. It follows from Step 1 that $g(\widehat{u}_{(0-3)}, u'_{(0-3)}) > g(u_{(0-3)}, u'_{(0-3)})$. Since the other three terms in (5.19) are not affected by the transition from u to $\widehat{u}$, we conclude that $g(\widehat{u}, u') > g(u, u')$. Hence $x_3 = 0$ in any best response.
Step 3: *In any best-response* $x_i = 0$ *for* $i > 2$. Assume by induction on k that $x_k = 0$, $x'_k = 0$. It is readily seen that the policy $(x_0, x_1, \dots, x_k = 0, x_{k+1}, \dots)$ with $x_{k+1} > 0$ is suboptimal, since $\widehat{u} = (x_0 - x_{k+1}, x_1 + x_{k+1}, \dots, \widehat{x}_k = x_{k+1}, \widehat{x}_{k+1} = 0, \dots)$ obviously obtains strictly higher TPS. Indeed, the deviating user benefits from the use of power level k as it did from power level $k+1$ (due to the induction assumption that $x'_k = 0$), and in addition it obtains a strictly positive benefit from additional power-1 transmissions. Hence,

$x_{k+1} = 0$. Since $x_i = 0$, $i > 2$ for any best response, there is no equilibrium point in which mobiles use power levels above two. □

Taking into account that $x_3 = 0$, it follows from the energy constraint (5.2) (which is met with equality) that $x_0 = (\beta - 1)x_2$ for any user policy. We next express the utility of a "deviating" user with such policy $u = (x_0 = (\beta - 1)x_2, x_1, x_2)$, where all others use a policy $u' = (x'_0, x'_1, x'_2)$.

$$\begin{aligned} TPS(u,u') &= \delta(1 - x_0) + (1-\delta)\big(x_2(x'_1 + x'_0) + x_1 x'_0\big) \\ &= \delta(1 - (\beta-1)x_2) + (1-\delta)\big(x_2(x'_1 + x'_0) + (1 - \beta x_2)x'_0\big) \\ &= \delta + (1-\delta)x'_0 + x_2(1-\delta)\left(x'_1 - (\beta-1)\big(x'_0 + \frac{\delta}{1-\delta}\big)\right). \end{aligned} \tag{5.20}$$

Define

$$A(x'_1, x'_0) = \left(x'_1 - (\beta-1)\big(x'_0 + \frac{\delta}{1-\delta}\big)\right). \tag{5.21}$$

Clearly, the sign of $A(x'_1, x'_0)$ would determine the best-response (BR) of the deviating user, as we summarize below.

$$\begin{cases} A(x'_1, x'_0) > 0: & x_0 = \frac{\beta-1}{\beta}, x_2 = \frac{1}{\beta}, x_1 = 0 \\ A(x'_1, x'_0) < 0: & x_0 = x_2 = 0, x_1 = 1 \\ A(x'_1, x'_0) = 0: & \text{Any strategy } (x_0, x_1, x_2) \text{ is BR.} \end{cases} \tag{5.22}$$

Using (5.22), we may explicitly characterize the symmetric equilibrium point, as we summarize in the next theorem. When the policies below result in non-integer numbers, mixed policies are used in the spirit of Theorem 5.2.10.

Theorem 5.3.2 *(i) The symmetric equilibrium point exists and is unique whenever $\delta > 1/2$ or $\beta < \frac{1}{\delta}$. It is given by*

$$\begin{cases} \delta \le \frac{1}{2}: & x_0 = x_2 = \frac{1-2\delta}{3(1-\delta)}, x_1 = \frac{1+\delta}{3(1-\delta)} \\ \delta > \frac{1}{2}: & x_0 = x_2 = 0, x_1 = 1. \end{cases} \tag{5.23}$$

When $\delta < 1/2$ and $\beta > \frac{1}{\delta}$, the two previous policies constitute both equilibria.

(ii) The corresponding TPS are given by

$$\begin{cases} \delta \le \frac{1}{2}: & TPS = \delta + 1 - 2\delta\frac{\beta-1}{2\beta-1}, \\ \delta > \frac{1}{2}: & TPS = \delta. \end{cases} \tag{5.24}$$

Proof (i) Consider first the case where $\delta > \frac{1}{2}$. For that case, $A(x'_1, x'_0) = \left(x'_1 - \big(x'_0 + \frac{\delta}{1-\delta}\big)\right) < \big(x'_1 - (x'_0 + 1)\big) \le 0$, which immediately leads to the best response policy of $x_0 = x_2 = 0, x_1 = 1$.

Consider next the case of $\delta \le \frac{1}{2}$ and the possible values for $A(x'_1, x'_0)$. Assume that $A(x'_1, x'_0) > 0$; then obviously $x'_1 \ge x'_0$; however, the best-response (5.22) in this case is such that $0 = x1 < x_0 = 1/2$. Hence $A(x_1', x_0') > 0$ does not lead to a symmetric equilibrium. Similarly, assume that $A(x_1', x_0') < 0$. This implies that $x_1' < x_0' + \frac{\delta}{1-\delta} \leq x_0' + 1$; the best-response in this case is such that $1 = x_1, x_0 = 0$. Hence in order to achieve a symmetric equilibrium we must have $\beta > \frac{1}{\delta}$. The remaining case is $A(x_1', x_0') = 0$. Since the deviating user is indifferent about its policy (as long as it uses power levels not greater than two), a symmetric equilibrium is obtained for $x_1 = x_0 + \frac{\delta}{1-\delta}$. Using the energy constraint, the last equation immediately implies that $x_0 = \frac{\beta-1}{2\beta-1}\frac{1-2\delta}{1-\delta}$, $x_2 = \frac{1}{2\beta-1}\frac{1-2\delta}{1-\delta}$, $x_1 = \frac{\beta-1}{2\beta-1}\frac{1-2\delta}{1-\delta} + \frac{\delta}{1-\delta}$.

(ii) For $\delta > \frac{1}{2}$, it is immediate that the TPS is δ, since users always transmit with a power level of one; the TPS in this case is thus equivalent to the probability of not facing an interferer. For the case of $\delta \leq \frac{1}{2}$, we substitute $A(x_1', x_0') = 0$ and the allocation rule (5.23) in (5.20) and obtain that $TPS = \delta + 1 - 2\delta\frac{\beta-1}{2\beta-1}$ which establishes the result. $\square$

The evolution of the power allocation at the symmetric equilibrium as a function of δ is summarized in Fig. 5.2, and the corresponding TPS is given in Fig. 5.3.

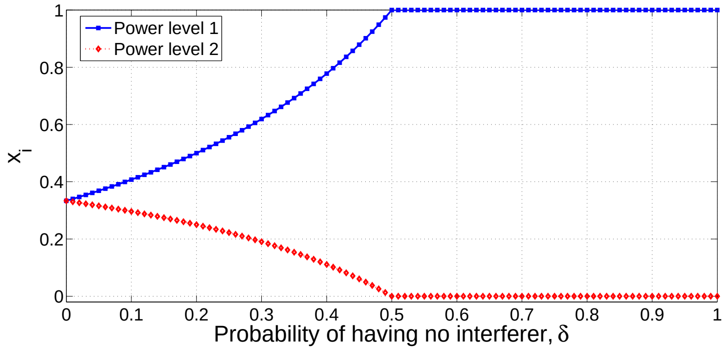


Figure 5.2: The distribution of power levels at the symmetric equilibrium as a function of the probability of having no interferer.

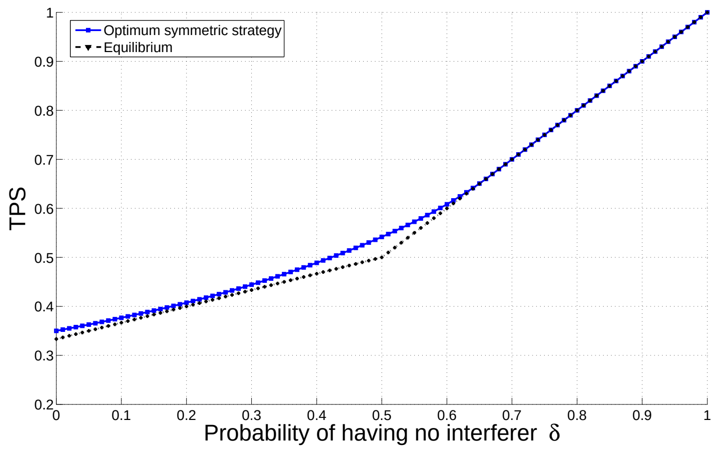


Figure 5.3: The TPS at the symmetric equilibrium as a function of the probability of having no interferer.

5.3.2 Efficiency Loss

Equipped with a complete characterization of both the symmetric optimal solution and the symmetric equilibrium point, we may compare the performance at both frameworks. A popular measure for comparison is the PoA [Roughgarden 2005], which corresponds in our case to the ratio between the TPS obtained in the team problem and the TPS at the symmetric equilibrium[3]. We emphasize that we do not consider here asymmetric working points.

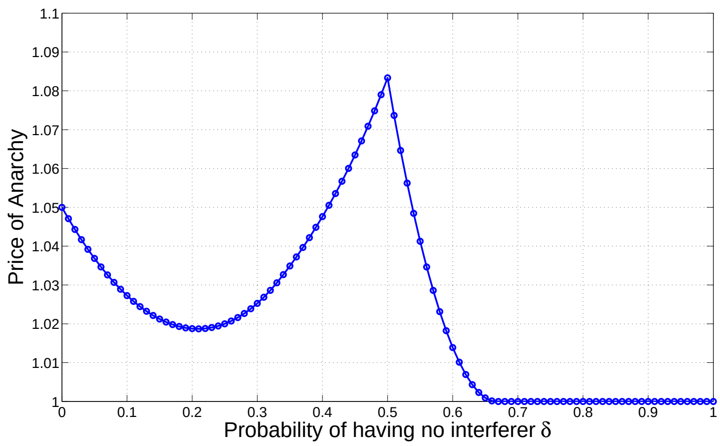


Figure 5.4: Efficiency loss as a function of δ. The y-axis is the ratio between the symmetric-optimal TPS and the symmetric equilibrium.

The PoA as a function of δ is depicted in Figure 5.4. It is seen that the efficiency loss is always smaller than 9 percent. An interesting direction for future work is to study the efficiency loss in cases where the energy available is larger (i.e., $K > N$) and examine whether users misuse the access energy.

5.3.3 Existence of Asymmetric Equilibria

We focused in preceding subsections on symmetric Nash equilibria. In this subsection we show that asymmetric equilibria exist in general. In view of (5.21), any set of policies (that use power levels less than 3), for which the *average* distribution of power levels among the user-population satisfies $n_1 = n_0 + \frac{\delta N}{1-\delta}$, leads to an equilibrium. Indeed, no user will benefit from deviating, as all (n_0, n_1, n_2) policies are in fact best responses. A particular case of the above are the symmetric equilibria obtained in Theorem 5.3.2. Based on this observation, it is possible to construct asymmetric equilibria as, for instance,

- A fraction $\frac{1+\delta}{3(1-\delta)}$ of the population use $n_1 = N$, $n_0 = n_2 = 0$. The remaining fraction $\frac{2-4\delta}{3(1-\delta)}$ use $n_0 = n_2 = \frac{N}{2}$, $n_1 = 0$.

[3] In general, the PoA corresponds to the ratio between the optimal solution and the *worst* Nash equilibrium. However, in our case, both the symmetric optimal solution and the symmetric equilibrium are unique.

In the present work, we do not focus on asymmetric equilibria, yet point to their existence. The numeric example above strongly relies on our characterization of the symmetric equilibrium. It remains to be verified whether additional asymmetric equilibria (which may lead to different TPS) do exist. The comprehensive analysis of asymmetric equilibria remains a challenging direction for future work.

5.4 Extensions to the model

We briefly mention how to adapt the analysis to variations on the initial model, and present some conclusions and future research directions.

5.4.1 Soft capture Network

Assume that if two stations transmit at the same power level then a given packet is successfully received with probability $a \leq 1/2$. Let $\overline{a} = 1 - a$. If powers are different then, as before, the packet transmitted with larger power is successful and the other is not. The objective to maximize is given by

$$
\begin{aligned}
g^{cap} &= \delta(N-n_0) + (1-\delta)\frac{\overline{a}}{N}\sum_{i=1}^{K}\sum_{j=0}^{i-1} n_i n_j + (1-\delta)\frac{a}{N}\sum_{i=1}^{K}\sum_{j=0}^{i} n_i n_j && (5.25)\\
&= \delta(N-n_0) + (1-\delta)\frac{1}{N}\Big(\sum_{i=1}^{K}\sum_{j=0}^{i-1} n_i n_j + a\sum_{i=1}^{K} n_i^2\Big) && (5.26)\\
&= \delta(N-n_0) - (1-\delta)n_0^2 a/N && (5.27)\\
&+ (1-\delta)\Big[\frac{1-2a}{N}\sum_{i=1}^{K}\sum_{j=0}^{i-1} n_i n_j + \frac{2a}{N}\Big(\sum_{i=1}^{K}\sum_{j=0}^{i-1} n_i n_j + \frac{1}{2}\sum_{i=0}^{K} n_i^2\Big) && (5.28)\\
&= \delta(N-n_0) - (1-\delta)\frac{a n_0^2}{N} + (1-\delta)\Big[\frac{1-2a}{N}\sum_{i=1}^{K}\sum_{j=0}^{i-1} n_i n_j + aN\Big], && (5.29)\\
& && (5.30)
\end{aligned}
$$

where we used (5.8).

Consider $a = 1/2$. In this case, g^{cap} equals $-\delta n_0 - (1-\delta)n_0^2/(2N)$ plus some constant that does not depend on u. For any δ, this utility is maximized at $n_0 = 0$ which means $n_1 = N$ and $n_i = 0$ for all $i \neq 1$.

The case $a < 1/2$ remains to be investigated in future work.

Asymptotic Analysis From eqs. 5.27 and 5.8 we can write the asymptotic TPS in this case as:

$$TPS^{cap} = \delta(1-x_0) + \frac{(1-\delta)}{2}\left(1 - (1-2a)\sum_{i=1}^{K} x_i^2 - x_0^2\right)$$

We have a convex optimization problem whose Lagrangian is

$$\mathcal{L}(\mathbf{x}) = \delta(1-x_0) + \frac{(1-\delta)}{2}\left(1-(1-2a)\sum_{i=1}^{K} x_i^2 - x_0^2\right) + \lambda\left(\sum_{i=0}^{K} x_i - 1\right) + \mu\left(\sum_{i=0}^{K} i x_i - 1\right) \tag{5.31}$$

$$\frac{\partial \mathcal{L}}{\partial x_0} = (\delta-1)x_0 - \delta + \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_i} = (\delta-1)(1-2a)x_i + \lambda + \mu i = 0 \quad i = 1,\dots,i^*$$

Let us assume only power levels up to 4 are used in the optimal solution. Then, calculating the values of the multipliers in order to satisfy the constraint, we can obtain

$$x_4 = \frac{1}{15}\frac{(-1+2a)(-5\delta+12\delta a-12a)}{(-5+6a)(1-\delta)(1-2a)}$$

which is always negative, meaning that power level 4 will not be used. In addition, since we still have $n_i \geq n_{i+1}$ for $i > 0$ we can conclude that only 3 power levels will also be used in the soft capture case.

5.4.2 Case $K \geq N$

Proposition 5.4.1 *We can obtain the socially optimal policy for $N, K > N$, starting from the optimal (already known) policy for $N, K' = N$ by recursively obtaining new policies with one extra energy unit by using once power level $j+1$ instead of j, and leaving all the remaining time slots unaltered.*

Let us consider a fixed N and increase K. For $N = K$, we know the optimal policy, in the social case, $u = n_0, n_1, n_2, n_3, n_4 = 0$. From it, we may construct a new policy using $K+1$ energy units, u', by using once power level $j+1$ instead of j, and leaving all the remaining time slots unaltered. In order to maximize the throughput, the level j must be chosen as $j = \arg\max n_j - n_{j+1}$. Define $d = n_j - n_{j+1}$, then $n_3 \leq d, \quad n_2 \leq 2d, \quad n_1 \leq 3d, \quad n_0 \leq 4d, \quad N \leq 10d$.

Suppose, by contradiction, there exists a better policy than u', $\widehat{u} = (\widehat{n}_0, \widehat{n}_1, ..., \widehat{n}_k)$. Conversely, we may construct new policies using K energy units, $\widehat{u}'$, from $\widehat{u}$, by using once power level j instead of $j+1$, and leaving all the remaining time slots unaltered. But given the optimality of u this implies $\widehat{n}_l - \widehat{n}_{l+1} \geq d+1 \quad \forall l$. Moreover, $\widehat{u}$ must use at least power level 4, or otherwise it could be constructed in the same way as u', contradicting the hypothesis. Then, $\widehat{n}3 \geq 1+d+\widehat{n}_4, \quad \widehat{n}_2 \geq 2d+2+\widehat{n}_4, \quad \widehat{n}_1 \geq 3+3d+\widehat{n}_4, \quad \widehat{n}_0 \geq 4+4d+\widehat{n}_4, N \geq 10+10d+4\widehat{n}_4$, getting a contradiction.

5.4.3 General Access Problems

Through the model we used for the power control problem, we intended to introduce a methodology that can be useful for general control of priority access. We briefly comment on some specific variations that may be needed in other network applications. In a general priority assignment context, one may again enumerate priority levels using the integers $\{0, 1, 2, \ldots\}$; an access request with priority $i \geq 1$ would prevail if it is the only request, or if all other requests are with lower priority. It may even be granted access (with some positive probability a) in the case that another request is made with the same priority level (as in the “soft-capture" model above). Yet, there may be a difference in the way that priority level 0 is treated, compared to the way that power level 0 is modeled in the power control problem. In the power control framework, when the transmission power is zero then transmission fails, in particular, for the following two cases: (i) there is no interference, or (ii) there is interference with another mobile that “transmits" with a power of zero. This need not be the situation in other priority assignment models. For example, the lowest priority (i.e., zero) can be interpreted as “best-effort" service in QoS-supporting network architectures.

To concretize our discussion, assume that a request with zero priority will be successful w.p.1 in case (i) above, and with positive probability a in case (ii) (i.e., the “soft-capture" rule includes priority zero as well). The expected utility is given by δN plus the third term of (5.29), which yields

$$g^{general} = \delta N + (1-\delta)\Big[\frac{1-2a}{N}\sum_{i=1}^{K}\sum_{j=0}^{i-1} n_i n_j + aN\Big].$$

Interestingly, the optimal and equilibria policies for any δ coincide with those obtained for the original power control problem with $\delta = 0$. Note that for $\delta = 1$ or for $a = 0.5$, the performance does not depend on the policy anymore (all policies are thus optimal).

5.4.4 Conclusions

We have considered the priority assignment problem that corresponds to “sparse" multiple access networks, in which pairwise interactions occur. We have provided an explicit solution for the team problem, and a complete characterization of the symmetric equilibrium in a noncooperative framework. Interestingly, the number of power levels that is used in the competitive setup is smaller than the corresponding number for the team problem (this holds for every δ). This phenomenon is counter-intuitive perhaps, as in many noncooperative networking scenarios, the users consume the network resources in a more aggressive way, compared to the socially-optimal point (e.g., in queuing networks, see [Hassin 1997]).

CHAPTER 6

Conclusions

In this thesis we have considered several scenarios of resource allocation techniques for multiuser uplink scenarios. Based on a general model allowing to consider different transmission technologies, we have formulated the power allocation problems with the goal of maximizing the throughput, either of the total system, or by each user in a selfish manner. The goal was both to quantify the achievable gains and to propose power allocation algorithms allowing to implement in practice these optimal solutions.

We have initially studied the optimal decoding order and power allocation for SIC receivers, both MMSE and matched filter, considering different, realistic channel models. We have shown that, for the former, the optimal detection order depends on the requested SINR, eventually weighted, while in the latter, only on the channel energies. The use of asymptotic tools from random matrix theory provide a neat framework for the analysis of SIC systems, allowing to consider different transmission technologies under a common system model. It has also been shown that, under certain conditions, the power allocation can be determined in a decentralized manner (by each user individually) when considering a high number of users in the network.

Next we considered the problem of resource allocation in fast fading MIMO channels with correlation. Our goal was to design power allocation algorithms while minimizing the amount of control signal from the BS, assuming to this end only CDIT (and CSIR). Two scenarios were studied: in the first single user decoding was performed at the base station, whereas in the second a random coordination signal was introduced allowing to perform interference cancellation decoding and determining the decoding order of each user, and thus significantly enhancing the system performance. A game theoretic setting was used to analyze both scenarios, determining the existence of equilibria, which effectively allows the mobiles to choose their power allocation policies in order to selfishly optimize their ergodic transmission rates. In addition, an iterative algorithm has been proposed to this end which is guaranteed to converge to the optimum if it does converge (extensive simulations indicate its convergence).

We then proceeded to consider an scenario in which mobile stations are able to connect simultaneously to several base stations (which may be equipped with different transmission technologies such as OFDM, MIMO or CDMA), communicating on nonoverlapping frequency bands. A cross-system power allocation algorithm has been studied in order to exploit the available cross-system diversity. For the optimum receiver, it was shown that a simple cross-layer algorithm, analogous to the water-filling algorithm, can be implemented at the central controller to schedule the powers of all the users in order to maximize the network capacity, and this can be done in a simple, iterative way, which generally converges to the optimum.

For the MF and MMSE receivers a water-filling solution can still be obtained by introducing two additional assumptions, which simplify the optimization problem but at the price of a performance loss that has to be evaluated in the situations of interest. For the typical scenarios considered in this paper, we saw that they were reasonable. The potential performance gain of cross-system diversity was shown to be important in several typical simulation setups. For instance, by simply using MMSE receivers at the base stations and uniform power allocation over the different systems, the mobile transmit power could be divided by a factor greater than 10 with respect to a standard network using the MF and hard handover power allocation scheme.

Finally we have considered a somewhat different scenario, the priority assignment problem in “sparse" multiple access networks, in which pairwise users interactions occur. We have provided an explicit solution for the team problem, and a complete characterization of the symmetric equilibrium in a noncooperative framework. Interestingly, the number of power levels that is used in the competitive setup is smaller than the corresponding number for the team problem (this holds for every δ). This phenomenon may appear counter-intuitive, as in many noncooperative networking scenarios, the users consume the network resources in a more aggressive way, compared to the socially-optimal point.

Future Research In this thesis we have presented a number of results on how to improve the uplink performance through proper resource allocation, specially when limited channel state information is available at the transmitters, and determining conditions under which it can be done in a distributed way.

In chapter 2 the results rely on simplified, separable channel model. The obtained decoding order is still optimal for more general channel models under certain assumptions, such as the high rate regime, but it would be interesting to study whether this is the case also in general.

In chapter 3 the use of a simple coordination signal was able to produce an important performance improvement by allowing the use of successive decoding at the BS. However, more sophisticated coordination signals could be considered, creating a closed loop approach in which it would also transfer information about the channels. In addition, the convergence of the proposed iterative algorithm was only obtained through simulation results, its theoretical analysis would add a significant result.

The results from chapter 4 could be extended by considering other performance metrics such as the outage probability in order to further characterize the benefits of the cross-system diversity. Extensions considering more heterogeneous networks might also be considered (e.g. introducing CDMA base stations with multiple antennas), since the Random Matrix theory results used could still be applied there. Finally, in order to consider a possible practical implementations of the scheme a number of issues would have to be considered, such as the BS coordination problem.

In chapter 5 a simpler, symmetric, channel model is assumed, based on some related networking problems. Several directions appear to extend the interesting results obtained here. First a fading channel could be considered, even a simple i.i.d. one, in order to be able to exploit some of the properties obtained here. Secondly, to consider general battery life-time N and general budget K and obtain complete characterization of both the team and game problems. Another challenging extension is to relax the assumption on sparsity an to consider interactions of more than two users and their consequences.

A.1 Optimum eigenvectors for decentralized MIMO MAC with double-sided correlation

To prove Theorem 3.3.1 for $u_k^{(SU)}$ we follow the same steps as [Soysal 2009] and use an additional argument due to the fact that the receive antenna can be correlated here. By definition $\mathbf{H}_\ell = \mathbf{R}^{\frac{1}{2}}\mathbf{\Theta}_\ell\mathbf{T}_\ell^{\frac{1}{2}} = \mathbf{U}_R\mathbf{D}_R^{\frac{1}{2}}\mathbf{U}_R^H\mathbf{\Theta}_\ell\mathbf{U}_\ell\mathbf{D}_\ell^{\frac{1}{2}}\mathbf{U}_\ell^H$, where $\mathbf{\Theta}_\ell$ is a zero-mean i.i.d. Gaussian identity covariance random matrix. Using the fact that multiplying $\mathbf{\Theta}_\ell$ by a unitary matrix does not change its joint distribution and the fact that $|\mathbf{UMU}^H + \mathbf{I}| = |\mathbf{M}+\mathbf{I}|$ for any unitary matrix $\mathbf{U}$ one can write:

$$\begin{aligned}
&\max_{\mathbf{Q}_k} u_k^{(SU)} = \\
&\max_{\mathbf{Q}_k} \mathbb{E}\left[\log_2\left|\mathbf{I}+\rho\sum_{\ell=1}^{K}\mathbf{U}_R\mathbf{D}_R^{\frac{1}{2}}\mathbf{U}_R^H\mathbf{\Theta}_\ell\mathbf{U}_\ell\mathbf{D}_\ell^{\frac{1}{2}}\mathbf{U}_\ell^H\mathbf{Q}_\ell\mathbf{U}_\ell\mathbf{D}_\ell^{\frac{1}{2}}\mathbf{U}_\ell^H\mathbf{\Theta}_\ell^H\mathbf{U}_R\mathbf{D}_R^{\frac{1}{2}}\mathbf{U}_R^H\right|\right] = \\
&\max_{\mathbf{Q}_k}\mathbb{E}\left[\log_2\left|\mathbf{I}+\rho\sum_{\ell=1}^{K}\mathbf{D}_R^{\frac{1}{2}}\mathbf{\Theta}_\ell\mathbf{D}_\ell^{\frac{1}{2}}\mathbf{U}_\ell^H\mathbf{Q}_\ell\mathbf{U}_\ell\mathbf{D}_\ell^{\frac{1}{2}}\mathbf{\Theta}_\ell^H\mathbf{D}_R^{\frac{1}{2}}\right|\right].
\end{aligned} \tag{A.1}$$

Then we can spectrally decompose the matrix $\mathbf{D}_\ell^{\frac{1}{2}}\mathbf{U}_\ell^H\mathbf{Q}_\ell\mathbf{U}_\ell\mathbf{D}_\ell^{\frac{1}{2}} = \tilde{\mathbf{U}}_\ell\tilde{\mathbf{D}}_\ell\tilde{\mathbf{U}}_\ell^H$ and write that

$$\begin{aligned}
\max_{\mathbf{Q}_k} u_k^{(SU)} &= \max_{\mathbf{Q}_k}\mathbb{E}\left[\log_2\left|\mathbf{I}+\rho\sum_{\ell=1}^{K}\mathbf{D}_R^{\frac{1}{2}}\mathbf{\Theta}_\ell\tilde{\mathbf{U}}_\ell\tilde{\mathbf{D}}_\ell\tilde{\mathbf{U}}_\ell^H\mathbf{\Theta}_\ell^H\mathbf{D}_R^{\frac{1}{2}}\right|\right] \\
&= \max_{\mathbf{Q}_k}\mathbb{E}\left[\log_2\left|\mathbf{I}+\rho\sum_{\ell=1}^{K}\mathbf{D}_R^{\frac{1}{2}}\mathbf{\Theta}_\ell\tilde{\mathbf{D}}_\ell\mathbf{\Theta}_\ell^H\mathbf{D}_R^{\frac{1}{2}}\right|\right].
\end{aligned} \tag{A.2}$$

We see that the function to be optimized depends on the eigenvectors $\tilde{\mathbf{U}}_k$ only through the power constraint $\mathrm{Tr}(\mathbf{Q}_\mathrm{k}) = \mathrm{Tr}(\tilde{\mathbf{U}}_\mathrm{k}^\mathrm{H}\mathbf{D}_\mathrm{k}^{-1}\tilde{\mathbf{U}}_\mathrm{k}\tilde{\mathbf{D}}_\mathrm{k}) \leq \mathrm{n_t}$. The matrix $\tilde{\mathbf{U}}_k$ can be chosen arbitrarily provided it meets the power constraint $\mathrm{Tr}(\mathbf{Q}_\mathrm{k}) \leq \mathrm{n_t}$. The choice $\tilde{\mathbf{U}}_k = \mathbf{I}$ is feasible since $\mathrm{Tr}(\mathbf{D}_\mathrm{k}^{-1}\tilde{\mathbf{D}}_\mathrm{k}) \leq \mathrm{Tr}(\tilde{\mathbf{U}}_\mathrm{k}^\mathrm{H}\mathbf{D}_\mathrm{k}^{-1}\tilde{\mathbf{U}}_\mathrm{k}\tilde{\mathbf{D}}_\mathrm{k}) \leq \mathrm{n_t}$. This shows that $\mathbf{Q}_k$ can be chosen without loss of optimality to be structured as: $\mathbf{Q}_k = \mathbf{U}_k\mathbf{D}_k^{-1}\tilde{\mathbf{D}}_k\mathbf{U}_k^H$. For the optimization of $v_k^{(OL)}$ one has to note that for each user $k \in \{1,2\}$, $\mathbf{Q}_k^{(1)}$ and $\mathbf{Q}_k(2)$ are optimized independently. For a given realization s of S, the optimum structure of $\mathbf{Q}_k^{(s)}$ for the interference-free channel follows from [Jorswieck 2004a]. For the other user re-use the derivation for $u_k^{(SU)}$ to conclude the proof.

A.2 Proof of Theorem 3.3.2

In the proof provided here we assumed for clarity $\mathbf{R} = \mathbf{I}$ but the result can be easily proved to hold in the general case. We want to derivate the utility function $\tilde{u}_k$ given by eq. (3.10). It turns out that the partial derivative with respect to $P_k(i)$ is the same as it would be if α and β would be assumed to be independent of these quantities. This result is useful because it allows us to cope with the convergence issue of the quantities α, β towards strict constants as the numbers of users and dimensions grow. Therefore, the main interest in the proposed derivation is that one does not need to assume α or β to be independent of $P_k(i)$. Otherwise the result can be obtained much more easily. We want to prove that the derivative of the approximated utility of user k can be expressed as: $\frac{\partial \gamma_{sum}}{\partial P_k(i)} = \frac{1}{n_r \ln 2} \frac{K\rho d_k(i)\alpha}{1 + K\rho P_k(i) d_k(i)\alpha}$. We have:

$$n_r \gamma_{sum} = \log_2 \left\{ \prod_{\ell,j} [1 + K\rho P_\ell(j) d_\ell(j) \alpha(P_k(i))] \right. \tag{A.3}$$

$$\left. \times \quad (1 + K\rho\beta(P_k(i)))^{n_r} e^{-n_t K^2 \rho \alpha(P_k(i))\beta(P_k(i))} \right\}. \tag{A.4}$$

Define $u \triangleq \prod_{\ell,j} [1 + K\rho P_\ell(j) d_\ell(j) \alpha(P_k(i))]$ and $v \triangleq (1 + K\rho\beta(P_k(i)))^{n_r} e^{-n_t K^2 \rho \alpha(P_k(i))\beta(P_k(i))}$. With these notations: $\frac{\partial n_r \gamma_{sum}}{\partial P_k(i)} = \frac{1}{\ln 2} \frac{1}{uv} \frac{\partial uv}{\partial P_k(i)}$. It turns out that $\frac{\partial uv}{\partial P_k(i)} = uv \times \frac{K\rho d_k(i)\alpha}{1 + K\rho P_k(i) d_k(i)\alpha}$. This is what we want to show. We want to derivate the function u. As u is a product of functions $u_{\ell,j}$, i.e. $u = \prod_{\ell,j} u_{\ell,j}$, its derivative u' can be written as: $u' = u \times \sum_{\ell,j} \frac{u'_{\ell,j}}{u_{\ell,j}}$. More precisely $u' = \underbrace{\prod_{\ell',j'} [1 + K\rho P_{\ell'}(j') d_{\ell'}(j')\alpha]}_{u} \sum_{\ell,j} \frac{N(\ell,j)}{1 + K\rho P_\ell(j) d_\ell(j)\alpha}$ where

$$N(\ell,j) = \left| \begin{array}{ll} K\rho P_\ell(j) d_\ell(j)\alpha' & \text{if } (\ell,j) \neq (k,i) \\ K\rho d_k(i)(\alpha + P_k(i)\alpha') & \text{if } (\ell,j) = (k,i). \end{array} \right.$$

Using a similar reasoning for v one can check that $v' = v \times K\rho \left[\frac{n_r \beta'}{1+K\rho\beta} - K n_t (\alpha'\beta + \alpha\beta') \right]$. Now using the relations proved in the previous steps we have that

$$\begin{aligned} &\tfrac{\partial uv}{\partial P_k(i)} = u'v + uv' \\ &= uv \times \left\{ \sum_{\ell,j} \tfrac{N(\ell,j)}{1+K\rho P_\ell(j) d_\ell(j)\alpha} + K\rho \left[\tfrac{n_r\beta'}{1+K\rho\beta} - K n_t(\alpha'\beta + \alpha\beta') \right] \right\} \\ &= uv \times \left\{ \sum_{(\ell,j)\neq(k,i)} \tfrac{K\rho P_\ell(j) d_\ell(j)\alpha'}{1+K\rho P_\ell(j) d_\ell(j)\alpha} + \tfrac{K\rho d_k(i)(\alpha + P_k(i)\alpha)'}{1+K\rho P_k(i) d_k(i)\alpha} + K\rho \left[\tfrac{n_r\beta'}{1+K\rho\beta} - K n_t(\alpha'\beta + \alpha\beta') \right] \right\} \end{aligned}$$

Now using the definitions of α and β (see eq. (3.11)) we find, after simplifications, the proposed expression for the derivative of γ_{sum}. Finally, by setting the derivative of $\mathcal{L}_{\lambda_k}(P_k(i))$ to zero we find equation (3.13).

B.1 Proof of Lemma 4.2.2

We want to derivate the argument of the maximum in equation (4.5) with respect to $\alpha_{k,s}$. First note from the system of equations (4.6) that r_t and q_t do not depend on $\alpha_{k,s}$ for all $t \neq s$. Based on this observation one just needs to consider the following auxiliary function:

$$\phi(\alpha_{k,s}) = \log_2 \left\{ \prod_{\ell=1}^{K} [1 + \gamma_\ell \alpha_{\ell,s} r(\alpha_k)] \times \prod_{j=1}^{N} \left(1 + \rho d_j^2 q(\alpha_{k,s})\right) \times e^{-K\rho r(\alpha_{k,s}) q(\alpha_{k,s})} \right\} \tag{B.1}$$

where we dropped the system index s and receiver subscript (R) for sake of clarity.

Define $u \triangleq \prod_{\ell=1}^{K} [1 + \gamma_\ell \alpha_{\ell,s} r(\alpha_{k,s})]$ and $v \triangleq \prod_{j=1}^{N} \left(1 + \rho d_j^2 q(\alpha_{k,s})\right) \times e^{-K\rho r(\alpha_{k,s}) q(\alpha_{k,s})}$. With these notations:

$$\frac{\partial \phi(\alpha_{k,s})}{\partial \alpha_{k,s}} = \frac{1}{\ln 2} \frac{1}{uv} \frac{\partial uv}{\partial \alpha_{k,s}}. \tag{B.2}$$

It turns out that $\frac{\partial (uv)}{\partial \alpha_k} = uv \times \frac{\gamma_k r}{1 + \gamma_k \alpha_{k,s} r}$. This is what we want to show.

We want to derivate the function u w.r.t. $\alpha_{k,s}$. As u is a product of functions u_ℓ, *i.e.* $u = \prod_{\ell=1}^{K} u_\ell$, its derivative u' can be written as $u' = u \times \sum_{\ell=1}^{K} \frac{u'_\ell}{u_\ell}$ where

$$u'_\ell = \left| \begin{array}{ll} \gamma_\ell \alpha_{\ell,s} r' & \text{if } \ell \neq k \\ \gamma_k (r + \alpha_{k,s} r') & \text{if } \ell = k. \end{array} \right. \tag{B.3}$$

Using a similar reasoning for v one can check that

$$v' = v \times \left[\sum_{j=1}^{N} \frac{\rho d_j^2 q'}{1 + \rho d_j^2 q} - K\rho(q' r + q r') \right]. \tag{B.4}$$

Now using the relations proved in the previous steps we have that

$$\frac{\partial (uv)}{\partial \alpha_{k,s}} = uv \times \underbrace{\left(\sum_{\ell=1}^{K} \frac{u'_\ell}{1 + \gamma_\ell \alpha_{\ell,s} r} + \sum_{j=1}^{N} \frac{\rho d_j^2 q'}{1 + \rho d_j^2 q} - K\rho(q' r + q r') \right)}_{\psi} \tag{B.5}$$

with ψ expanding as

$$\psi = \sum_{\ell \neq k} \frac{\gamma_\ell \alpha_{\ell,s} r'}{1+\gamma_\ell \alpha_{\ell,s} r} + \frac{\gamma_k (r + \alpha_{k,s} r')}{1+\gamma_k \alpha_{k,s} r} + \sum_{j=1}^{N} \frac{\rho d_j^2 q'}{1+\rho d_j^2 q} - K\rho(q'r + qr'). \quad \text{(B.6)}$$

$$\text{(B.7)}$$

Now by observing that

$$\begin{cases} \sum_{\ell \neq k} \frac{\gamma_\ell \alpha_{\ell,s} r'}{1+\gamma_\ell \alpha_{\ell,s} r} = \left(K\rho q - \frac{\gamma_k \alpha_k}{1+\gamma_k \alpha_{k,s} r} \right) r' \\ \sum_{j=1}^{N} \frac{\rho d_j^2 q'}{1+\rho d_j^2 q} = K\rho q' r \end{cases} \quad \text{(B.8)}$$

we find that

$$\psi = \frac{\gamma_k r}{1+\gamma_k \alpha_{k,s} r}, \quad \text{(B.9)}$$

which concludes the proof.

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